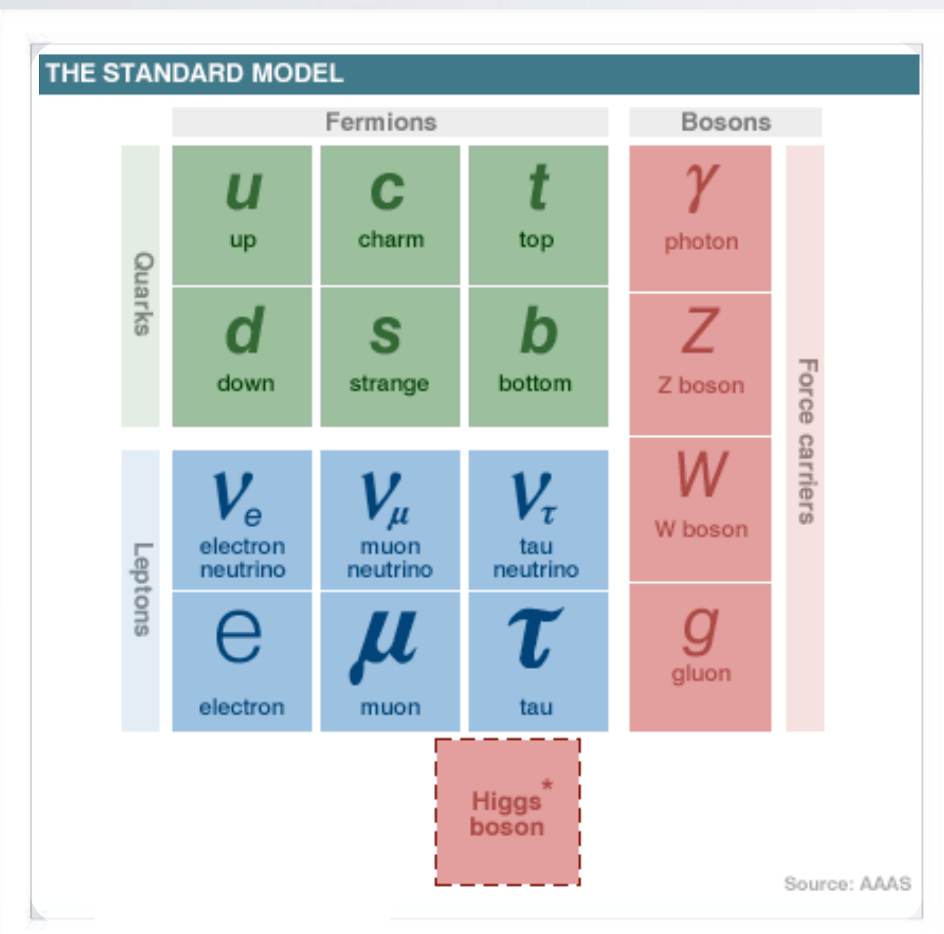
The Geometric Standard Model Effective Field Theory

Michael Trott, NBI/CERN

The Standard model ...

- The SM, an SU(3) xSU(2)xU(1) ***linearly realized gauge theory*** :

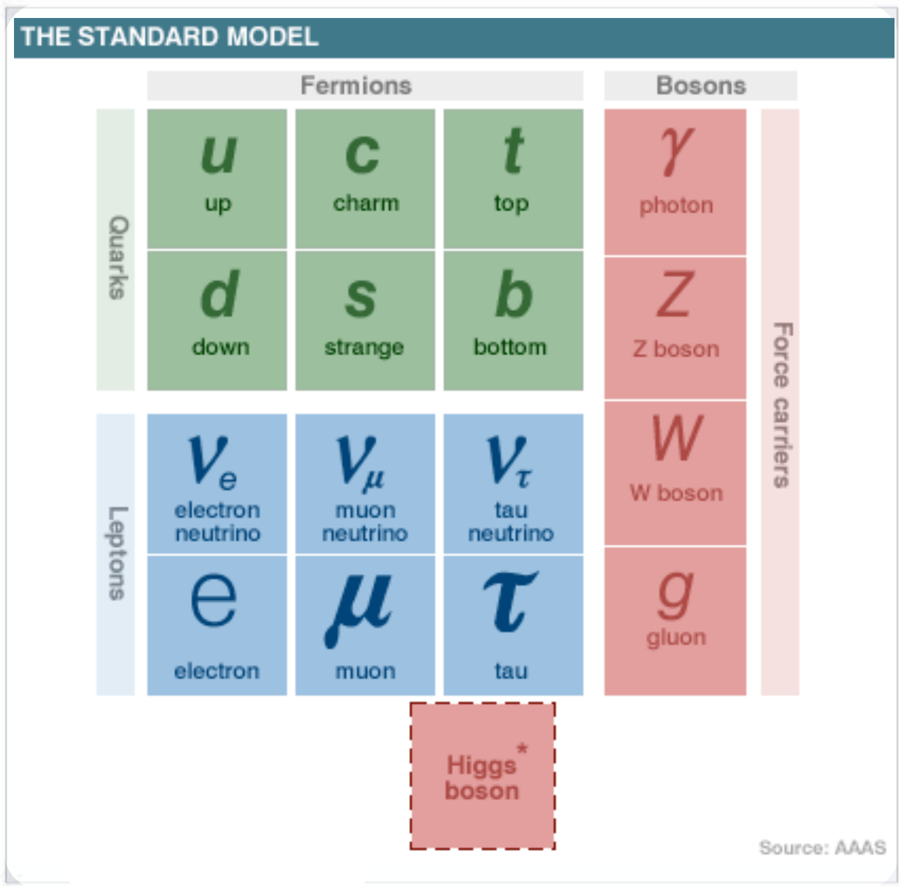
$$
\begin{aligned}
\mathcal{L}_{\rm SM} = & -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \overline{\psi}\, i\not{D}\, \psi \\
& - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - \left[H^{\dagger j}\overline{d}\,Y_d\,q_j + \widetilde{H}^{\dagger j}\overline{u}\,Y_u\,q_j + H^{\dagger j}\overline{e}\,Y_e\,l_j + \text{h.c.}\right],
\end{aligned}
$$



The Standard model EFT

- An SU(3) xSU(2)xU(1) ***linearly realised EFT***:

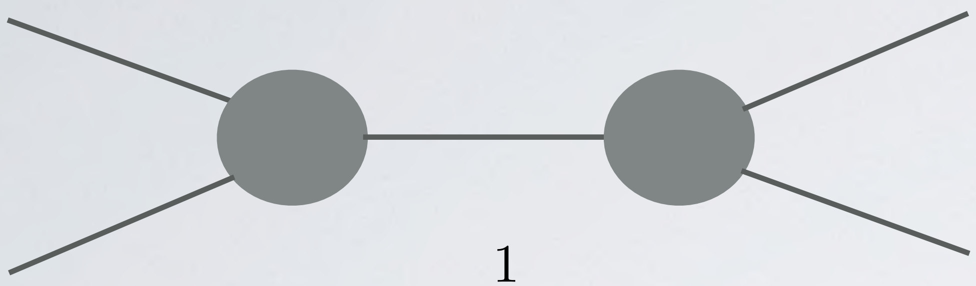
$$\mathcal{L}_{\rm SM} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \overline{\psi}\, i \not{D}\, \psi$$
$$- \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 - \left[H^{\dagger j} \overline{d}\, Y_d\, q_j + \widetilde{H}^{\dagger j} \overline{u}\, Y_u\, q_j + H^{\dagger j} \overline{e}\, Y_e\, l_j + \text{h.c.} \right],$$



$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \cdots$$

- (red) Glashow 1961, Weinberg 1967 (Salam 1967)
- (blue) Weinberg 1979, Wilczek and Zee 1979
- (green) Leung, Love, Rao 1984, Buchmuller Wyler 1986, Grzadkowski, Iskrzynski, Misiak,Rosiek 2010

When you do measurements below a particle threshold



$$\sim \frac{1}{s - m^2 + i\Gamma(s)\, m}$$

IF the collision probe does not reach $\sim m^2_{heavy}$

THEN observable's dependence on that scale simplified

- Taylor expand in LOCAL functions (operators)

$$\langle\rangle \sim O^0_{SM} + \frac{f_1(s,t,u)}{M^2_{heavy}} + \frac{f_2(s,t,u)}{M^4_{heavy}} + \cdots$$

This is the core idea of EFT interpretations of the data.

IR operator form

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + ..., \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

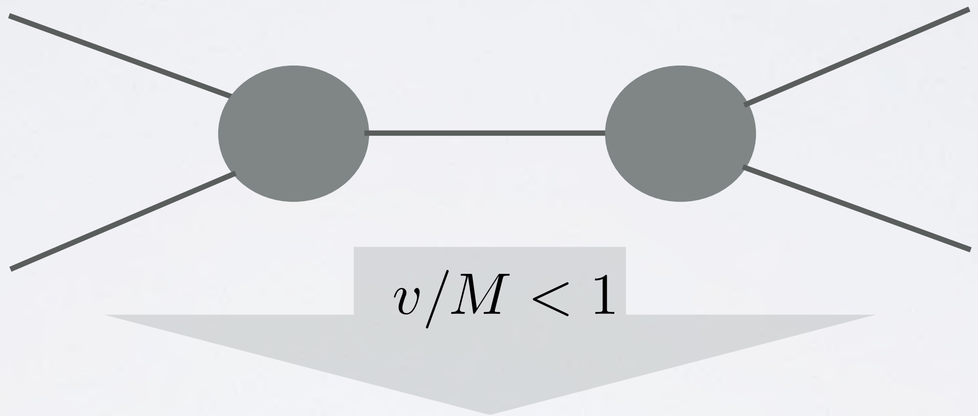
UV dependent Wilson coefficient
and suppression scale

The SMEFT as a consistent theory.

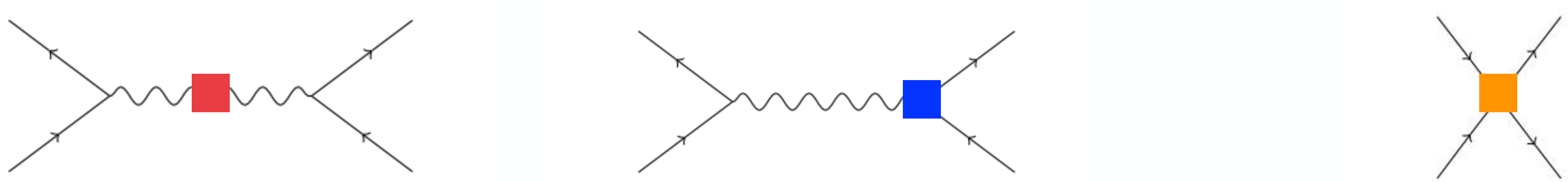
$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots, \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

- A theory with a tower of composite operator forms, useful (by assumption and definition) when interfacing with “decoupling” situation experimentally



$$Q_{HWB}, Q_{HD}, Q_{H\ell}^{(1)}, Q_{H\ell}^{(3)}, Q_{Hq}^{(1)}, Q_{Hq}^{(3)}, Q_{He}, Q_{Hu}, Q_{Hd}, Q_{\ell\ell}$$

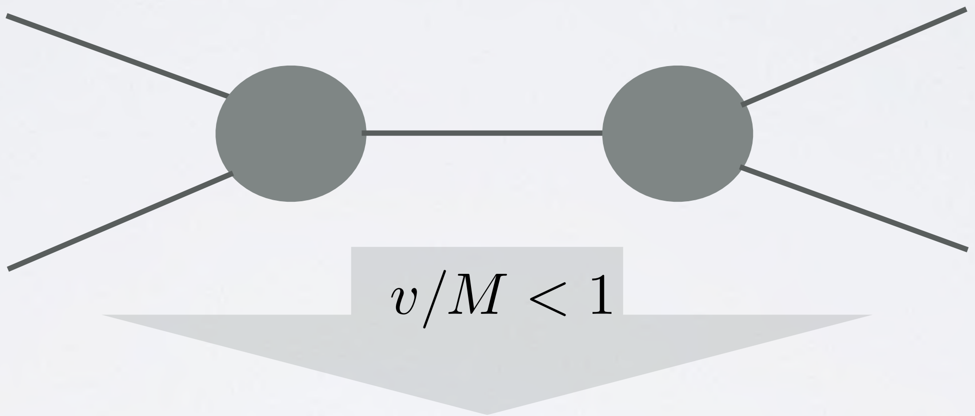


$\mathcal{L}_{SMEFT} \neq \mathcal{L}_{SM} + \mathcal{L}_{NP}$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + ..., \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

- A theory with a tower of composite operator forms, useful (by assumption and definition) when interfacing with “decoupling” situation experimentally



Decoupling theorem:T. Appelquist, J. Carazzone, Phys. Rev. D11, 2856 (1975

For any 1PI Feynman graph with external vector mesons only but containing internal fermions, when all external momenta (i.e. p^2) are small relative to m^2, then apart from coupling constant and field strength renormalization the graph will be suppressed by some power of m relative to a graph with the same number of external vector mesons but no internal fermions.[25]

$\mathcal{L}_{SMEFT} \neq \mathcal{L}_{SM} + \mathcal{L}_{NP}$

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + ..., \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

- Matching forces the SMEFT to *reproduce the IR behaviour* of a NP model. Only for a limited momentum regime, for some lower scale measurements.

- The SMEFT is a different theory than any particular NP model. Matching does not equate the theories. Consider the counterterms (Z) for renormalisation

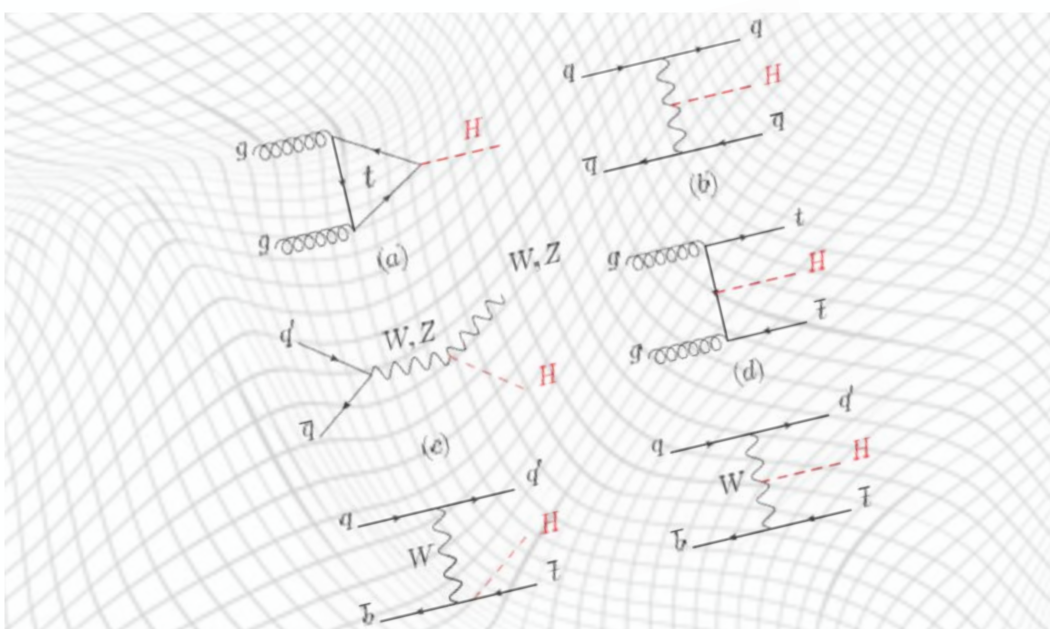
$$Z_{SMEFT} \neq Z_{SM} + Z_{NP}$$

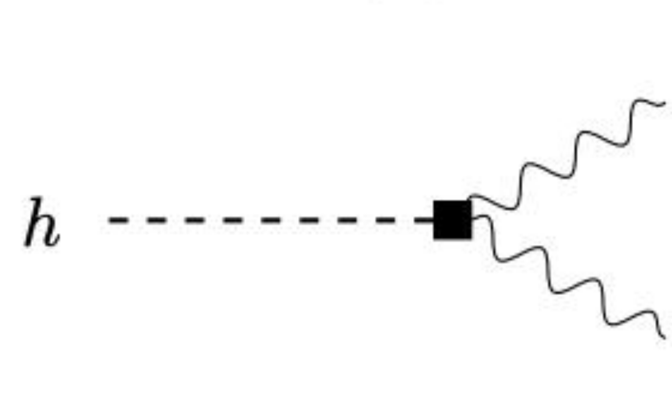
SMEFT is a consistent field theory, it is ***not*** the NP model.
We want to understand SMEFT and use it to interface with the data

Premise: SMEFT is a stand alone theory.

- Q’s: What is the core physics of the SMEFT? Is there any?
  Is this field theory too complicated to study bottom up?

- A: The physics of the SMEFT near resonances is interacting fields on a curved higgs field space. Using this idea, the theory is dramatically simplified.

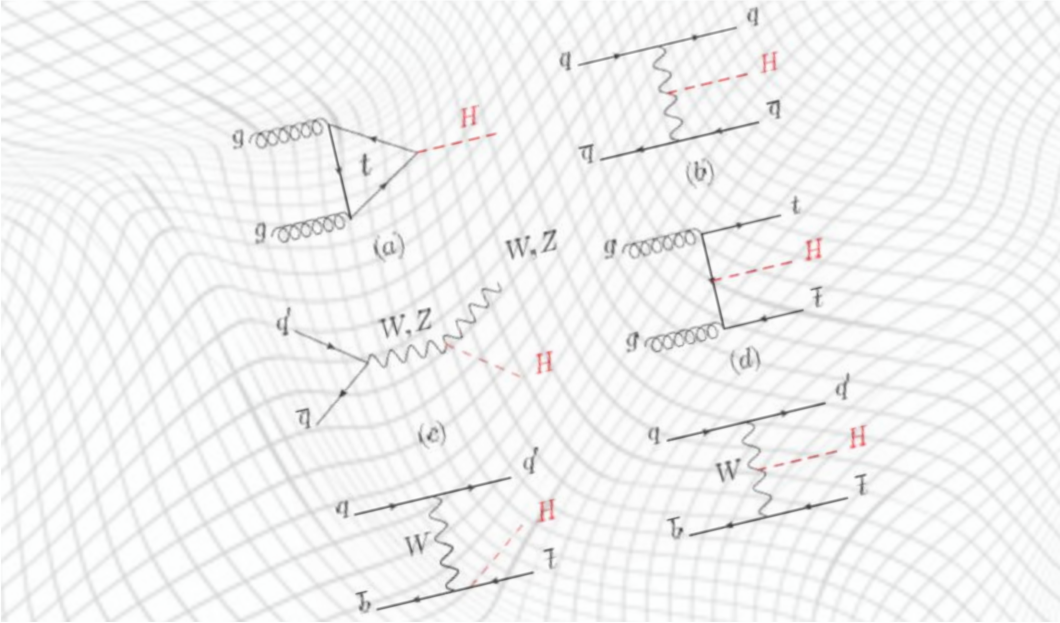


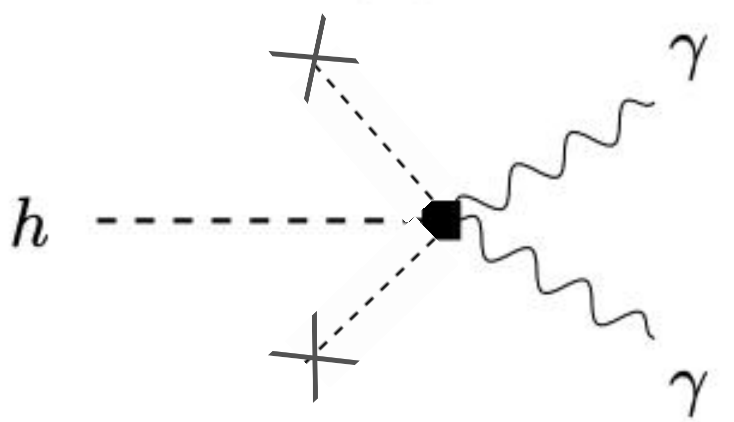


Premise: SMEFT is a stand alone theory.

- Q's: What is the core physics of the SMEFT? Is there any?
  Is this field theory too complicated to study bottom up?

- A: The physics of the SMEFT near resonances is interacting fields on a curved higgs field space. Using this idea, the theory is dramatically simplified.

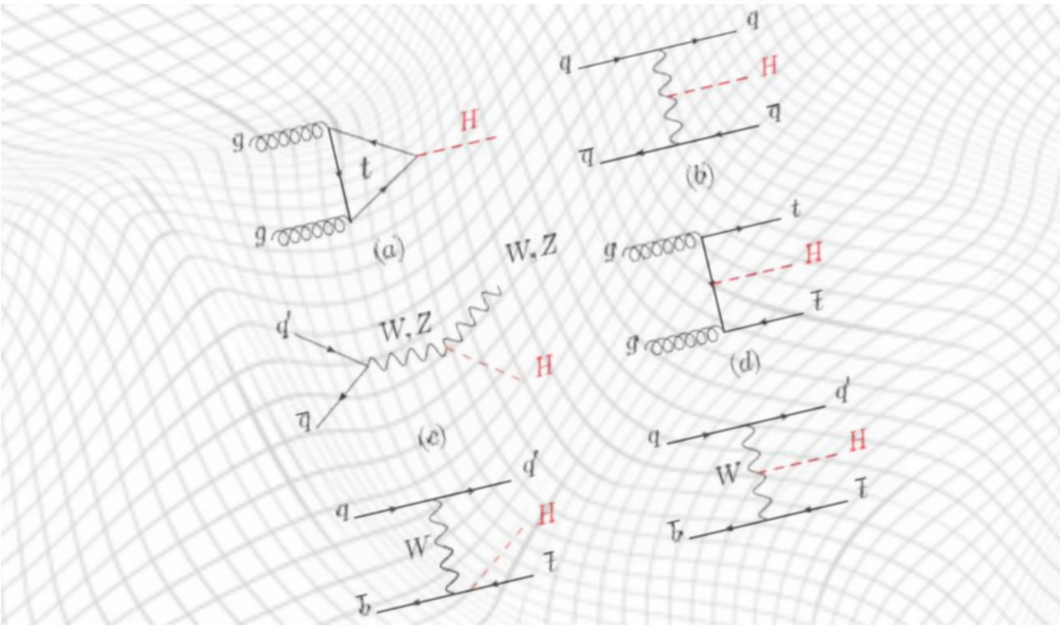


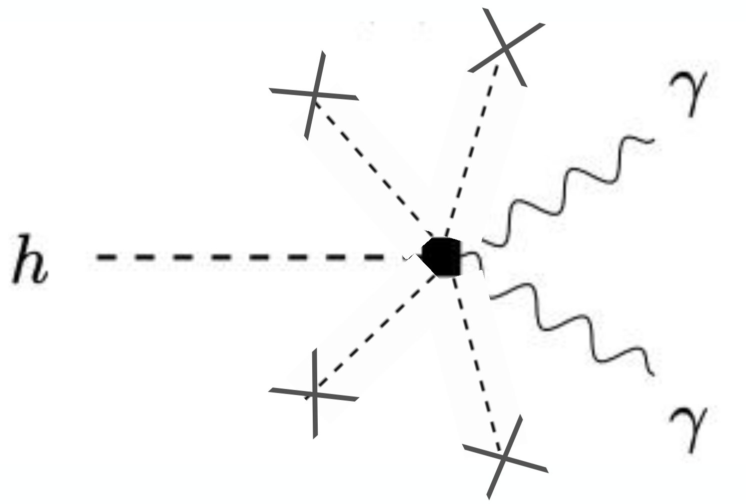


Premise: SMEFT is a stand alone theory.

- Q's: What is the core physics of the SMEFT? Is there any?
  Is this field theory too complicated to study bottom up?

- A: The physics of the SMEFT near resonances is interacting fields on
  a curved higgs field space. Using this idea, the theory is dramatically simplified.

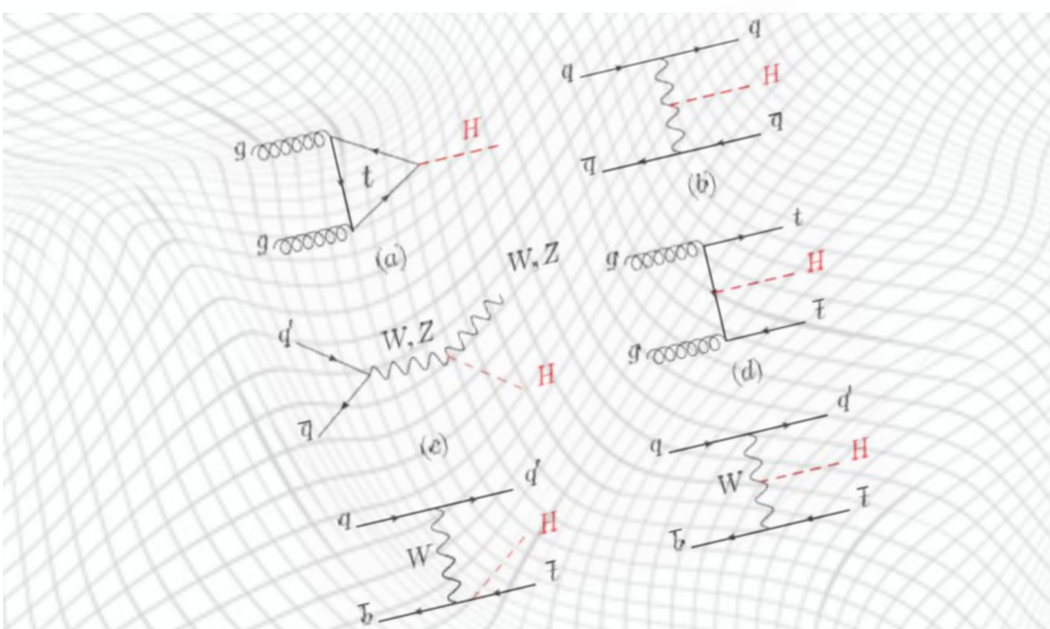


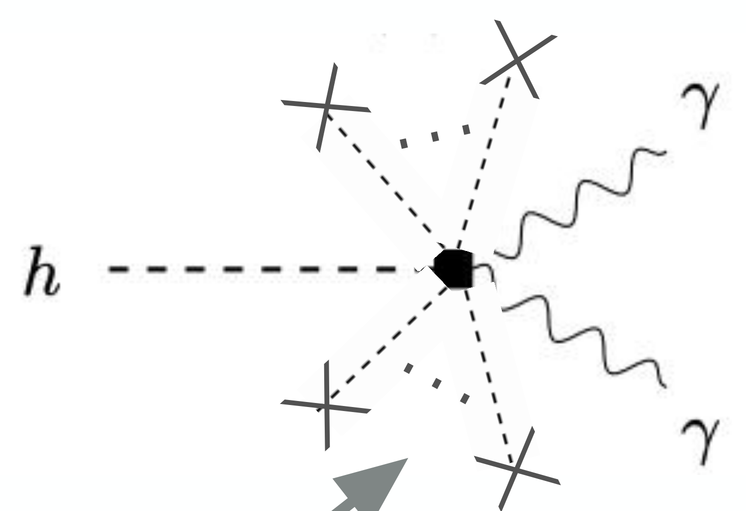


Premise: SMEFT is a stand alone theory.

- Q's: What is the core physics of the SMEFT? Is there any?
  Is this field theory too complicated to study bottom up?

- A: The physics of the SMEFT near resonances is interacting fields on a curved higgs field space. Using this idea, the theory is dramatically simplified.



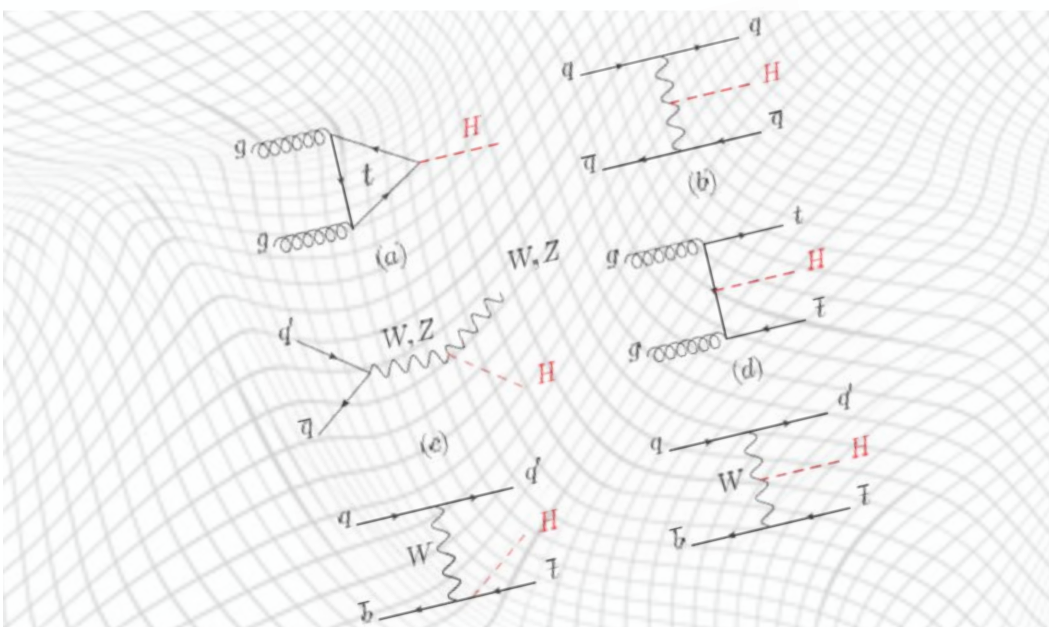


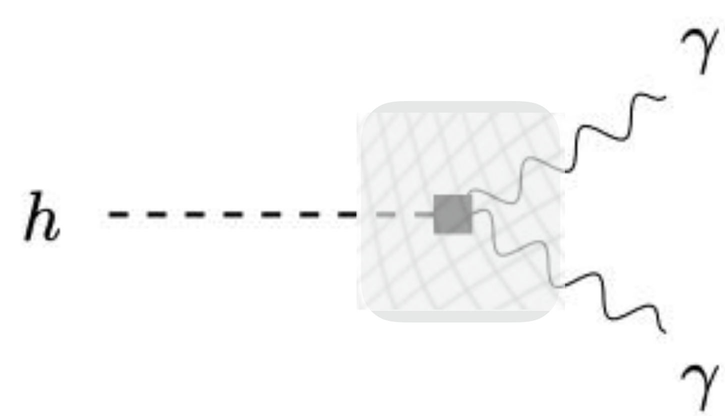
Powers of $\frac{H^\dagger H}{\Lambda^2}$
and symmetry generators

Premise: SMEFT is a stand alone theory.

- Q's: What is the core physics of the SMEFT? Is there any?
  Is this field theory too complicated to study bottom up?

- A: The physics of the SMEFT near resonances is interacting fields on a curved higgs field space. Using this idea, the theory is dramatically simplified.





Curved SMEFT spaces: scalar fields

- Curved SMEFT field space manifest in background field formulation

In general terms: G. A. Vilkovisky, Nucl. Phys. B234 (1984) 125.

Metric on Higgs field space, SM a **FLAT** field space

$$\mathcal{L}_{\rm scalar,kin} = \frac{1}{2} h_{IJ}(\phi) \left(D_\mu \phi\right)^I \left(D^\mu \phi\right)^J ,$$

Where $H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\Box} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

Small perturbations so positive semi-definite Matrix and unique square root

(sqrt) Metric in SMEFT, a *curved* field space

$$R^I_{JKL} \neq 0$$

1002.2730 Burgess, Lee, Trott
1511.00724 Alonso, Jenkins, Manohar
1605.03602 Alonso, Jenkins, Manohar

Curved SMEFT space: gauge fields

- Similarly in the gauge coupling space a curved field space

Metric on gauge field space, SM a **FLAT** field space

$$\mathcal{L}_{\text{gauge,kin}} = -\frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu},$$

Where $\mathcal{W}^A = (W^1, W^2, W^3, B)$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1+\tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1+\tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1+\tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1+\tilde{C}_{HB} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

1803.08001 Helset, Paraskevas, Trott
1909.08470 Corbett, Helset, Trott

(sqrt) Metric in SMEFT, a *curved* field space

Generalisation for composite ops

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots, \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

$$\mathcal{L}_{\rm SMEFT} = \sum_i f_i(\alpha \cdots) G_i(I, A \cdots),$$

Derivative expansion

Composite operator form
With minimal scalar field
coordinate dependence

Vev expansion

Scalar field coordinate dependence
And insertions of symmetry generators

$D^\mu \phi$

Mixes expansions, but grouped with derivative forms.

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu\phi)^I \, \delta(D_\nu\phi)^J}\Bigg|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu\Phi)^K, \bar\psi\sigma^\mu\psi, \bar\psi\psi\cdots\}$

- Limited number of such connections for up to three point functions

$$V(\phi) \qquad h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}, \quad k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A,$$
$$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$

With fermions $\quad Y(\phi)\bar\psi_1\psi_2, \quad L_{I,A}(\phi)\bar\psi_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar\psi_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$

Gluon fields $\quad k_{\mathcal{AB}}(\phi)G^{\mathcal{A}}_{\mu\nu}G^{\mathcal{B},\mu\nu}, \quad k_{\mathcal{ABC}}(\phi)G^{\mathcal{A}}_{\nu\mu}G^{\mathcal{B},\rho\nu}G^{\mathcal{C},\mu\rho}, \quad c(\phi)\bar\psi_1\sigma^{\mu\nu}T_{\mathcal{A}}\psi_2G^{\mathcal{A}}_{\mu\nu}.$

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu \phi)^I \, \delta(D_\nu \phi)^J}\Bigg|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu \Phi)^K, \bar{\psi}\sigma^\mu \psi, \bar{\psi}\psi \cdots\}$

- Limited number of such connections for up to three point functions

This is a non trivial fact proven for: $F = \{H, \psi, \mathcal{W}^{\mu\nu}\}$ via the following:

$$D^2 F \;\Rightarrow\; \boxed{\rm EOM} \text{ and higher-points,}$$

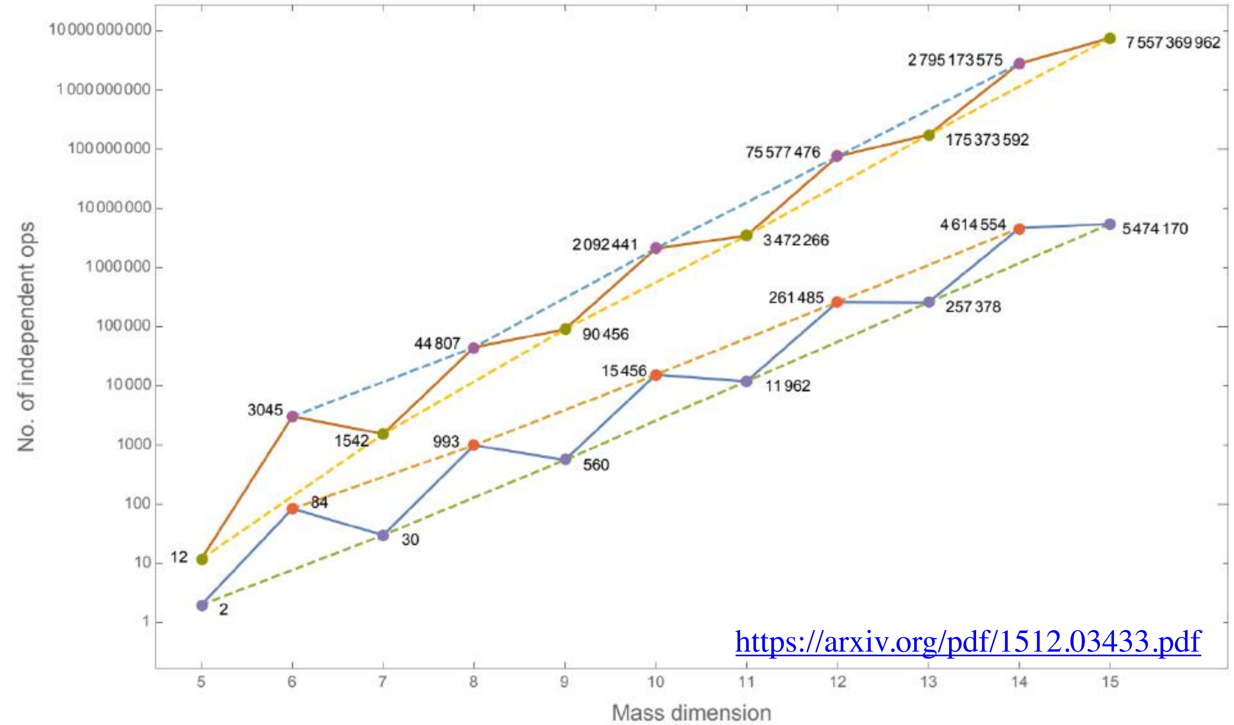
2001.01453 Helset, Martin, Trott

$$f(H)(D_\mu F_1)(D_\nu F_2) D_{\{\mu\nu\}} F_3 \;\Rightarrow\; \boxed{\rm EOM} \text{ and higher-points.}$$

$$f(\phi)\, F_1\, (D_\mu F_2)\, (D_\mu F_3) \Rightarrow (D_\mu f(\phi))\, (D_\mu F_1)\, F_2\, F_3 + \frac{1}{2}(D^2 f(\phi))\, F_1\, F_2\, F_3 + \boxed{\rm EOM},$$

The connections can be defined to all orders

2001.01453 Helset, Martin, Trott



- Growth in operator forms in connections

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

- Growth in operator forms from Hilbert series

  https://arxiv.org/abs/1503.07537

  https://arxiv.org/abs/1510.00372

  https://arxiv.org/pdf/1512.03433.pdf

  https://arxiv.org/abs/1706.08520

rather overwhelming…

- Number of operator forms saturate. This is due to reducing possible generator insertions on the Higgs manifold

$$T^a_{ij}T^a_{k\ell} = \frac{1}{2}\left(\delta_{i\ell}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{k\ell}\right)$$

Generators on scalar SMEFT space

- To think in a unified gauge space manifold need generators (reformulate SM generators)

$$(D^\mu \phi)^I = (\partial^\mu \delta^I_J - \tfrac{1}{2}\mathcal{W}^{A,\mu}\tilde{\gamma}^I_{A,J})\phi^J$$

$$\tilde{\epsilon}^A{}_{BC} = g_2\, \epsilon^A{}_{BC}, \qquad \text{with } \tilde{\epsilon}^1{}_{23} = +g_2,$$

$$\tilde{\gamma}^I_{A,J} = \begin{cases} g_2\, \gamma^I_{A,J}, & \text{for } A = 1,2,3 \\ g_1 \gamma^I_{A,J}, & \text{for } A = 4. \end{cases}$$

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

(this last one also “i”)

1803.08001 Helset, Paraskevas, Trott

- Some interesting math here, we also define $\Gamma^I_{A,K} = \gamma^I_{A,J}\, \gamma^J_{4,K}$

$$\Gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \Gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma^I_{3,J} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^I_{4,J} = -\mathbb{I}_{4\times 4}.$$

Generators on scalar SMEFT space

- The mapping of operator forms works via: 2001.01453 Helset, Martin, Trott

$$
\begin{aligned}
H^\dagger \sigma_a H &= -\frac{1}{2}\phi_I \Gamma^I_{a,J}\phi^J, \\
H^\dagger \, i\overleftrightarrow{D}^\mu H &= -\phi_I \, \gamma^I_{4,J}(D^\mu\phi)^J = (D^\mu\phi)_I \, \gamma^I_{4,J}\phi^J, \\
H^\dagger \, i\overleftrightarrow{D}^\mu_a H &= -\phi_I \, \gamma^I_{a,J}(D^\mu\phi)^J = (D^\mu\phi)_I \, \gamma^I_{a,J}\phi^J, \\
2\tilde{H}^\dagger D^\mu H &= \tilde{\phi}_I(-\Gamma^I_{4,J} + i\,\gamma^I_{4,J})\,(D^\mu\phi)^J.
\end{aligned}
$$

It is useful to "real" the SM symmetry representation on the scalar manifold for lots of reasons. Makes more manifest possible contractions

$$
\phi_I \Gamma^I_{A,J}\phi^J \neq 0, \qquad \phi_I \gamma^I_{a,J}\phi^J = \phi_I \gamma^I_{4,J}\phi^J = 0.
$$

- All orders results follow, for example:

$$
\begin{aligned}
h_{IJ} = &\left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+2}\left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right]\delta_{IJ} \\
&+ \frac{\Gamma^I_{A,J}\phi_K \Gamma^K_{A,L}\phi^L}{2}\left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right).
\end{aligned}
$$

Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Field space connection for W,Z coupling to fermion pairs $(D^\mu\phi)^I\,\bar\psi\,\Gamma_\mu\psi$

$$
\begin{aligned}
\mathcal{Q}^{1,(6+2n)}_{\substack{H\psi\\pr}} &= (H^\dagger H)^n H^\dagger\, i\overleftrightarrow{D}^\mu H \bar\psi_p\gamma_\mu\psi_r,\\
\mathcal{Q}^{3,(6+2n)}_{\substack{H\psi\\pr}} &= (H^\dagger H)^n\, H^\dagger\, i\overleftrightarrow{D}^\mu_a H \bar\psi_p\gamma_\mu\sigma_a\psi_r,\\
\mathcal{Q}^{2,(8+2n)}_{\substack{H\psi\\pr}} &= (H^\dagger H)^n (H^\dagger\sigma_a H)\, H^\dagger\, i\overleftrightarrow{D}^\mu H \bar\psi_p\gamma_\mu\sigma_a\psi_r,\\
\mathcal{Q}^{\epsilon,(8+2n)}_{\substack{H\psi\\pr}} &= \epsilon^a_{bc}\,(H^\dagger H)^n\,(H^\dagger\sigma_c H)\, H^\dagger\, i\overleftrightarrow{D}^\mu_b H \bar\psi_p\gamma_\mu\sigma_a\psi_r.
\end{aligned}
$$

Not that many op forms. Closed form field space connection.

$$
\begin{aligned}
L^{\psi,pr}_{J,A} = &-(\phi\,\gamma_4)_J\delta_{A4}\sum_{n=0}^{\infty} C^{1,(6+2n)}_{\substack{H\psi\\pr}}\left(\frac{\phi^2}{2}\right)^n - (\phi\,\gamma_A)_J(1-\delta_{A4})\sum_{n=0}^{\infty} C^{3,(6+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n\\
&+\frac{1}{2}(\phi\,\gamma_4)_J(1-\delta_{A4})\left(\phi_K\Gamma^K_{A,L}\phi^L\right)\sum_{n=0}^{\infty} C^{2,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n\\
&+\frac{\epsilon^A_{BC}}{2}(\phi\,\gamma_B)_J\left(\phi_K\Gamma^K_{C,L}\phi^L\right)\sum_{n=0}^{\infty} C^{\epsilon,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n.
\end{aligned}
$$

Notice the clean form due to generator structure and real fields.

Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Off shell operators contributing to three points $(D_\mu\phi)^I \sigma_A (D_\nu\phi)^J \mathcal{W}^A_{\mu\nu}$

$$
\begin{aligned}
Q^{(8+2n)}_{HDHB} &= i\,(H^\dagger H)^{n+1}(D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}, \\
Q^{(8+2n)}_{HDHW} &= i\delta_{ab}(H^\dagger H)^{n+1}(D_\mu H)^\dagger \sigma^a (D_\nu H) W_b^{\mu\nu}, \\
Q^{(8+2n)}_{HDHW,2} &= i\epsilon_{abc}(H^\dagger H)^{n}(H^\dagger \sigma^a H)(D_\mu H)^\dagger \sigma^b (D_\nu H) W_c^{\mu\nu}, \\
Q^{(10+2n)}_{HDHW,3} &= i\delta_{ab}\delta_{cd}(H^\dagger H)^{n}(H^\dagger \sigma^a H)(H^\dagger \sigma^c H)(D_\mu H)^\dagger \sigma^b (D_\nu H) W_d^{\mu\nu}.
\end{aligned}
$$

This connection saturates last in op dimension. This is due to EOM reduction. No entries at dim 6 in Warsaw basis.

$$
\begin{aligned}
k^A_{IJ}(\phi) = &-\frac{1}{2}\gamma^I_{4,J}\delta_{A4}\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHB}\left(\frac{\phi^2}{2}\right)^{n+1} - \frac{1}{2}\gamma^I_{A,J}(1-\delta_{A4})\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHW}\left(\frac{\phi^2}{2}\right)^{n+1} \\
&- \frac{1}{8}(1-\delta_{A4})\left[\phi_K \Gamma^K_{A,L}\phi^L\right]\left[\phi_M \Gamma^K_{B,L}\phi^N\right]\gamma^I_{B,J}\sum_{n=0}^{\infty} C^{(10+2n)}_{HDHW,3}\left(\frac{\phi^2}{2}\right)^{n} \\
&+ \frac{1}{4}\epsilon_{ABC}\left[\phi_K \Gamma^K_{B,L}\phi^L\right]\gamma^I_{C,J}\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHW,2}\left(\frac{\phi^2}{2}\right)^{n}.
\end{aligned}
$$

All orders weak mass eigenstate relations

- Weak eigenstates

Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \sqrt{g}^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

1909.08470 Corbett, Helset, Trott

Generator transform

$$\gamma^{I}_{C,J} = \frac{1}{2}\tilde{\gamma}^{I}_{A,J}\sqrt{g}^{AB} U_{BC}.$$

Field space metrics
(Now known to all orders)

Rotations

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix} \quad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^{J} = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^{K} = \{\Phi^{-}, \Phi^{+}, \chi, h\}$$

$$\alpha^{A} = \{g_2\, g_2, g_2, g_1\}, \qquad \mathcal{W}^{A} = \{W_1, W_2, W_3, B\},$$

$$\beta^{C} = \left\{\frac{g_2\,(1-i)}{\sqrt{2}}, \frac{g_2\,(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c^2_{\overline{\theta}} - s^2_{\overline{\theta}}), \frac{2\, g_1\, g_2}{\sqrt{g_1^2+g_2^2}}\right\}, \qquad \mathcal{A}^{C} = (\mathcal{W}^{+}, \mathcal{W}^{-}, \mathcal{Z}, \mathcal{A})\,.$$

What else could you write? Nothing that generalises to all orders.

Dim 6 SMEFT EW Lagrangian terms

- EW sector parameters redefined in the SMEFT

$$\begin{bmatrix} \mathcal{W}^3_\mu \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}\, v_T^2\, C_{HWB} \\ -\frac{1}{2}\, v_T^2\, C_{HWB} & 1 \end{bmatrix} \begin{bmatrix} \cos\overline{\theta} & \sin\overline{\theta} \\ -\sin\overline{\theta} & \cos\overline{\theta} \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix},$$

Mass redefinitions

$$M_W^2 = \frac{\overline{g}_2{}^2 v_T^2}{4},$$
$$M_Z^2 = \frac{v_T^2}{4}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{8} v_T^4 C_{HD} (\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{2} v_T^4 \overline{g}_1 \overline{g}_2 C_{HWB}.$$

Mixing angle redefinitions

$$\sin\overline{\theta} = \frac{\overline{g}_1}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 + \frac{v_T^2}{2} \frac{\overline{g}_2}{\overline{g}_1} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$
$$\cos\overline{\theta} = \frac{\overline{g}_2}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 - \frac{v_T^2}{2} \frac{\overline{g}_1}{\overline{g}_2} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$

Interactions to remaining SM fields via:

$$D_\mu = \partial_\mu + i\, \frac{\overline{g}_2}{\sqrt{2}} \left[\mathcal{W}^+_\mu T^+ + \mathcal{W}^-_\mu T^-\right] + i\, \overline{g}_Z \left[T_3 - \overline{s}^2 Q\right] \mathcal{Z}_\mu + i\, \overline{e}\, Q\, \mathcal{A}_\mu,$$

$$\overline{e} = \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} \left[1 - \frac{\overline{g}_1 \overline{g}_2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} v_T^2 C_{HWB}\right]$$

$$\overline{g}_Z = \sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} v_T^2 C_{HWB}$$

$$\overline{s}^2 = \sin^2\overline{\theta} = \frac{\overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2 (\overline{g}_2{}^2 - \overline{g}_1{}^2)}{(\overline{g}_1{}^2 + \overline{g}_2{}^2)^2} v_T^2 C_{HWB}.$$

1312.2014 Alonso, Jenkins, Manohar, Trott

All orders SM Lagrangian parameters

- Low n-point interactions of fields are parameterised in terms of couplings,

2001.01453 Helset, Martin, Trott

$$\bar{g}_2 = g_2 \sqrt{g}^{11} = g_2 \sqrt{g}^{22},$$
$$\bar{g}_Z = \frac{g_2}{c^2_{\theta_Z}} \left(c_{\bar{\theta}} \sqrt{g}^{33} - s_{\bar{\theta}} \sqrt{g}^{34} \right) = \frac{g_1}{s^2_{\theta_Z}} \left(s_{\bar{\theta}} \sqrt{g}^{44} - c_{\bar{\theta}} \sqrt{g}^{34} \right),$$
$$\bar{e} = g_2 \left(s_{\bar{\theta}} \sqrt{g}^{33} + c_{\bar{\theta}} \sqrt{g}^{34} \right) = g_1 \left(c_{\bar{\theta}} \sqrt{g}^{44} + s_{\bar{\theta}} \sqrt{g}^{34} \right),$$

- Masses

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4} \sqrt{h_{11}}^2 \bar{v}_T^2, \qquad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4} \sqrt{h_{33}}^2 \bar{v}_T^2 \qquad \bar{m}_A^2 = 0.$$

- Mixing angles:

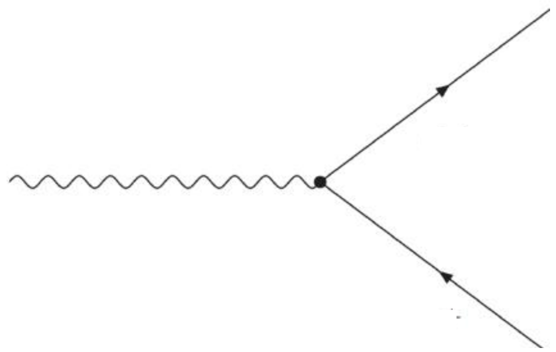
$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})}{g_2(\sqrt{g}^{33} c_{\bar{\theta}} - \sqrt{g}^{34} s_{\bar{\theta}}) + g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})},$$
$$s^2_{\bar{\theta}} = \frac{(g_1 \sqrt{g}^{44} - g_2 \sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2 g_1 g_2 \sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

(Interesting way to think of the Weinberg angle)

GeoSMEFT a friend with benefits

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

The all orders coupling in the SMEFT is a sum of two field space connections.

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \sqrt{g}^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

$\bar{\psi} i \not{D} \psi$:with a consistent change weak to mass eigenstates in SMEFT

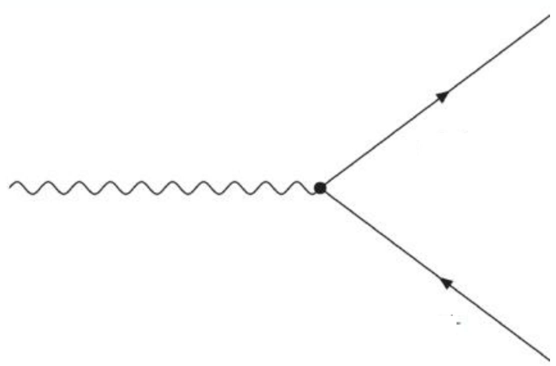
Added to this is the scalar, fermion connection (with a background field expectation)

$$L^{\psi_R}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,R} \gamma_\mu \sigma_A \psi_{r,R})$$
$$L^{\psi_L}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,L} \gamma_\mu \sigma_A \psi_{r,L})$$

GeoSMEFT a friend with benefits

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



$\mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A})$

Consider a $W^\pm, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^I_{C,4})\bar{v}_T,$$

Compact all $\bar{v}_T/\Lambda$ orders answer!

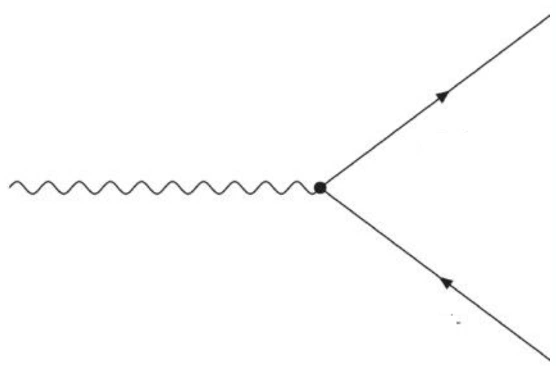
Here we have introduced the generators:

$$\bar{\tau}_{1,2} = \frac{\bar{g}_2}{\sqrt{2}} \frac{\sigma_1 \pm i\sigma_2}{2}, \quad \bar{\tau}_3 = \bar{g}_Z(T_3 - s^2_{\theta_Z} Q_\psi), \quad \bar{\tau}_4 = \bar{e} Q_\psi.$$

GeoSMEFT a friend with benefits

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\boldsymbol{\gamma}_{C,4}^{I})\bar{v}_T,$$

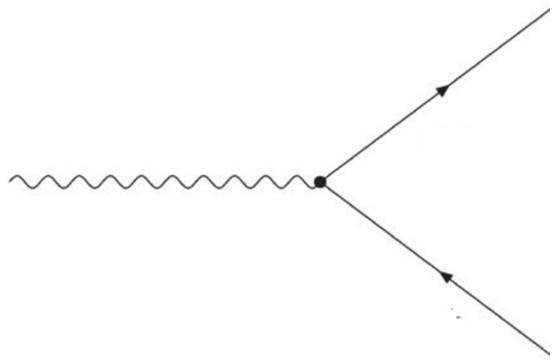
The coupling of the canonically normalised mass eigenstate fields is then

$$\begin{aligned}
\langle \mathcal{Z}|\bar{\psi}_p \psi_r\rangle &= \frac{\bar{g}_Z}{2}\,\bar{\psi}_p \,\not{\epsilon}_{\mathcal{Z}} \left[(2 s_{\theta_Z}^2 Q_\psi - \sigma_3)\delta_{pr} + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi,pr}\rangle + \bar{v}_T \langle L_{3,4}^{\psi,pr}\rangle\right] \psi_r,\\
\langle \mathcal{A}|\bar{\psi}_p \psi_r\rangle &= -\bar{e}\,\bar{\psi}_p \,\not{\epsilon}_{\mathcal{A}}\, Q_\psi\, \delta_{pr}\, \psi_r,\\
\langle \mathcal{W}_{\pm}|\bar{\psi}_p \psi_r\rangle &= -\frac{\bar{g}_2}{\sqrt{2}}\bar{\psi}_p(\not{\epsilon}_{\mathcal{W}^{\pm}})\, T^{\pm}\left[\delta_{pr} - \bar{v}_T \langle L_{1,1}^{\psi,pr}\rangle \pm i \bar{v}_T \langle L_{1,2}^{\psi,pr}\rangle\right] \psi_r.
\end{aligned}$$

Rather compact result.

GeoSMEFT a friend with benefits

- Can build up observable quantities, such as a decay width.



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\boldsymbol{\gamma}_{C,4}^{I})\bar{v}_T,$$

- Two body decay widths:

$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_{\psi}^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$\bar{\Gamma}_{W\to\bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi}\sqrt{\bar{m}_W^2}|g_{\rm eff}^{W,\psi}|^2\left(1-\frac{4\bar{M}_{\psi}^2}{\bar{m}_W^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}^2\, Q_{\psi}-\sigma_3)\delta_{pr} + \bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle + \sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right]$$

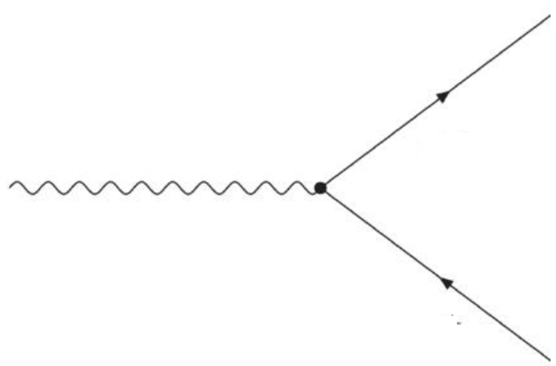
$$g_{\rm eff}^{W,q_L} = -\frac{\bar{g}_2}{\sqrt{2}}\left[V_{\rm CKM}^{pr} - \bar{v}_T\langle L_{1,1}^{q_L,pr}\rangle \pm i\bar{v}_T\langle L_{1,2}^{q_L,pr}\rangle\right],$$

$$g_{\rm eff}^{W,\ell_L} = -\frac{\bar{g}_2}{\sqrt{2}}\left[U_{\rm PMNS}^{pr,\dagger} - \bar{v}_T\langle L_{1,1}^{\ell_L,pr}\rangle \pm i\bar{v}_T\langle L_{1,2}^{\ell_L,pr}\rangle\right],$$

- Many further contributions and radiative corrections constructible from two and three point functions

GeoSMEFT a friend with benefits

- Can build up observable quantities, such as a decay width.



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

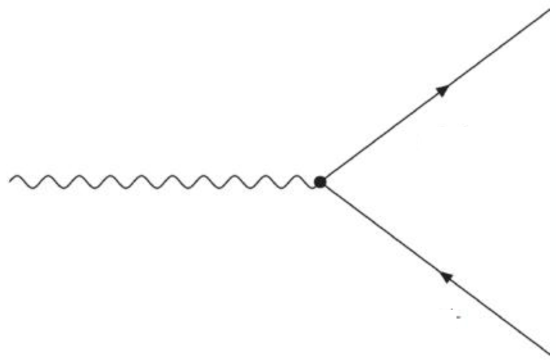
$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p\gamma_\mu\bar{\tau}_A\psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p\gamma_\mu\sigma_A\psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^{I}_{C,4})\bar{v}_T,$$

Or?

- Many further contributions and radiative corrections constructible from two and three point functions

GeoSMEFT a friend with benefits

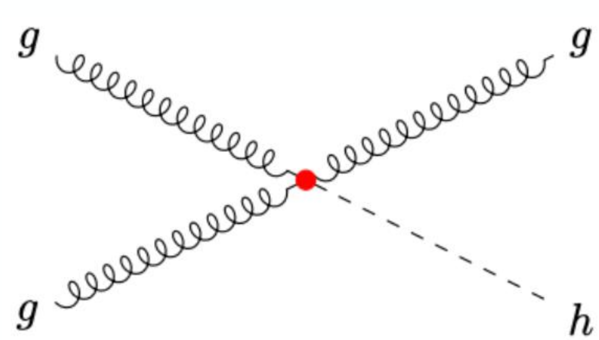
- Can build up observable quantities, such as a decay width.

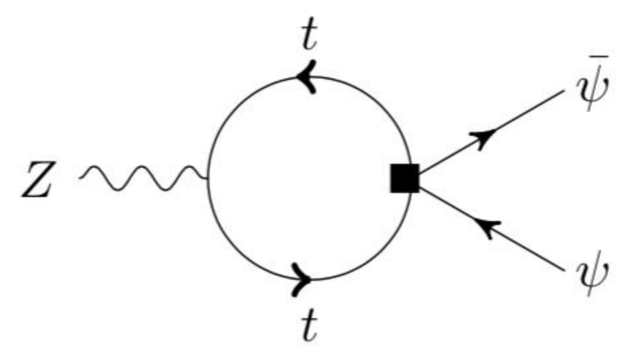


Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^{I}_{C,4})\bar{v}_T,$$

- Not all physics is derivable from two and three point functions





- How to incorporate such higher n-point effects is the key challenge.

GeoSMEFT where is the body?

- Note these integration by parts steps were used

$$
\begin{aligned}
& f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \\
= & - f(H)\left[(D^2 F_1)(D_\nu F_2) + (D_\mu F_1)(D_\mu D_\nu F_2) + (D_\mu D_\nu F_1)(D_\mu F_2) + (D_\nu F_1)(D^2 F_2)\right](D_\nu F_3) \\
& - (D_\mu f(H))\left[(D_\mu F_1)(D_\nu F_2) + (D_\nu F_1)(D_\mu F_2)\right](D_\nu F_3)
\end{aligned}
$$

$$
f(\phi)\,F_1\,(D_\mu F_2)\,(D_\mu F_3) \Rightarrow (D_\mu f(\phi))\,(D_\mu F_1)\,F_2\,F_3 + \frac{1}{2}(D^2 f(\phi))\,F_1\,F_2\,F_3 + \boxed{\mathrm{EOM}},
$$

These steps were critical to reducing the number of connections for two and three point functions. This just fails for four points and higher.

One knows that there are an infinite set of higher derivative terms lurking In higher n points, dependent on $\{D_\mu\phi^I, D_{\{\mu,\nu\}}\phi^I, D_{\{\mu,\nu,\rho\}}\phi^I, \cdots\}$,

This is a problem for measurements away from SM resonances.

GeoSMEFT based loop corrections?

- The simplicity of the results for two and three point functions points to radiative corrections being more elegant than expected.

  The renormalisation follows the dependence on the Wilson coefficients.

- Do we have hints of this yet? Yes.

  $\mathcal{L}_{GF}$

- Background field gauge fixing with preserved background Gauge invariance 1803.08001 Helset, Paraskevas, Trott

$$\mathcal{L}_{\rm GF} = -\frac{\hat{g}_{AB}}{2\,\xi}\mathcal{G}^A\,\mathcal{G}^B,$$

$$\mathcal{G}^X \equiv \partial_\mu \mathcal{W}^{X,\mu} - \tilde{\epsilon}^X_{\;CD}\hat{\mathcal{W}}^C_\mu \mathcal{W}^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\,\hat{h}_{IK}\,\tilde{\gamma}^K_{C,J}\hat{\phi}^J.$$

- Gauge fixing confusion directly solved generalising to GeoSMEFT

  Further exploration of gauge fixing based on this idea: 1812.11513 Misiak et al

GeoSMEFT based loop corrections?

- Will this simplify the NLO SMEFT radiative correction program?(Yes)

  Immediate BFM Ward Identities have already been derived:

$$\frac{\delta \Gamma[\hat{F},0]}{\delta \hat{\alpha}^B} = 0.$$

Background field gauge transformation

1909.08470 Corbett, Helset, Trott

$$0 = \left(\partial^\mu \delta^A_B - \tilde{\epsilon}^A_{\;BC}\, \hat{\mathcal{W}}^{C,\mu}\right) \frac{\delta \Gamma}{\delta \hat{\mathcal{W}}^\mu_A} - \frac{\tilde{\gamma}^I_{B,J}}{2} \hat{\phi}^J \frac{\delta \Gamma}{\delta \hat{\phi}^I} + \sum_j \left(\bar{f}_j \bar{\Lambda}^j_{B,i} \frac{\delta \Gamma}{\delta \bar{f}_i} - \frac{\delta \Gamma}{\delta f_i} \Lambda^i_{B,j} f_j\right).$$

Photon identities:

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^{4\mu} \delta \hat{\mathcal{A}}^{Y\nu}}, \qquad 0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^{4\mu} \delta \hat{\Phi}^I}.$$

$$\Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{L,\rm SMEFT}(k^2) = 0, \qquad \Sigma^{\hat{\mathcal{A}},\hat{\mathcal{A}}}_{T,\rm SMEFT}(0) = 0.$$

One loop behaviour is being checked - it looks promising

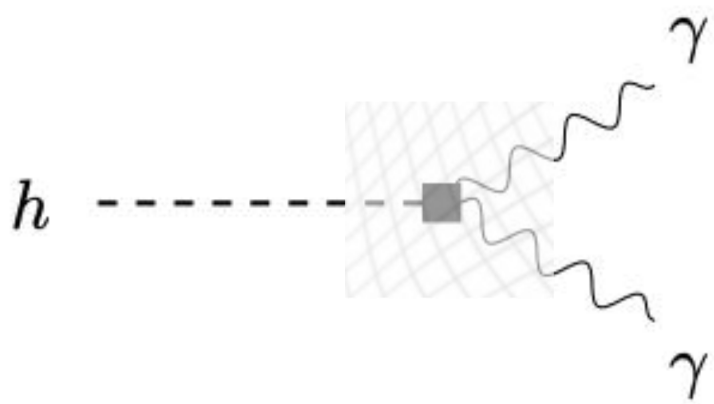
Z identities:

Geometric mass

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^{3\mu} \delta \hat{\mathcal{A}}^{Y\nu}} - \bar{M}_Z \frac{\delta^2 \Gamma}{\delta \hat{\Phi}^3 \delta \hat{\mathcal{A}}^{Y\nu}},$$

$$0 = \partial^\mu \frac{\delta^2 \Gamma}{\delta \hat{\mathcal{A}}^{3\mu} \delta \hat{\Phi}^I} - \bar{M}_Z \frac{\delta^2 \Gamma}{\delta \hat{\Phi}^3 \delta \hat{\Phi}^I} + \frac{\bar{g}_Z}{2} \frac{\delta \Gamma}{\delta \hat{\Phi}^4} \left(\sqrt{h}_{[4,4]} \sqrt{h}^{[3,3]} - \sqrt{h}_{[4,3]} \sqrt{h}^{[4,3]}\right) \delta^3_I - \frac{\bar{g}_Z}{2} \frac{\delta \Gamma}{\delta \hat{\Phi}^4} \left(\sqrt{h}_{[4,4]} \sqrt{h}^{[3,4]} - \sqrt{h}_{[4,3]} \sqrt{h}^{[4,4]}\right) \delta^4_I,$$

GeoSMEFT for the Higgs



- SMEFT diphoton decay

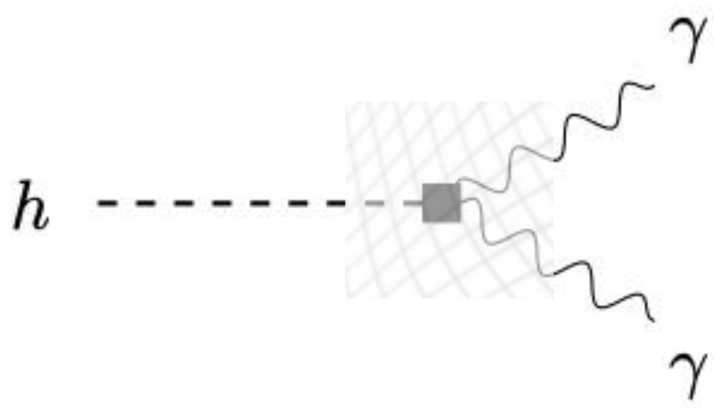
$$\langle h|\mathcal{A}(p_1)\mathcal{A}(p_2)\rangle = -\langle hA^{\mu\nu}A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right],$$

One loop result known and counter term structure through tree level ops in g_{AB} metric did pave the way.

https://arxiv.org/abs/1505.02646, https://arxiv.org/abs/1507.03568

- Open question for years. What is the detailed difference between squaring the dimension 6 correction, and the full dimension 8 result?

GeoSMEFT for the Higgs



- SMEFT diphoton decay

$$\langle h|\mathcal{A}(p_1)\mathcal{A}(p_2)\rangle = -\langle hA^{\mu\nu}A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right],$$

One loop result known and counter term structure through tree level ops in g_{AB} metric did pave the way.

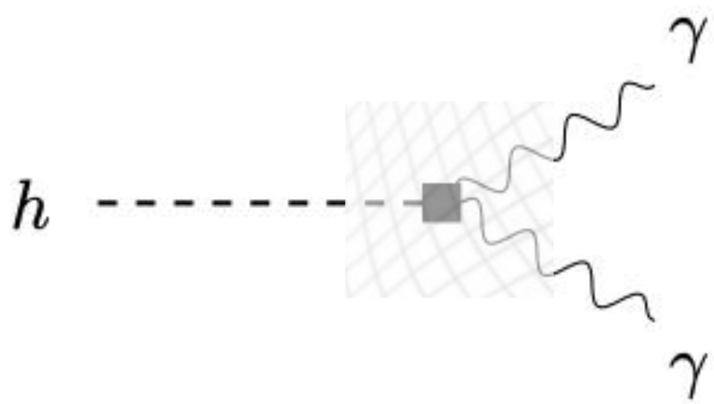
https://arxiv.org/abs/1505.02646, https://arxiv.org/abs/1507.03568

- Open question for years. What is the detailed difference between squaring the dimension 6 correction, and the full dimension 8 result?
- Now Solved! Using the expression above, just expand.

"Naive square" proportional to $$\left|\mathcal{A}_{SM}^{h\gamma\gamma}\right|^2 + 2\,\mathrm{Re}(\mathcal{A}_{SM}^{h\gamma\gamma})\,\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}} + \langle h|\gamma\gamma\rangle^2_{\mathcal{L}^{(6)}}$$

Where: $$\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}} = \left[\frac{g_2^2\,\tilde{C}_{HB}^{(6)} + g_1^2\,\tilde{C}_{HW}^{(6)} - g_1\,g_2\,\tilde{C}_{HWB}^{(6)}}{(g_1^2+g_2^2)\,\bar{v}_T}\right]$$

GeoSMEFT for the Higgs



- SMEFT diphoton decay

$$\langle h|\mathcal{A}(p_1)\mathcal{A}(p_2)\rangle = -\langle hA^{\mu\nu}A_{\mu\nu}\rangle \frac{\sqrt{h}^{44}}{4}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right],$$

One loop result known and counter term structure through tree level ops in g_{AB} metric did pave the way.

https://arxiv.org/abs/1505.02646, https://arxiv.org/abs/1507.03568

- Open question for years. What is the detailed difference between squaring the dimension 6 correction, and the full dimension 8 result?
- Now Solved! Using the expression above, just expand.

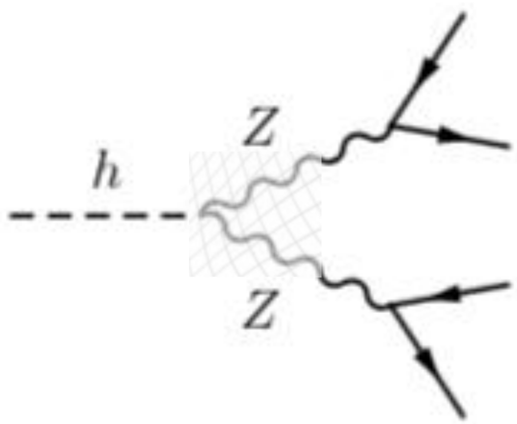
Correct result:

First self isolation paper: Hays, Helset, Martin Trott - to appear

$$\left|\mathcal{A}_{SM}^{h\gamma\gamma}\right|^2 + 2\,\mathrm{Re}(\mathcal{A}_{SM}^{h\gamma\gamma})(1+\langle\sqrt{h}^{44}\rangle_{\mathcal{L}^{(6)}})\,\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}} + (1+4\,\bar{v}_T\,\mathrm{Re}(\mathcal{A}_{SM}^{h\gamma\gamma}))\,(\langle h|\gamma\gamma\rangle_{\mathcal{L}^{(6)}})^2,$$

$$+ 2\,\mathrm{Re}(\mathcal{A}_{SM}^{h\gamma\gamma})\left[\frac{g_2^2\,\tilde{C}_{HB}^{(8)} + g_1^2\,(\tilde{C}_{HW}^{(8)} - \tilde{C}_{HW,2}^{(8)}) - g_1\,g_2\,\tilde{C}_{HWB}^{(8)}}{(g_1^2+g_2^2)\,\bar{v}_T}\right].$$

GeoSMEFT for the Higgs

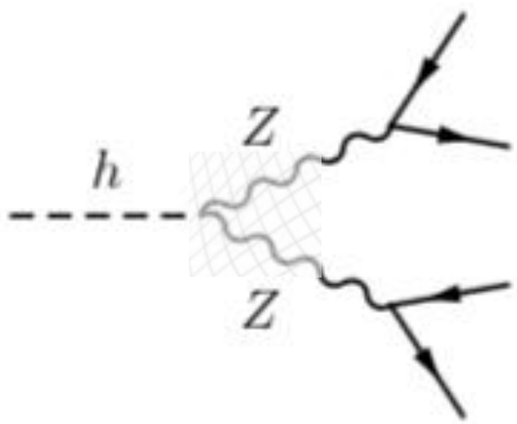


- All orders SMEFT higgs coupling to $W^{\pm}, Z$

$$\langle h|\mathcal{Z}(p_1)\mathcal{Z}(p_2)\rangle = -\frac{\sqrt{h}^{44}}{4}\bar{g}_Z^2\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{c^4_{\theta_Z}}{g_2^2} - 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{c^2_{\theta_Z}\,s^2_{\theta_Z}}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{s^4_{\theta_Z}}{g_1^2}\right]\langle h\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu}\rangle$$
$$+\sqrt{h}^{44}\frac{\bar{g}_Z^2}{2}\left[\langle\frac{\delta h_{33}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{33}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{Z}_\mu\mathcal{Z}^\mu\rangle$$
$$+\sqrt{h}^{44}\bar{g}_Z^2\bar{v}_T\left[\langle k^3_{34}\rangle\frac{c^2_{\theta_Z}}{g_2} - \langle k^4_{34}\rangle\frac{s^2_{\theta_Z}}{g_1}\right]\langle\partial^\nu h\mathcal{Z}_\mu\mathcal{Z}^{\mu\nu}\rangle,$$

$$\langle h|\mathcal{W}(p_1)\mathcal{W}(p_2)\rangle = -\frac{\sqrt{h}^{44}}{2}\bar{g}_2^2\left[\langle\frac{\delta g_{11}(\phi)}{\delta\phi_4}\rangle\frac{1}{g_2^2}\right]\langle h\mathcal{W}_{\mu\nu}\mathcal{W}^{\mu\nu}\rangle$$
$$+\sqrt{h}^{44}\bar{g}_2^2\left[\langle\frac{\delta h_{11}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{11}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{W}_\mu\mathcal{W}^\mu\rangle$$
$$+2\sqrt{h}^{44}\frac{\bar{g}_2^2}{g_2}\frac{\bar{v}_T}{4}\left[i\,\langle k^1_{42}\rangle - \langle k^2_{42}\rangle\right]\langle(\partial^\mu h)(\mathcal{W}^+_{\mu\nu}W^\nu_- + \mathcal{W}^-_{\mu\nu}W^\nu_+)\rangle.$$

GeoSMEFT for the Higgs



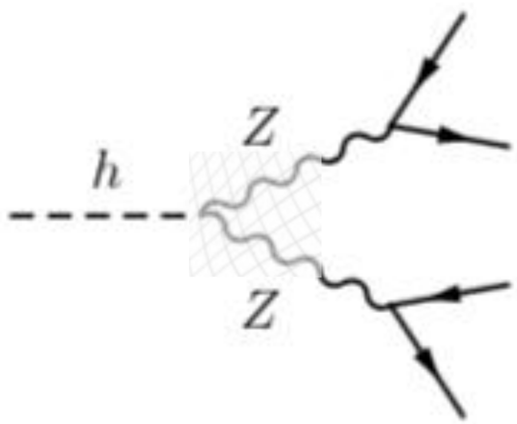
- All orders SMEFT higgs coupling to $W^{\pm}, Z$

$$
\begin{aligned}
\langle h|\mathcal{Z}(p_1)\mathcal{Z}(p_2)\rangle = & -\frac{\sqrt{h}^{44}}{4}\bar{g}_Z^2\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{c_{\theta_Z}^4}{g_2^2} - 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{c_{\theta_Z}^2\,s_{\theta_Z}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{s_{\theta_Z}^4}{g_1^2}\right]\langle h\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu}\rangle \\
& + \sqrt{h}^{44}\frac{\bar{g}_Z^2}{2}\left[\langle\frac{\delta h_{33}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{33}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{Z}_\mu\mathcal{Z}^\mu\rangle \\
& + \sqrt{h}^{44}\bar{g}_Z^2\bar{v}_T\left[\langle k_{34}^3\rangle\frac{c_{\theta_Z}^2}{g_2} - \langle k_{34}^4\rangle\frac{s_{\theta_Z}^2}{g_1}\right]\langle\partial^\nu h\mathcal{Z}_\mu\mathcal{Z}^{\mu\nu}\rangle,
\end{aligned}
$$

← SM like kinematics

$$
\begin{aligned}
\langle h|\mathcal{W}(p_1)\mathcal{W}(p_2)\rangle = & -\frac{\sqrt{h}^{44}}{2}\bar{g}_2^2\left[\langle\frac{\delta g_{11}(\phi)}{\delta\phi_4}\rangle\frac{1}{g_2^2}\right]\langle h\mathcal{W}_{\mu\nu}\mathcal{W}^{\mu\nu}\rangle \\
& + \sqrt{h}^{44}\bar{g}_2^2\left[\langle\frac{\delta h_{11}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{11}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{W}_\mu\mathcal{W}^\mu\rangle \\
& + 2\sqrt{h}^{44}\frac{\bar{g}_2^2}{g_2}\frac{\bar{v}_T}{4}\left[i\,\langle k_{42}^1\rangle - \langle k_{42}^2\rangle\right]\langle(\partial^\mu h)(\mathcal{W}_{\mu\nu}^{+}\mathcal{W}_{-}^{\nu} + \mathcal{W}_{\mu\nu}^{-}\mathcal{W}_{+}^{\nu})\rangle.
\end{aligned}
$$

GeoSMEFT for the Higgs



- All orders SMEFT higgs coupling to $W^{\pm}, Z$

$$\langle h|\mathcal{Z}(p_1)\mathcal{Z}(p_2)\rangle = -\frac{\sqrt{h}^{44}}{4}\bar{g}_Z^2\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{c_{\theta_Z}^4}{g_2^2} - 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{c_{\theta_Z}^2\, s_{\theta_Z}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{s_{\theta_Z}^4}{g_1^2}\right]\langle h\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu}\rangle$$

$$+\sqrt{h}^{44}\frac{\bar{g}_Z^2}{2}\left[\langle\frac{\delta h_{33}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{33}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{Z}_\mu\mathcal{Z}^\mu\rangle$$

← SM like kinematics

$$+\sqrt{h}^{44}\bar{g}_Z^2\bar{v}_T\left[\langle k_{34}^3\rangle\frac{c_{\theta_Z}^2}{g_2} - \langle k_{34}^4\rangle\frac{s_{\theta_Z}^2}{g_1}\right]\langle\partial^\nu h\mathcal{Z}_\mu\mathcal{Z}^{\mu\nu}\rangle,$$

Anomalous kinematic population correction factor - not many!

$$\langle h|\mathcal{W}(p_1)\mathcal{W}(p_2)\rangle = -\frac{\sqrt{h}^{44}}{2}\bar{g}_2^2\left[\langle\frac{\delta g_{11}(\phi)}{\delta\phi_4}\rangle\frac{1}{g_2^2}\right]\langle h\mathcal{W}_{\mu\nu}\mathcal{W}^{\mu\nu}\rangle$$

$$+\sqrt{h}^{44}\bar{g}_2^2\left[\langle\frac{\delta h_{11}(\phi)}{\delta\phi_4}\rangle\left(\frac{\bar{v}_T}{2}\right)^2 + \langle h_{11}(\phi)\rangle\frac{\bar{v}_T}{2}\right]\langle h\mathcal{W}_\mu\mathcal{W}^\mu\rangle$$

$$+2\sqrt{h}^{44}\frac{\bar{g}_2^2}{g_2}\frac{\bar{v}_T}{4}\left[i\,\langle k_{42}^1\rangle - \langle k_{42}^2\rangle\right]\langle(\partial^\mu h)(\mathcal{W}^+_{\mu\nu}\mathcal{W}^\nu_- + \mathcal{W}^-_{\mu\nu}\mathcal{W}^\nu_+)\rangle.$$

GeoSMEFT and flat directions

- Deep irony of the GeoSMEFT. As soon as people started to use the full SMEFT - immediately data analysis indicated this hidden structure.

<u>Key early one going in right direction: Han,Skiba 0412166</u>

They found flat directions in LEP data. We now know due an invariance.

Weak eigenstates

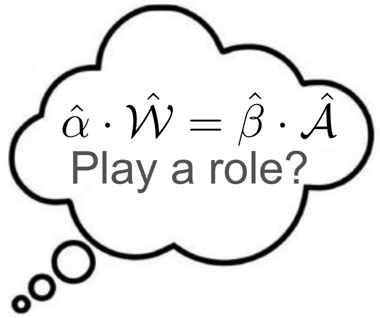
Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \sqrt{g}^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

1909.08470 Corbett, Helset, Trott

Field space metrics

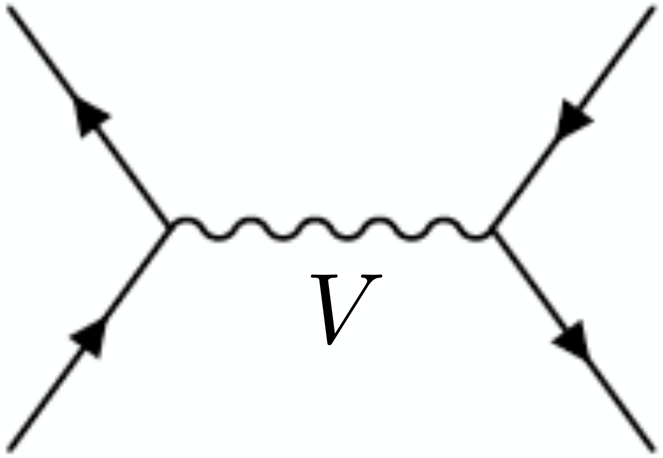
Rotations



EWPD flat directions

- Flat directions due the invariance, fundamentally its $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$

arXiv:1701.06424 Reparameterization! Ilaria Brivio, MT



$$(V, g) \leftrightarrow \left(V' \left(1+\epsilon\right), g' \left(1-\epsilon\right)\right),$$

$\bar{\psi}\psi \to \bar{\psi}\psi$ scattering has a reparamatrization invariance

- LEP data can't see EOM equivalent to parameters cancelling in $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$

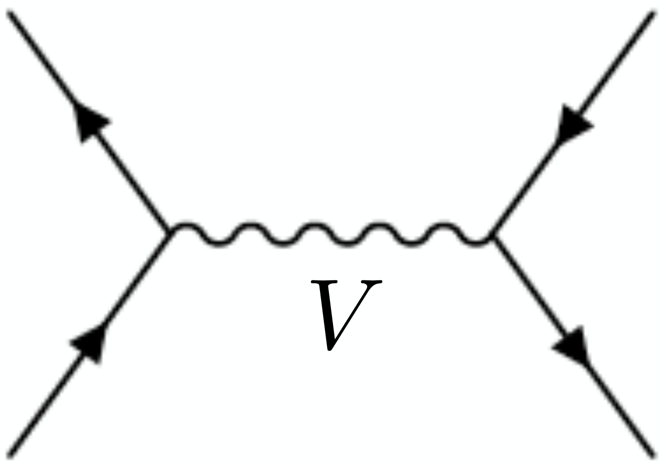
$$\langle \mathsf{y}_h\, g_1^2 Q_{HB} \rangle_{S_R} = \langle \sum_{\substack{\psi_\kappa = u,d,\\ q,e,l}} \mathsf{y}_k\, g_1^2\, \overline{\psi}_\kappa\, \gamma_\beta \psi_\kappa\, (H^\dagger\, i \overleftrightarrow{D}_\beta H) + \frac{g_1^2}{2}\left(Q_{H\Box} + 4 Q_{HD}\right) - \frac{1}{2} g_1\, g_2\, Q_{HWB} \rangle_{S_R},$$

$$\langle g_2^2 Q_{HW} \rangle_{S_R} = \langle g_2^2\, (\overline{q}\, \tau^I \gamma_\beta q + \overline{l}\, \tau^I \gamma_\beta l)\, (H^\dagger\, i \overleftrightarrow{D}^I_\beta H) + 2\, g_2^2\, Q_{H\Box} - 2\, g_1\, g_2\, \mathsf{y}_h\, Q_{HWB} \rangle_{S_R}.$$

EWPD flat directions

- Flat directions due the invariance, fundamentally its $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$

arXiv:1701.06424 Reparameterization! Ilaria Brivio, MT



$$(V, g) \leftrightarrow (V' (1+\epsilon), g' (1-\epsilon)) \, ,$$

$\bar{\psi}\psi \to \bar{\psi}\psi$ scattering has a reparamatrization invariance

- Flat directions in many data sets project onto EOM equivalents to what cancels in the invariant $\hat{\alpha} \cdot \hat{\mathcal{W}} = \hat{\beta} \cdot \hat{\mathcal{A}}$ $\left(\tilde{C}_{HB}, \tilde{C}_{HW}\right)$

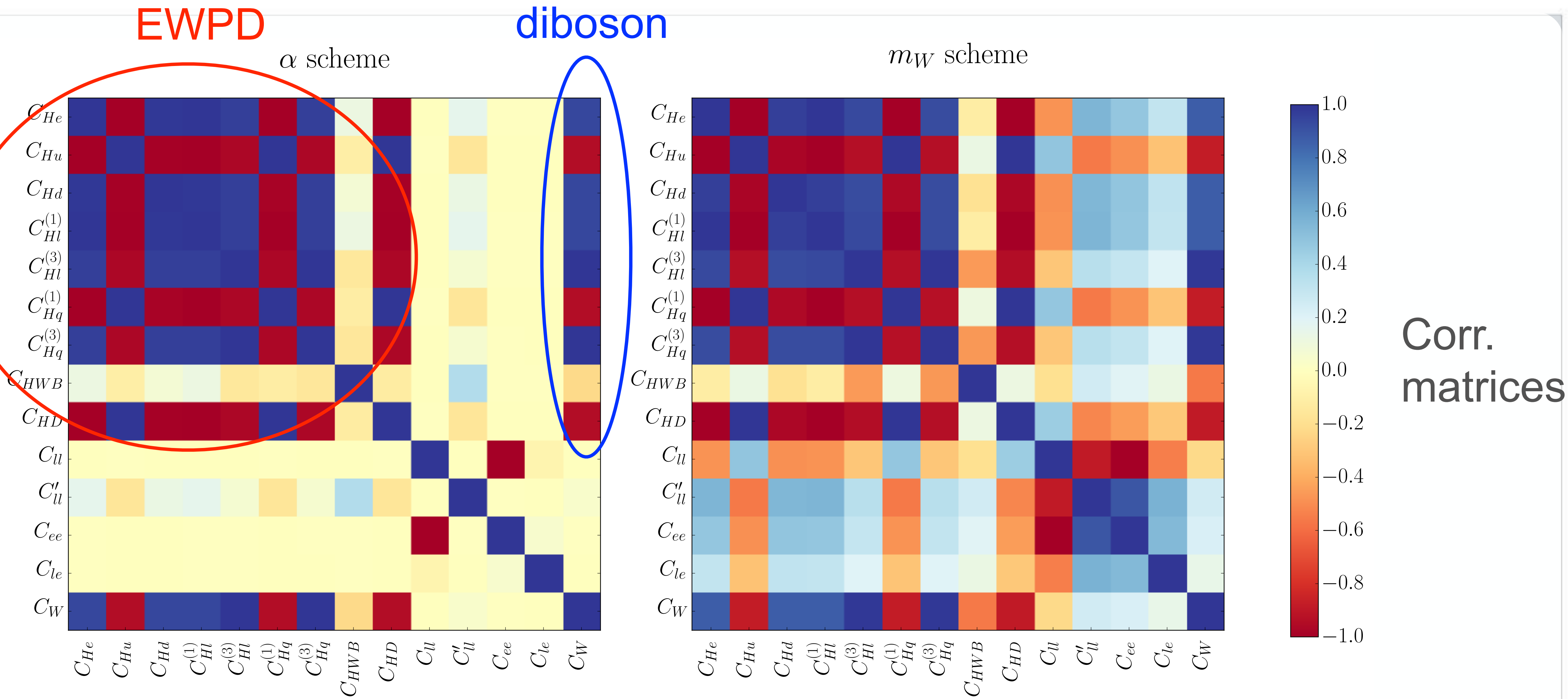
$$w_1^\alpha = -w_B - 2.59\, w_W \qquad w_1^{mw} = -w_B - 2.48\, w_W$$

$$w_2^\alpha = -w_B + 4.31\, w_W \, , \qquad w_2^{mw} = -w_B + 4.40\, w_W \, .$$

be careful and keep all operators!

- Input scheme independent.

Flat directions reflect a consistent analysis



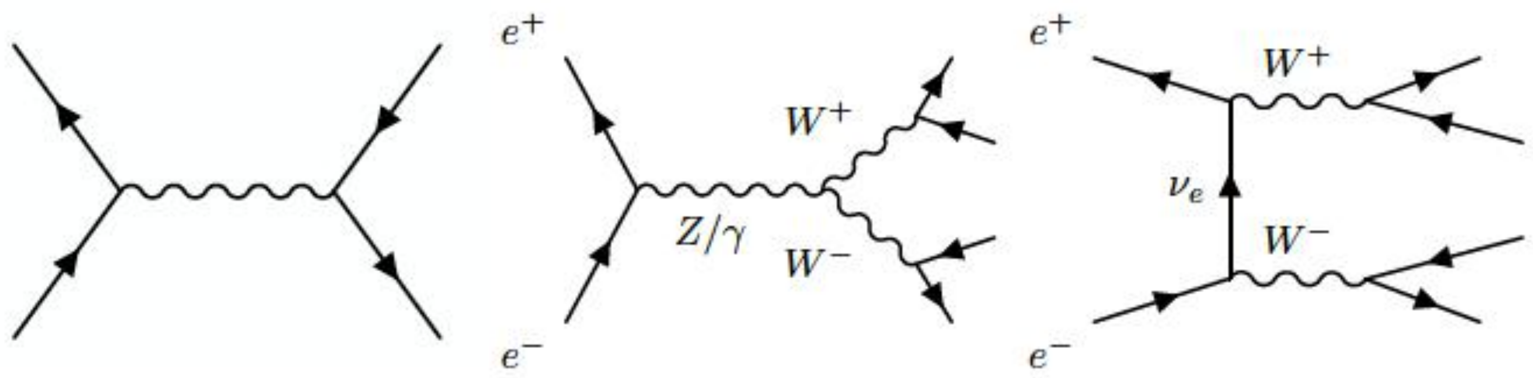
- Global analysis of data from PEP, PETRA, TRISTAN, SpS, Tevatron, SLAC, LEPI and LEP II
- Correlation matrices in a likelihood for the SMEFT (before higgs data)

$$L(C) = \frac{1}{\sqrt{(2\pi)^n |V|}} \exp\left(-\frac{1}{2}\left(\hat{O}-\bar{O}\right)^T V^{-1}\left(\hat{O}-\bar{O}\right)\right),$$

1502.02570, 1508.05060, Berthier, MT ,1606.06693 Berthier, Bjorn, MT , arXiv:1701.06424 Brivio, MT

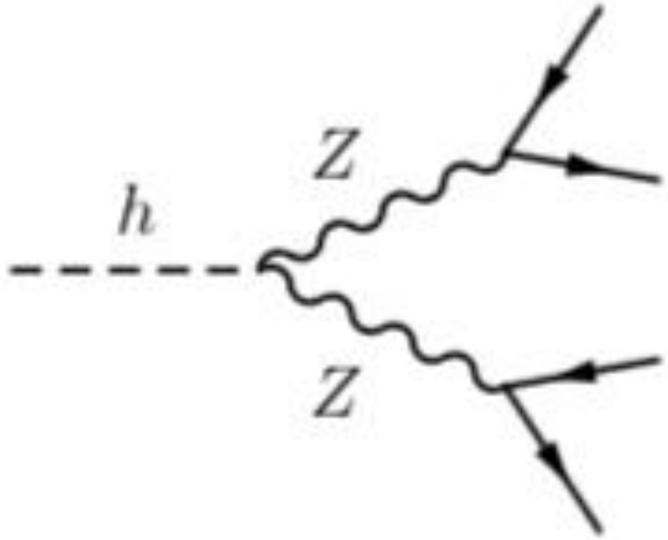
SMEFT reparameterization invariance

- Must combine data sets in a well defined SMEFT, so no matter what operator basis you choose you get consistent results



Breaks the invariance. This channel dominant at LEP2

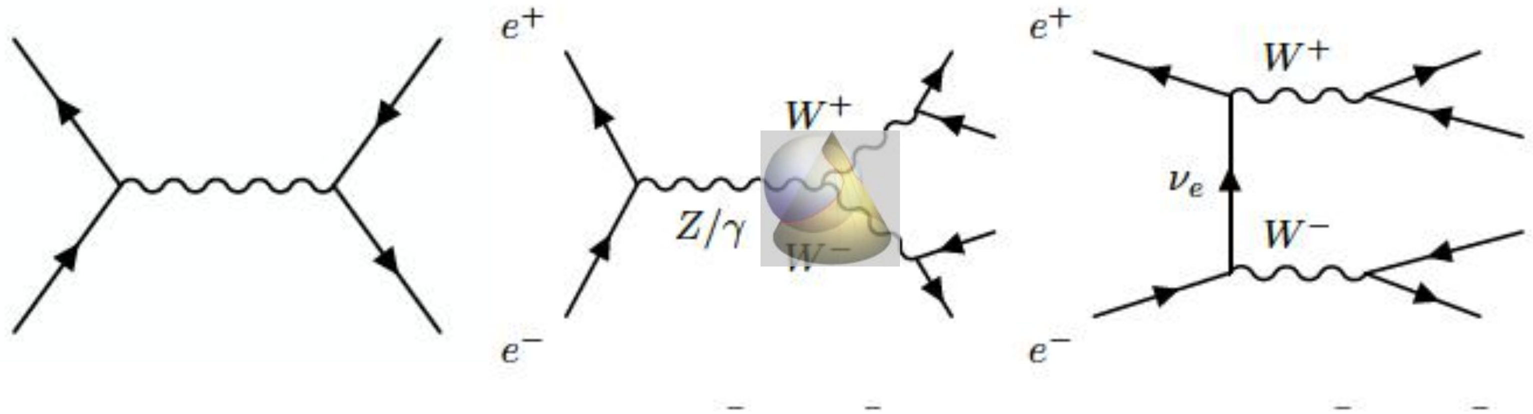
- *Precision Higgs physics data will compete* and we need to combine it consistently



Breaks the invariance.
Probes the scalar and gauge metric connections directly.

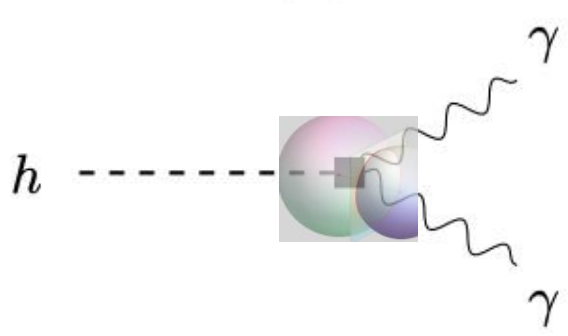
SMEFT reparameterization invariance

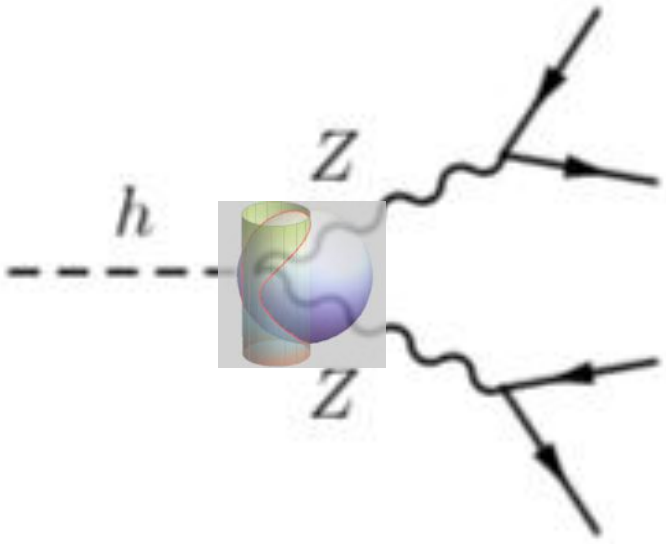
- Must combine data sets in a well defined SMEFT, so no matter what operator basis you choose you get consistent results



This channel dominant at LEP2

- *Precision Higgs physics data will compete* and we need to combine it consistently





Conclusions.

Higgs physics is the physics of curved field space.