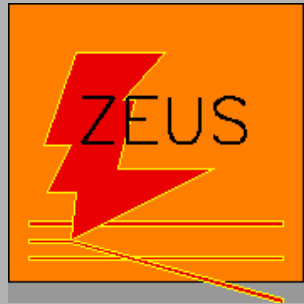


# PIC2010 – QCD 2

Precision QCD tests and  $\alpha_s$  measurements  
(at HERA)



Ian C. Brock  
University of Bonn

Karlsruhe  
02/09/2010



# Why test QCD?

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- Everyone knows QCD is the theory of the strong interaction
- Everyone knows that a particle and an antiparticle have the same mass
- Everyone knows that (after the big bang) there is the same amount of matter and antimatter in the universe
  - or not?
- It is important not to take such things for granted and continue to test even established theories

# Why test QCD and measure $\alpha_s$ ?

- With the start of the LHC we hope to discover life beyond the Standard Model
- The  $pp$  cross section is many orders of magnitude larger than the interesting new physics cross sections
- QCD backgrounds are dominant in many processes
- $\alpha_s$  is the parameter (apart from masses) that fixes QCD!
- Calculations and precise tests are hard as  $\alpha_s$  is large

# Outline

- Measurements of  $\alpha_s$ 
  - How can one measure  $\alpha_s$ ?
  - Low-energy ( $\tau$ ,  $\Upsilon$ ) + LEP measurements
  - HERA measurements
  - Averages
- Other precision QCD tests
  - What one has to worry about
  - Selected results
- Summary

# How to measure $\alpha_S$ ?

- Measure the leptonic branching fraction of the  $\tau$  lepton!

- Obvious?

- 3 colours lead to leptonic BR of 20%

- QCD corrections lead to:

e:  $(17.85 \pm 0.05)\%$ ,  $\mu$ :  $(17.36 \pm 0.05)\%$

- From this extract  $\alpha_S$

$$R_\tau = \frac{\Gamma(\tau \rightarrow \text{hadrons})}{\Gamma(\tau \rightarrow l \nu_l \nu_\tau)}$$

- Using properties of hadronic system leads to further improvement:

$$\alpha_S(m_\tau) = 0.330 \pm 0.014$$

Refs in S. Bethke  
EPJ C64:(2009) 689,  
arXiv:0908.1135



# Running of $\alpha_s$

- Running coupling satisfies renormalisation group equation(RGE):

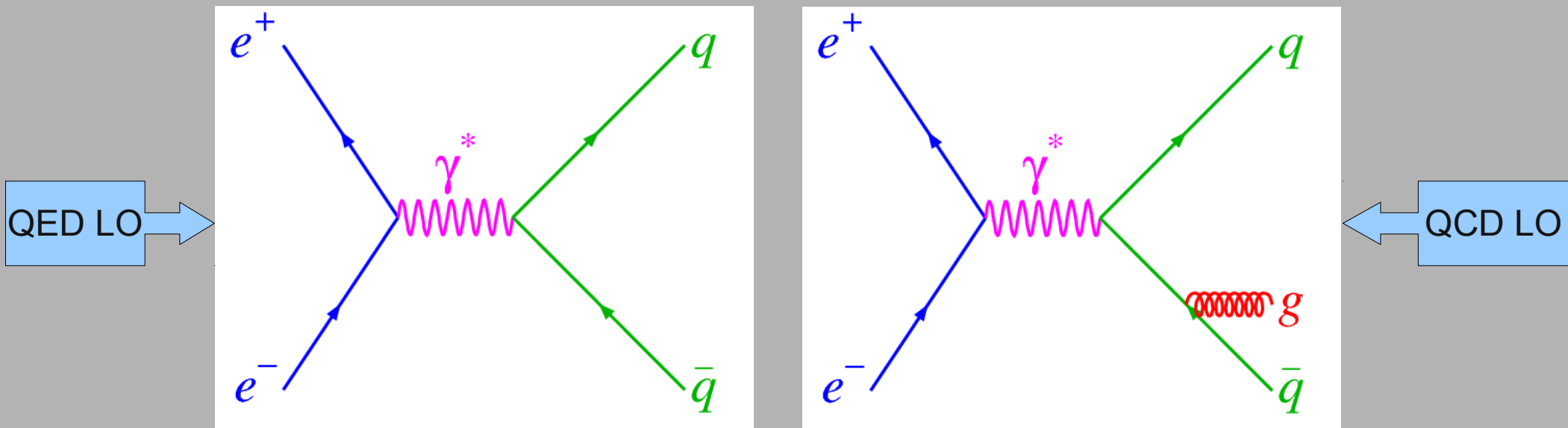
$$\mu^2 \frac{d \alpha_s}{d \mu^2} = \beta(\alpha_s) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \dots)$$

- 1-loop approximation ( $\beta_1=0$ )

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2) \beta_0 \ln(Q^2/\mu^2)} \quad \text{or} \quad \alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)}$$

# LO, NLO, NNLO and all that

- What is leading order (LO)?

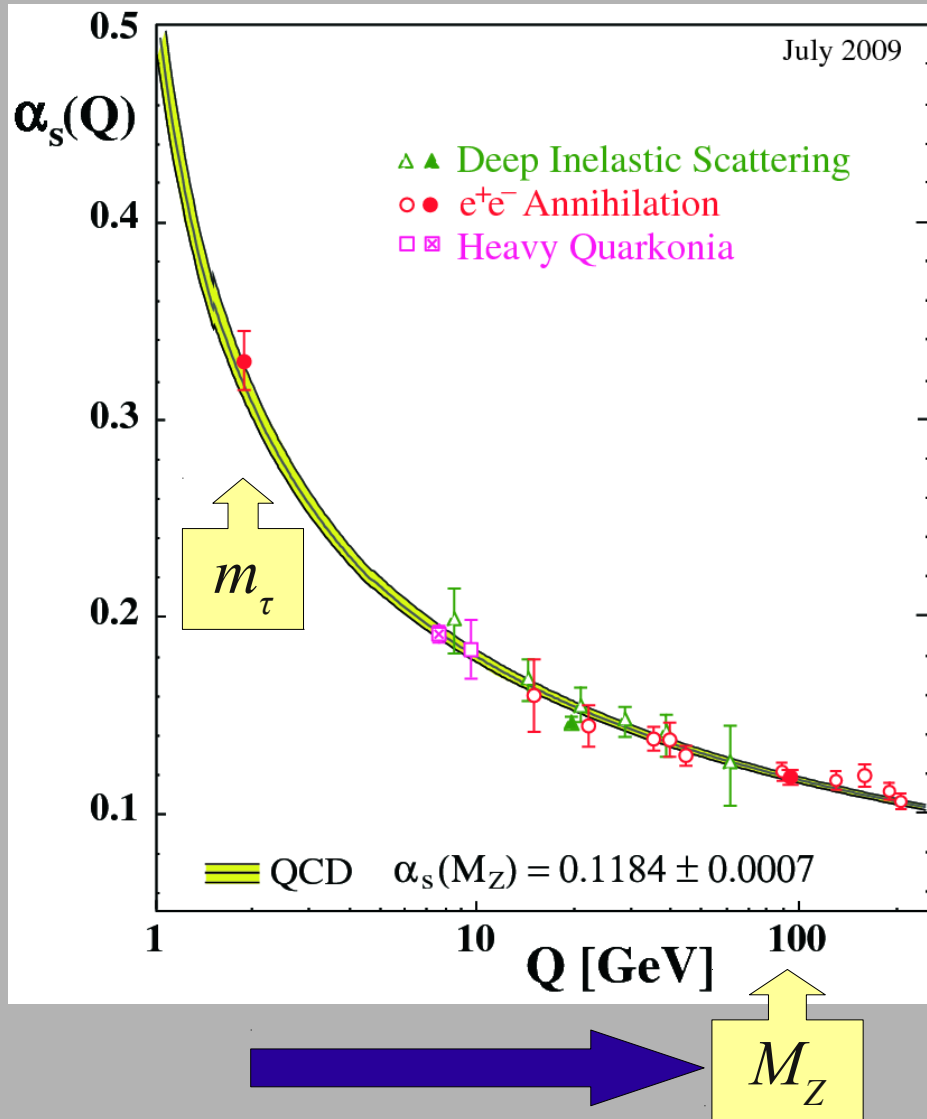


- Usually defined as lowest relevant order in  $\alpha_S$
- In this talk NLO means calculations to  $\alpha_S^2$
- NNLO = next-to-next-to-leading order:  $N^2LO$
- NNNLO = next-to-next-to-next-to...:  $N^3LO$

# LO, NLO, NNLO and all that

- LO: calculations exist (by definition) for all processes
- NLO: exist for many processes
  - but not always in the form of a MC
- NNLO: few results mostly for inclusive kinematics
- N<sup>3</sup>LO: very few, e.g. 4-loop running coupling
- Summing/including leading logs helps precision:
  - NLL etc.

# Running of $\alpha_s$



- Is a precision of 4% at  $m_\tau$  competitive?
  - $\alpha_s$  runs and error goes down!
  - Error goes as

$$\Delta \alpha_s(Q^2) / \alpha_s(Q^2) \sim \alpha_s(Q^2)$$

- Swim to  $M_Z$ :

$$\alpha_s(M_Z) = 0.1197 \pm 0.0016$$

N<sup>3</sup>LO

# How to measure $\alpha_s$ ?

- $\Upsilon(1S) \rightarrow ggg$  is proportional to  $\alpha_s^3$ 
  - but significant theoretical uncertainties
- Look at ratio:  
 $\text{BR}(\Upsilon(1S) \rightarrow \gamma ggg) / \text{BR}(\Upsilon(1S) \rightarrow ggg)$ 
  - Slightly more obvious?
  - Many systematics cancel:

$$\alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}$$

NLO

N. Brambilla et al.  
Phys. Rev. D75 (2007) 074014  
arXiv:hep-ph/0702079  
CLEO Collab.  
Phys. Rev. D74 (2006) 012003  
arXiv:hep-ex/0512061

# How to measure $\alpha_s$ ?

- Take  $m(Y(2S)) - m(Y(1S))$
- Adjust  $u, d, s$  masses to give correct light mesons masses
- Let lattice gauge theory do the work for you!

$$\alpha_s(M_Z) = 0.1183 \pm 0.0008$$

Lattice

C.T.H. Davies et al., HPQCD Collab., Phys.Rev. D78  
(2008) 114507; arXiv:0807.1687 [hep-lat]

# How to measure $\alpha_s$ ?

- Take lots of data with an  $e^+e^-$  collider, i.e. LEP (c.m. energy 90 GeV)
- Measure event shapes in hadronic events

$$\alpha_s(M_Z) = 0.1224 \pm 0.0039$$

NNLO

- Include  $\alpha_s$  in the electroweak fits

$$R_Z = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow l^+ l^-)}$$

$$\alpha_s(M_Z) = 0.1193^{+0.0028}_{-0.0027} \pm 0.0005$$

N<sup>3</sup>LO

- Go back and re-analyse JADE (PETRA) data including latest theory

$$\alpha_s(M_Z) = 0.1172 \pm 0.0051$$

NNLO

# How to measure $\alpha_s$ ?

- Take lots of data with an *ep* collider, i.e. HERA
- Use PDF data and its development as a function of  $Q^2$ 
  - See also earlier talk (K. Lipka)

$$\alpha_s(M_Z) = 0.1142 \pm 0.0023$$

N<sup>3</sup>LO

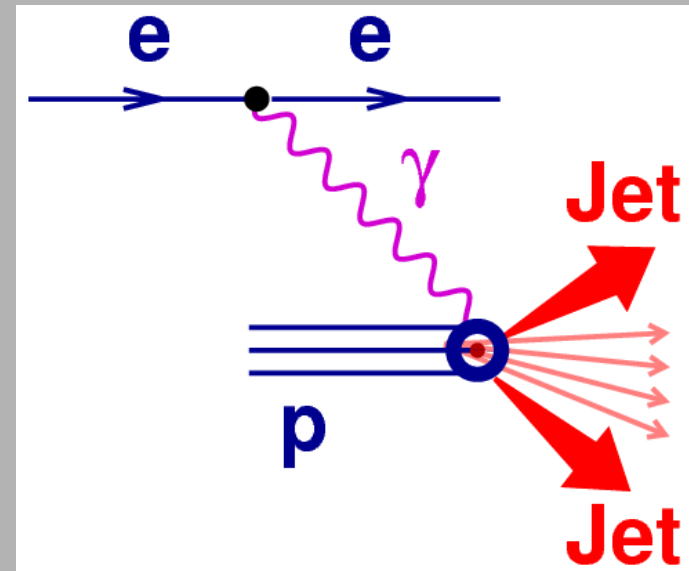
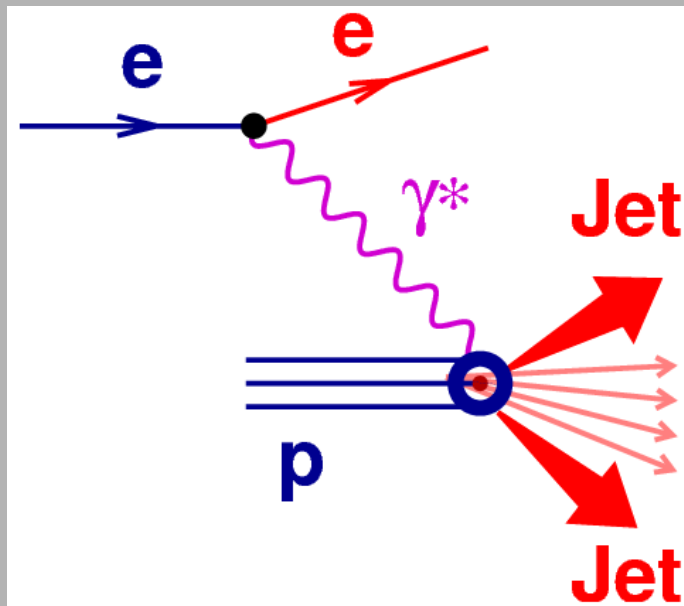
- Look at jet cross sections and ratios of cross sections

J. Blümlein, H. Böttcher and A. Guffanti  
Nucl. Phys. B774 (2007) 182; hep-ph/0607200



# Jets and Kinematics at HERA

- Measurements in both DIS and photoproduction (PhP) are used to determine  $\alpha_S$



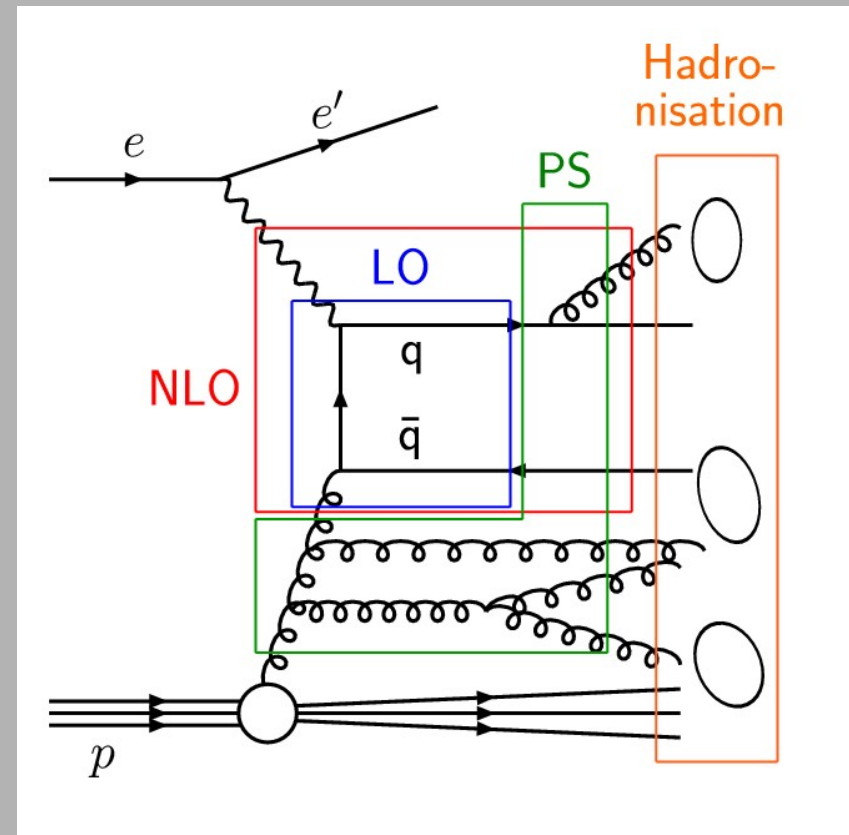
- Cross-section ratios are a good way to reduce systematics

# From jets to $\alpha_s$

- QCD factorisation theorem allows perturbative from non-perturbative contributions to cross-sections to be separated:

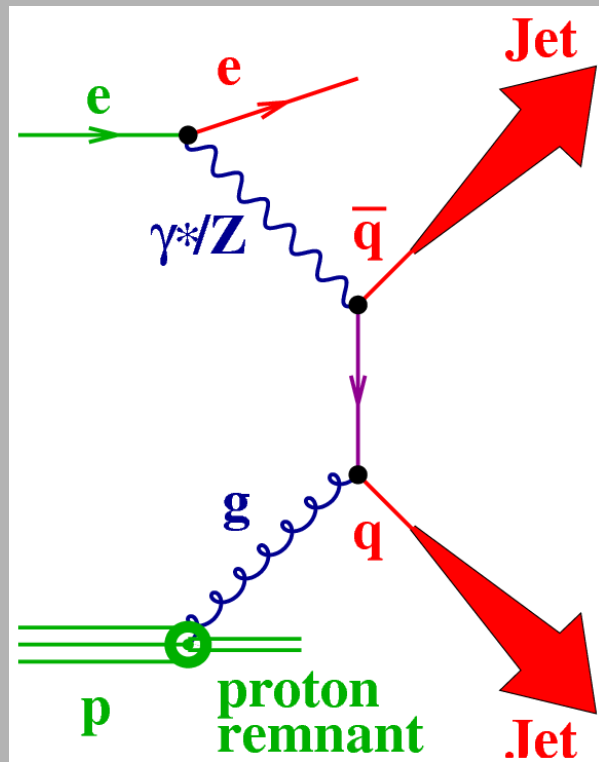
$$d\sigma_{\text{jet}} = \sum_{a=q, \bar{q}, g} \int dx f_a(x, \mu_F) \cdot d\hat{\sigma}_a(x, \alpha_s(\mu_R), \mu_R, \mu_F)$$

- $f_a$ : parton density
- $d\hat{\sigma}_a$ : subprocess cross-section

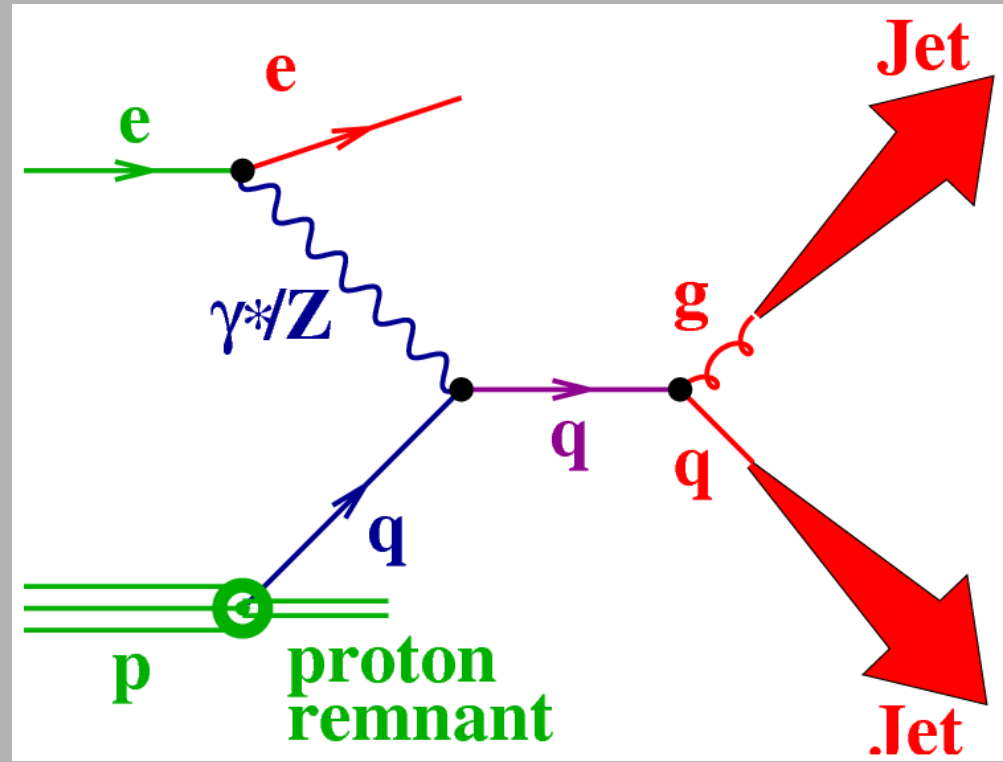


# Jets in DIS

- Sources of (di)jet production:



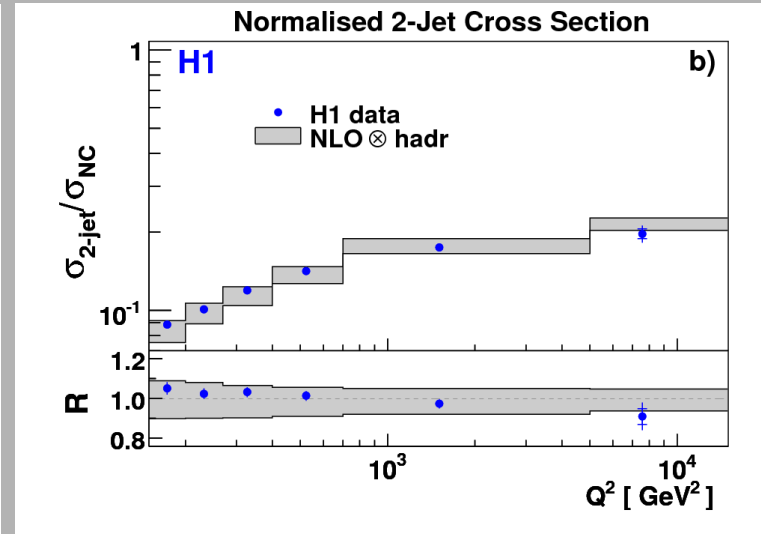
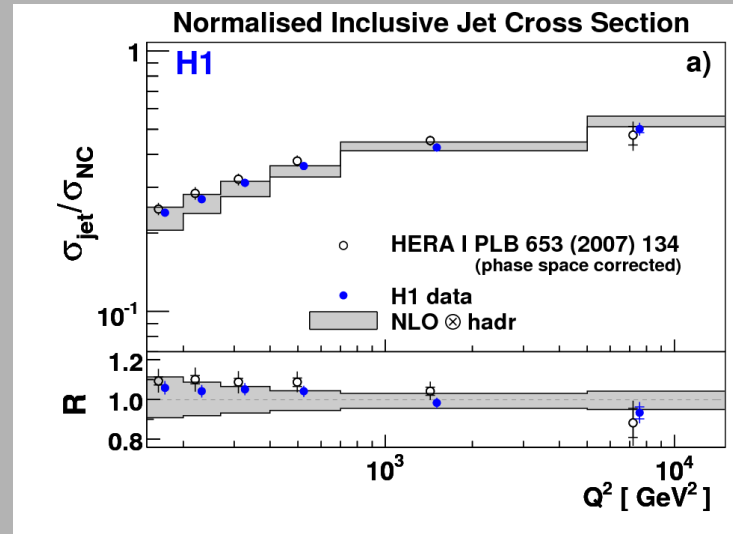
Boson-gluon fusion



QCD Compton

# Jets at high $Q^2$

- DIS events with 1,2 and 3 jets and their ratios
- Compare cross-sections as a function of  $Q^2$ ,  $P_T^{\text{jet}}$  and  $\xi$  with NLO predictions  $\xi = x_{\text{Bj}}(1 + M_{jj}/Q^2)$
- Normalise to NC cross-section



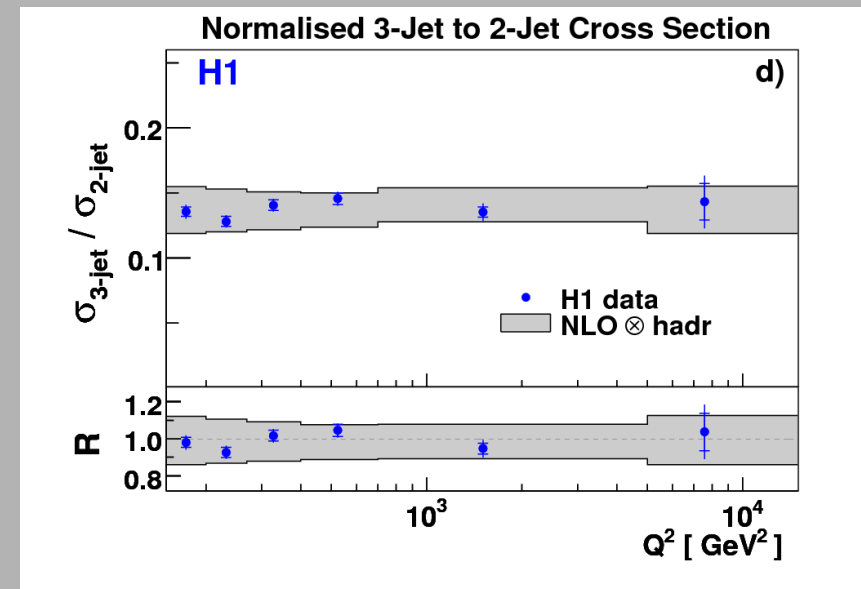
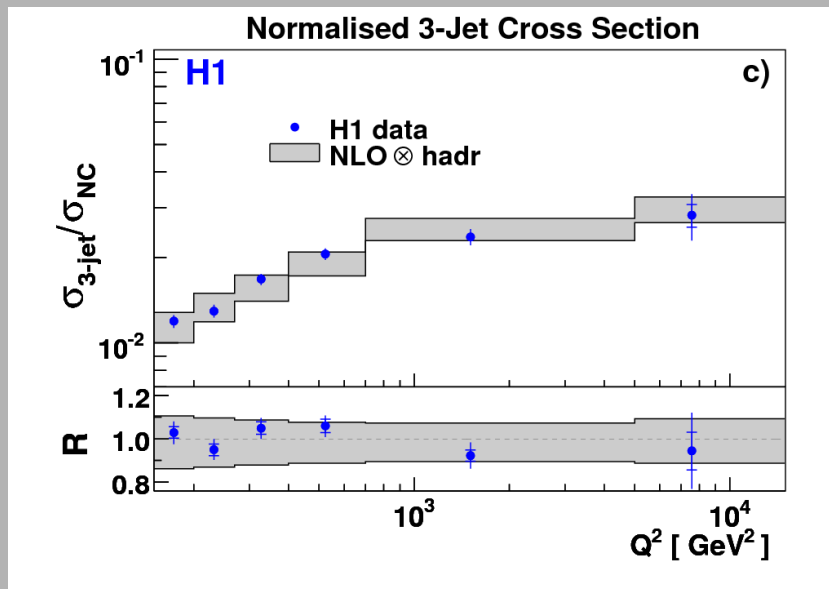
395 pb<sup>-1</sup>, Event:  
 $150 < Q^2 < 15000 \text{ GeV}^2$   
 $0.2 < y < 0.7$

Jet:  
 $P_T > 7(5) \text{ GeV}$   
 $-0.8 < \eta_{\text{lab}} < 2.0$   
 $k_T$  algorithm in BF

H1 Collab, Eur Phys J. C 65 (2010) 363

# Jets at high $Q^2$

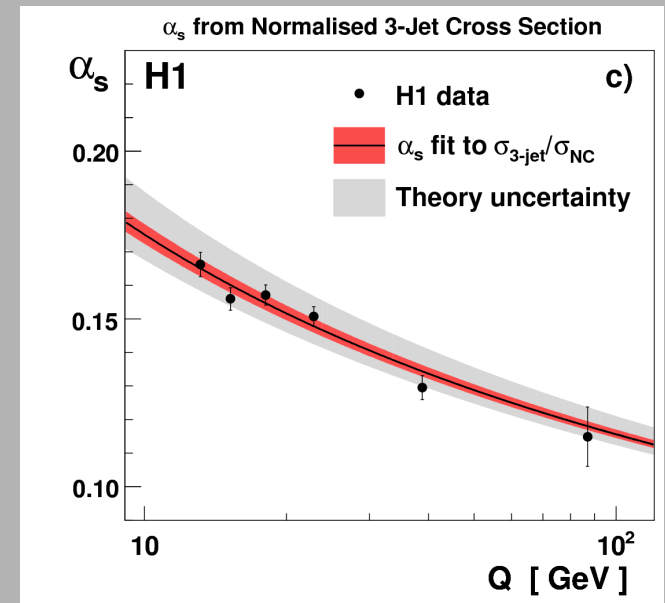
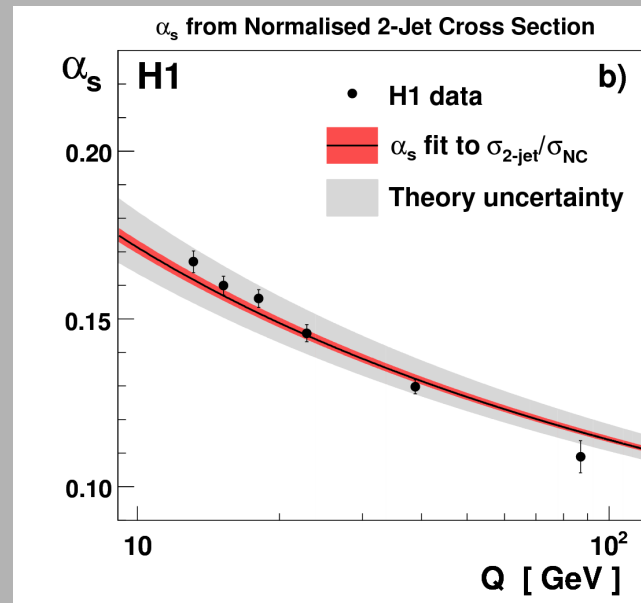
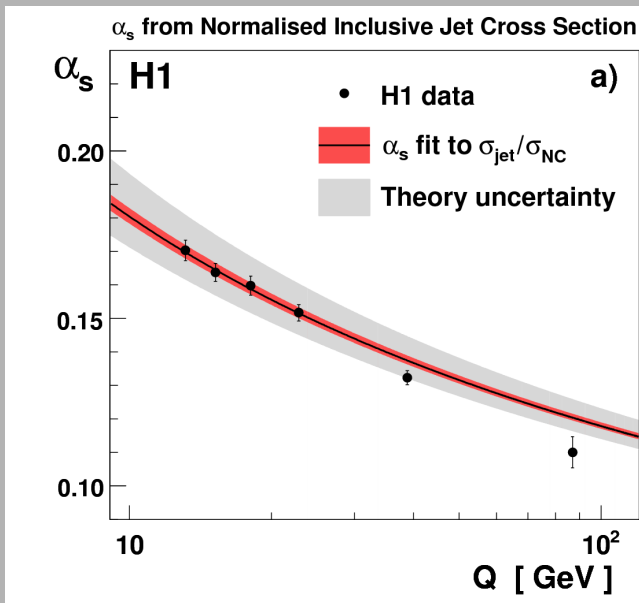
- Plenty of statistics even in 3-jet channel
- NLO uncertainties at 10% level
- Good agreement with predictions over whole  $Q^2$  range



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# Jets at high $Q^2$

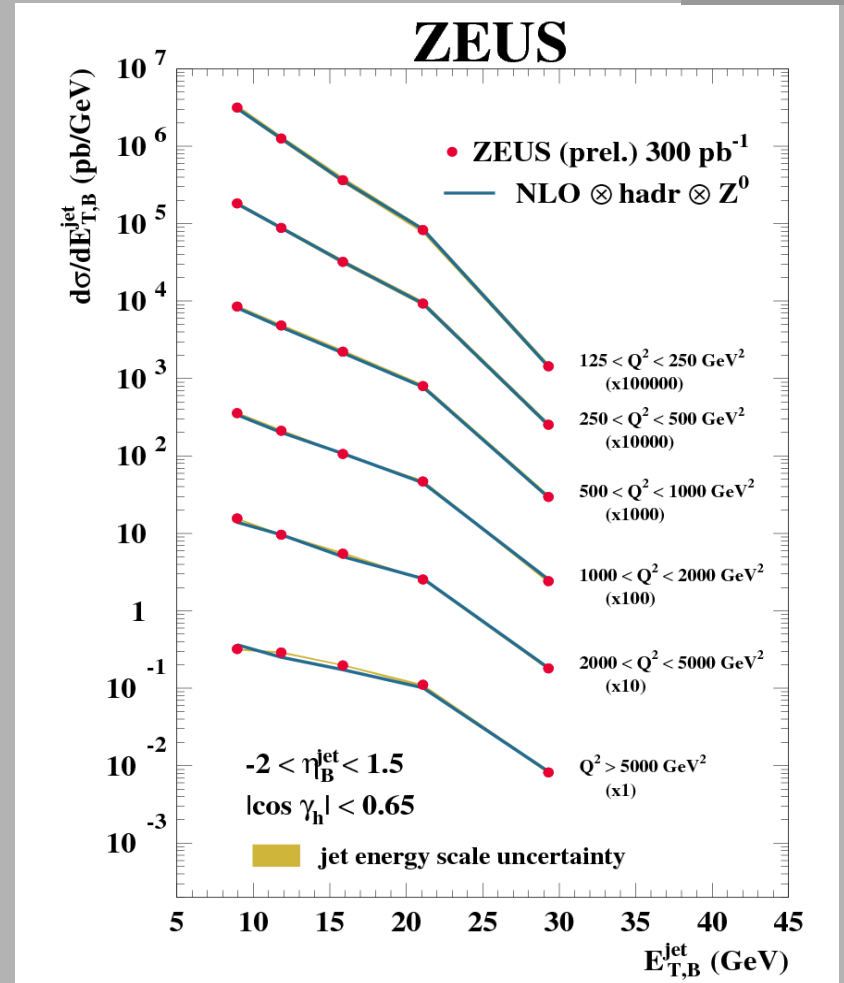
- Each cross-section and cross-section ratio can be used to derive  $\alpha_s$  as a function of the scale
- Running of  $\alpha_s$  (within a single experiment) clearly seen



H1 Collab, Eur Phys J. C 65 (2010) 363

# Jets at high $Q^2$

- Inclusive jet cross-sections:  
 $Q^2, E_T$  (BF)
- Compare with NLO predictions
- Good agreement over whole measured range



NLO

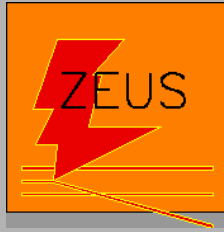
$$\alpha_S(M_Z) = 0.1208 \pm 0.0007 (\text{stat.})_{-0.0031}^{+0.0036} (\text{exp.}) \pm 0.0022 (\text{th.})$$

ZEUS Collab, ZEUS-prel-10-002

300 pb<sup>-1</sup>, Event:  
 $Q^2 > 125 \text{ GeV}^2$   
 $|\cos \gamma_h| < 0.65$

Jet:  
 $E_T > 8 \text{ GeV}$   
 $-2 < \eta_{\text{BF}} < 1.5$   
 $k_T$  algorithm in BF

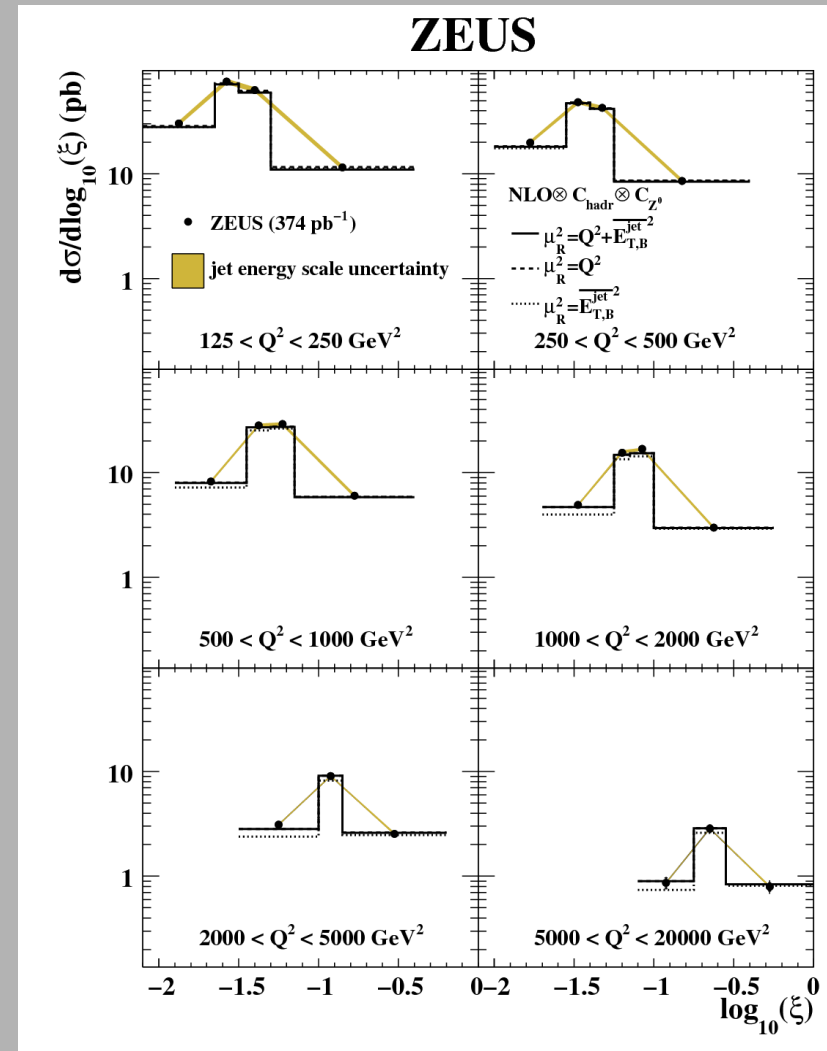
# Dijets in NC DIS



- NLO predictions using NLOJET++
- Gluon fraction substantial up to  $Q^2 \sim 500 \text{ GeV}^2$
- Theory uncertainty  $\sim 5\text{-}10\%$
- PDF sensitivity

374 pb<sup>-1</sup>, Event:  
 $125 < Q^2 < 20000 \text{ GeV}^2$   
 $0.2 < y < 0.6$

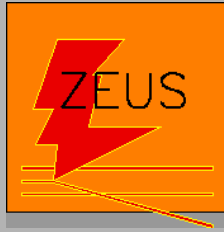
Jet:  
 $E_T > 8 \text{ GeV}$  (BF)  
 $-1 < \eta_{\text{lab}} < 2.5$   
 $k_T$  algorithm in BF



ZEUS Collab, ZEUS-pub-10-005



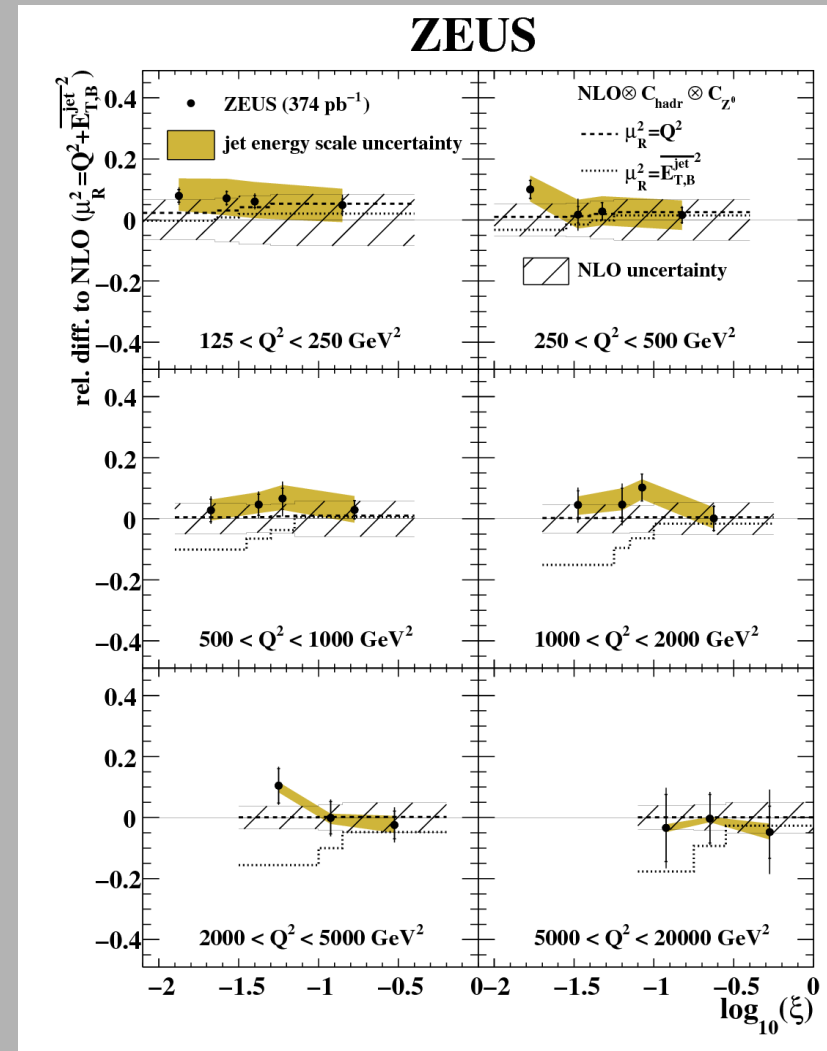
# Dijets in NC DIS



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374 pb<sup>-1</sup>, Event:  
 $125 < Q^2 < 20000 \text{ GeV}^2$   
 $0.2 < y < 0.6$

Jet:  
 $E_T > 8 \text{ GeV (BF)}$   
 $-1 < \eta_{\text{lab}} < 2.5$   
 $k_T$  algorithm in BF



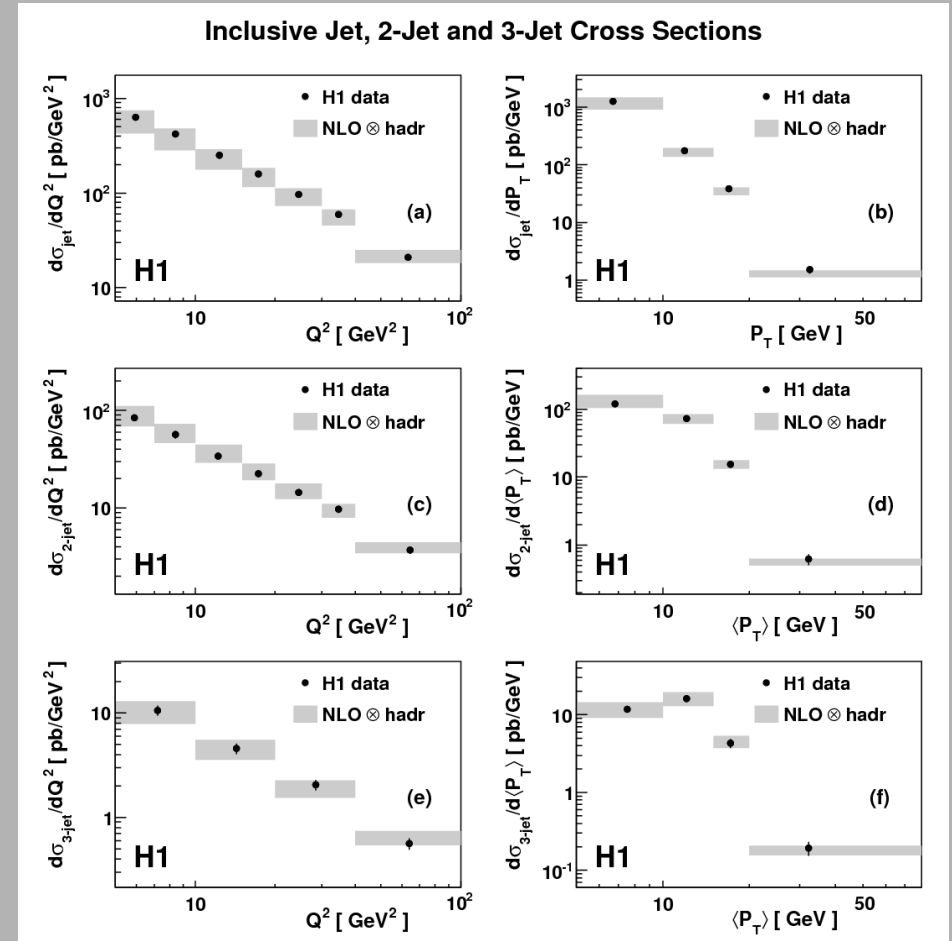
ZEUS Collab, ZEUS-pub-10-005

# Jets at low $Q^2$

- Look at distributions as a function of  $Q^2$ ,  $P_T^{\text{jet}}$  and  $\xi$
- Good description of data by NLO predictions

43.5 pb<sup>-1</sup>, Event:  
 $5 < Q^2 < 100 \text{ GeV}^2$   
 $0.2 < y < 0.7$

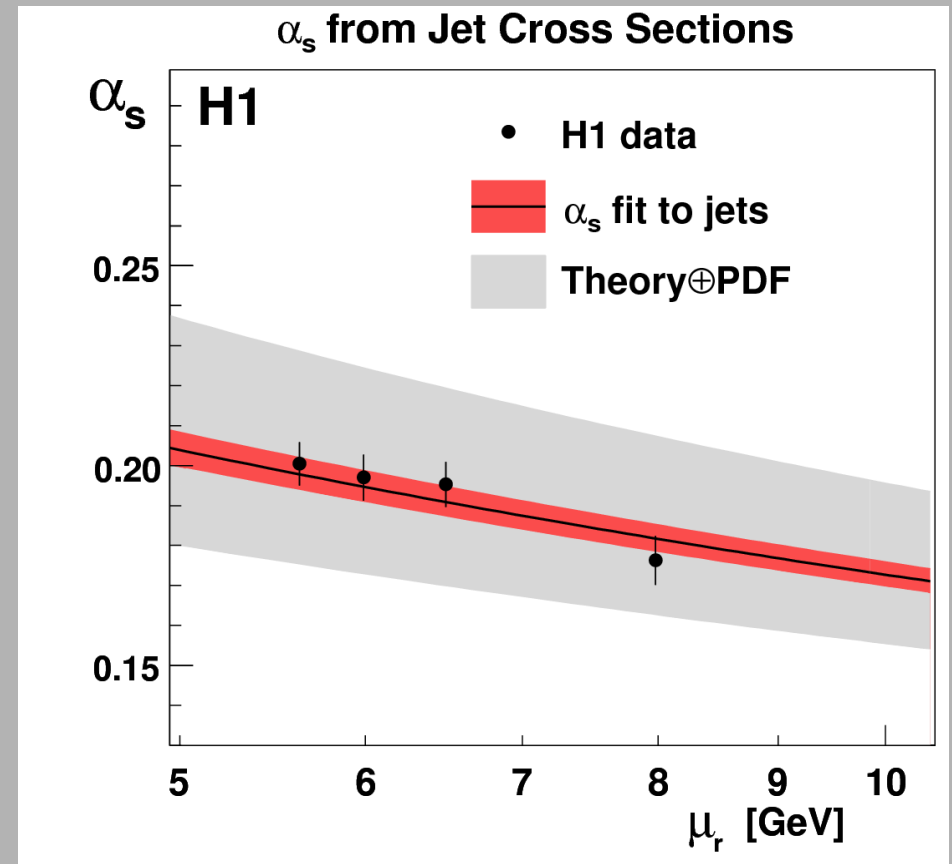
Jet:  
 $P_T > 5 \text{ GeV}$   
 $-1 < \eta_{\text{lab}} < 2.5$   
 $k_T$  algorithm in BF



H1 Collab, Eur Phys J. C67 (2010) 1

# Jets at low $Q^2$

- Use measured jet cross-sections to extract  $\alpha_s$
- Simultaneous fit of inclusive, dijet and trijet measurements

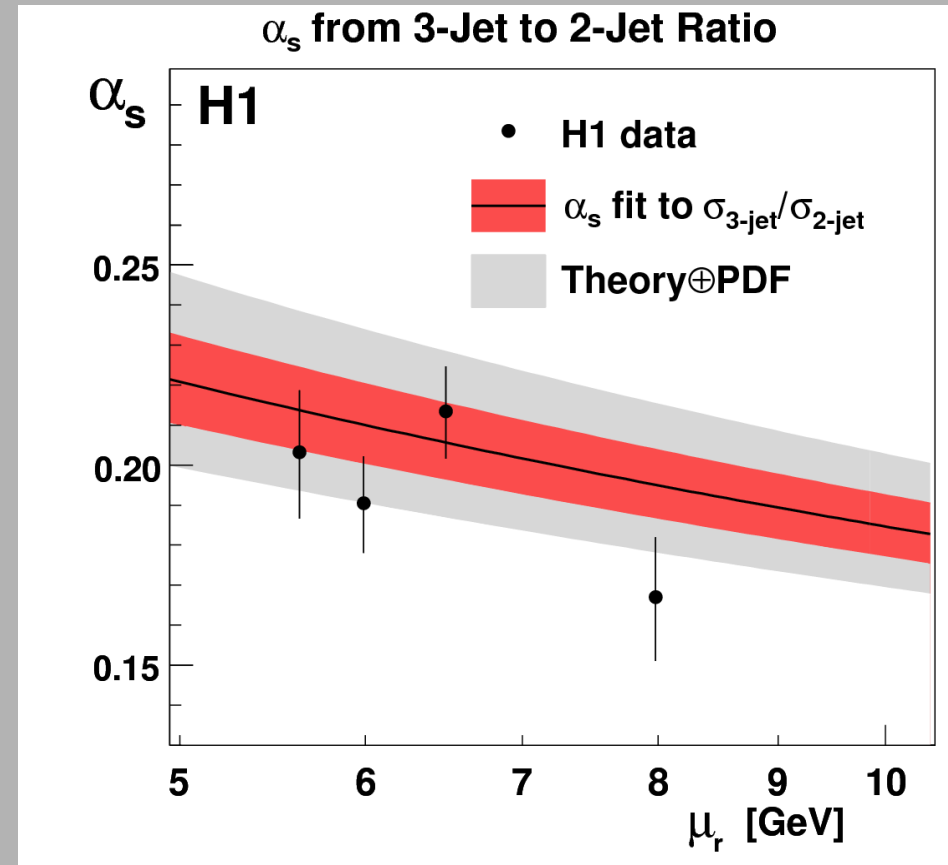
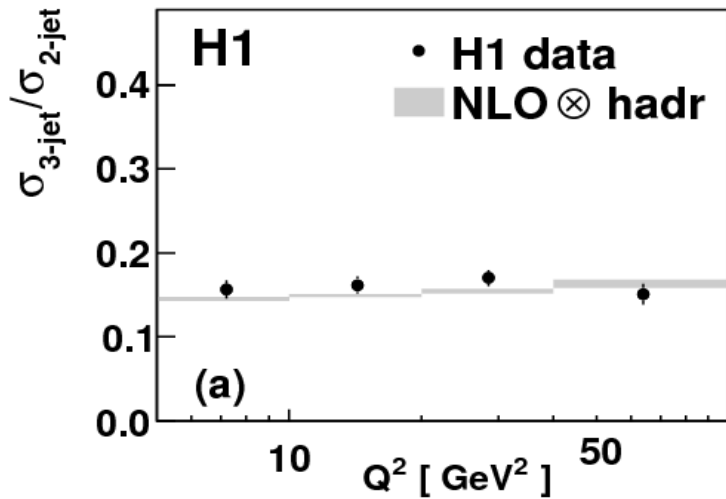


$$\alpha_s(M_Z) = 0.1160 \pm 0.0014 (\text{exp.})_{-0.0079}^{+0.0094} (\text{th.})$$

NLO

# Jets at low $Q^2$

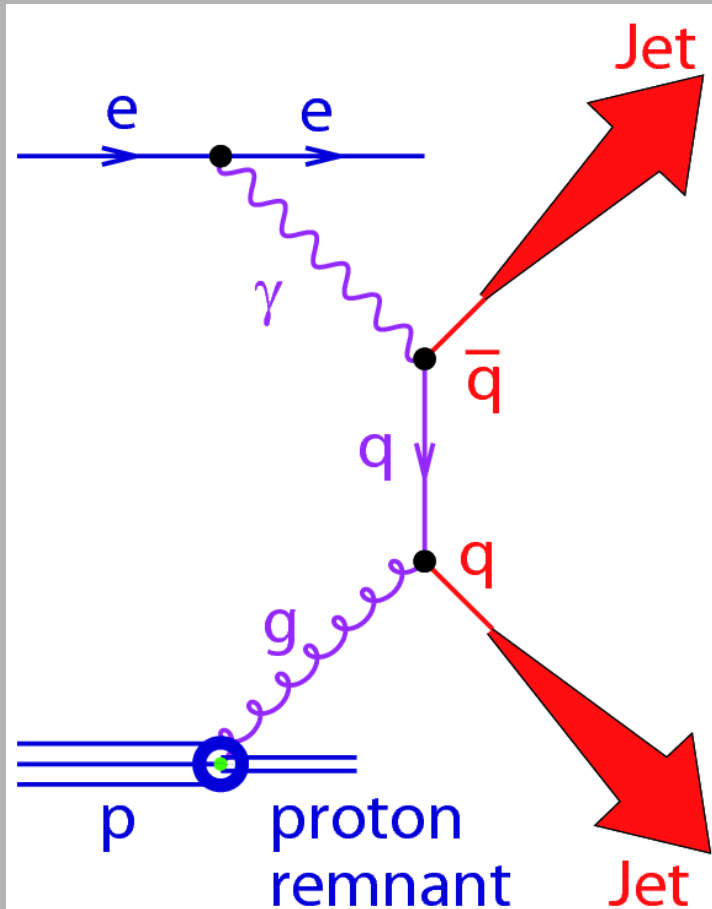
- Shame that precise exp. measurement has large theory error
- Using 3-jet/2-jet ratio reduces theory error



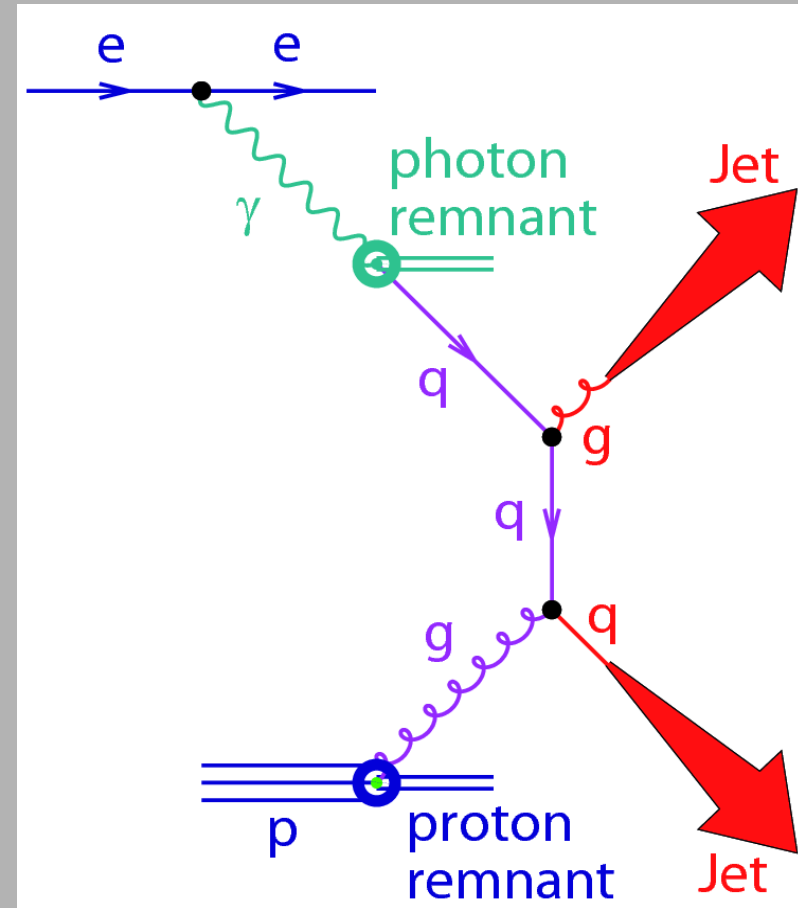
NLO

$$\alpha_S(M_Z) = 0.1215 \pm 0.0032 (\text{exp.})_{-0.0059}^{+0.0067} (\text{th.})$$

# Jets in Photoproduction

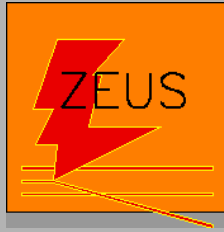


Direct photoproduction

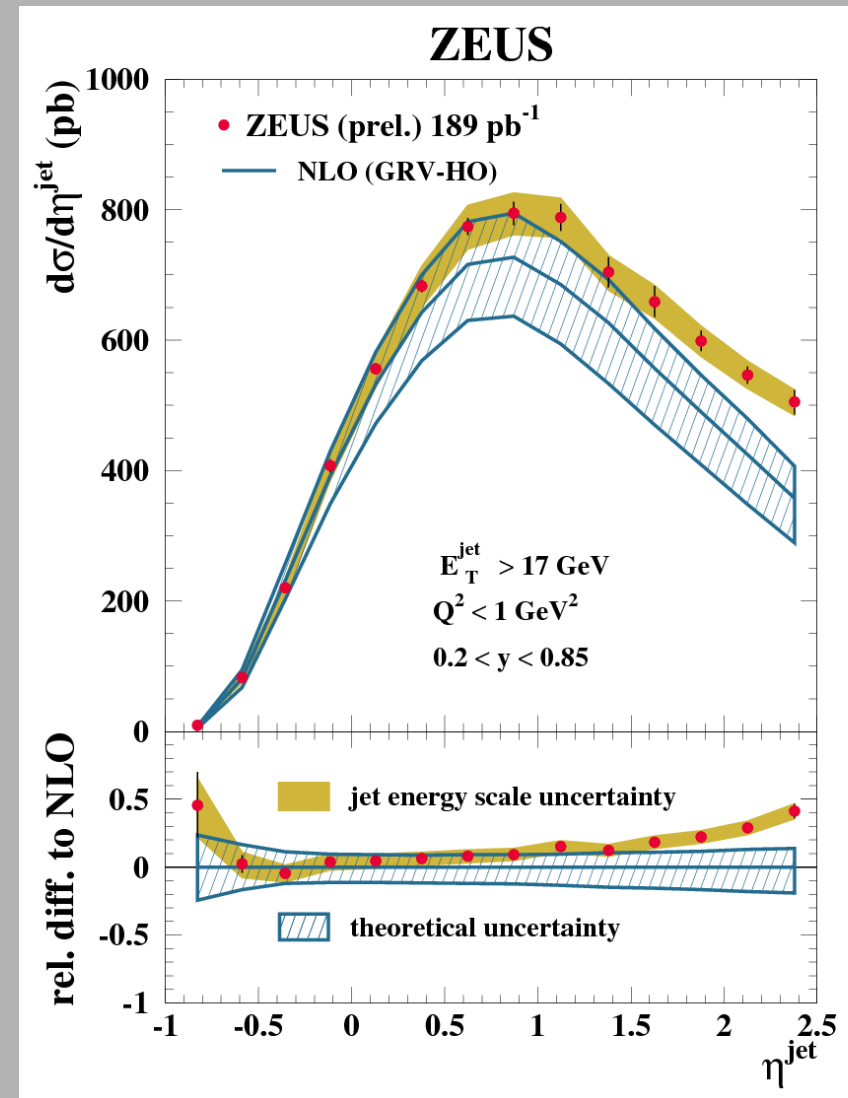


Resolved photoproduction

# Jets in Photoproduction



- High  $E_T$  inclusive jet cross sections used to extract  $\alpha_S$
- Proton and photon PDFs play a role
- Non-perturbative effects (underlying event) are also relevant



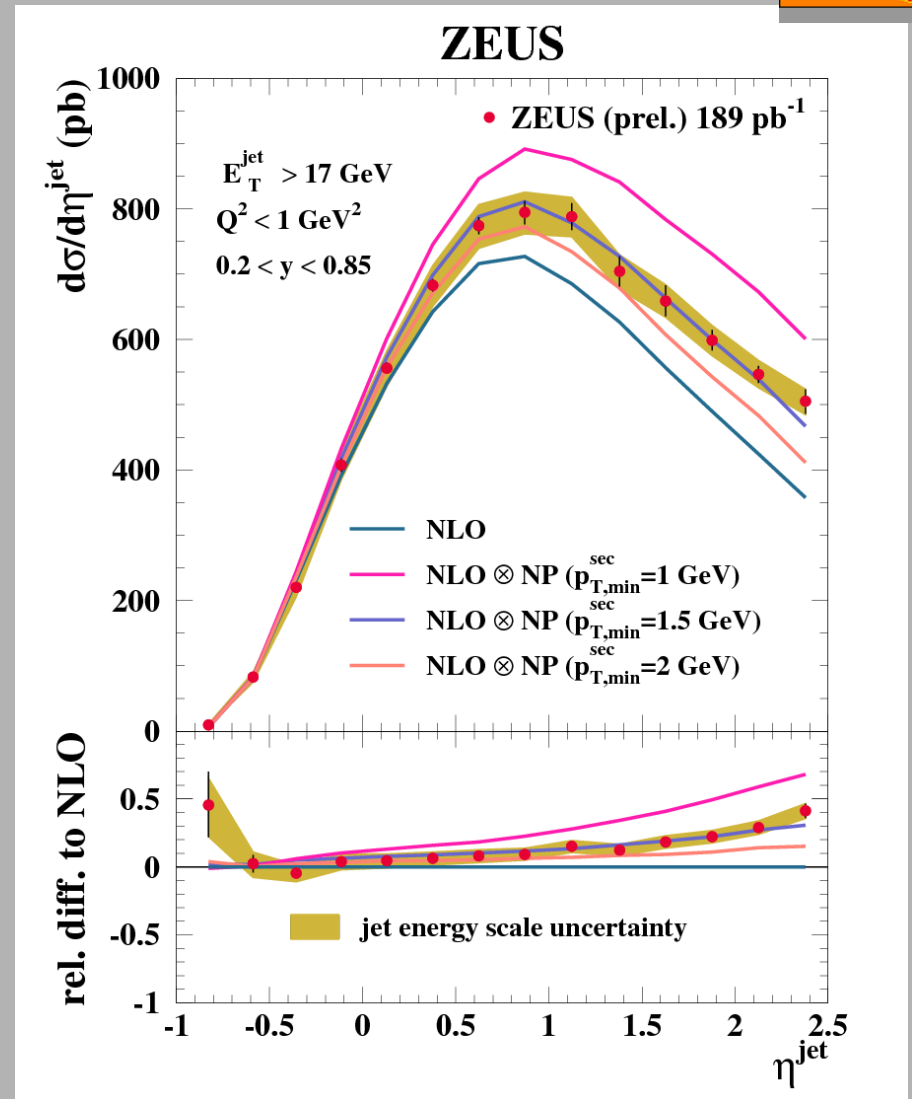
$$\eta = -\ln \tan \theta/2$$

ZEUS Collab, ZEUS-prel-10-003

# Jets in Photoproduction



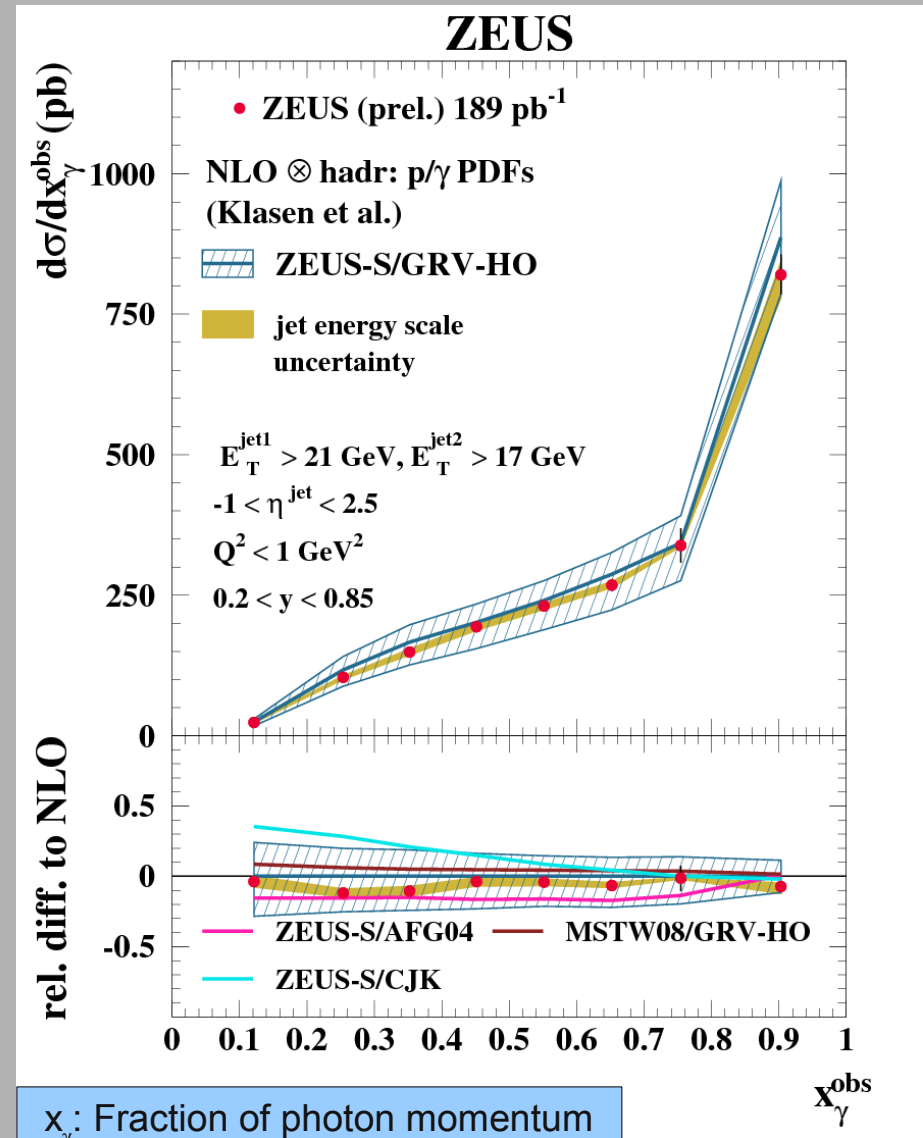
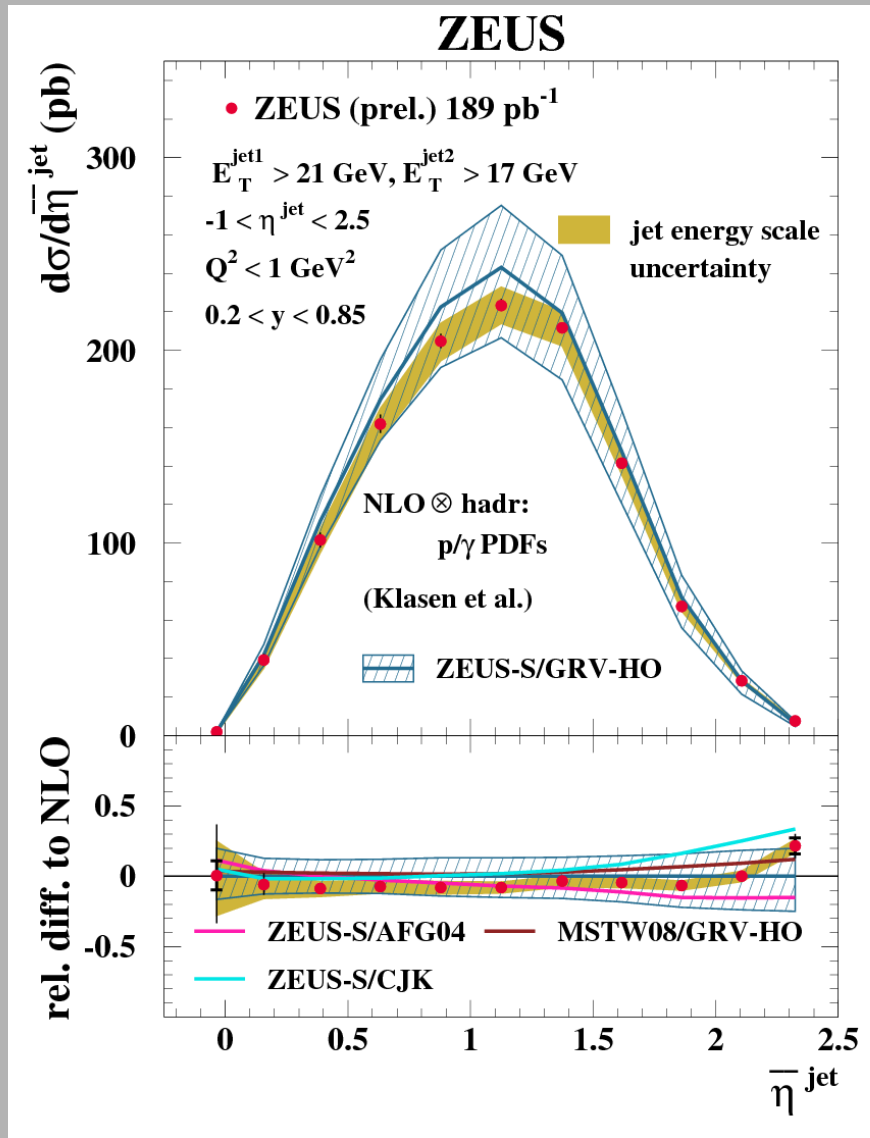
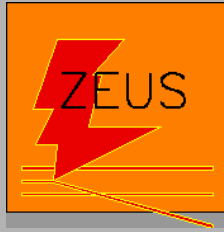
- Include non-perturbative effects using PYTHIA-MI
  - much better agreement at large  $\eta$
- Size of effect also much reduced for higher  $E_T^{\text{jet}}$



$$\alpha_S(M_Z) = 0.1160_{-0.0023}^{+0.0024} (\text{exp})_{-0.0033}^{+0.0044} (\text{th})$$

NLO

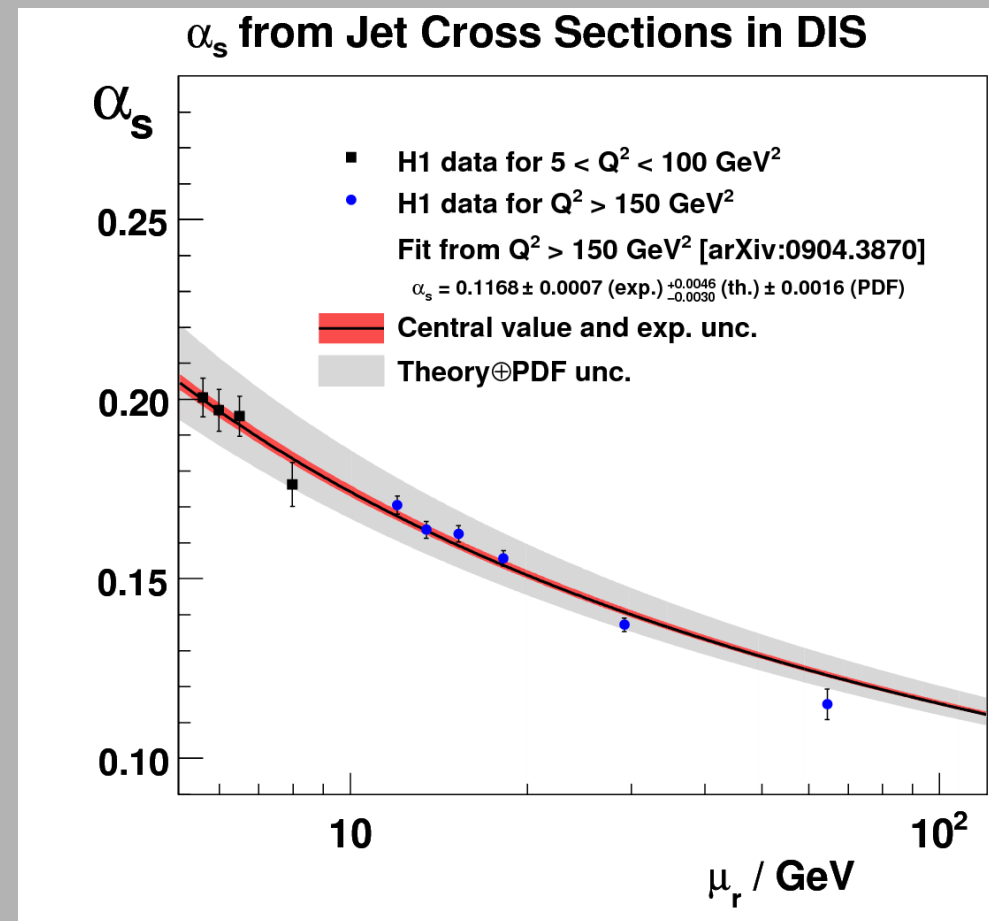
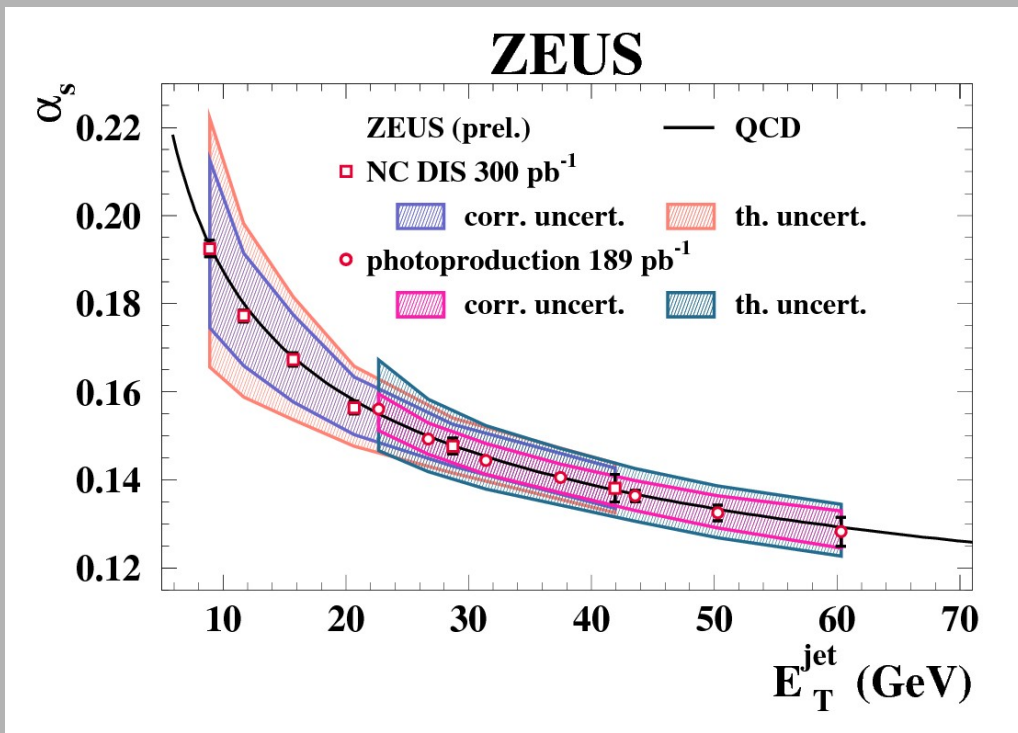
# Dijets in Photoproduction



$x_\gamma$ : Fraction of photon momentum carried by interacting parton

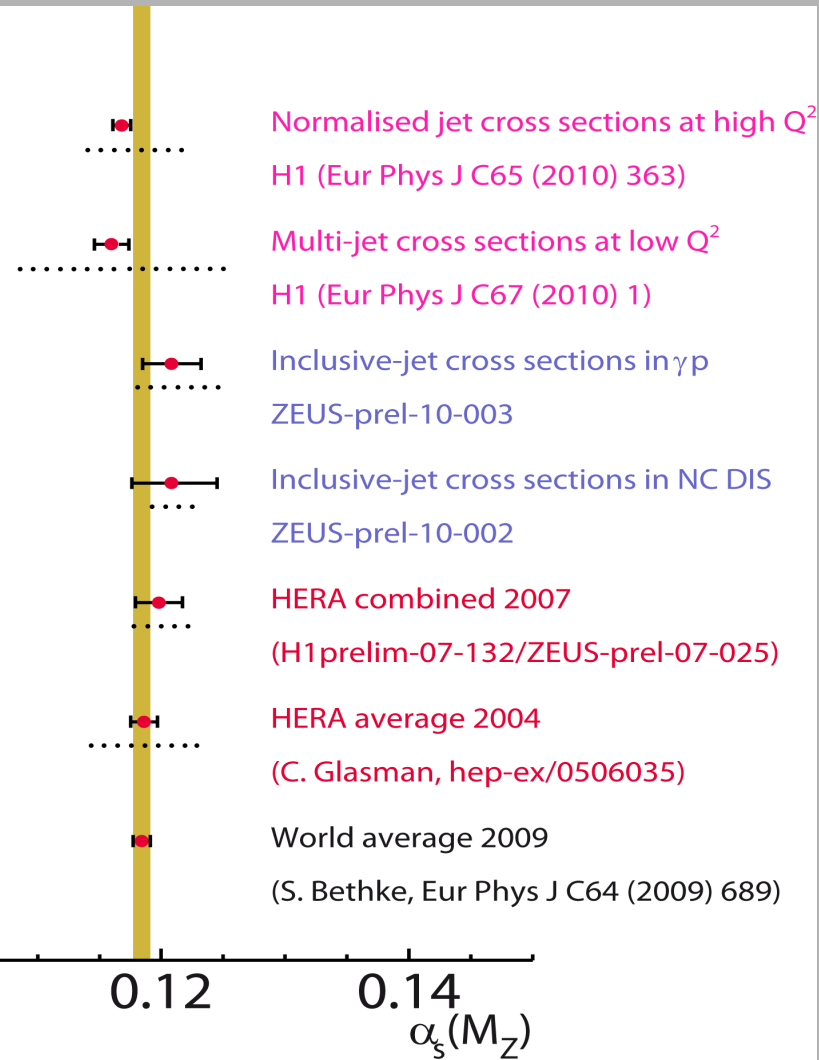


# HERA $\alpha_s$ measurements



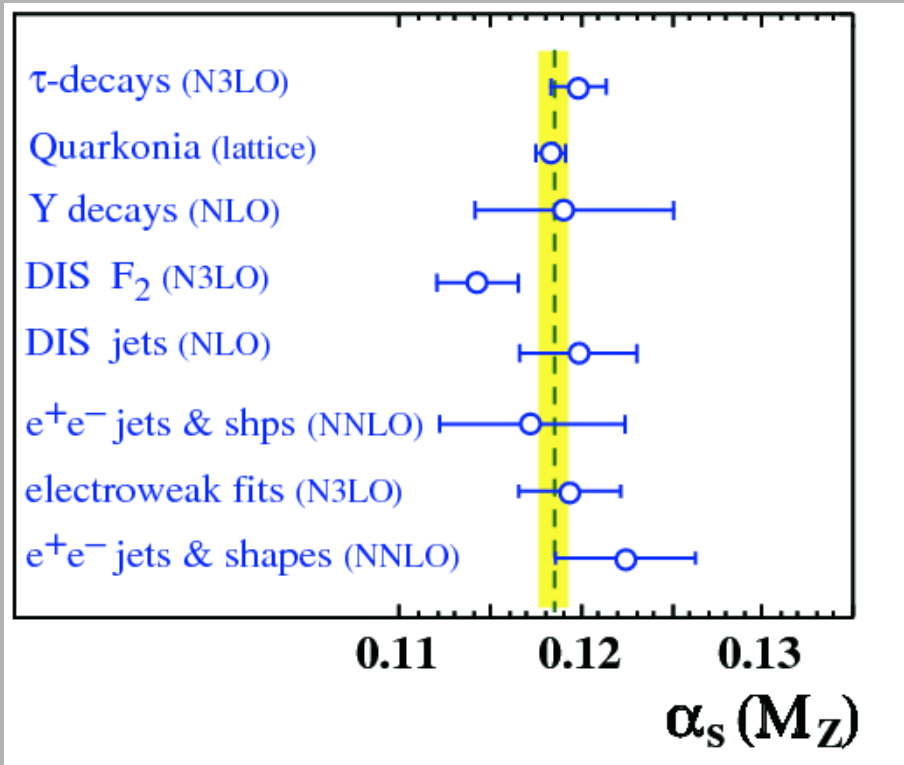
- DIS and photoproduction measurements in good agreement with each other

# HERA $\alpha_s$ measurements summary

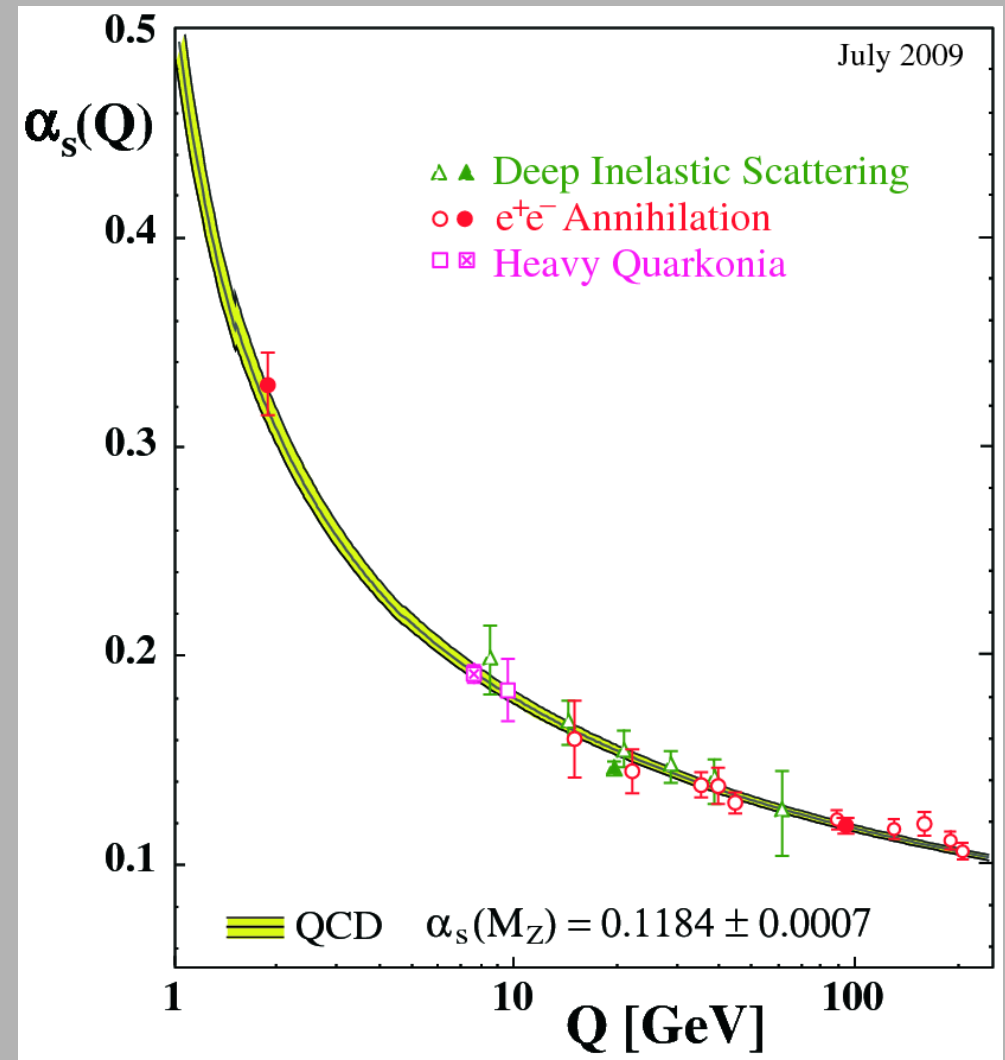


- Compare several precise determinations of  $\alpha_s$
- Trade off between statistics and theoretical uncertainties clearly visible

# $\alpha_s$ measurements summary

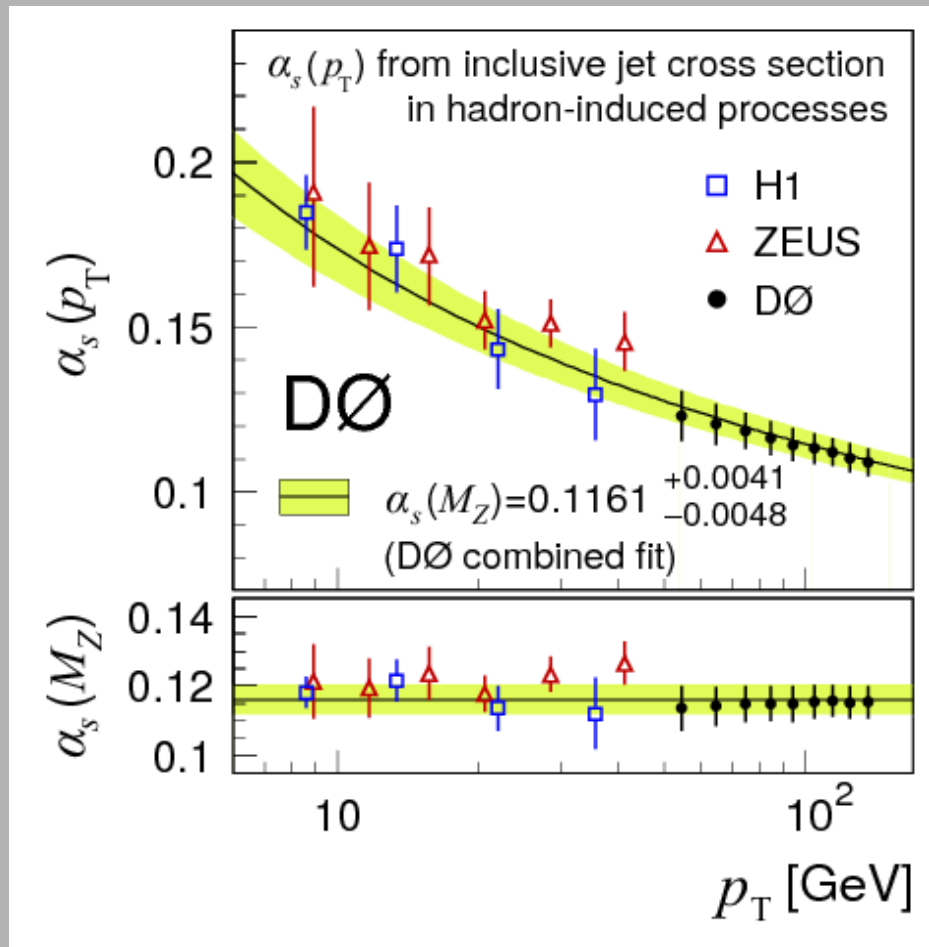


- Most effort on NLO MC generators expended for LHC; similar effort for HERA would be very welcome!



$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$

# Running at the Tevatron

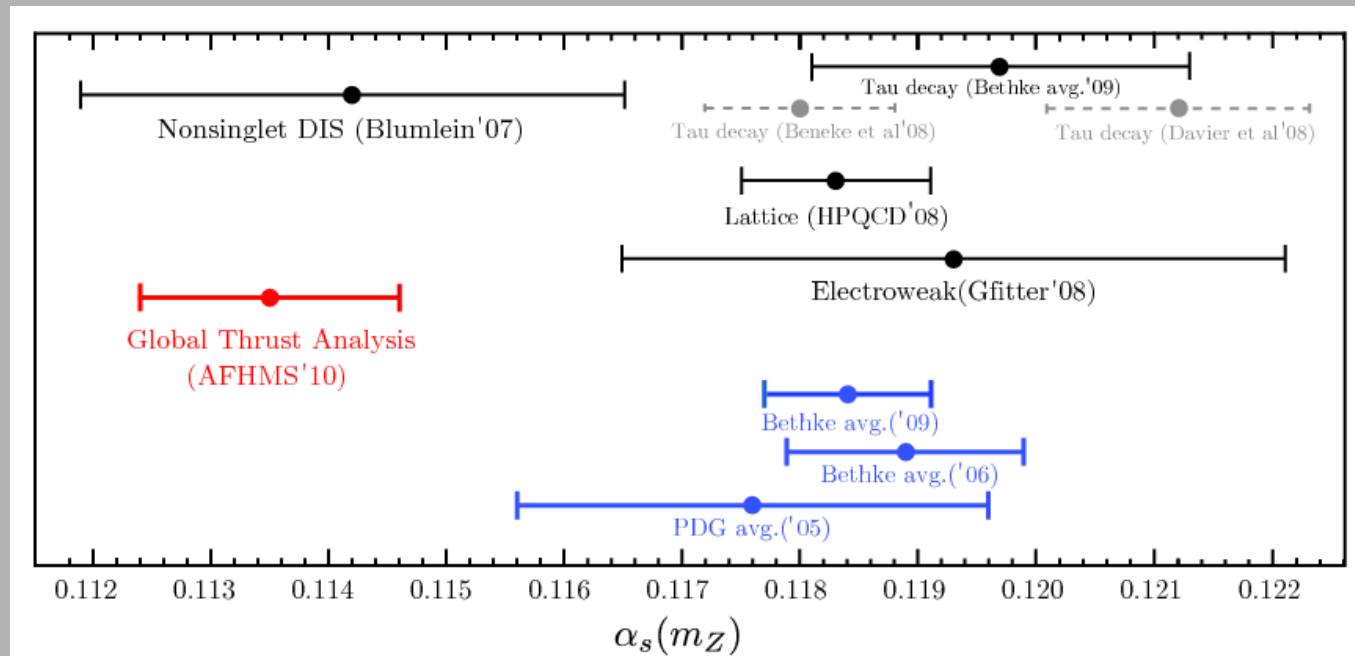


- Careful attention paid to avoid circular reasoning!
  - $g(x)$  and  $\alpha_s$  are often correlated
- DØ errors are dominated by correlated experimental uncertainties
- Complementarity of HERA and Tevatron kinematic ranges

DØ Collab.  
Phys.Rev.D80 (2009) 111107  
arXiv:0911.2710

# $\alpha_s$ measurements summary

- Recent determination using soft collinear effective theory and only thrust:



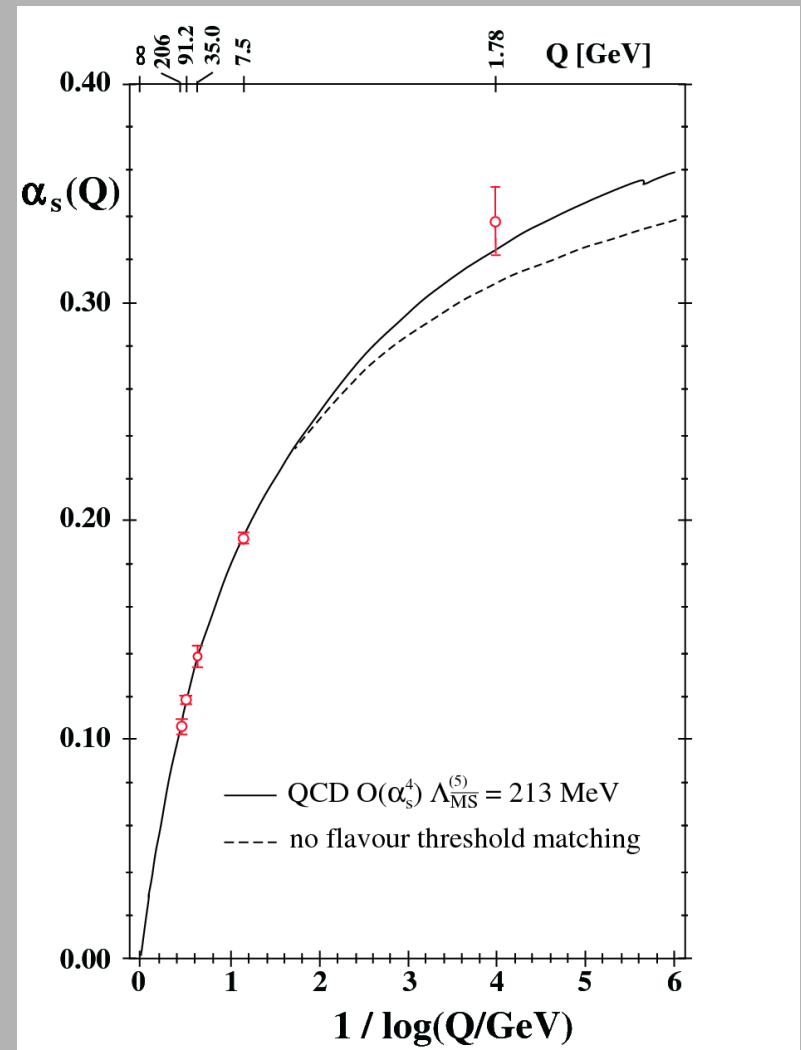
N<sup>3</sup>LL'

- Not included in current world average (data is already in LEP event shape)

R. Abbate et al., arXiv:1006.3080

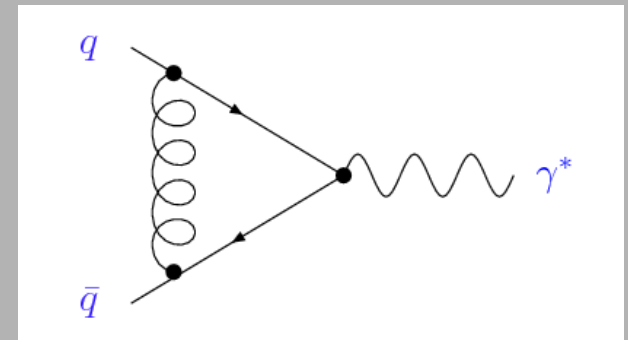
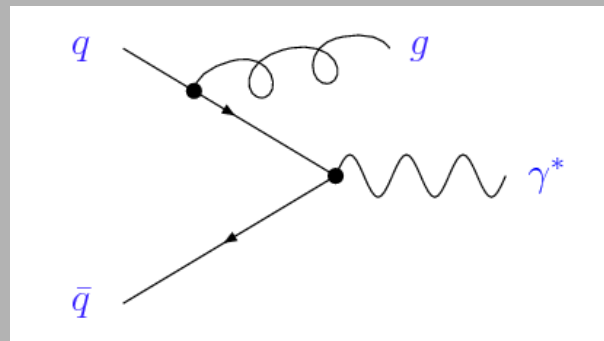
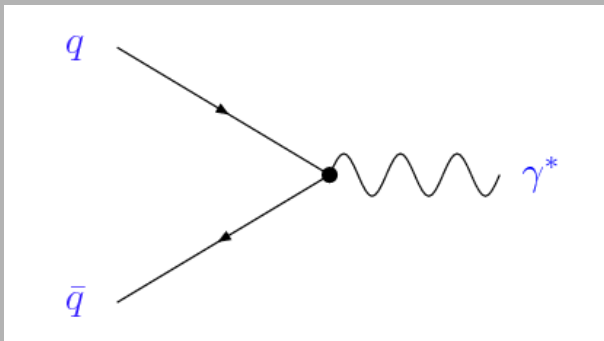
# Does $\alpha_S$ run as expected?

- For selected measurements look at  $\alpha_S$  as a function of  $1 / \log Q$
- $\alpha_S(Q) \rightarrow 0$  as  $Q \rightarrow \infty$
- Demonstrate the validity of the concept of asymptotic freedom
- “Threshold matching” also necessary



# Precision QCD tests

- What do we have to worry about?
  - $\alpha_s$  is not small
  - Leading order calculations are often/usually not sufficient
  - Divergences!



- Improve if summed over all orders, but...
  - Absorb most of remaining infinities in renormalisation
- Buzzwords: soft and collinear divergences

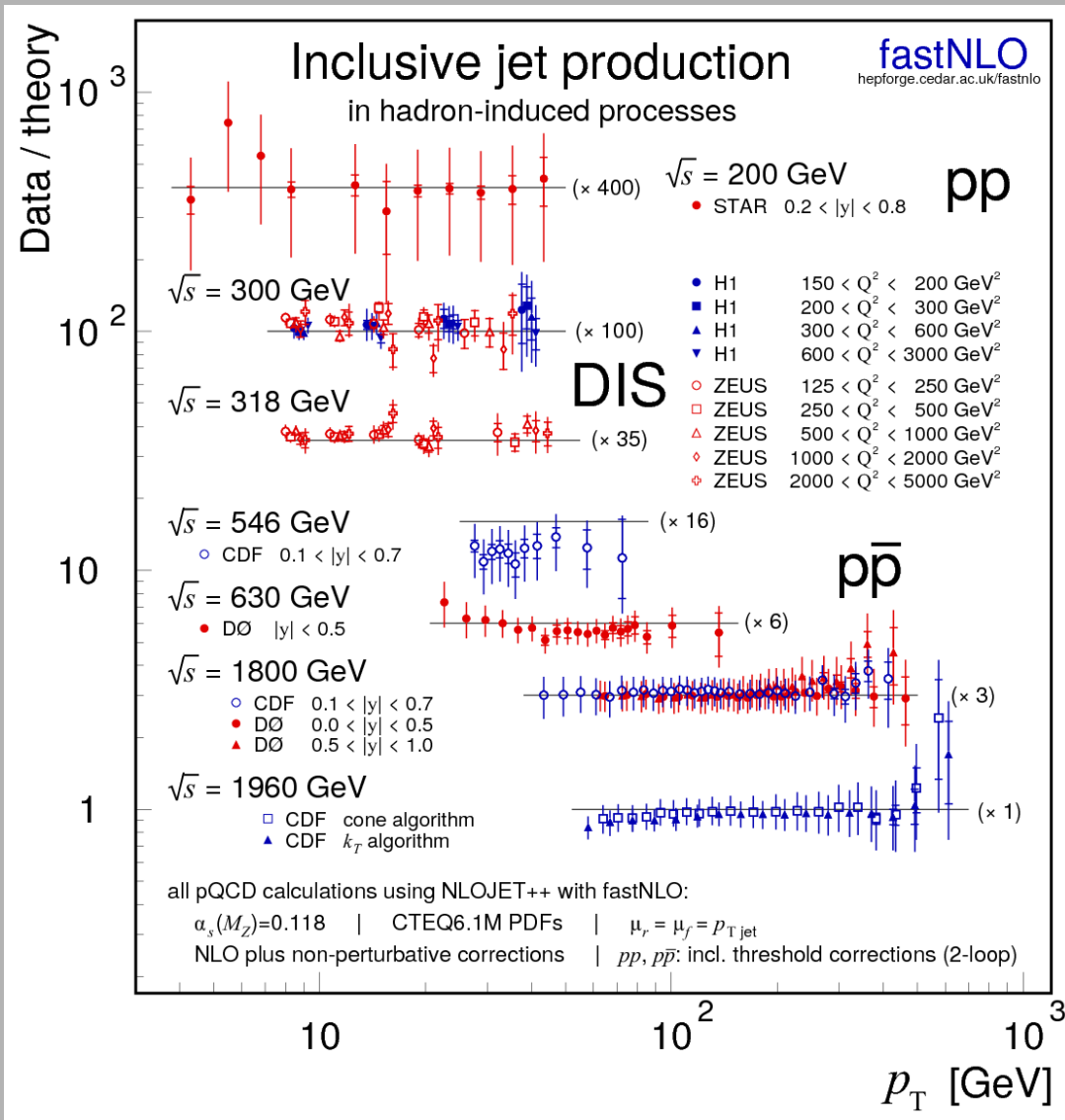
# Precision QCD tests

- Inclusive quantities without initial-state hadrons best suited for precision tests (can be calculated to higher order):
  - $\tau$  decay rates
  - Z width
- Infrared safe quantities:
  - Event shape distributions
  - Jet cross-sections
- Unsafe quantities:
  - Hardest QCD particle
  - Require absence of radiation (rapidity gaps etc.)
  - Particle multiplicity

Even here convergence not as fast as expected!



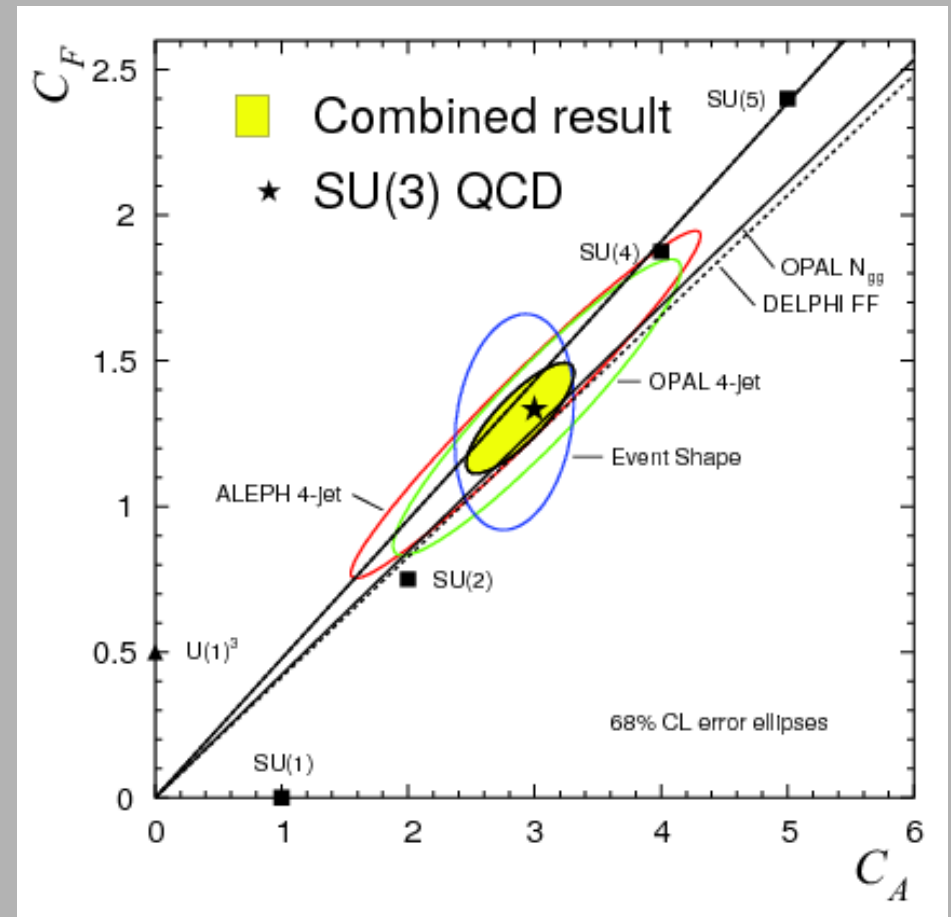
# Inclusive jet production



- Compare data with NLO predictions for different process and kinematics
- Remarkably good and consistent agreement seen

# Is QCD the right theory?

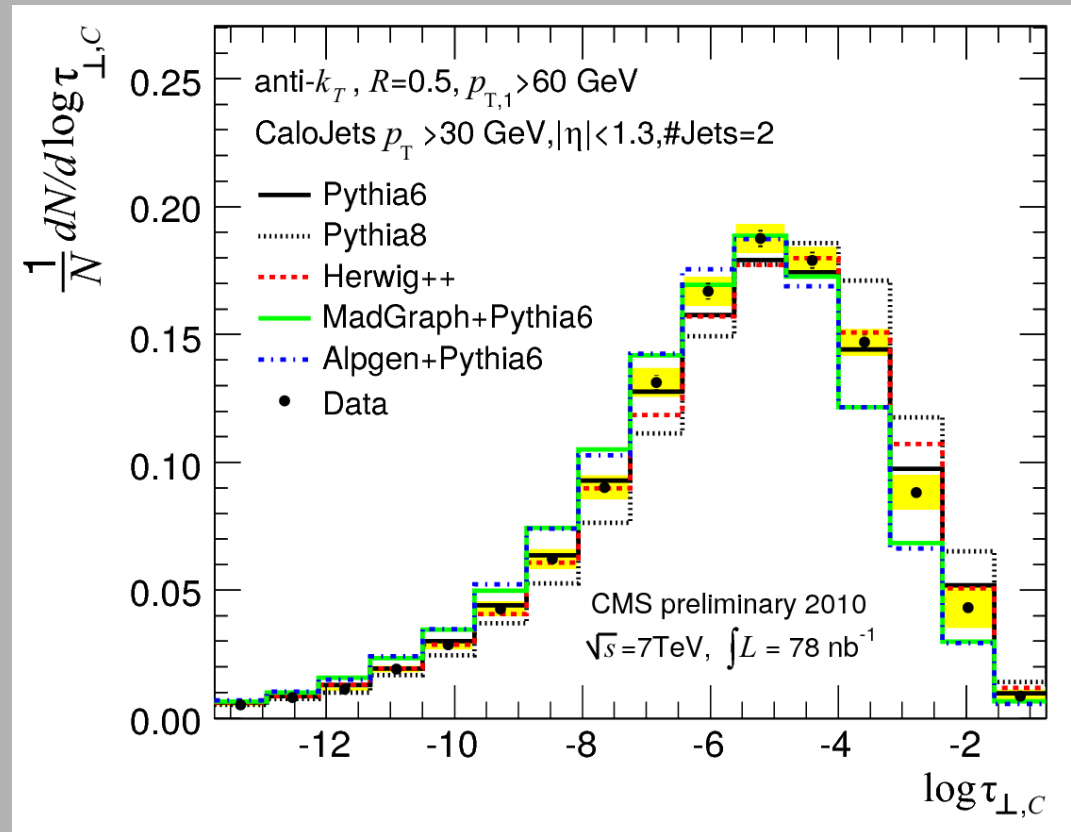
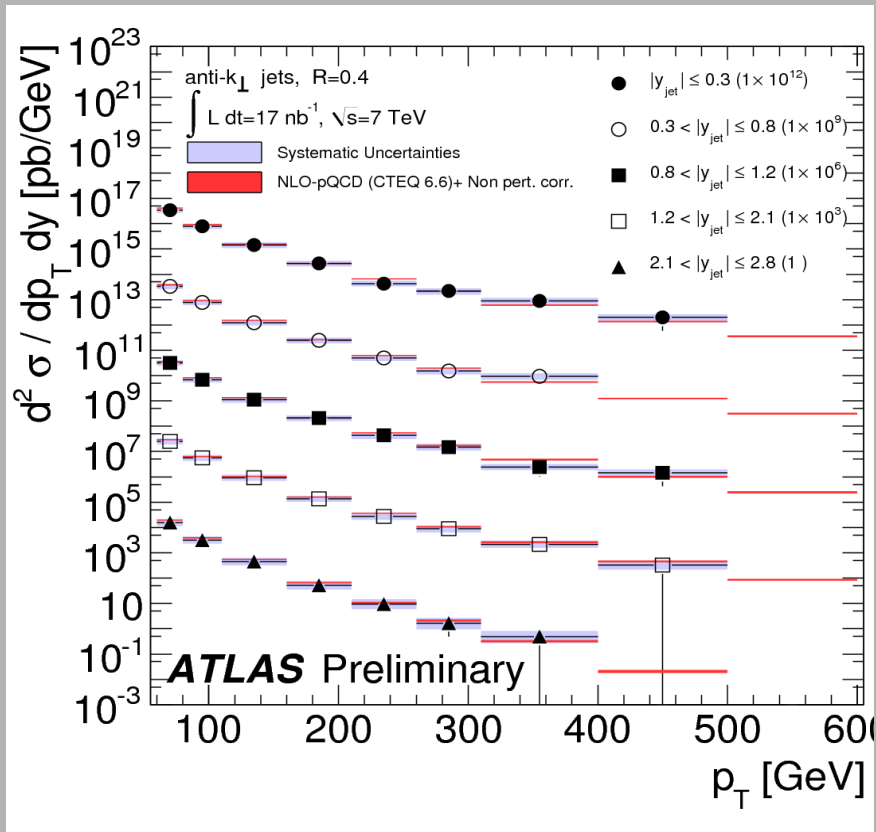
- LEP result from 2004
- $e^+e^-$  collisions
- Look at
  - event shapes
  - $q\bar{q}gg$  final state
- Nice demonstration of consistency of data with SU(3)



$C_F$ ,  $C_A$ : colour factors

# QCD and LHC

- LHC already has first results on QCD tests
- Huge cross-sections means  $100 \text{ nb}^{-1}$  are enough to make comparisons with theory



# Summary

- $\alpha_s$  measured with an accuracy of 0.6%
- Many different methods, colliders, experiments give in general very consistent results
  - DIS (and thrust) determinations tend to be a bit lower
- Running of  $\alpha_s$  seen within single experiments
- Trend consistent with expectations from asymptotic freedom
- Further precision QCD tests show good agreement between data and predictions
- LHC has entered the game!

# Backup

# Theory uncertainty

- Assess theory uncertainty by requiring physical observable to be independent of scale for a given order of calculation

$$\frac{d}{d \ln \mu^2} \sigma_{pp \rightarrow X} = O(\alpha_S^{l+1})$$

Equation motivates commonly adopted approach of varying renormalisation and factorisation scale by  $\frac{1}{2}$  and 2

# Running of $\alpha_s$

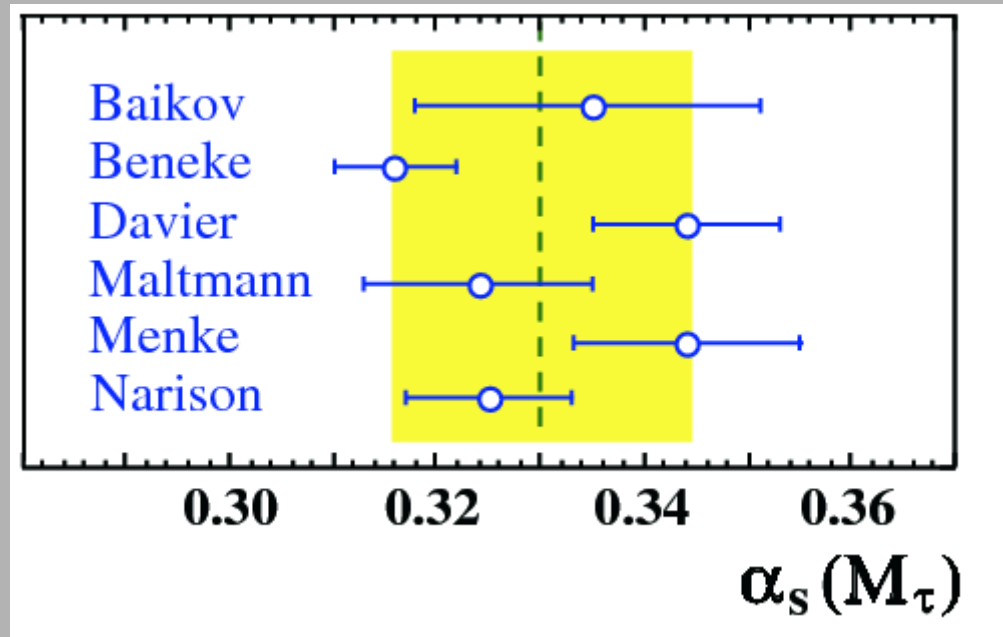
- Running coupling satisfies renormalisation group equation(RGE):

$$\begin{aligned}\mu^2 \frac{d\alpha_s}{d\mu^2} &= \beta(\alpha_s) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \dots) & C_F &\equiv (N_c^2 - 1)/(2N_c) = 4/3 \\ b_0 &= (11C_A - 4n_f T_R)/(12\pi) & C_A &\equiv N_c = 3 \\ &= (33 - 2n_f)/(12\pi) & T_R &= 1/2 \\ b_1 &= (153 - 19n_f)/(24\pi^2)\end{aligned}$$

- 1-loop approximation ( $\beta_1=0$ )

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2) \beta_0 \ln(Q^2/\mu^2)} \quad \text{or} \quad \alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)}$$

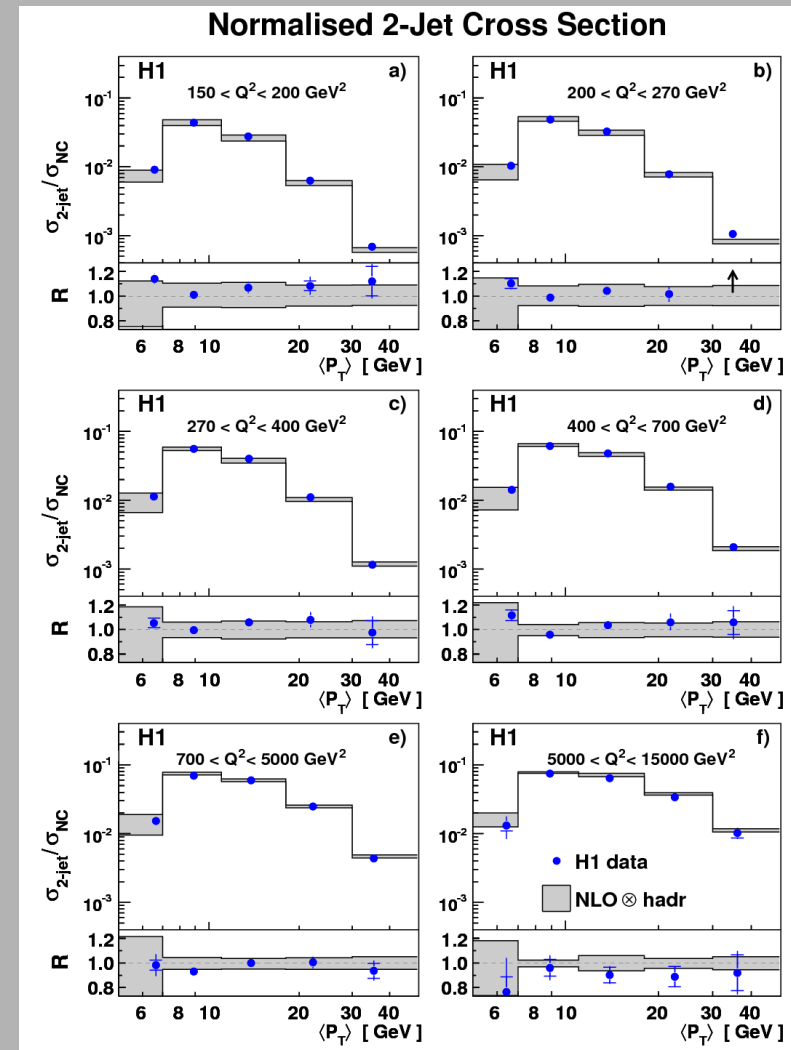
# $\alpha_S$ from $\tau$ measurements





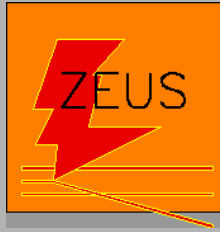
# Jets at high $Q^2$

- Compare average  $P_T^{\text{jet}}$  distribution in different  $Q^2$  ranges
- Again good description by NLO prediction

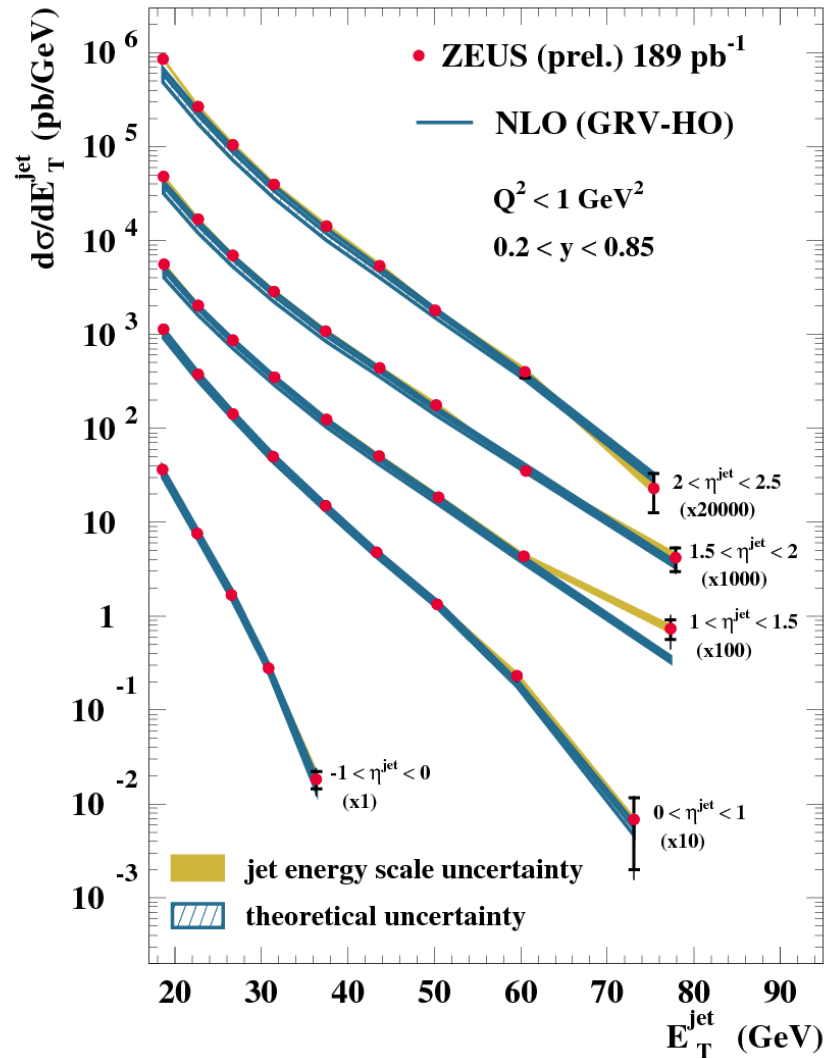


H1 Collab, Eur Phys J. C 65 (2010) 363

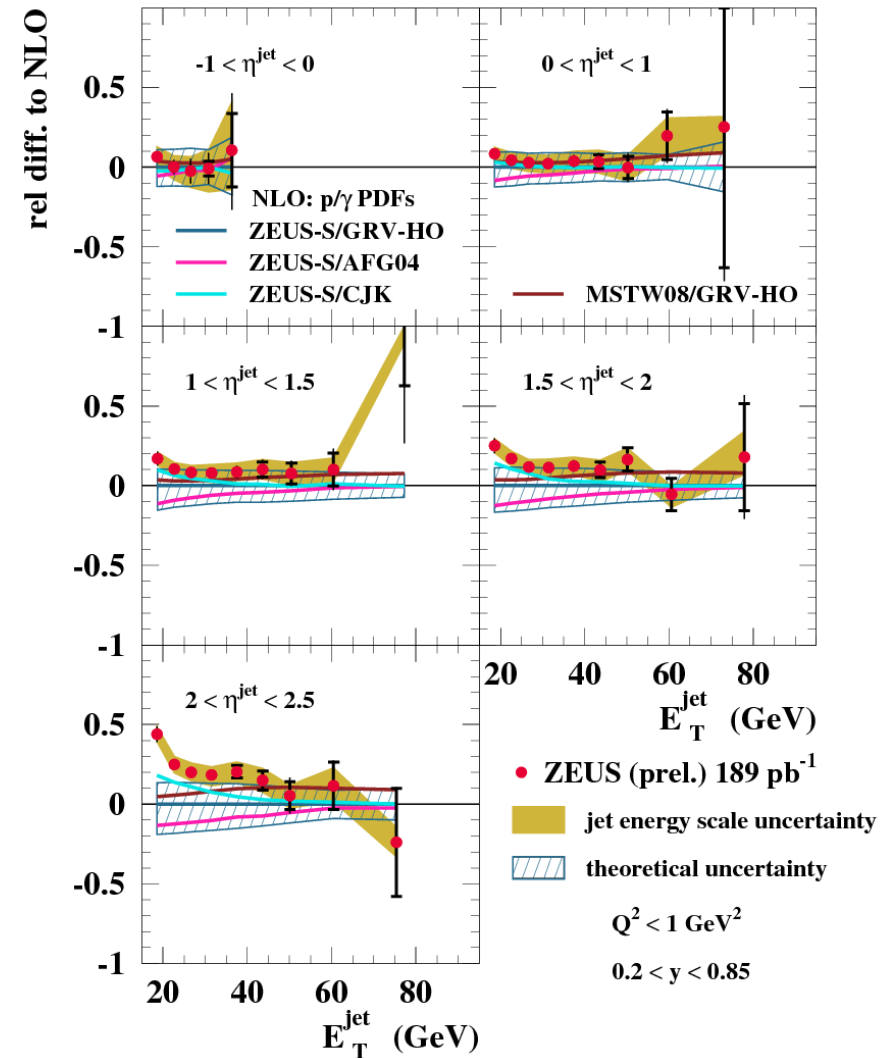
# Jets in Photoproduction



ZEUS

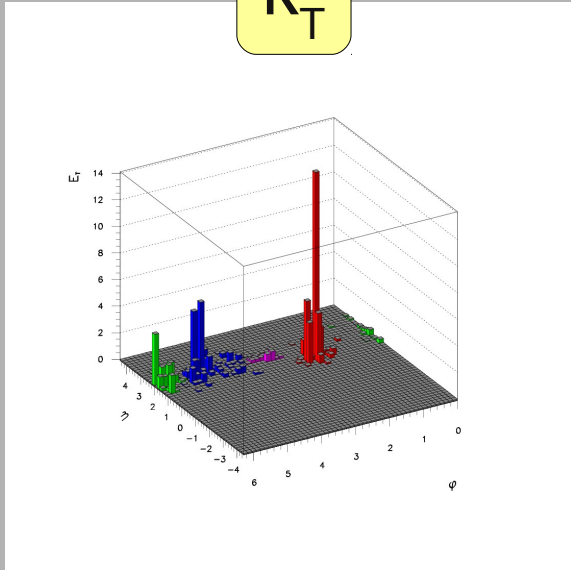


ZEUS

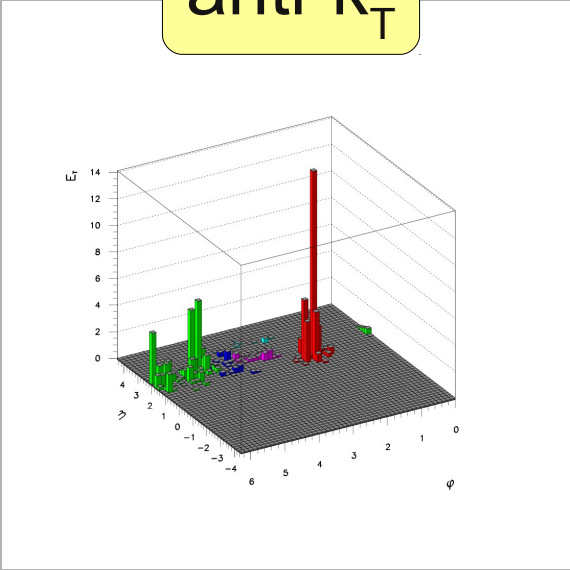


# Jet algorithms

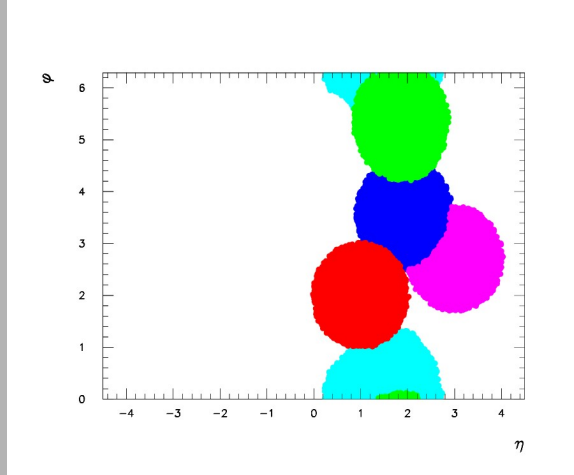
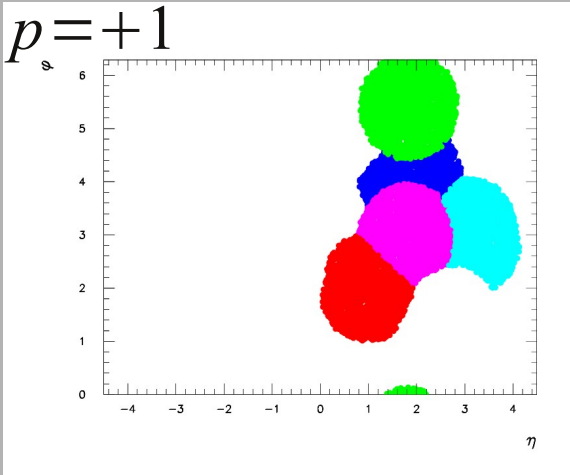
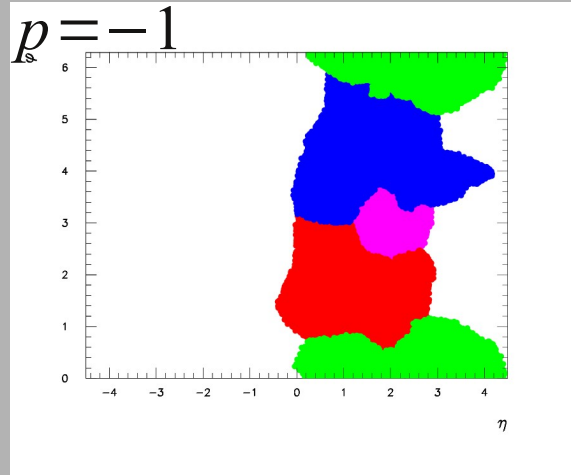
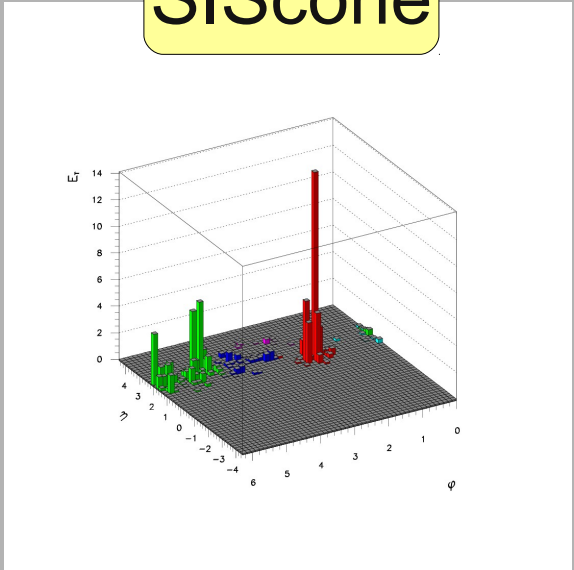
$k_T$



anti- $k_T$

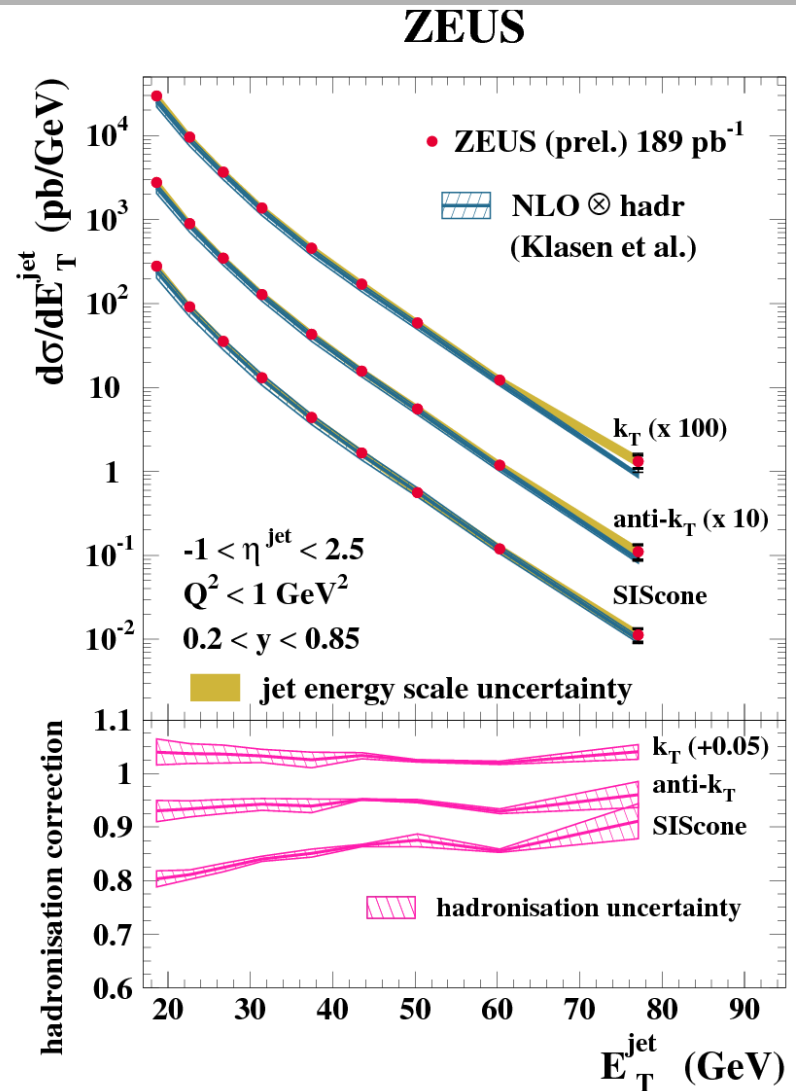
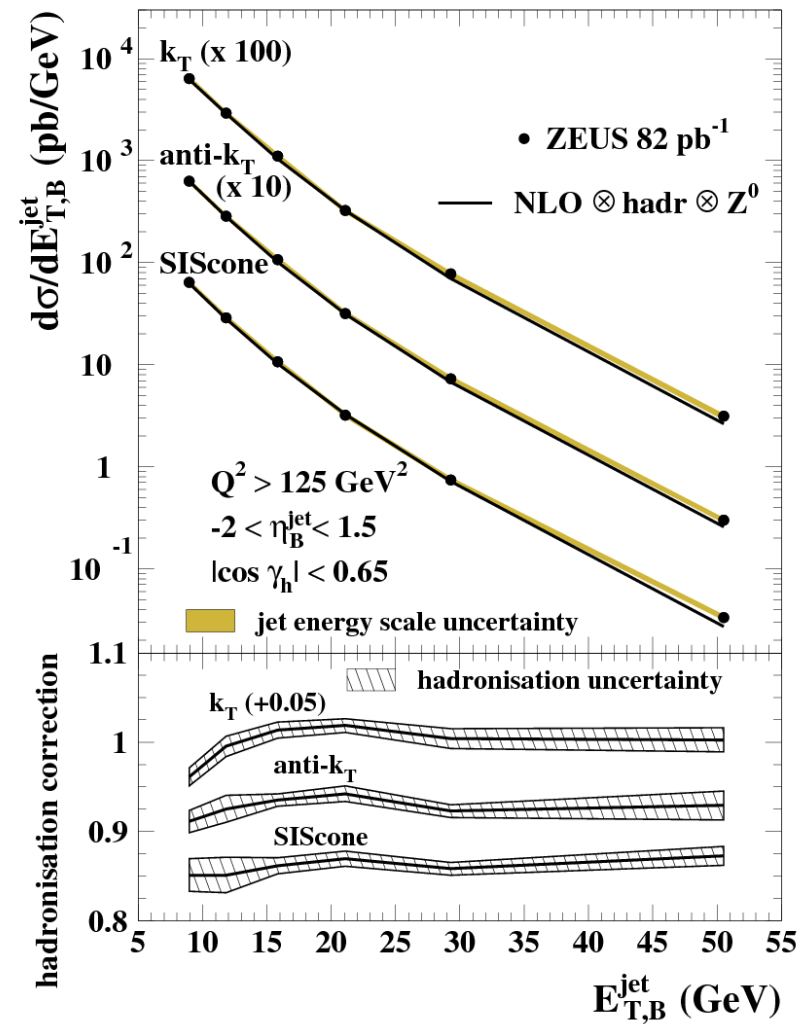


SIScone

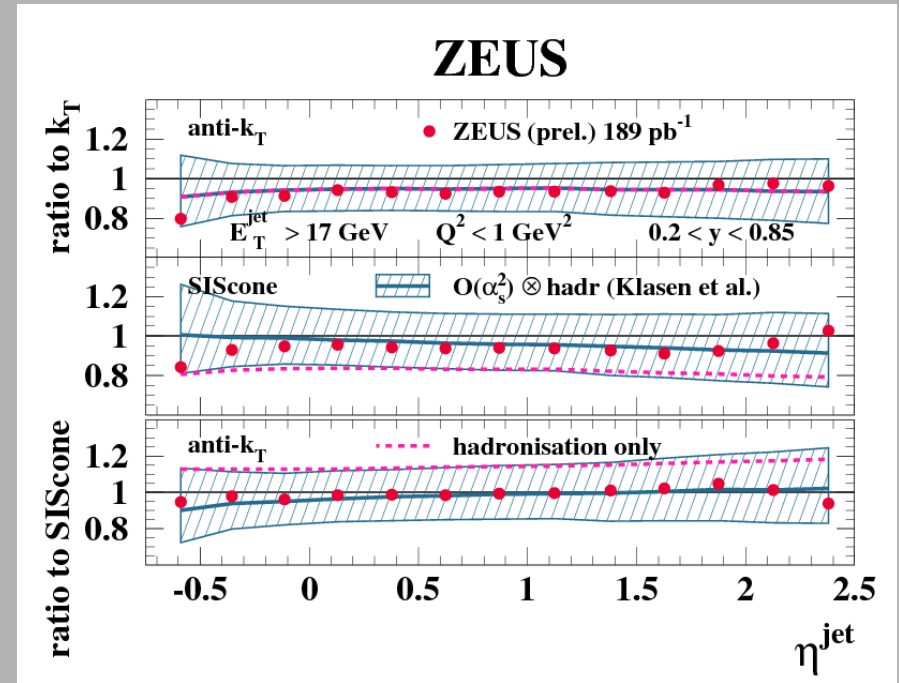
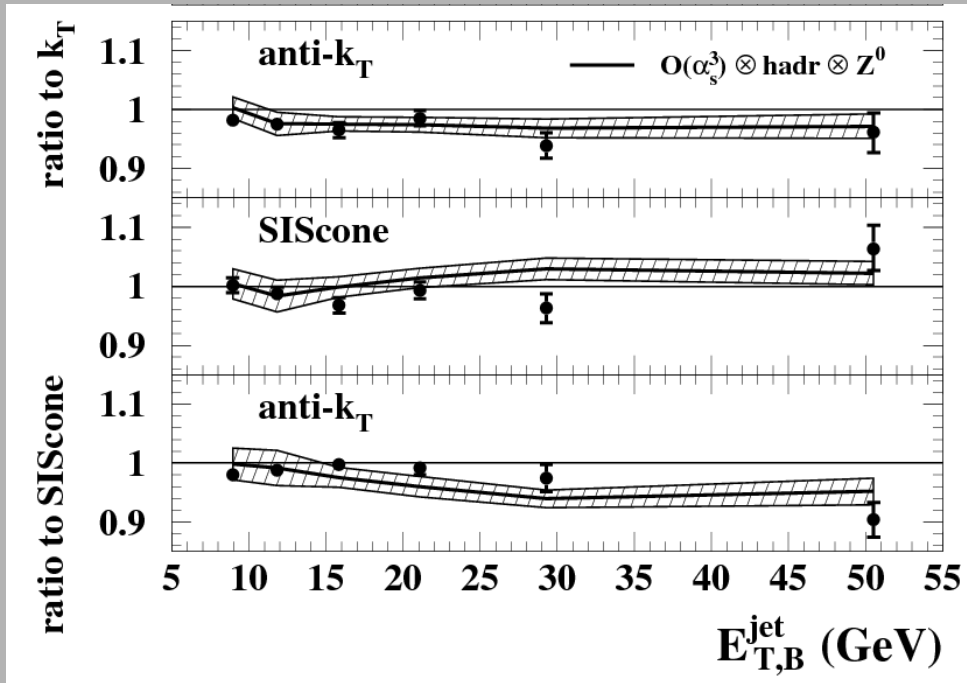


$$d_{ij} = \min[(E_T^i)^{2p}, (E_T^j)^{2p}] \Delta R^2 / R^2$$

# Jet algorithms



# Cross-section ratios



- Measured cross-sections with different algorithms similar
- pQCD calculations account adequately for differences in algorithms