

Proton Structure Functions at HERA

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For H1 and ZEUS Collaborations

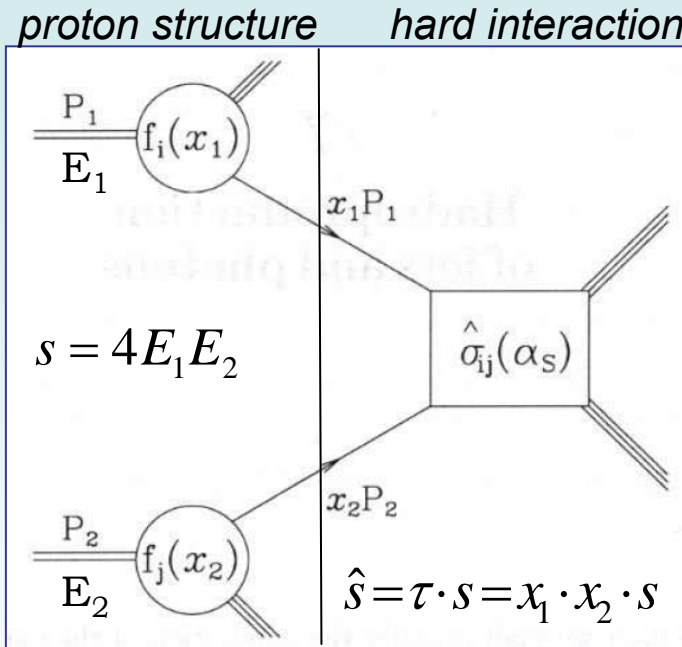
Physics in Collision 2010 Karlsruhe



HERA Combined Results



Proton-Proton Collisions at high energies



Structure: $f_i(x) = q_i(x, Q^2), g(x, Q^2)$

Parton Distribution Functions (PDF):

probability density finding a parton in a proton carrying its momentum fraction x at scale Q^2

Hard 2-parton interaction calculable in pQCD

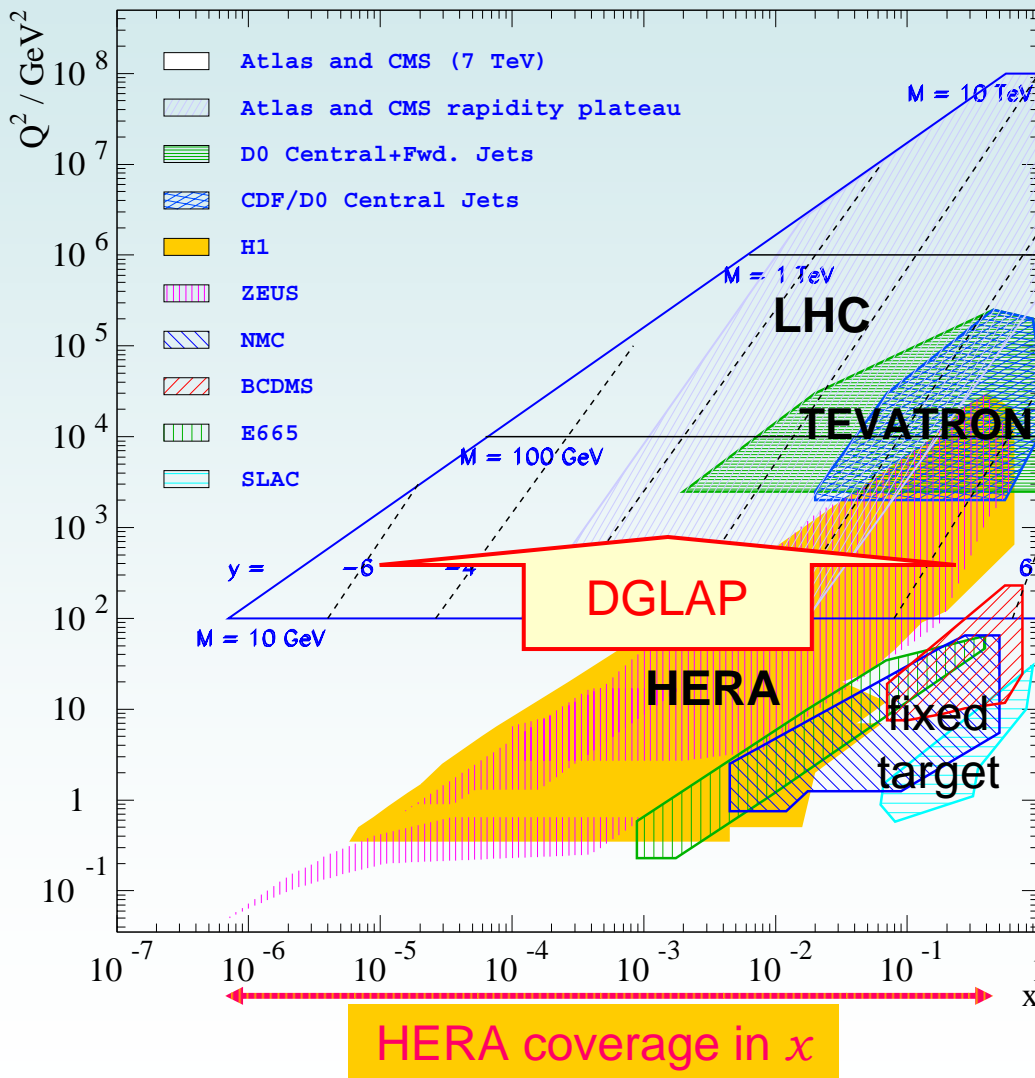
Factorization: PDF \otimes hard sub-process ME

$$\sigma(s) = \sum_{i,j} \int_{\tau_0}^1 \frac{d\tau}{\tau} \left[\frac{dL_{ij}(\mu_F^2)}{d\tau} \right] \cdot \left[\hat{s} \cdot \hat{\sigma}_{ij}(\alpha_S(\mu)) \right]$$

$$\tau \cdot \left(\frac{dL_{ij}}{d\tau} \right) \propto \int_0^1 dx_1 dx_2 \left[\left(x_1 f_i(x_1, \mu_F^2) \cdot x_2 f_j(x_2, \mu_F^2) \right) + (1 \leftrightarrow 2) \right] \delta(\tau - x_1 x_2)$$

Precise PDFs needed!

Kinematics of HEP experiments



Parton density functions determined experimentally

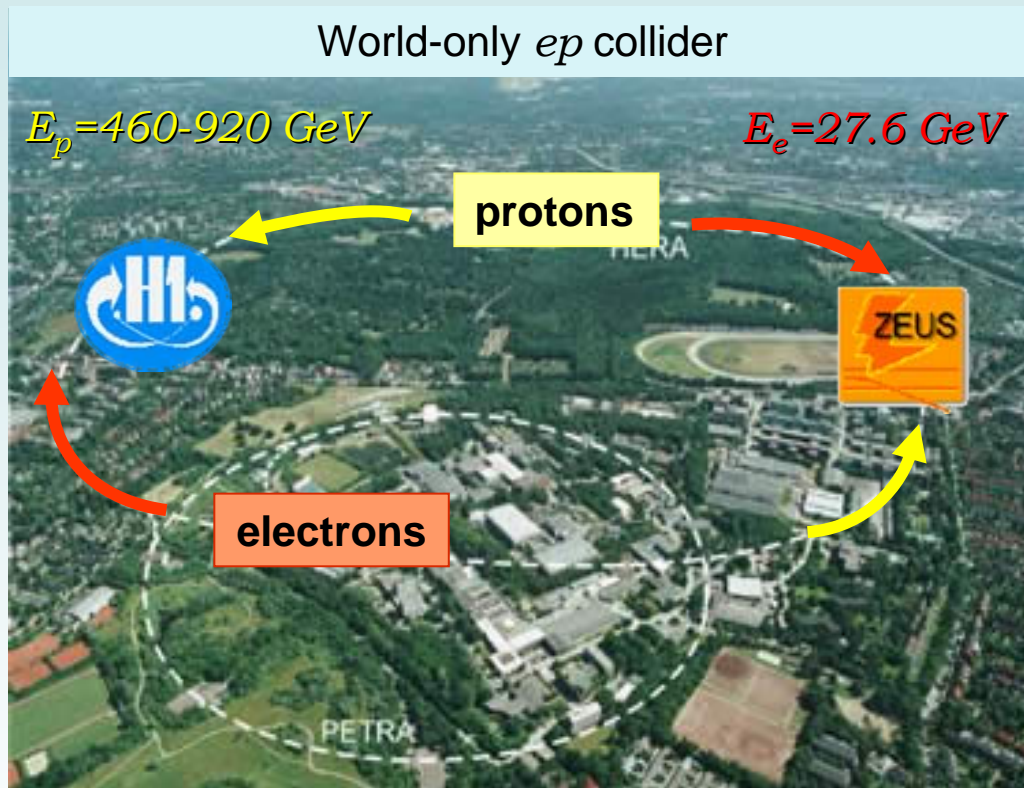
HERA Measurements:

covers most of the (x, Q^2) plane, best constrain at low, medium x

From HERA to kinematics of Tevatron, LHC:

Evolution in Q^2 via DGLAP

HERA: unique tool to study the proton



- HERA I : 1992-2000
 - HERA II: 2003-2007
 - collider experiments
- H1 & ZEUS, $\sqrt{s}_{max} = 318 \text{ GeV}$
- integrated Luminosity
 $\sim 0.5 \text{ fb}^{-1}$ experiment

Central
Tracker

HERA: unique tool to study the proton

World-only ep collider

$E_p = 460-920 \text{ GeV}$

$E_e = 27.6 \text{ GeV}$

protons

ZEUS

electrons

Calorimeter

Central Tracker

Calorimeter

Central Tracker

- HERA I : 1992-2000

- HERA II: 2003-2007

- collider experiments

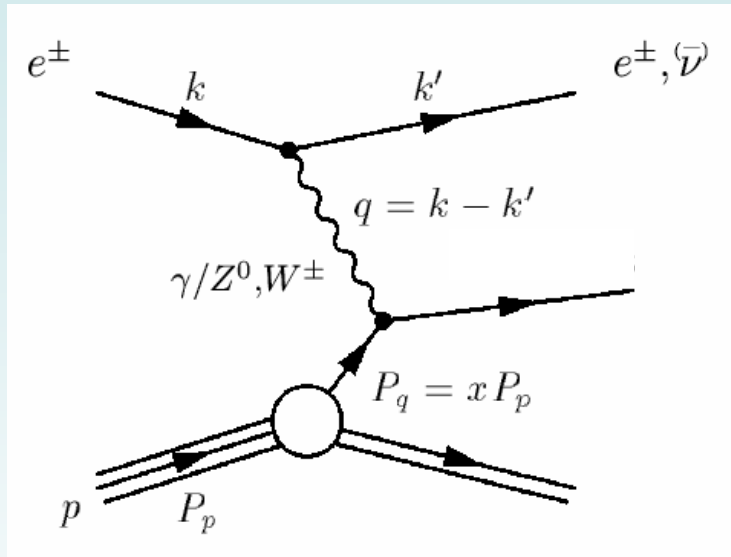
H1 & ZEUS, $\sqrt{s}_{max} = 318 \text{ GeV}$

- integrated Luminosity

$\sim 0.5 \text{ fb}^{-1}$ experiment

Inclusive Deep Inelastic Scattering

DIS: tool to study the proton



Kinematics:

$Q^2 = -q^2$ *photon virtuality*

$x = -q^2 / 2p \cdot q$ *Bjorken scaling variable*

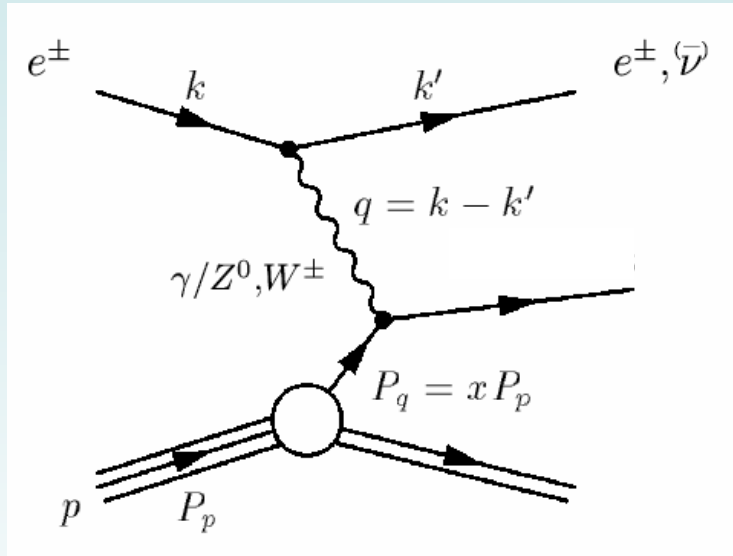
$y = p \cdot q / p \cdot k$ *transferred energy fraction*

$s = (k + p)^2$ *center of mass energy*

$Q^2 = sxy$

Inclusive Deep Inelastic Scattering

DIS: tool to study the proton



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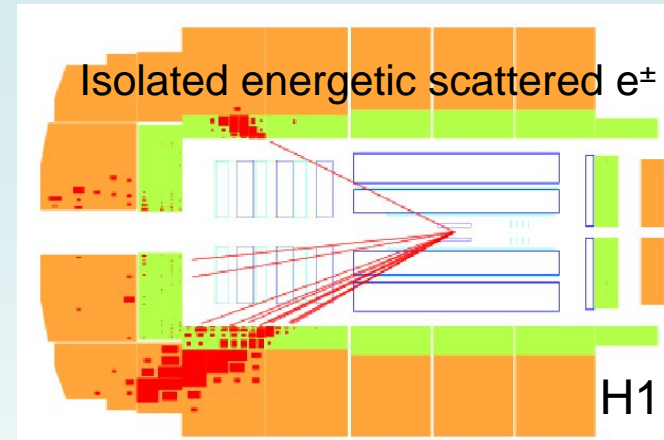
$x = -q^2 / 2p \cdot q$ Bjorken scaling variable

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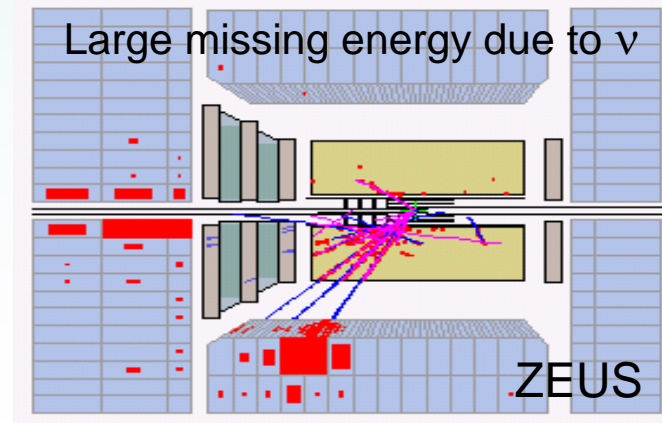
$s = (k+p)^2$ center of mass energy

$Q^2 = sxy$

γ, Z : Neutral Current $ep \rightarrow e X$



W^\pm : Charged Current $ep \rightarrow \nu X$



DIS and proton structure

➤ Neutral Current: $e^\pm p \rightarrow e^\pm X$

$$\frac{d^2 \sigma_{NC}^\pm}{dx dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} \left[Y_+ \tilde{F}_2(x, Q^2) \mp Y_- x\tilde{F}_3(x, Q^2) - y^2 \tilde{F}_L(x, Q^2) \right] \quad Y_\pm \equiv 1 \pm (1-y)^2$$

\tilde{F}_2 dominant contribution

QPM: $\{F_2, F_2^{\gamma Z}, F_2^Z\} = x \sum_q \{e_q^2, 2e_q v_q, v_q^2 + a_q^2\} (q + \bar{q})$

$x\tilde{F}_3$ γZ interference at $Q^2 \sim m_Z^2$

QPM: $\{xF_3^{\gamma Z}, xF_3^Z\} = 2x \sum_q \{e_q a_q, v_q a_q\} (q - \bar{q})$

\tilde{F}_L directly sensitive to the gluon in QCD: $F_L(x, Q^2) \sim x\alpha_s g(x, Q^2)$

DIS and proton structure

➤ **Neutral Current: $e^\pm p \rightarrow e^\pm X$**

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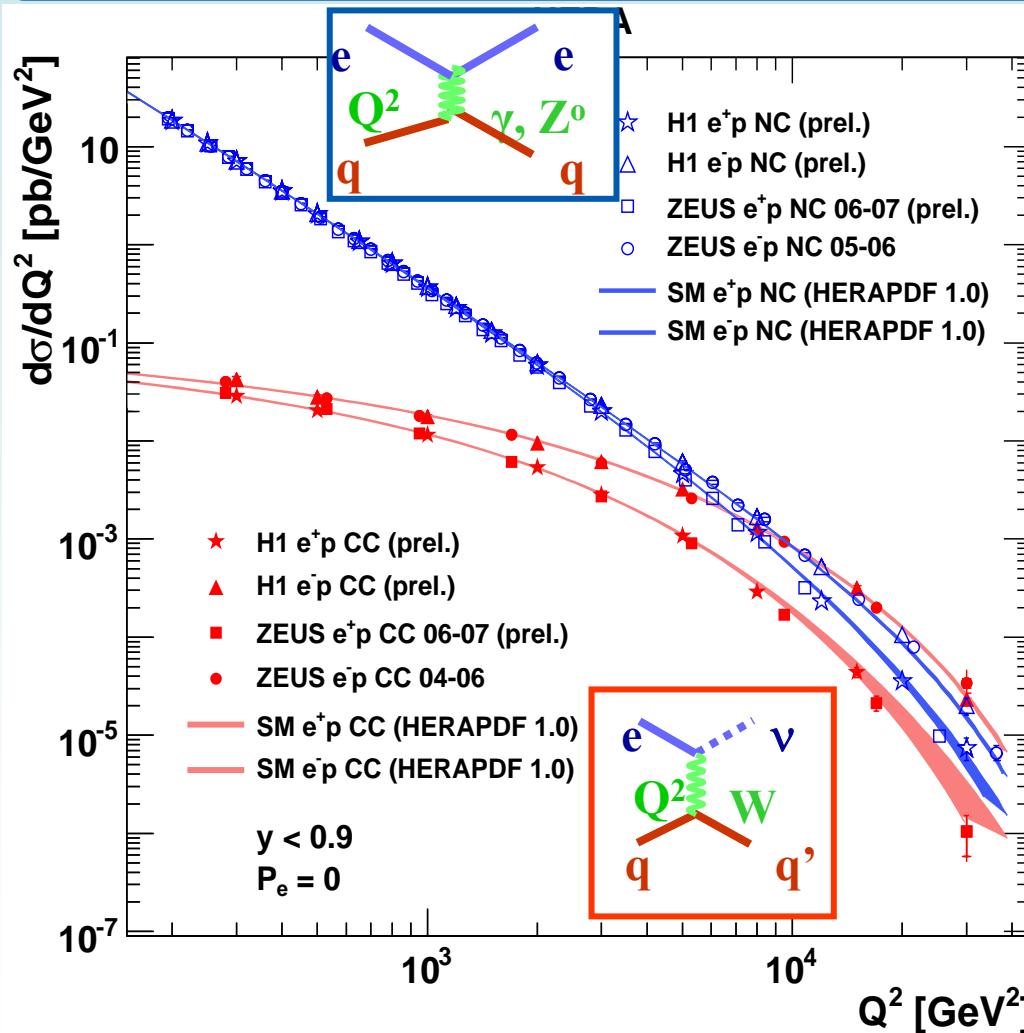
➤ **Charged Current: $e^\pm p \rightarrow \nu X$**

$$\frac{d^2 \sigma_{CC}^\pm}{dx dQ^2} \propto \frac{G_F^2}{4\pi x} \left[\frac{m_W^2}{Q^2 + m_W^2} \right]^2 \left[Y_+ \tilde{W}_2(x, Q^2) \mp Y_- x \tilde{W}_3(x, Q^2) - y^2 \tilde{W}_L(x, Q^2) \right]$$

σ_{CC}^+ $\propto x[(\bar{u} + \bar{c}) + (1-y)^2(d + s)]$ sensitive to d -quark at high x

σ_{CC}^- $\propto x[(u + c)] + (1-y)^2(\bar{d} + \bar{s})]$ sensitive to u -quark at high x

DIS cross sections vs Q^2



Neutral Current:

- small Q^2 : γ exchange
- high Q^2 : Z/γ interference:
 - constructive in e^-
 - destructive in e^+

Charged Current:

- e^-u enhanced
- e^+d suppressed

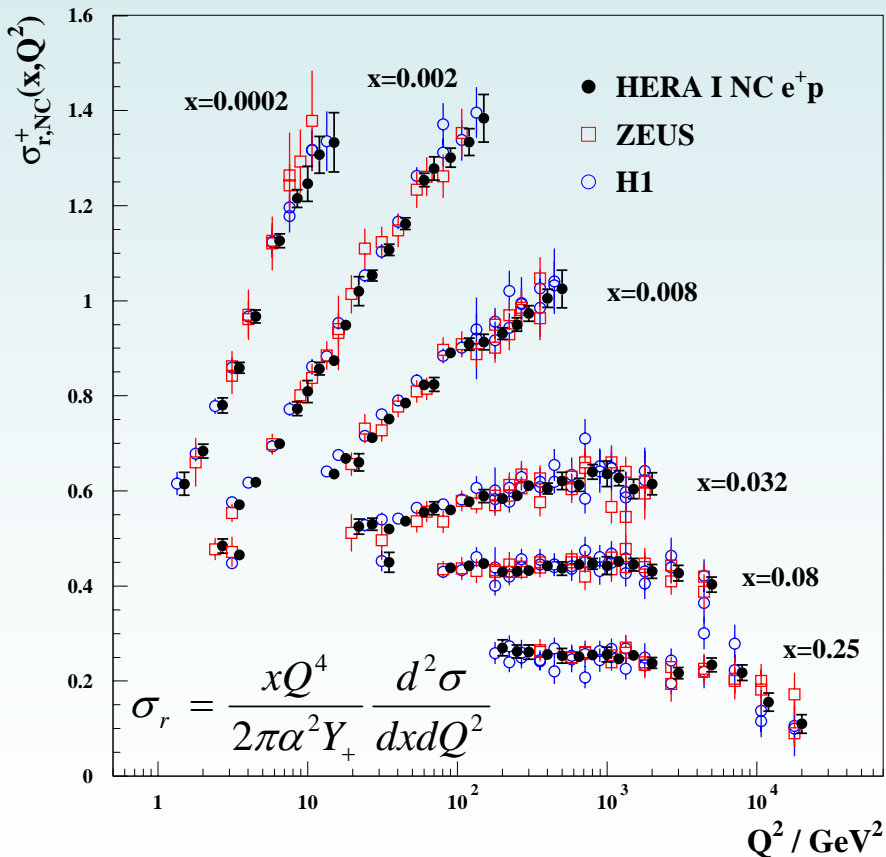
Electroweak unification at $Q^2 \sim M_W^2$

Ultimate precision DIS: combined HERA Data

Published in *JHEP 01 (2010) 109* : complete HERA I data, $\mathcal{L} \sim 115 \text{ pb}^{-1}$

e.g. NC cross section vs Q^2 : 6 bins in x

H1 and ZEUS



H1 and ZEUS data averaged:

- global fit of 1402 measurements
- 110 sources of systematic errors
- account for systematic correlations (cross-calibration of experiments)
- total uncertainty:
1-2% for $Q^2 < 500 \text{ GeV}^2$
- covered kinematics:

$$10^{-7} < x < 0.65,$$

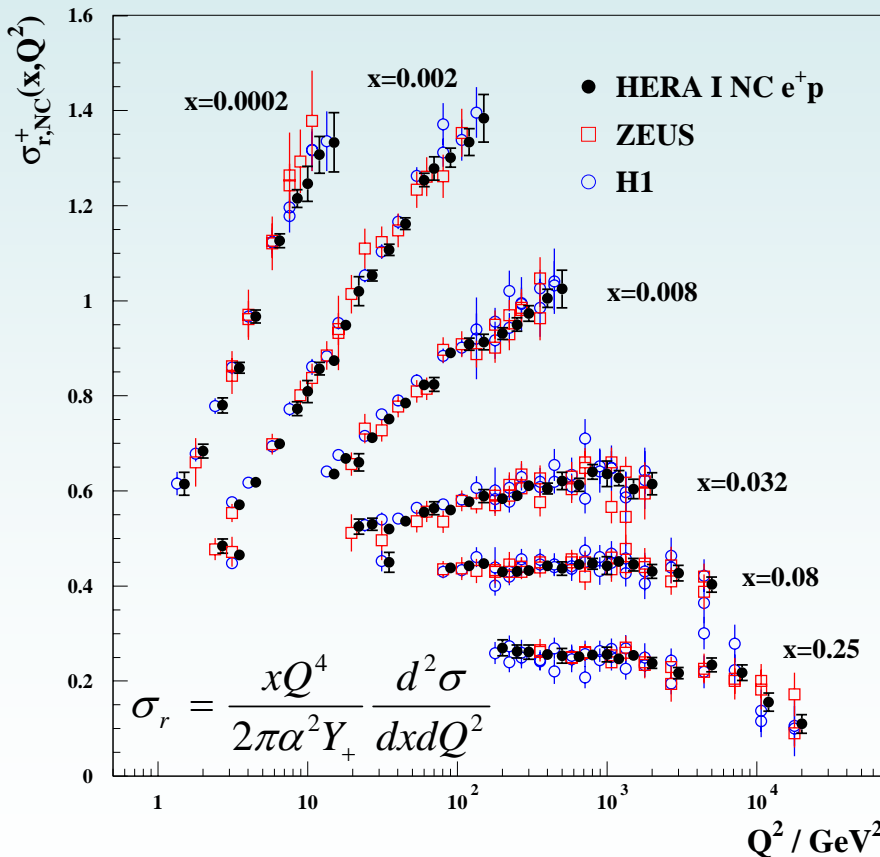
$$0.05 < Q^2 < 30000 \text{ GeV}^2$$

Ultimate precision DIS: combined HERA Data

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H1 and ZEUS



Reduced cross section:

$$\sigma_r = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

Scaling violations in F_2 :

at small x : F_2 rises with Q^2
 gluon splits into quark pair,
 $\Rightarrow \gamma$ resolves the quark-pair

$$\text{QCD: } \frac{\partial F_2}{\partial \ln Q^2} \propto \alpha_s(Q^2) x g(x, Q^2)$$

PDFs from inclusive NC and CC cross sections: $g(x)$ from scaling violations

Determination of Parton Density Functions

Structure Function Factorization: for an exchange-Boson V

$$F_2^V(x, Q^2) = \sum_i \int_x^1 dz \cdot C_2^{V,i}\left(\frac{x}{z}, Q^2, \mu\right) f_i(x, \mu_F)$$

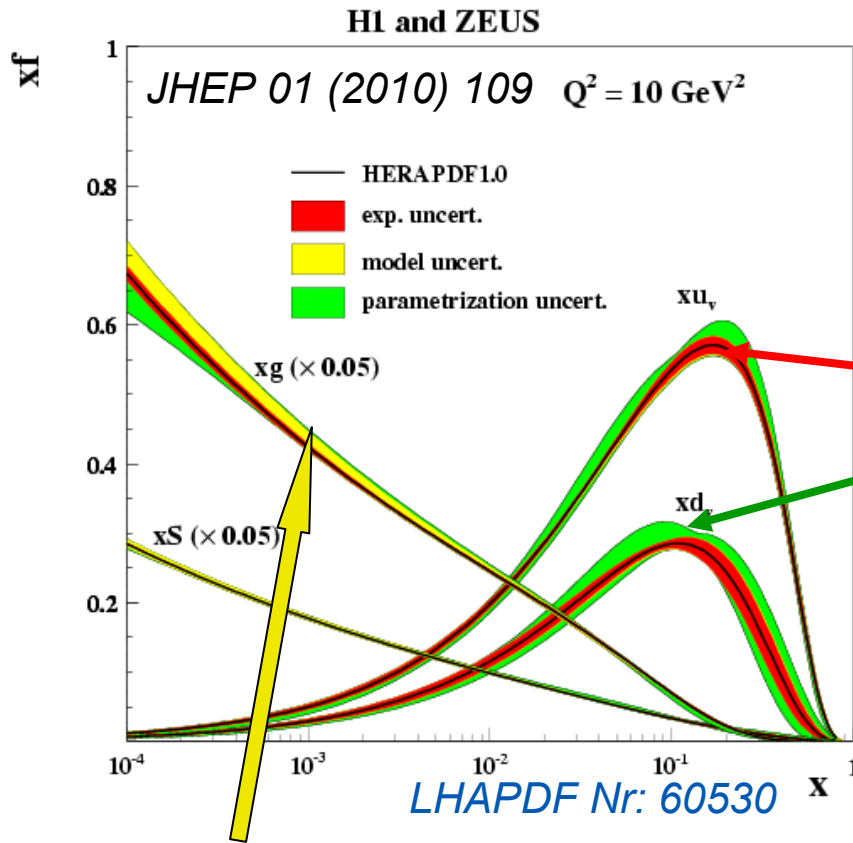
x -dependence of PDFs is not yet calculable in QCD:

- **parameterize at a starting scale** Q_0^2 : $f(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$
- **evolve these PDFs using DGLAP equations** to $Q^2 > Q_0^2$
- **construct structure functions** from PDFs and coefficient functions:
predictions for every data point in (x, Q^2) – plane
- χ^2 - **fit** to the experimental data

Global PDF Fit Groups: use data from different experiments,
best coverage at high x

HERAPDF: only H1 and ZEUS data: consistent data sample,
proper error correlation best precision at low and medium x

HERAPDF1.0: NLO PDF, VFNS



Model assumptions:

$$Q_0^2 = 1.9 \text{ GeV}^2, \alpha_s(M_Z) = 0.1176$$

$$m_c = 1.4 \text{ GeV}; m_b = 4.75 \text{ GeV}; f_s(Q_0^2) = 0.31$$

10 parameter fit, NLO DGLAP

Heavy quarks: massive

Variable Flavour Number Scheme

Scales: $\mu_r = \mu_f = Q^2$

Experimentally very precise

Parameterization at starting scale:

$$xg(x) = A_g x^{B_g} (1-x)^{C_g}$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2)$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

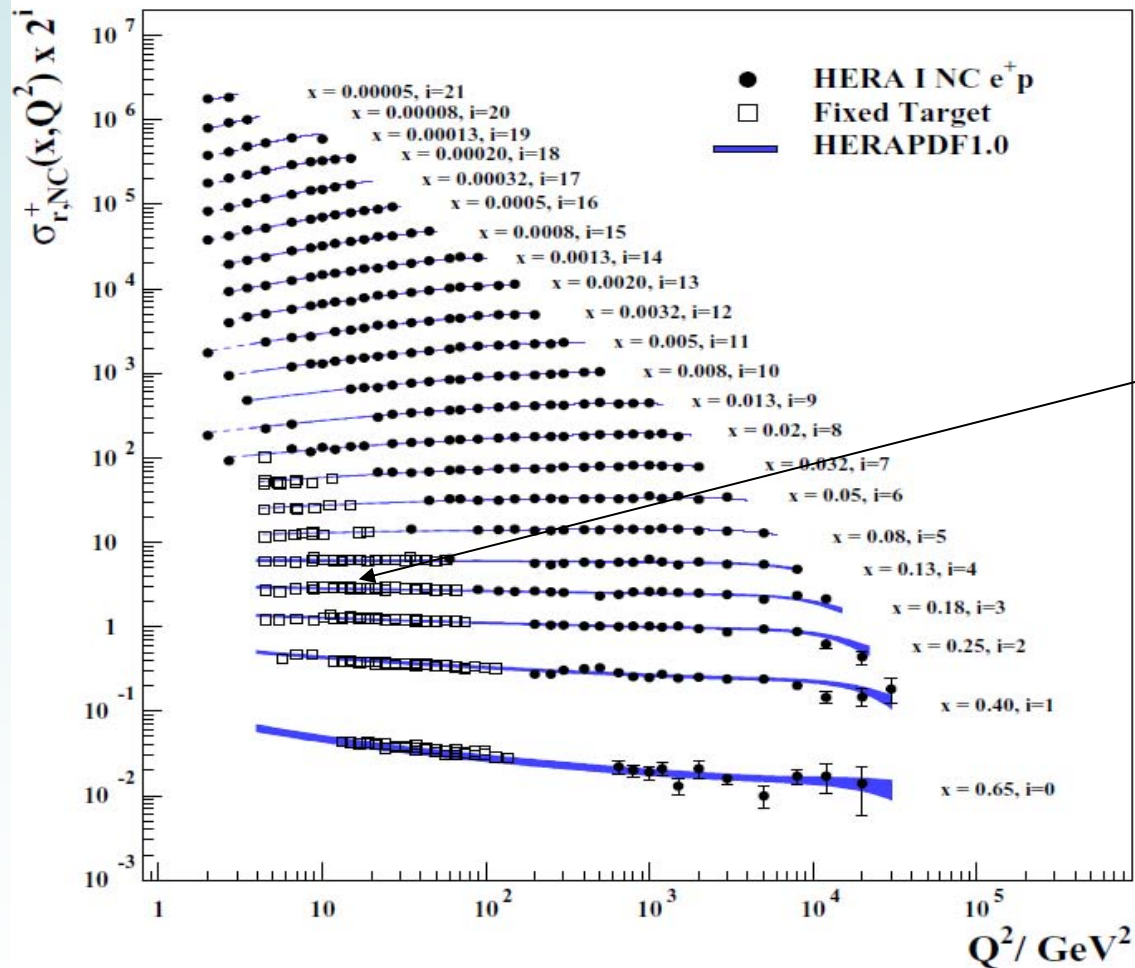
$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}}$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}$$

+ sum rules...

HERAPDF1.0 vs NC data

H1 and ZEUS

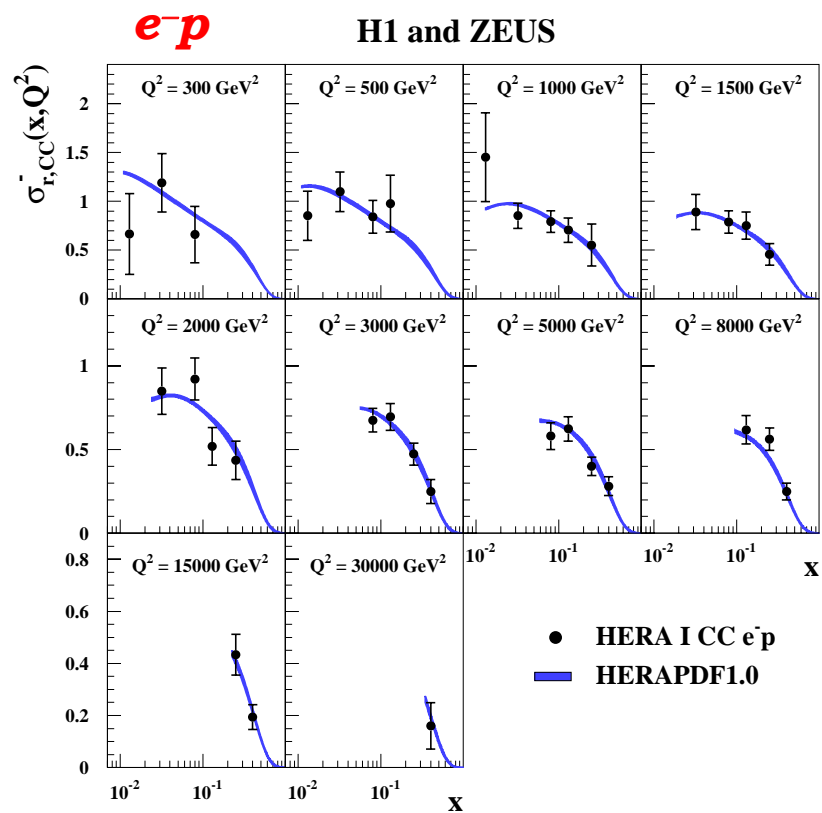
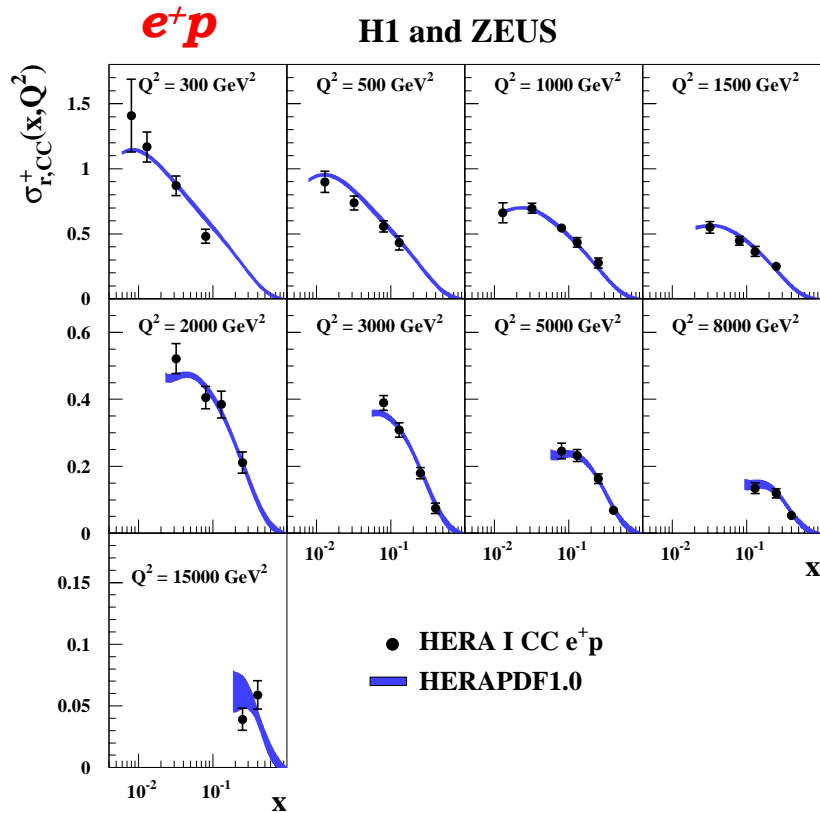


Only H1+ZEUS data included in the fit

Fixed targeted data

QCD \otimes HERAPDF1.0 describes the cross sections very well everywhere

HERAPDF1.0 vs CC data



QCD \otimes HERAPDF1.0 describes the cross sections very well everywhere

Heavy Quarks and PDF Fits

Factorization:
$$F_2^V(x, Q^2) = \sum_i \int_x^1 dz \cdot C_2^{V,i}\left(\frac{x}{z}, Q^2, \mu\right) f_i(x, \mu_F)$$

i - number of active flavours in the proton, what about heavy c and b ?

QCD analysis of the proton structure: **treatment of heavy quarks essential**

Fixed Flavour Number Scheme (FFNS) : i fixed

charm (beauty) quarks massive, produced in Boson-Gluon Fusion (BGF)

only light flavours in the proton: $i = 3$ (4)

Problem: expected to be less precise at $Q^2 \gg m_{HQ}^2$

Variable Flavour Number Scheme (VFNS) : i variable

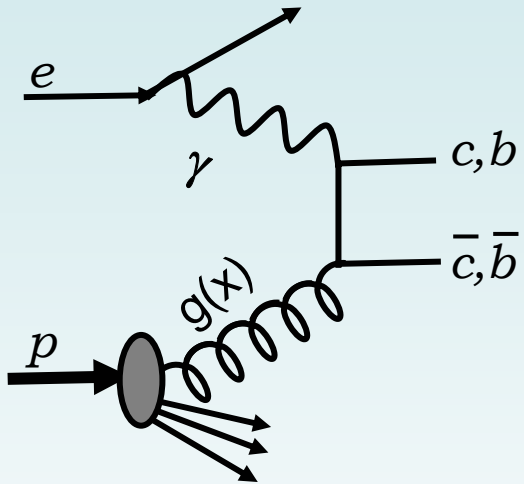
- *Zero Mass*: all flavours massless. Breaks down at $Q^2 \sim m_{HQ}^2$

- *Generalized Mass*: matched scheme, expect appropriate description at all Q^2 , different implementations available and used by global fit groups

Use HQ measurements to test different HQ treatment in PDF Fits

Heavy Quark Structure Functions

Heavy Quarks in ep Scattering produced in Boson-Gluon Fusion

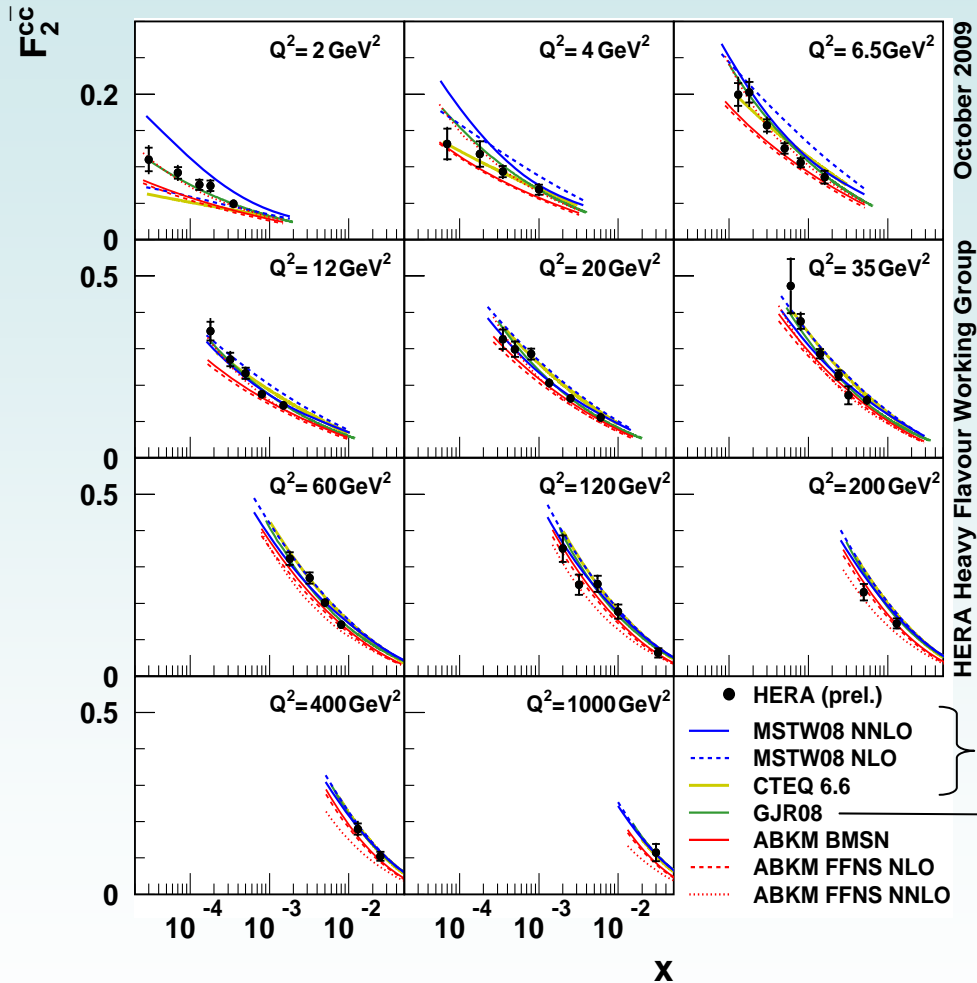


- Heavy: charm and beauty
 $m_c \sim 1.5 \text{ GeV}, m_b \sim 5 \text{ GeV},$
- Contribution to total DIS cross section
charm: up to 30% at high Q^2

- Gluon directly involved: cross-check of $g(x)$ from scaling violations
- Measure HQ structure functions: direct test of HQ schemes in PDF fits

$$\text{Charm structure function: } \sigma^{cc} \propto F_2^{cc}(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L^{cc}(x, Q^2)$$

Charm at HERA: test HQ schemes in PDFs



HERA Charm Measurement:
H1 + ZEUS

9 measurements

different charm tag methods

51 systematic error sources

correlations accounted for

Precision 5 - 10%

Generalized mass VFNS

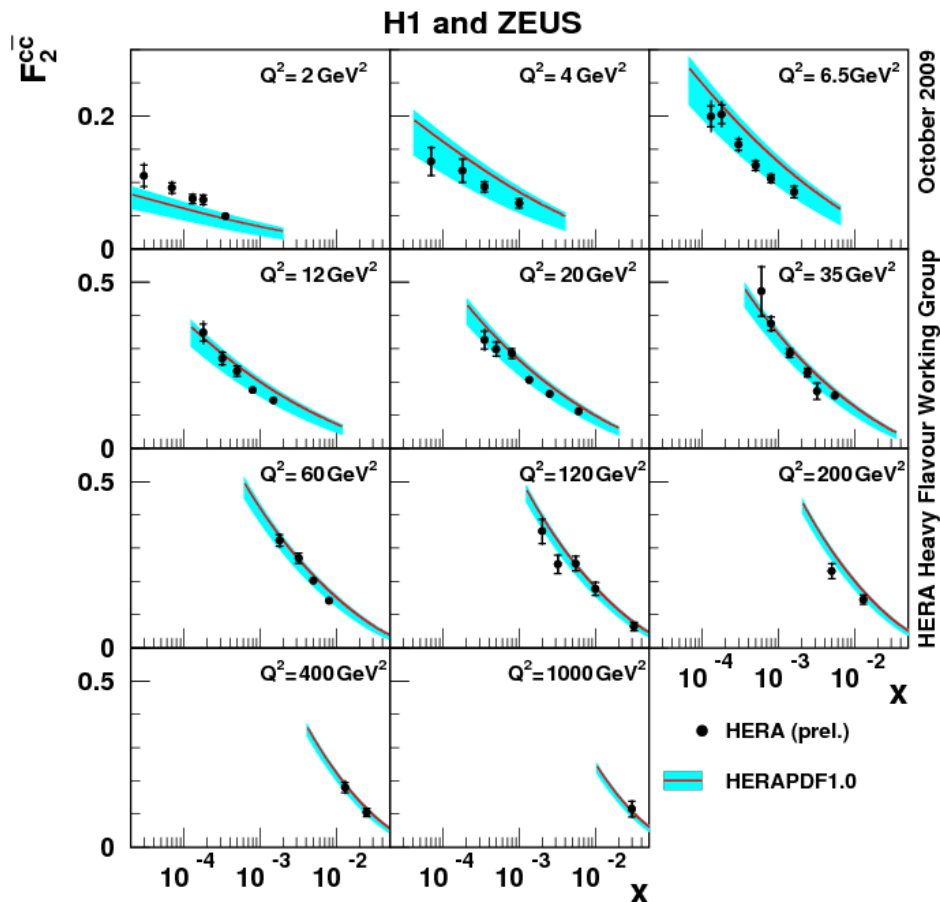
FFNS

FFNS

Precision of data similar to differences in calculations

⇒ potential to discriminate

Charm at HERA: test choice of m_c in PDF



F_2^{cc} data not included in HERAPDF1.0

F_2^{cc} consistent with HERAPDF1.0 prediction

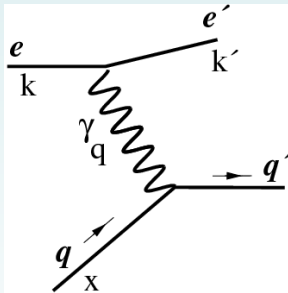
Direct access to the gluon: F_L

Photon-Parton Scattering: $\frac{d^2\sigma}{dx dQ^2} \propto (\sigma_T + \frac{2(1-y)}{Y_+} \sigma_L), Y_+ = 1 + (1-y)^2$

Structure functions: $F_2 \sim (\sigma_T + \sigma_L), F_L \sim \sigma_L$

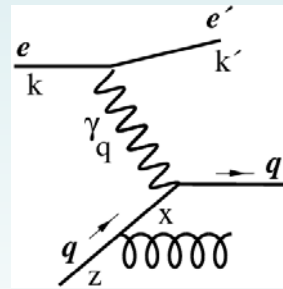
Angular momentum conservation: spin $\frac{1}{2}$ quark absorbs spin-1 photon

QPM

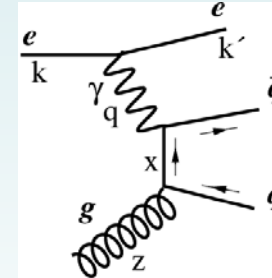


quark helicity $\pm \frac{1}{2}$, $F_L=0$

QCD



off-shell quarks may absorb longitudinal photons



$$\text{QCD: } F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[\frac{16}{3} F_2 + 8 \sum_q e_q^2 \left(1 - \frac{x}{z}\right) z g(z) \right]$$

quarks
radiating a gluon

gluons
splitting into quarks

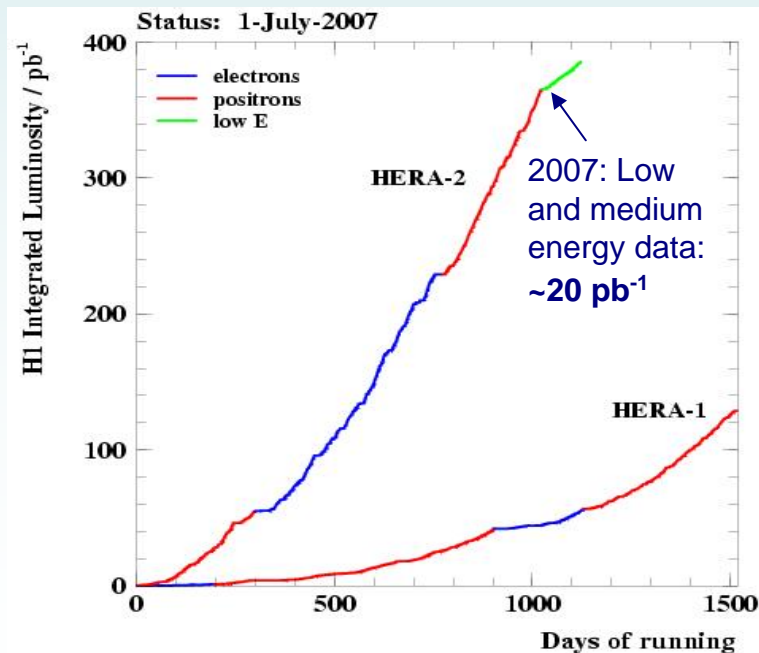
Measurement of F_L

Reduced cross section: $\sigma_r = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$

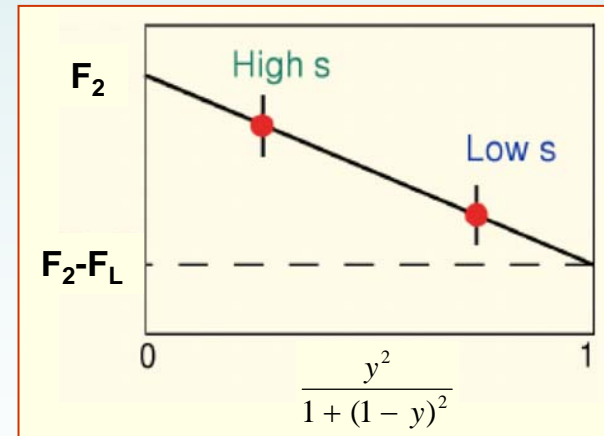
Idea: measure σ_r for same (Q^2, x) at different y (different \sqrt{s}): $y = Q^2 / xs$

Vary proton beam energy:

Lower $E_p := 460, 575 \text{ GeV}$



Rosenbluth plot:

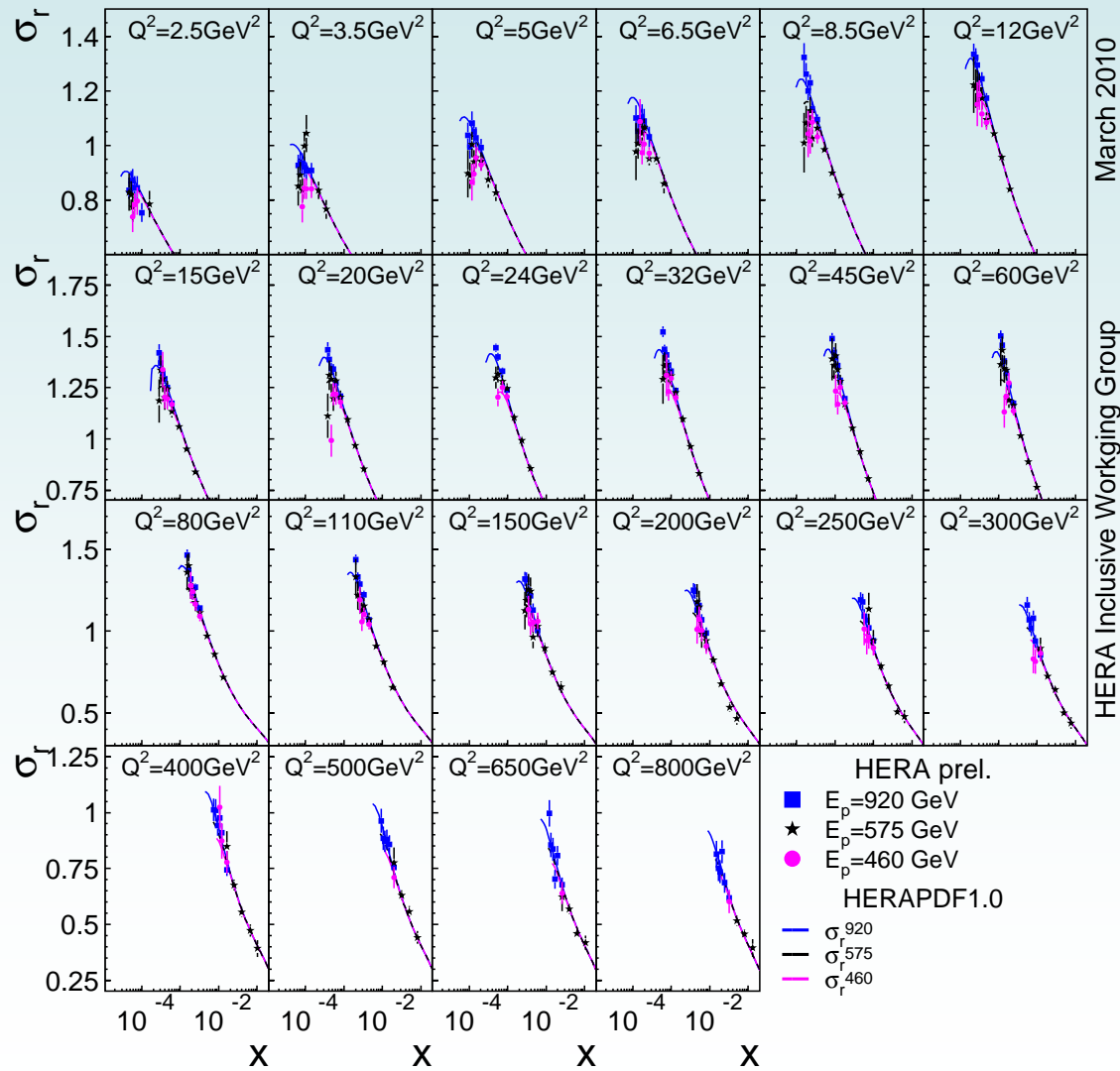


at same (x, Q^2)

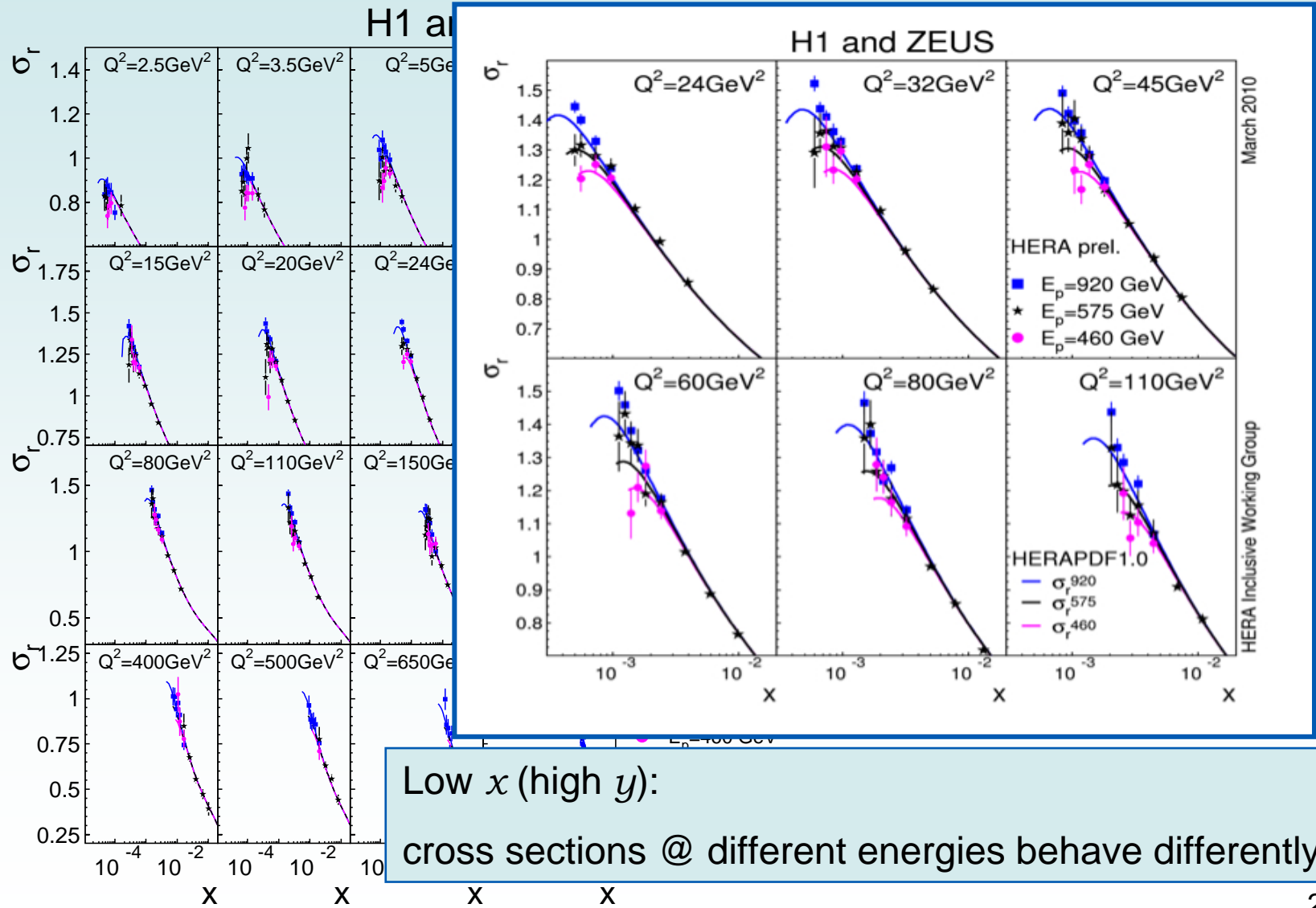
Intercept: F_2 , Slope: F_L

Combination of lower energy data

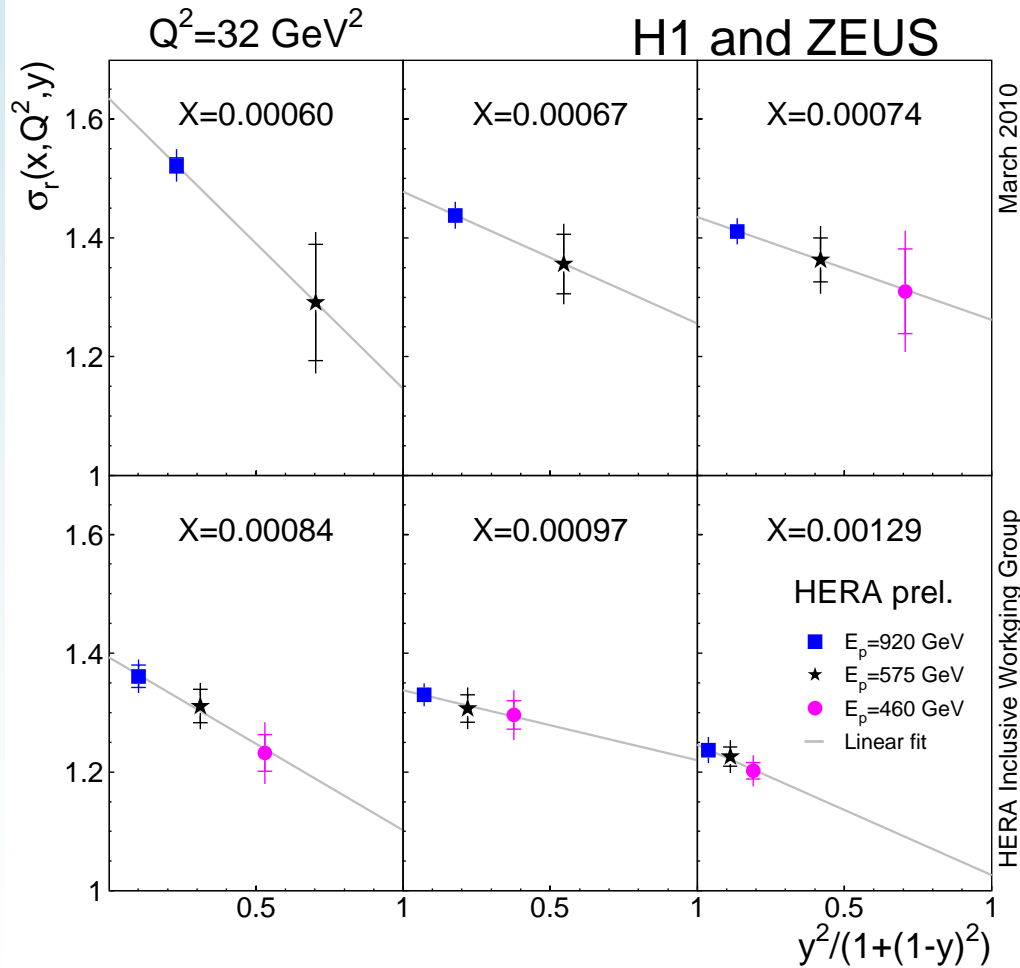
H1 and ZEUS



Combination of lower energy data



Extraction of F_L



Rosenbluth plot:
measurements @ same Q^2, x

$E_p=920 \text{ GeV}$

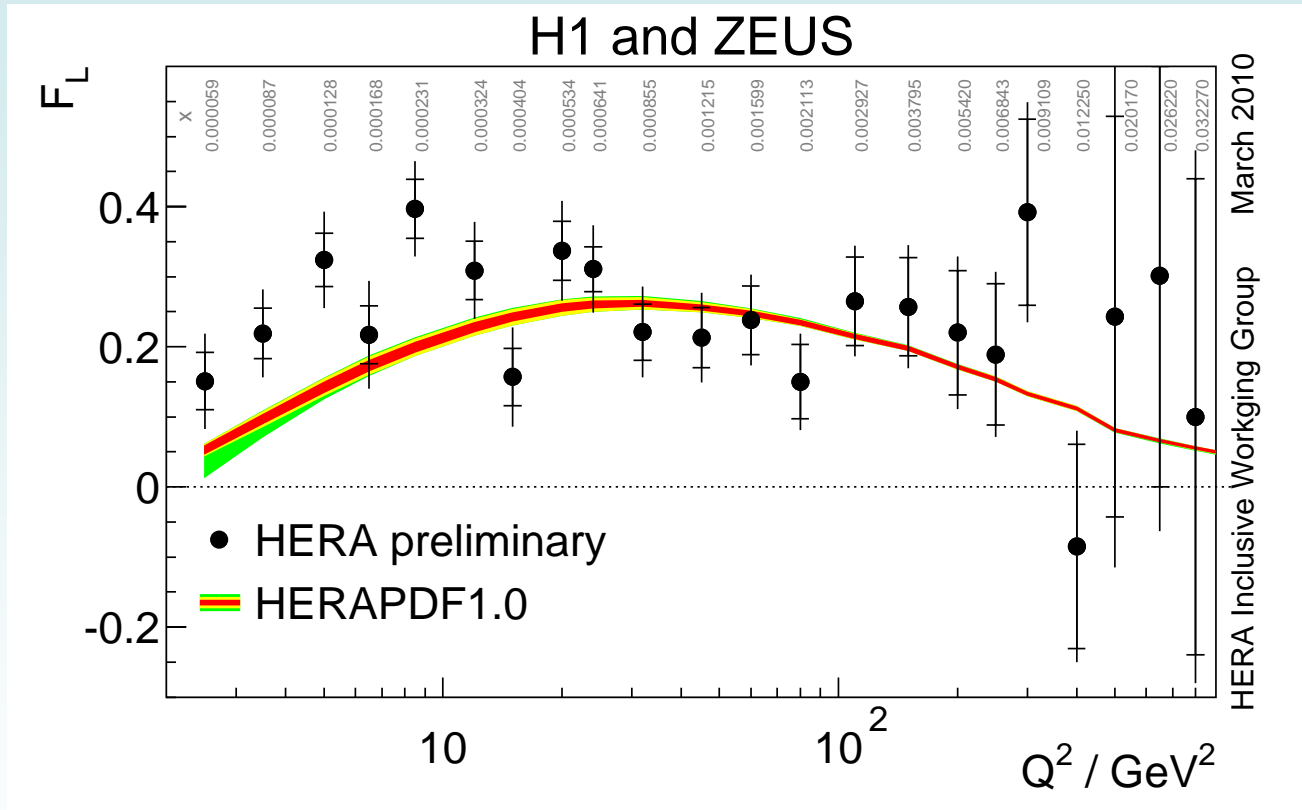
$E_p=575 \text{ GeV}$

$E_p=460 \text{ GeV}$

Intercept: F_2

Slope: F_L

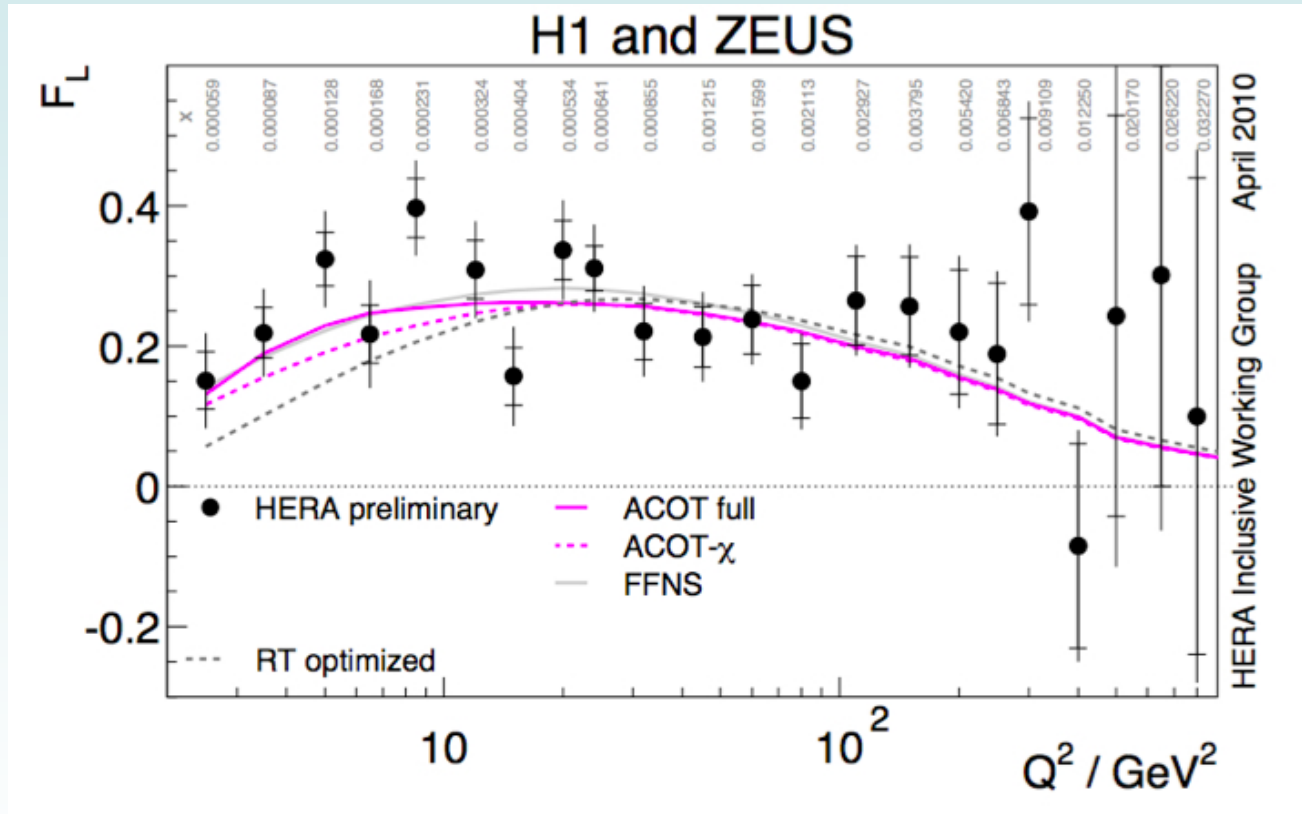
Measured F_L vs QCD \otimes HERAPDF1.0



Combined F_L in general consistent with QCD prediction (HERAPDF1.0)
 Low Q^2 : QCD prediction tend to underestimate the measurement

F_L and heavy quark treatment in PDFs

Combined F_L vs HERAPDF fits with different heavy flavour treatment



F_L data have sensitivity to treatment of heavy quarks in the PDF fit

Progress in HERAPDF

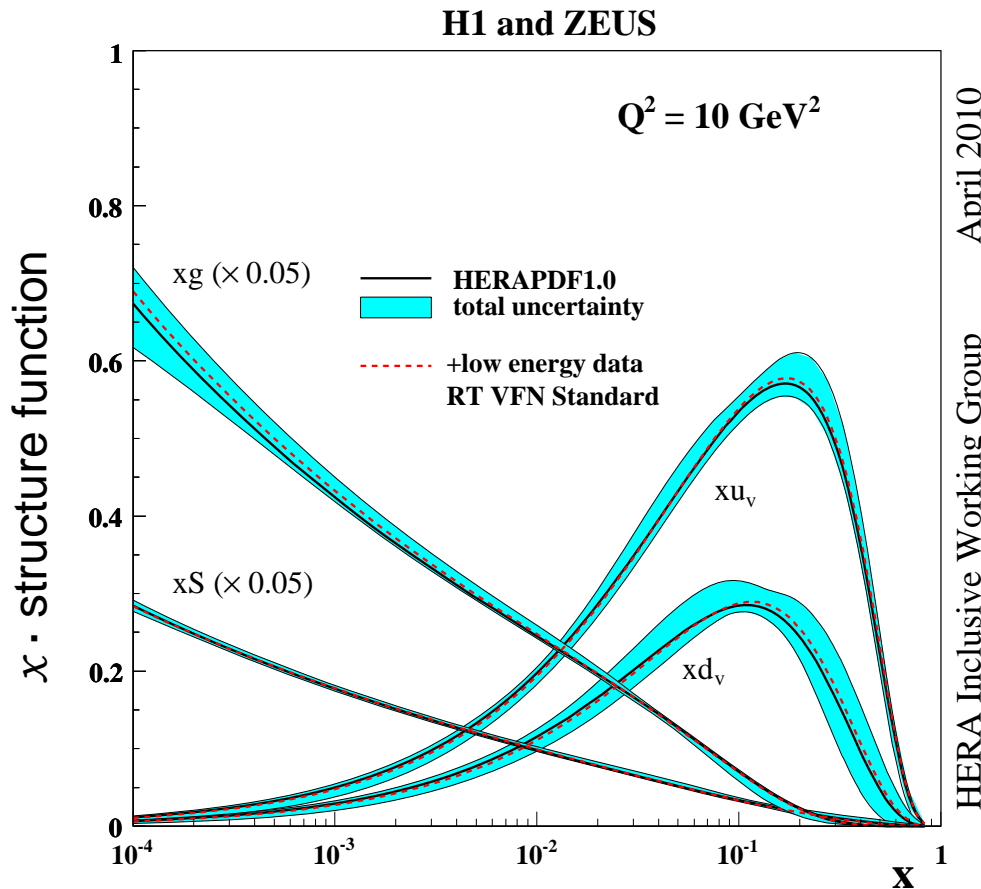
adding lower energy data

adding charm data

performing fits at NNLO

adding high Q^2 , high x data

Low Energy Data in the PDF Fit



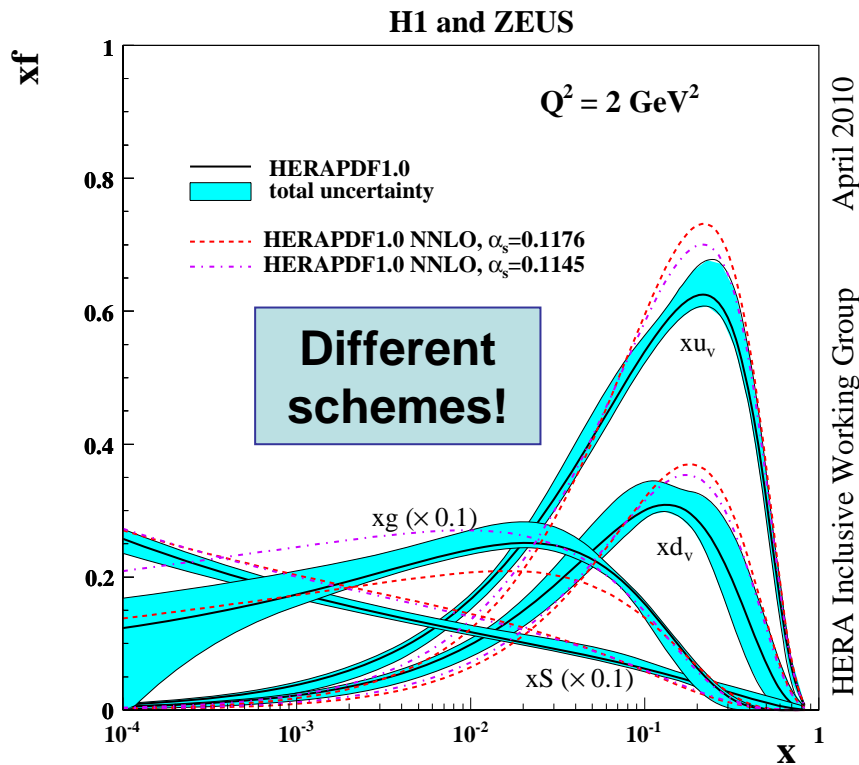
Lower energy data
sensitive to F_L (gluon!)

HERAPDF1.0 Settings

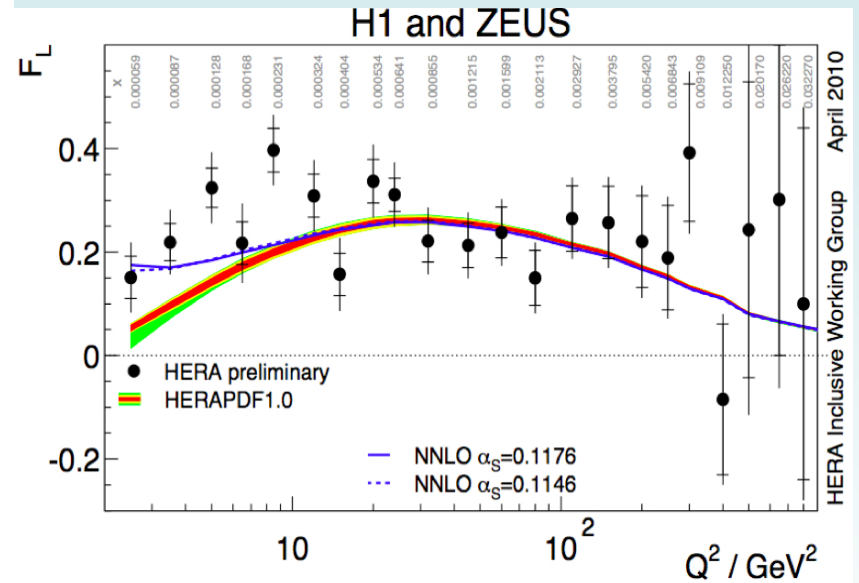
PDF using lower energy cross sections agree well with HERAPDF1.0

Does not explain difference at low Q^2 in F_L

HERA PDF Fits at NNLO



NNLO fit predicts different F_L shape



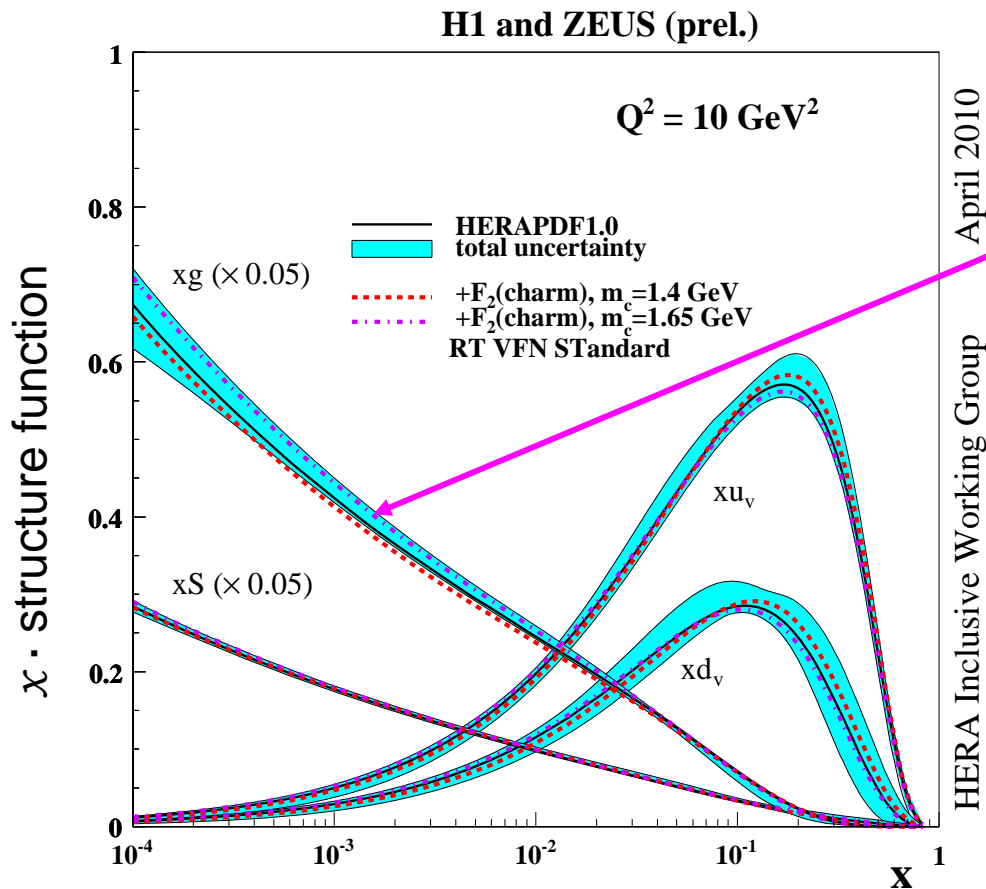
First HERA PDF Fits at NNLO:

lhpdf grids available https://www.desy.de/h1zeus/combined_results/

NNLO has impact on F_L at low Q^2

Charm Structure Function in the PDF Fit

Charm Data in the PDF Fit: $m_c=1.4 \text{ GeV}$ and $m_c=1.65 \text{ GeV}$ compared

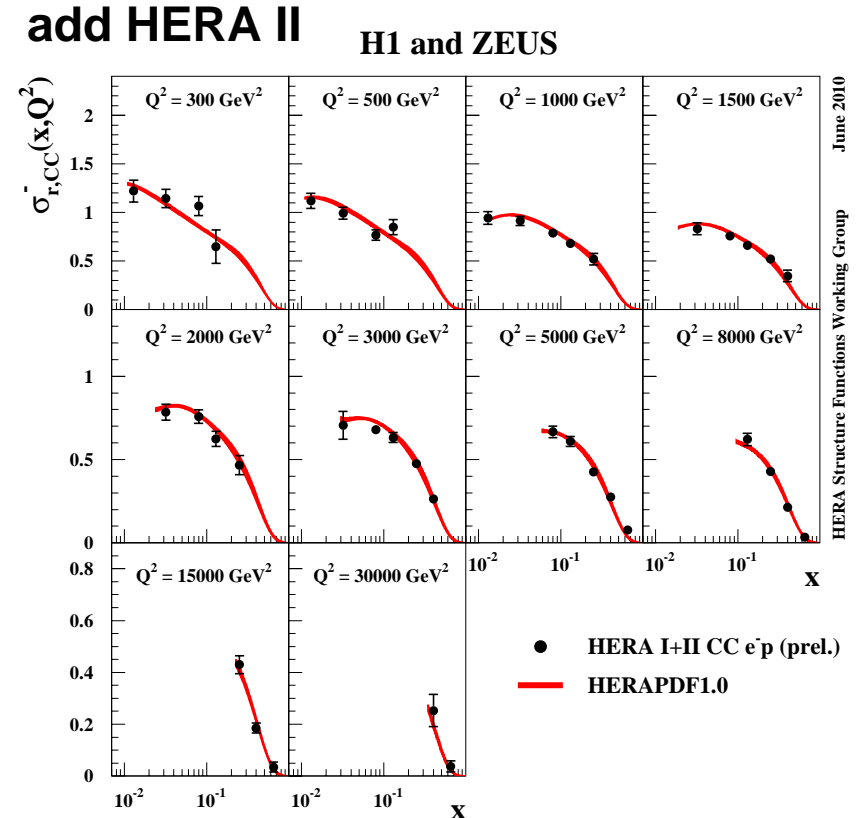
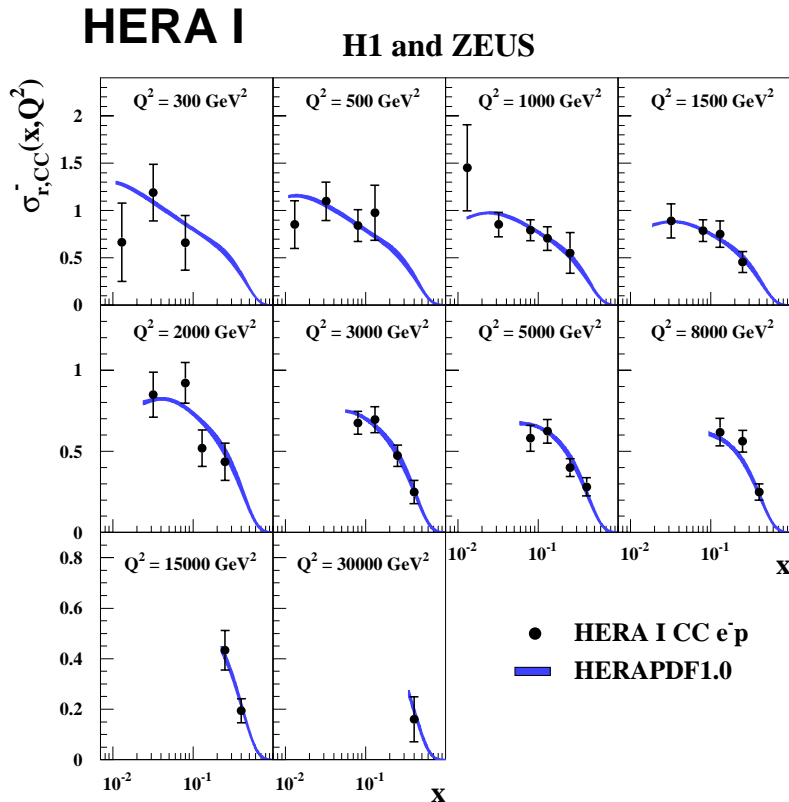


Better fit using
 $m_c=1.65 \text{ GeV}$

preference for
steeper gluon

HERAPDF1.0 vs CC data

Part of HERA II data added (not included in HERAPDF1.0)



HERA Structure Functions Working Group
June 2010

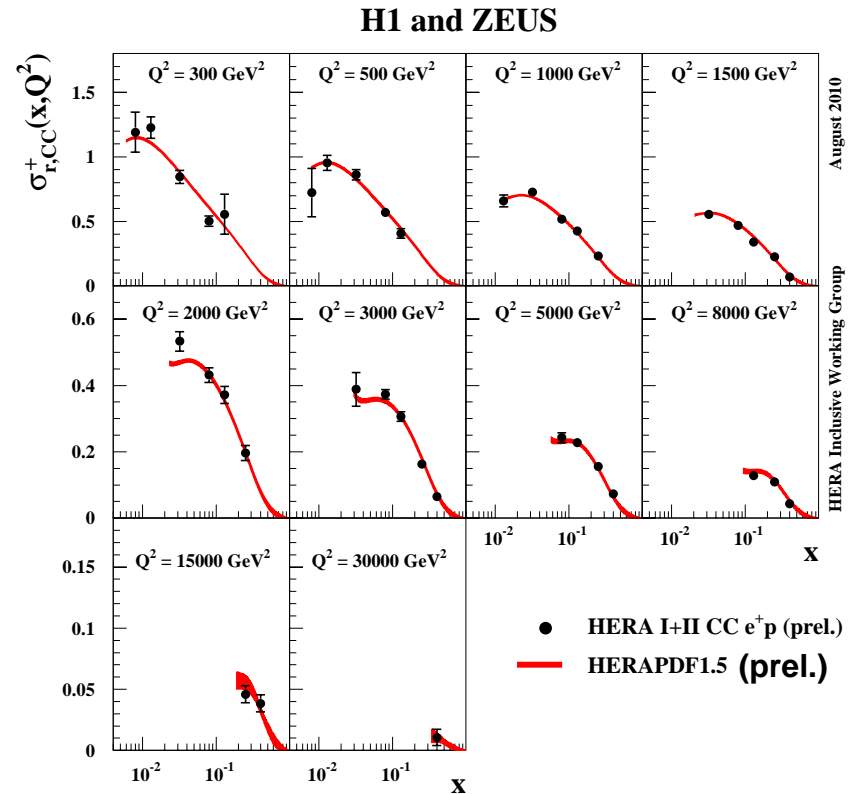
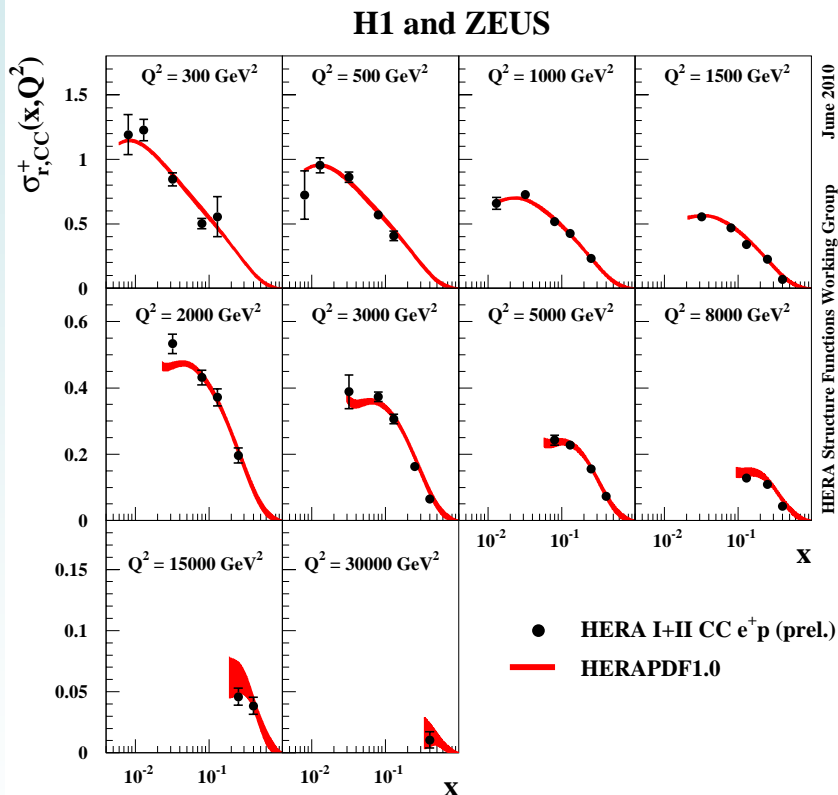
QCD \otimes HERAPDF1.0 describes data very well

Better precision at high x high Q^2 : **better constrained valence**

Combined e^+p CC cross section vs HERAPDF

HERAPDF1.0: HERA I data

+ part of HERA II data : HERAPDF1.5



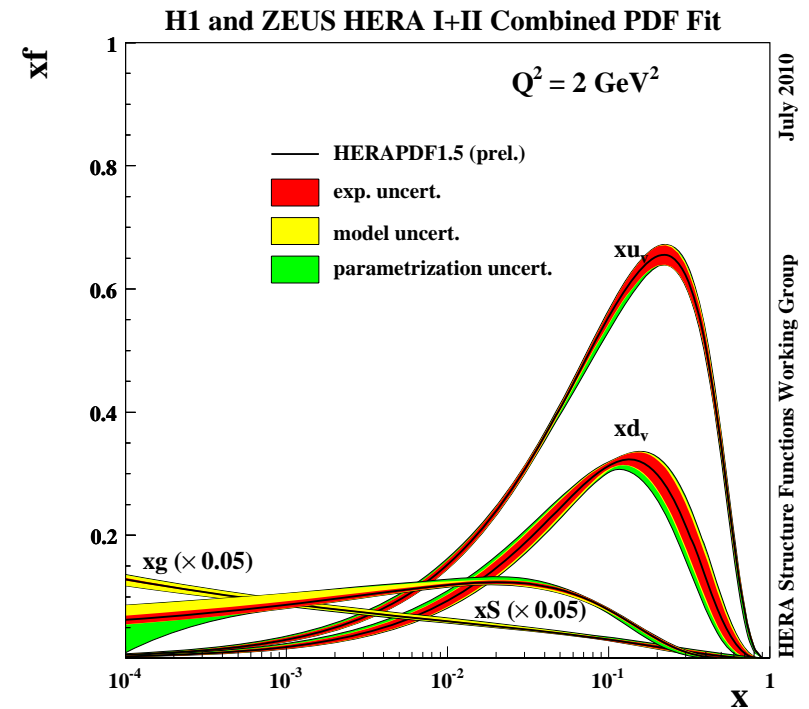
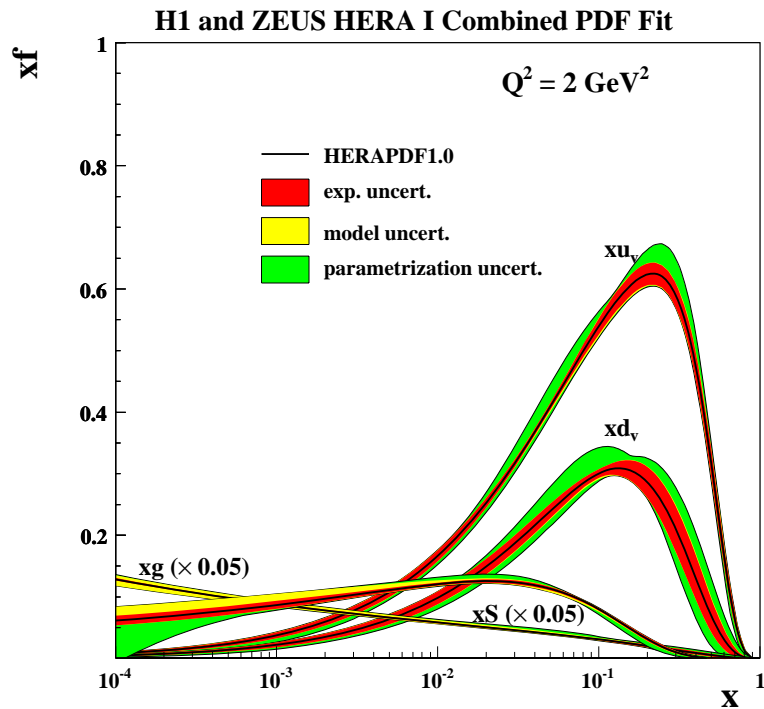
QCD \otimes HERAPDF1.0 describes data very well

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High Q^2 data in the PDF Fit

HERAPDF1.0: HERA I data

+ part of HERA II data : HERAPDF1.5



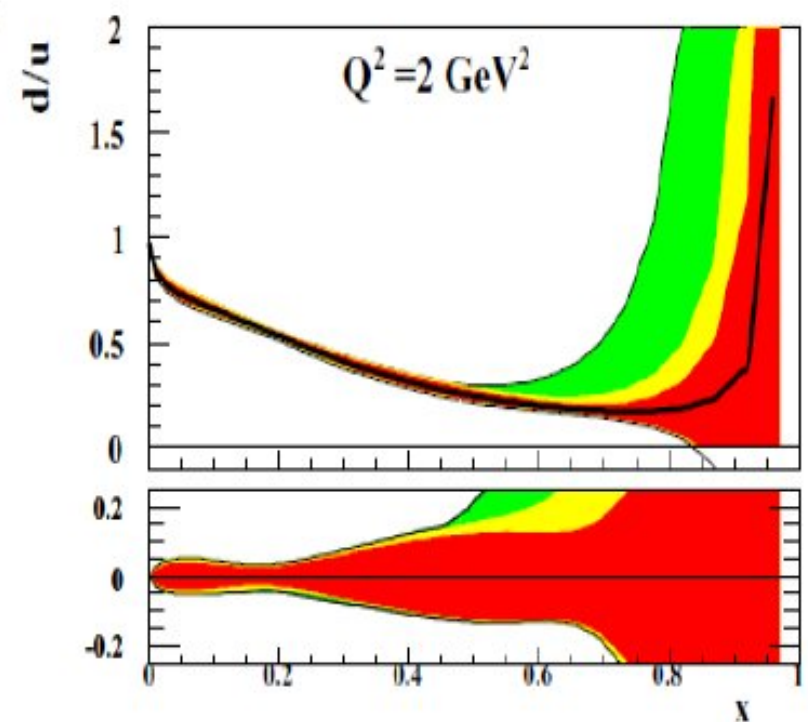
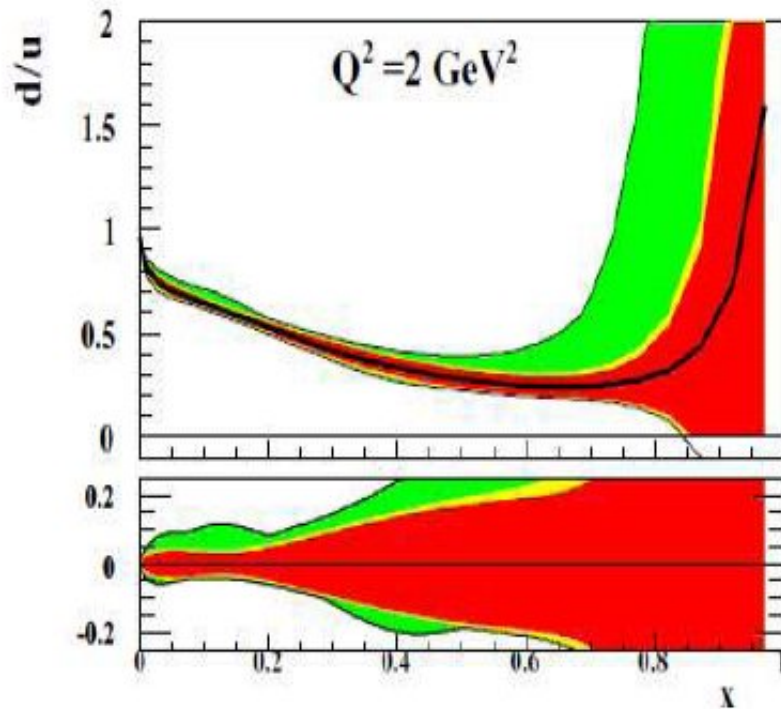
Valence much better constrained at high x :

- Reduced uncertainty (parameterization uncertainty remarkably smaller)
- Softer sea distribution

High Q^2 data in the PDF Fit

HERAPDF1.0: HERA I data

+ part of HERA II data : HERAPDF1.5

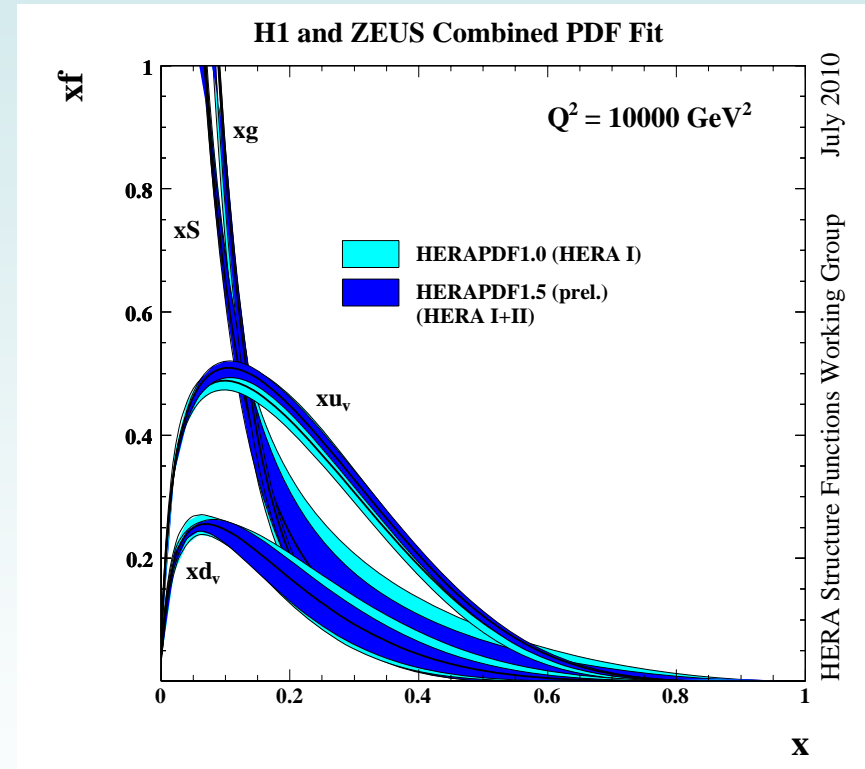
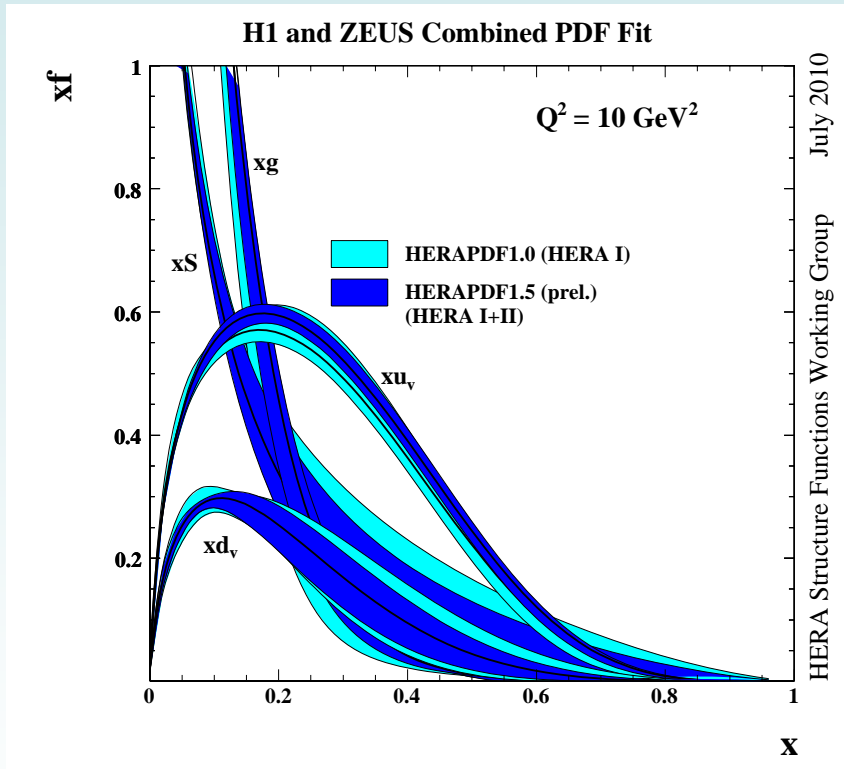


Valence much better constrained at high x :

- Reduced uncertainty (parameterization uncertainty remarkably smaller)
- Increased precision of d/u - ratio

HERAPDF1.5 vs HERAPDF1.0

On the linear scale!

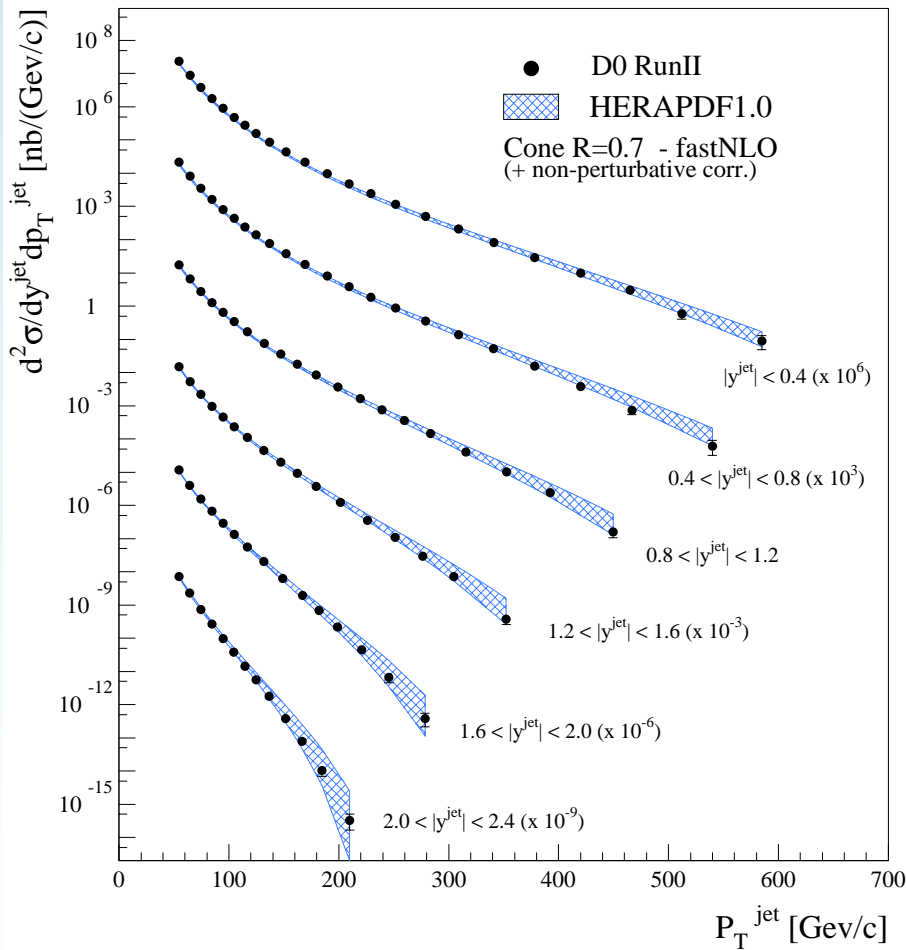


Reduced uncertainty for LHC predictions!

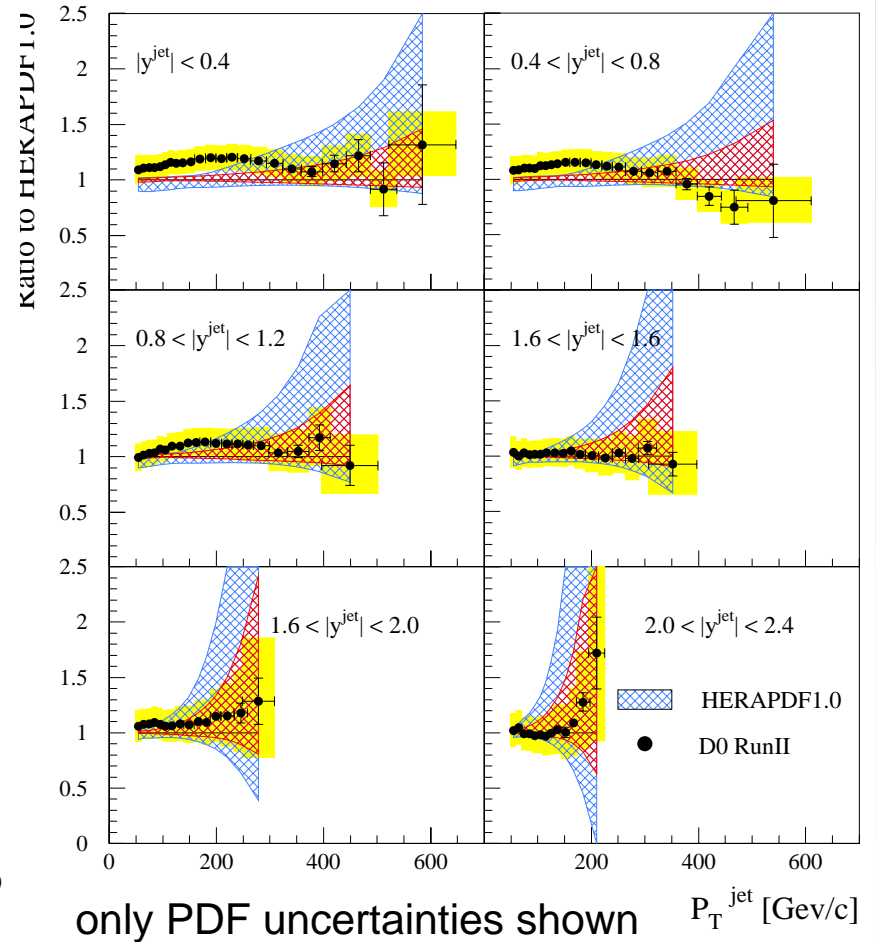
Parton Densities from HERA to TEVATRON and the LHC

HERAPDF1.0 vs Jets at TEVATRON

Tevatron Jet Cross Sections

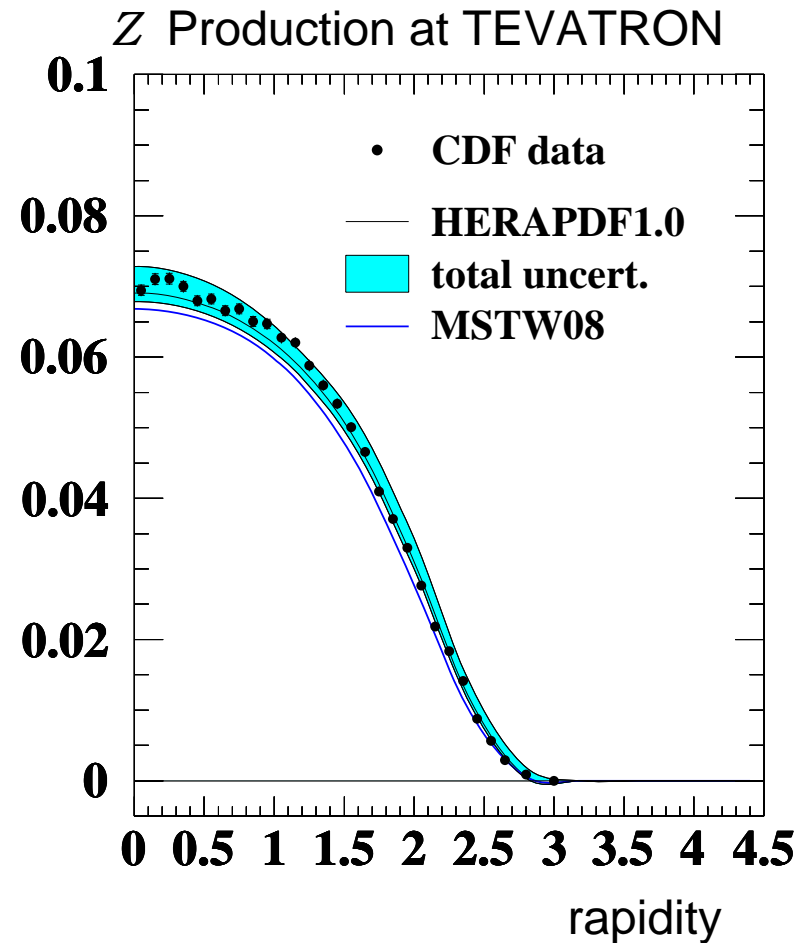
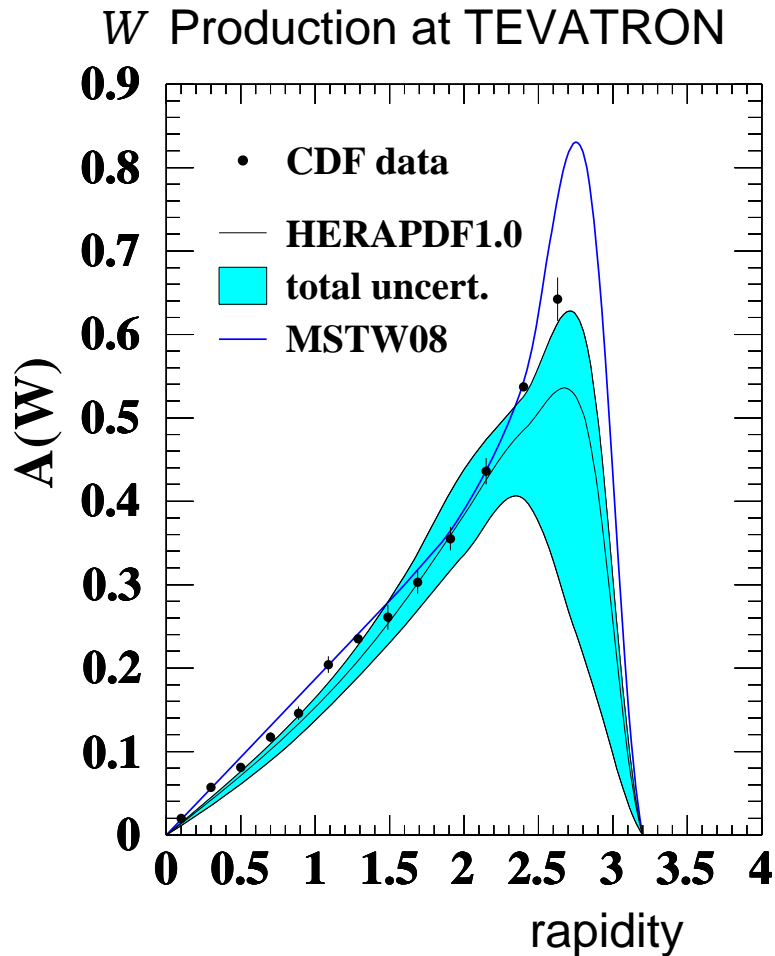


Tevatron Jet Cross Sections



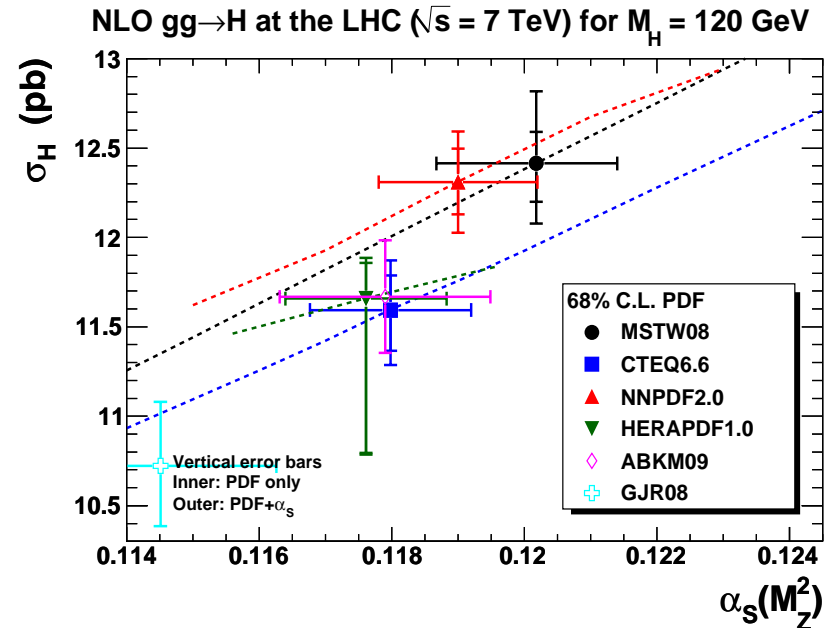
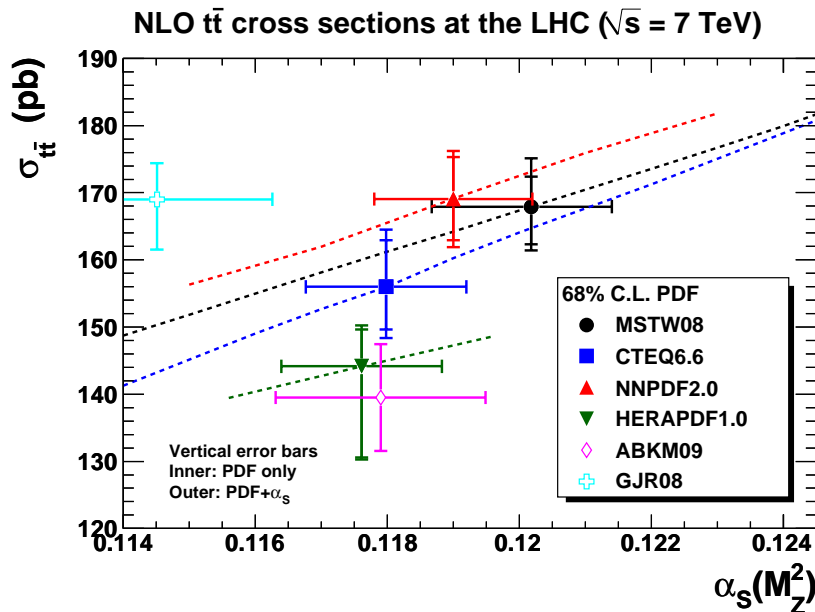
Predictions based on HERAPDF1.0 in agreement with TEVATRON data

W/Z Production at TEVATRON



Predictions based on HERAPDF1.0 in agreement with TEVATRON data

Benchmarking PDFs: LHC cross sections



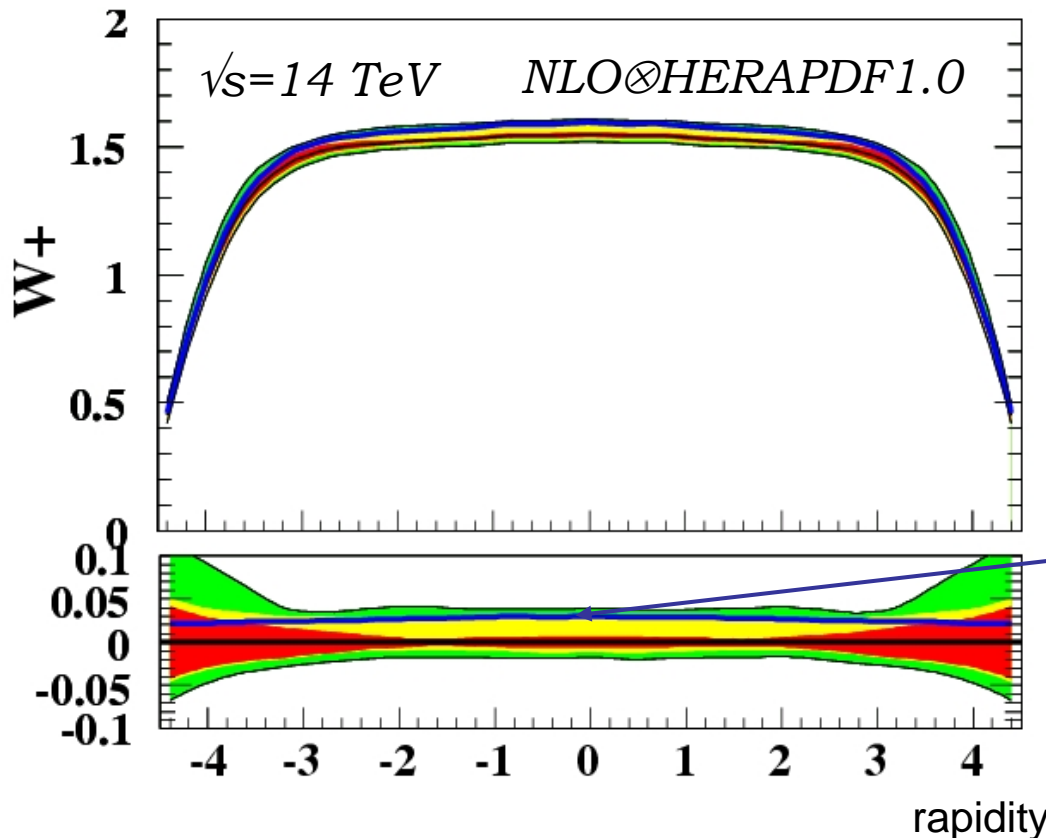
HERAPDF one of the major players in benchmarking activity

HERAPDF1.0 provides realistic uncertainty for LHC cross sections

dominant uncertainty (parameterization) not accounted for in most global Fit groups

Charm at HERA and W/Z at LHC

choice of $m_c=1.65$ raises W/Z cross-section predictions at the LHC by $\sim 3\%$



Prediction using $m_c=1.4 \text{ GeV}$

Error band: PDF uncertainty

Experimental

Model

Parametrization

Shift in central value due to $m_c=1.65$ in PDF

Larger $m_c \rightarrow$ more gluons, less charm \rightarrow more light quarks \rightarrow larger σ_W

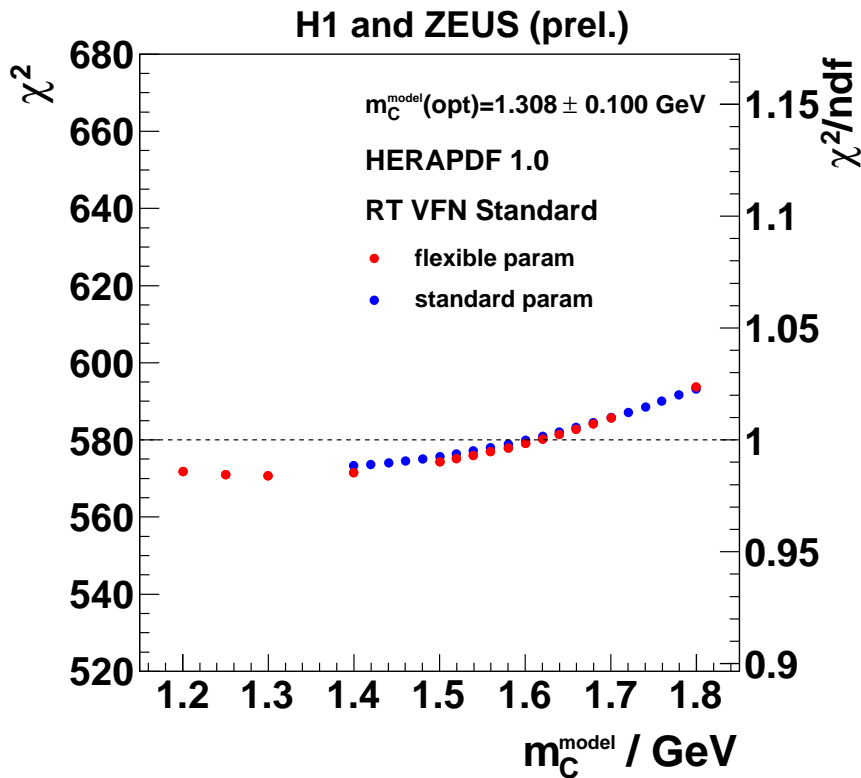
Does matter for luminosity @ LHC !

Charm mass as a model parameter in PDF

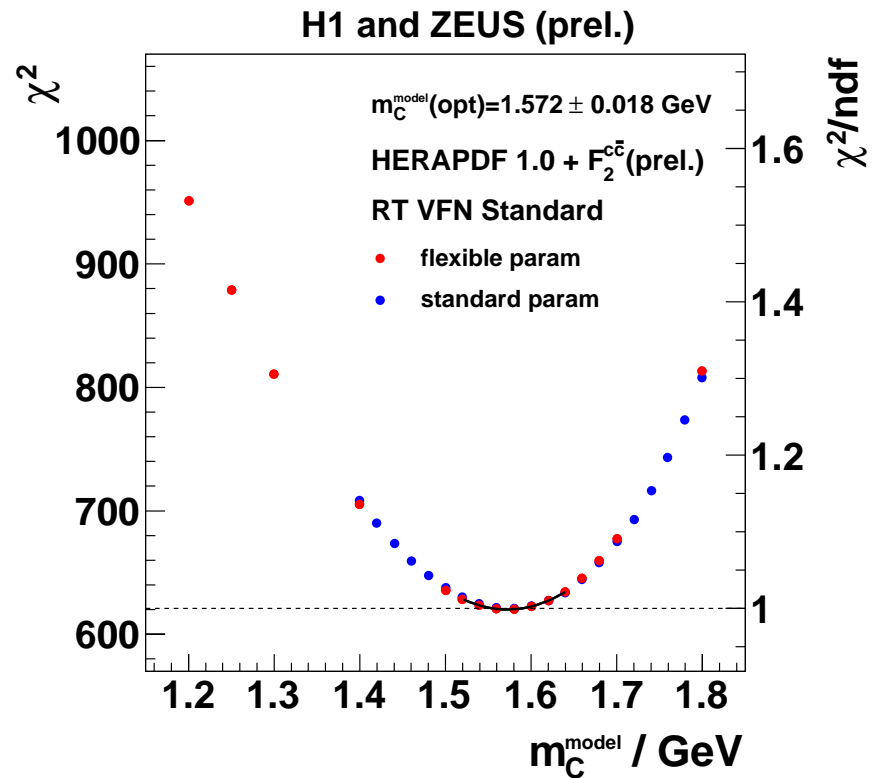
Value of m_c in PDF Fit: how sensitive are HERA structure functions?

Vary m_c^{model} in the Fit to inclusive DIS

Vary m_c^{model} in the PDF Fit to $F_2 + F_2^c$



F_2 not very sensitive to m_c^{model}



F_2^c sensitive to m_c^{model}

Charm at HERA and W/Z at LHC

Value of m_c^{model} : how different for various HQ schemes in PDF Fits?

Scan the value of m_c^{model} as PDF fit parameter for different schemes

HERAPDF can test different

heavy quark schemes:

GMVFNS (MSTW, CTEQ)

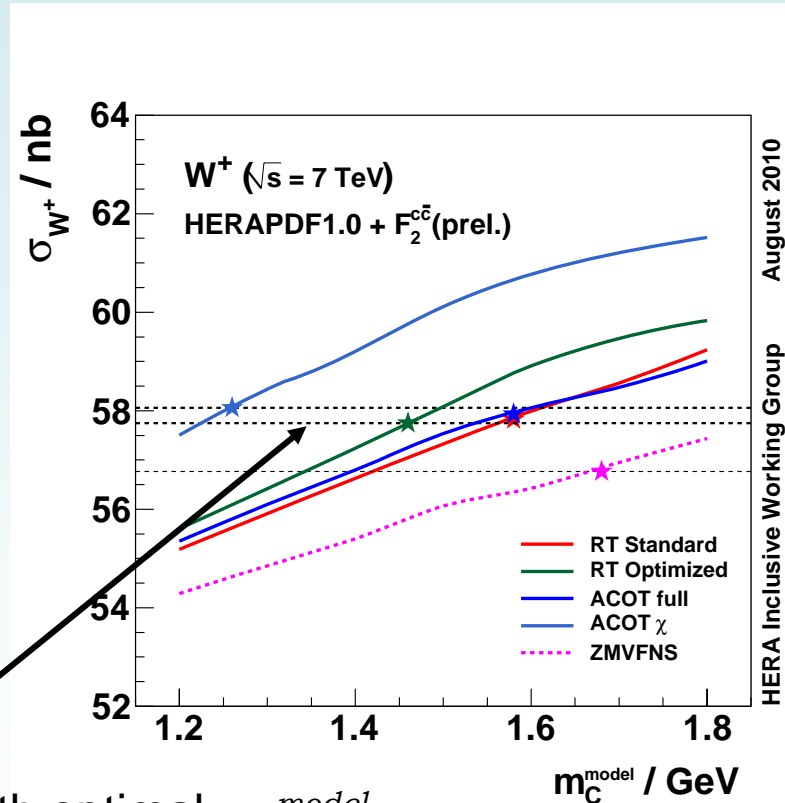
Zero-Mass (used in NNPDF)

Prediction for W cross section:

★ optimal m_c^{model} using $F_2 + F_2^c$

1% spread using different

implementations of GMVFNS with optimal m_c^{model}



Uncertainty significantly reduced

Summary

- Ultimate knowledge of proton structure comes from DIS at HERA
- Combinations of H1 and ZEUS provide increasing precision
- HERAPDF1.0 best PDF measurements at medium and low x
- HERA performs PDF fits using Low Energy and charm data
- HERA NNLO PDF fits available
- High Q^2 data improves precision of the valence at high x
- HERAPDF has a visible impact on LHC physics

Back up

Proton collisions at the LHC

LHC: p - p collisions at $\sqrt{s} = 7, 10, 14$ TeV

Goal @ LHC: Higgs and new physics

Main challenge: Background suppression

Main Background: QCD

Hard processes > 80% gluon-gluon fusion

Cross section $\sim |g(x)|^2$

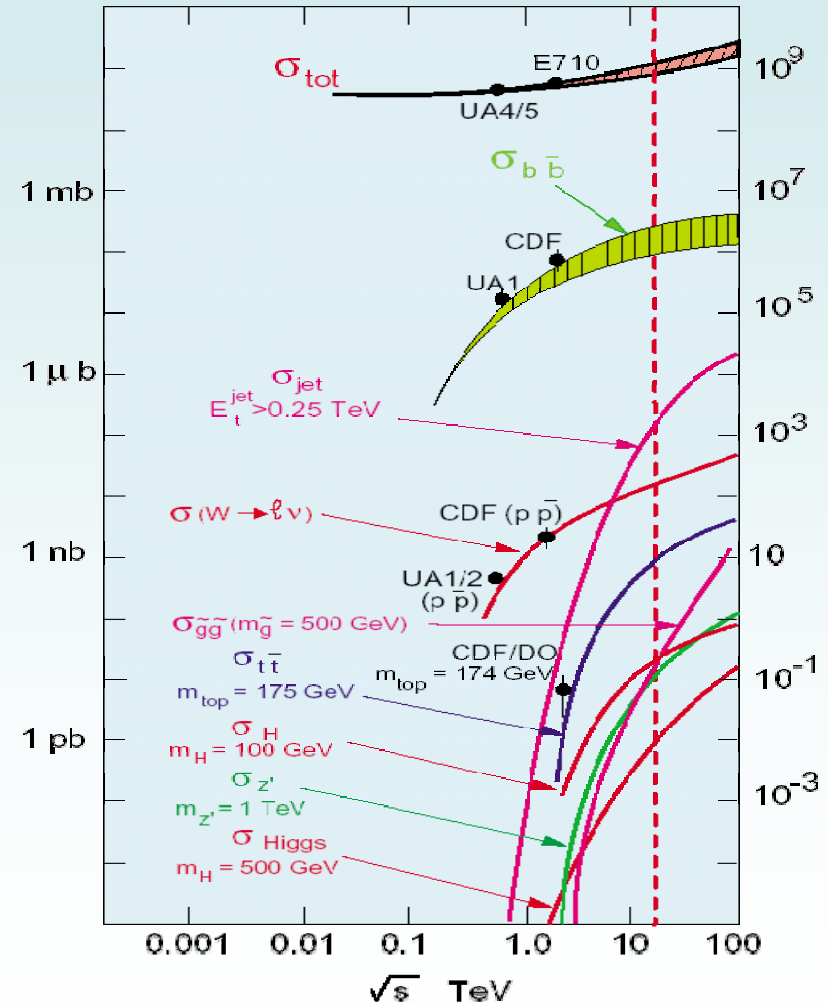
Precision of the gluon density essential!

Luminosity: e.g. $u\bar{d} \rightarrow W^+ \rightarrow l^+ \nu_l$

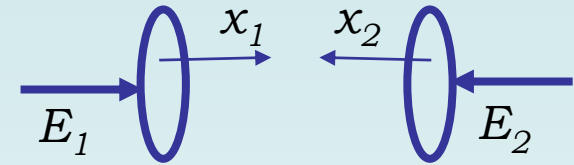
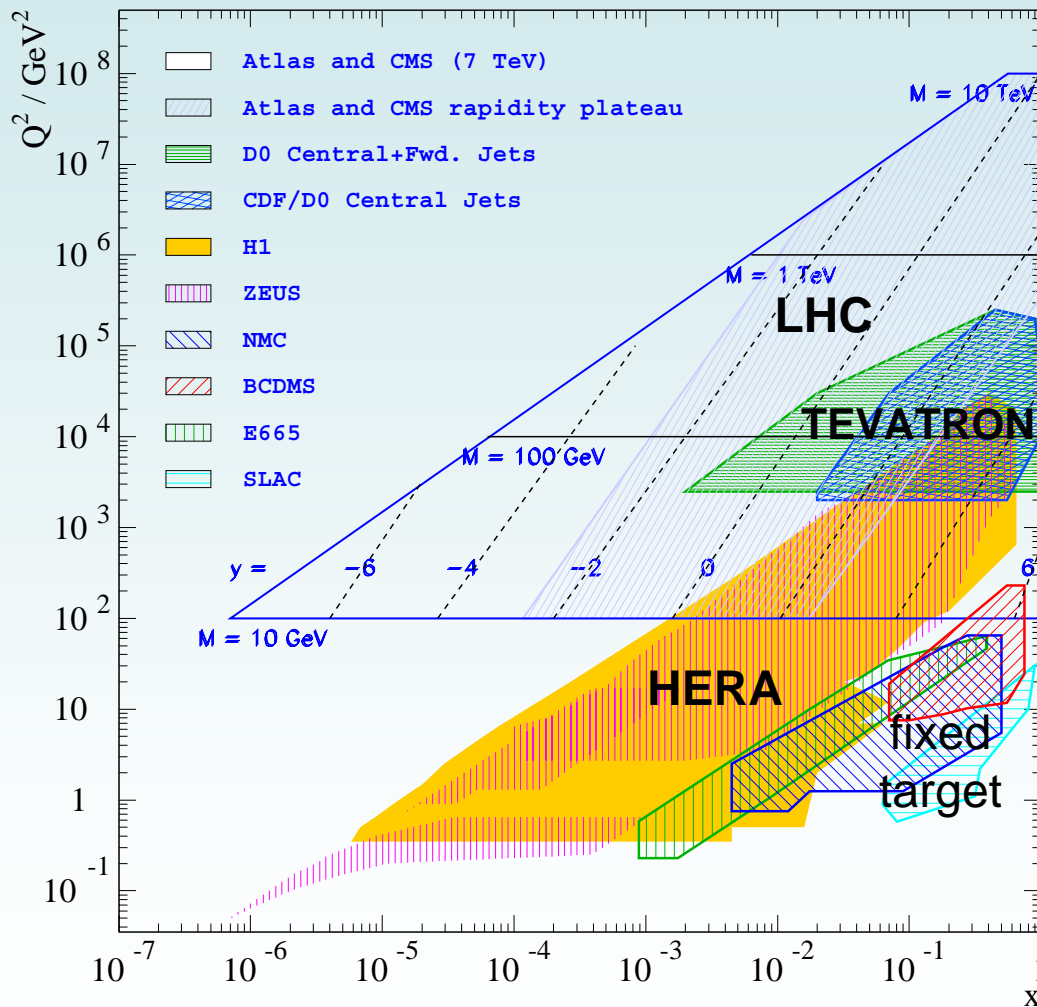
Precision of light quark densities essential!

Key issue: understanding of the proton

Rate and cross sections of pp collisions



Kinematics of collision experiments



Center-of-mass energy

$$s = 4 E_1 E_2$$

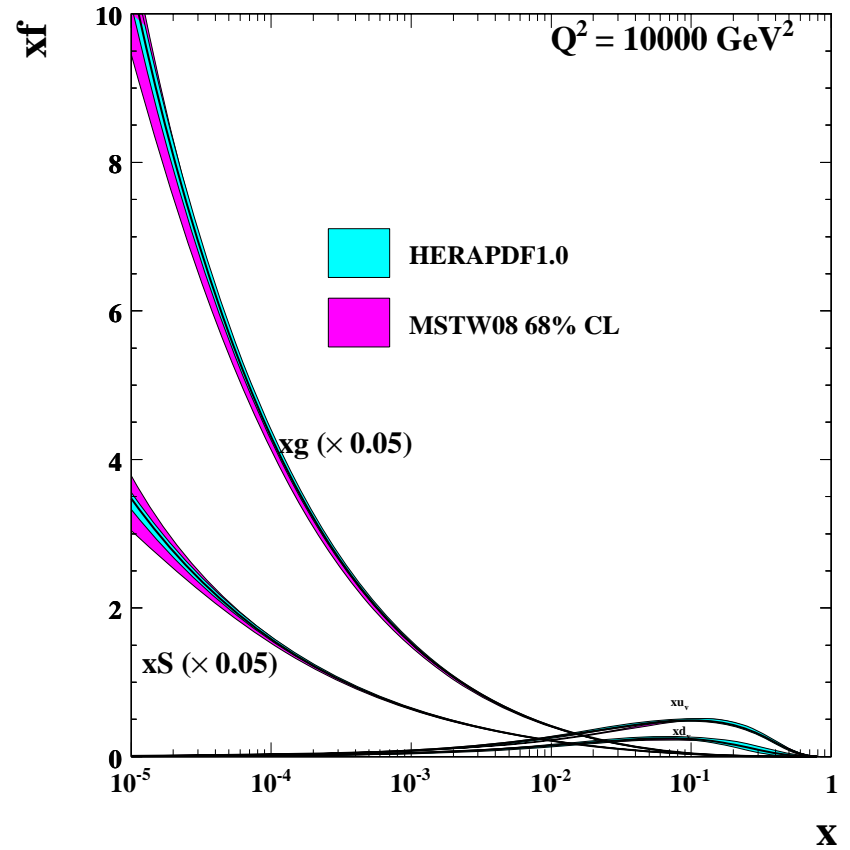
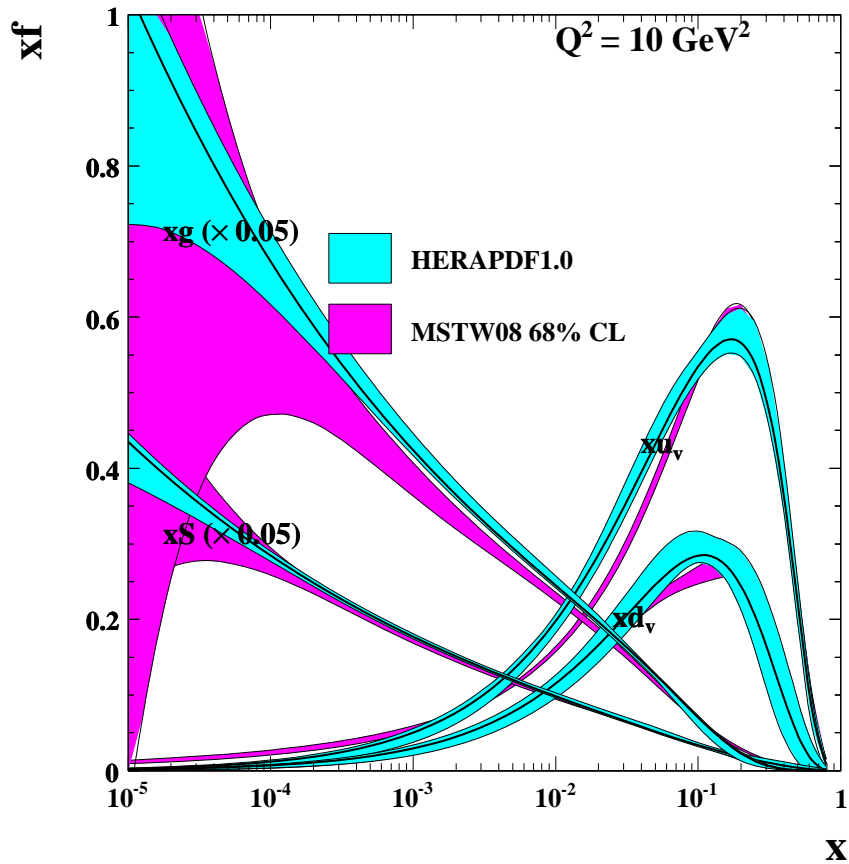
$$\hat{s} = x_1 \cdot x_2 \cdot s \geq M^2$$

Energy scale $M=Q$

$$x_{1,2} = \left(\frac{M}{\sqrt{s}} \right) \cdot \exp(\pm y)$$

y - rapidity

HERAPDF1.0 vs other PDF set



HQ Contribution to the Proton Structure

Can be determined experimentally: e.g. “charm structure function”:

$$F_2^{cc} \propto \frac{Q^2 \cdot \alpha_s}{m_c^2} \int \frac{dx}{x} \cdot e_c^2 g(x_g, Q^2) \cdot C(\dots)$$

- use and combine different charm tagging methods

measure cross sections of charm and beauty production in DIS:

$$\sigma^{cc} \propto F_2^{cc}(x, Q^2) - \frac{y^2}{1 + (1 - y)} F_L^{cc}(x, Q^2)$$

- Direct test of different schemes of HQ treatment in PDF fits
- Can be included in the full QCD analysis of DIS cross sections
 - additional constrain on the gluon density in the proton
 - reduce parameterization uncertainty

Combination Procedure

Minimized value:

$$\chi^2(\vec{m}, \vec{b}) = \sum_i \frac{\left(m^i - \sum_j \gamma_j^i m^i b_j - \mu^i\right)^2}{\left(\delta_{i,stat} \mu^i\right)^2 + \left(\delta_{i,unc} m^i\right)^2} + \sum_j b_j^2$$

μ^i measured value at point i

δ_i statistical, uncorrelated systematic error

γ_j^i – correlated systematic error

b_j – shift of correlated systematic error sources

m^i – true value (corresponds to $\min \chi^2$)

Measurements performed sometimes in slightly different range of (x, Q^2)

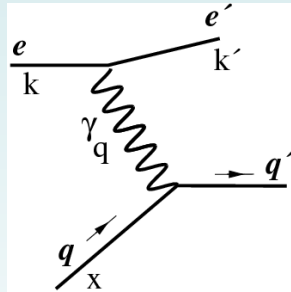
swimming to the common (x, Q^2) grid via NLO QCD in massive scheme

Direct access to the gluon: F_L

Photon-Parton Scattering: $\frac{d^2\sigma}{dx dQ^2} \propto (\sigma_T + \frac{2(1-y)}{Y_+} \sigma_L), Y_+ = 1 + (1-y)^2$

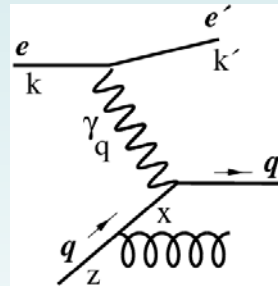
Angular momentum conservation: spin $\frac{1}{2}$ quark absorbs spin -1 photon

QPM

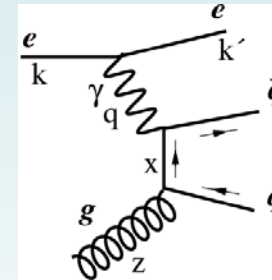


quark helicity $\pm \frac{1}{2}$, $F_L = 0$

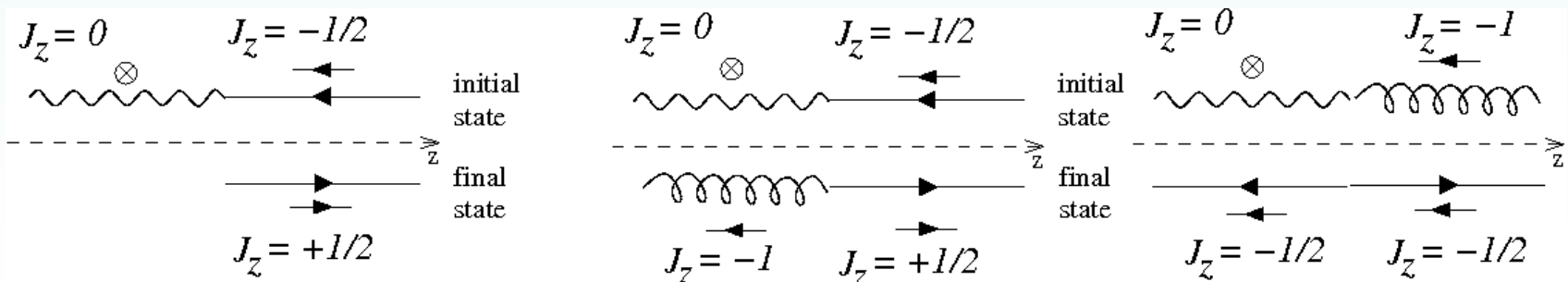
QCD



off-shell quarks may absorb longitudinal photons

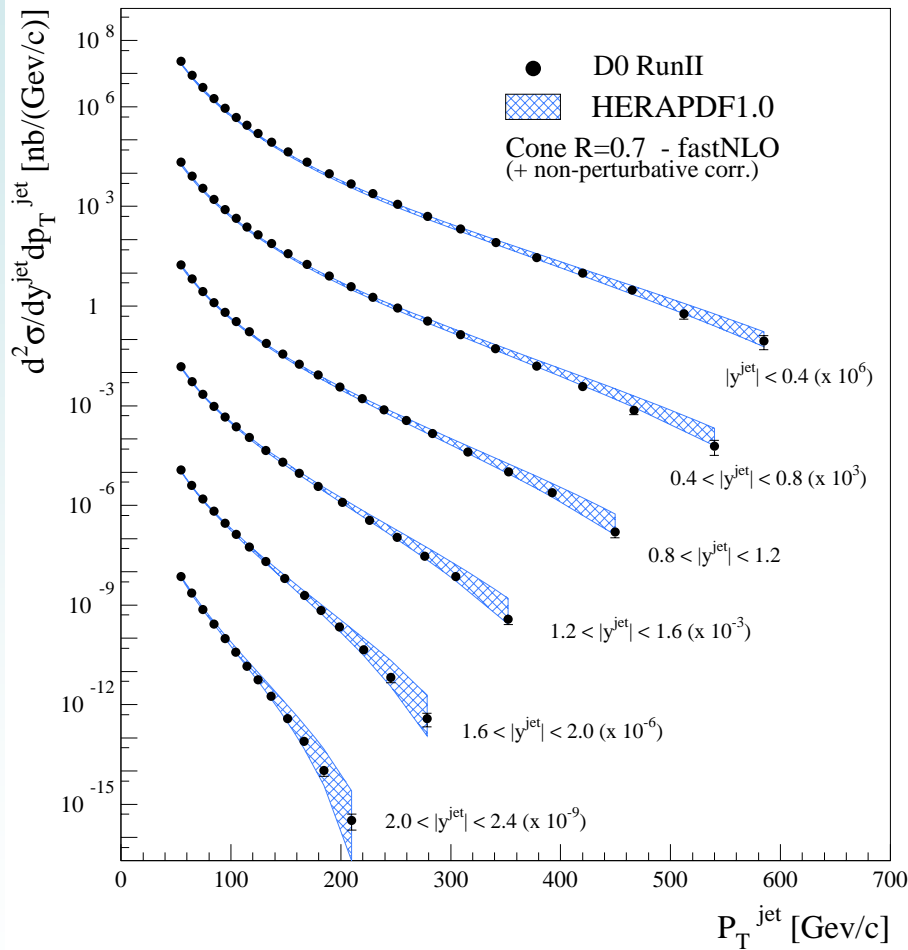


Scattering of longitudinally polarized photons on quarks in helicity frame

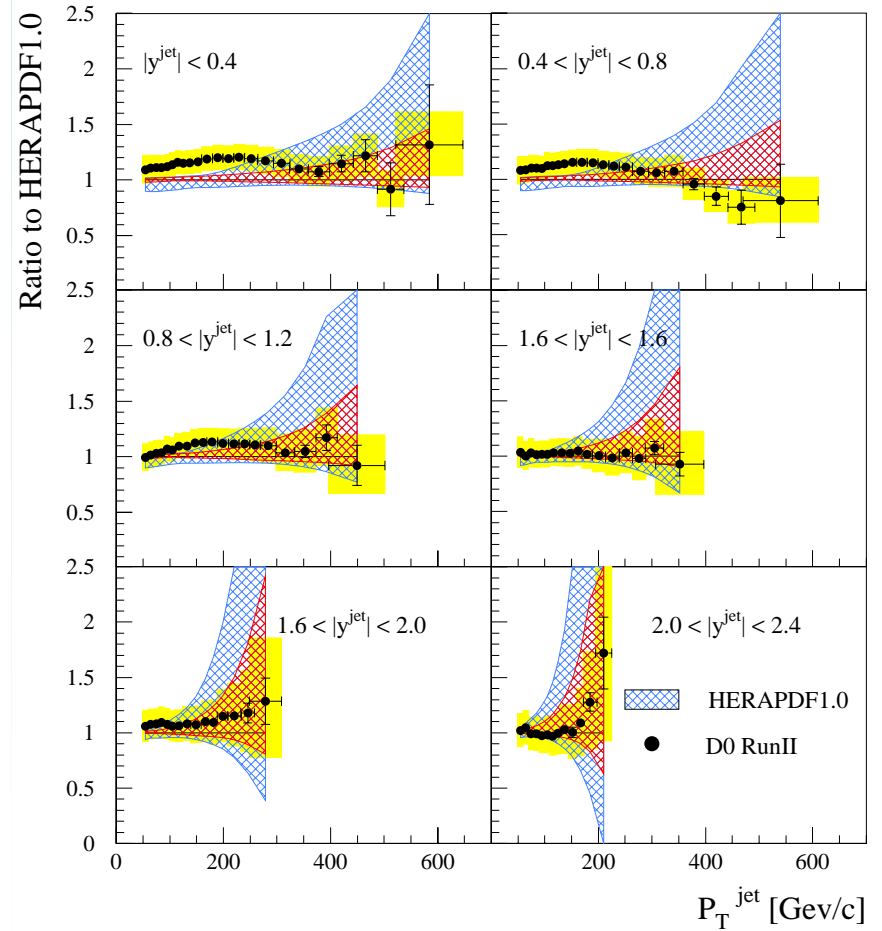


HERAPDF1.0 vs Jets at TEVATRON

Tevatron Jet Cross Sections

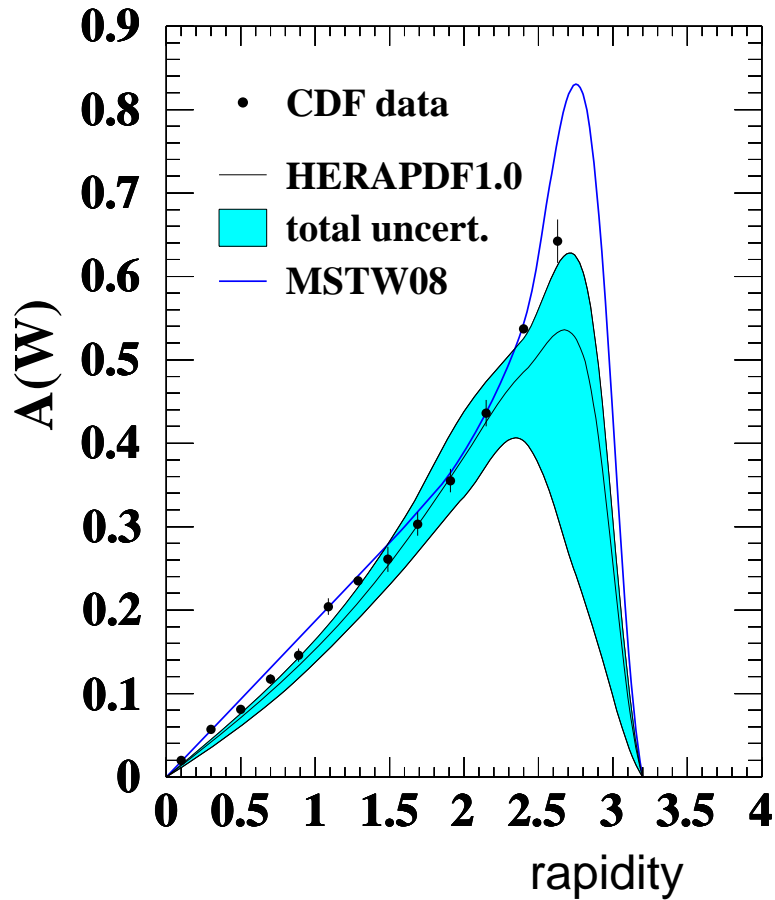


Tevatron Jet Cross Sections



W/Z Production at TEVATRON

W Production at TEVATRON



Z Production at TEVATRON

