# B decays and CKM 

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PHYSICS

## Outline

## CKM overview

Determination of $\left|V_{\mathrm{d}}\right|$ :

- $\mathrm{B} \rightarrow \mathrm{D} l v$ and $\mathrm{B} \rightarrow \mathrm{D}^{*} l v$ decays

Determination of $\left|V_{b b}\right|$ :

- inclusive $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v$ branching fraction measurements
- exclusive $\mathrm{B} \rightarrow(\pi, \rho) l v$ decays
- $\mathrm{B}^{+} \rightarrow \tau^{+} v$


## B decays and CKM

CKM matrix parametrizes the mixing of quark flavours via weak interaction

- Unitary by construction, implying non-trivial relationships between elements

$$
V=\left(\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right)
$$



- Angles related to CP violation measurements
- Lengths of sides related to magnitudes of CKM elements (i.e. CP conserving measurements)


## B decays and CKM

CP conserving B physics measurements probe the range of $3^{\text {rd }}$ generation CKM elements through both tree and one-loop processes
$B \rightarrow X_{u} l v$
$\mathrm{B} \rightarrow \pi, \rho l v$ and
$\mathbf{B}^{+} \rightarrow l^{+} \mathbf{v}$
$\mathrm{B}_{\mathrm{d},}$ mixing and
B $\rightarrow \mathbf{X}_{\mathrm{d},} \gamma$
$\downarrow$
a

$\beta$

"Redundant" determinations using theoretically and experimentally independent methods

- Validate methodology and can be interpreted in context of new physics models

$$
\mathbf{B} \rightarrow \mathbf{X}_{\mathbf{c}} l \mathbf{v} \text { and } \mathbf{B} \rightarrow \mathbf{D}^{(*)} l v
$$

## Determining CKM elements

Semileptonic B decays give direct access to CKM matrix elements $\left|\mathbf{V}_{\mathrm{cb}}\right|$ and $\left|\mathrm{V}_{\mathrm{cb}}\right|$ :

$$
\begin{gathered}
M\left(M_{Q \bar{q}} \rightarrow X_{q^{\prime} \bar{q}} l \bar{v}\right)=-i \frac{G_{F}}{\sqrt{2}} V_{q^{\prime} Q} L^{\mu} H_{\mu} \\
L^{\mu}=\bar{u}_{l} \gamma^{\mu}\left(1-\gamma_{5}\right) v_{v} \quad H_{\mu}=\langle X| \bar{q}^{\prime} \gamma_{\mu}\left(1-\gamma_{5}\right) Q|M\rangle
\end{gathered}
$$

- Challenge is to understand hadronic current (lattice QCD, HQET etc)

- Independent theoretical approaches for inclusive (OPE) and exclusive B decay processes (form factors)


## $\left|\mathrm{V}_{\mathrm{cb}}\right|$ from exclusive decays

Exclusive $\left|\mathrm{V}_{\mathrm{cb}}\right|$ determinations are based on $\mathrm{B} \rightarrow \mathrm{D} l v$ and $\mathrm{B} \rightarrow \mathrm{D}^{*} l y$ differential decay rate measurements

- Limitation is knowledge of $\mathrm{B} \rightarrow \mathrm{D}^{(*)}$ form factors:


$$
\begin{gathered}
\frac{d \Gamma}{d w}\left(\bar{B} \rightarrow D \ell \bar{\nu}_{\ell}\right)=\frac{G_{F}^{2}}{48 \pi^{3} \hbar} M_{D}^{3}\left(M_{B}+M_{D}\right)^{2}\left(w^{2}-1\right)^{3 / 2} \\
\frac{d \Gamma}{d w}\left(\bar{B} \rightarrow D^{*} \ell \bar{\nu}_{\ell}\right)=\left.\frac{G_{F}^{2}}{48 \pi}\right|^{2}\left|V_{c b}\right|^{2}(w) \\
\omega=n_{D^{*}}^{3}\left(w^{2}-1\right)^{1 / 2} P(w) v_{D^{(*)}}
\end{gathered}
$$

Form factors become unity at zero-recoil in heavy quark limit; corrections computed on lattice
$\left|\mathrm{V}_{\mathrm{c}}\right|$ extracted by extrapolating the differential decay rate to $\mathrm{w}=1$

- Requires assumption about shape of form factor:
- BABAR/Belle use parametrization characterized by form factor slope parameter $\boldsymbol{\rho}^{\mathbf{2}}$ (and $\mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{2}}$ in $\mathrm{D}^{*}$ decays)


## $\left|\mathrm{V}_{\mathrm{cb}}\right|$ from $\mathrm{B} \rightarrow \mathrm{D} / v$

BABAR measurement of $\mathrm{B} \rightarrow \mathrm{D} l v$ based on $460 \times 10^{6}$ BB pairs using exclusive reconstruction of the accompanying hadronic $B$ decay

- Background from $\mathrm{B} \rightarrow \mathrm{D}^{*} l v$ due to missing slow $\pi^{0}$
- Extract differential branching fraction from fit to missing mass spectrum in 10 bins of $w$



Most precise determination of $\mathrm{B} \rightarrow \mathrm{D} l v$ branching fraction to date

$$
\begin{aligned}
\mathrm{B}\left(\mathrm{~B} \rightarrow \mathrm{D} l^{+} v\right) & =(2.17 \pm 0.06 \pm 0.09) \% \\
\mathrm{G}(1)\left|\mathrm{V}_{\text {c }}\right| & =(43.0 \pm 1.9 \pm 1.4) \times 10^{-3} \\
\rho^{2} & =1.20 \pm 0.09 \pm 0.04
\end{aligned}
$$

## Methodology



$$
\begin{aligned}
& m_{\mathrm{ES}} \equiv \sqrt{E_{\text {beam }}^{* 2}-\mathrm{p}_{B}^{* 2}} \\
& \Delta E^{*} \equiv E_{B}^{*}-E_{\text {beam }}^{*}
\end{aligned}
$$

Advantage:

- Improved knowledge of signal kinematics, missing energy and suppression of combinatorial backgrounds
Disadvantage:
- Low tag reconstruction efficiency


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\end{aligned}
$$

## $\left|\mathrm{V}_{\mathrm{cb}}\right|$ from $\mathrm{B}^{0} \rightarrow \mathrm{D}^{*-l^{+} v}$

New untagged $\mathrm{B}^{0} \rightarrow \mathrm{D}^{*-} l^{+} v$ measurement based on $711 \mathrm{fb}^{-1}$ of Belle data

- Consider only $\mathrm{D}^{*+} \rightarrow \mathrm{D}^{0} \pi^{+} ; \mathrm{D}^{0} \rightarrow \mathrm{~K}^{-} \pi^{+}$ (and charge conjugate) modes, with no tag $B$ reconstruction
- about $120,000 \mathrm{~B}^{0} \rightarrow \mathrm{D}^{*-} l^{+} v$ decays selected in total
- Extract branching fraction from fits to $w$ and angles $\left(\cos \theta_{l}, \cos \theta_{\mathrm{v}}, \chi\right)$ characterizing the $\mathrm{D}^{*}$ decay
http://belle.kek.jp/results/summer10/dstlnu/


$$
\begin{aligned}
& \mathrm{B}\left(\mathrm{~B}^{0} \rightarrow \mathrm{D}^{*-} l^{+} v\right)=(4.56 \pm 0.03 \pm 0.26) \% \\
& \mathrm{~F}(1)\left|\mathrm{V}_{\text {b }}\right|=34.5 \pm 0.2 \pm 1.0 \\
& \rho^{2}=1.214 \pm 0.034 \pm 0.009 \\
& \mathrm{R}_{1}(1)=1.401 \pm 0.034 \pm 0.018 \\
& \mathrm{R}_{2}(1)=0.864 \pm 0.024 \pm 0.008
\end{aligned}
$$

## Measurement of $\left|\mathrm{V}_{\mathrm{lb}}\right|$

$$
\Gamma\left(\bar{B} \rightarrow X_{u} \ell \bar{\nu}\right)=\frac{G_{F}^{2}\left|V_{u b}\right|^{2} m_{b}^{5}}{192 \pi^{3}}\left[1+\mathcal{O}\left(\alpha_{s}\right)+\mathcal{O}\left(1 / m_{b}^{2}\right)+\text { h.c. }\right]
$$

Challenge for $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v$ determination due to background from CKM-favored $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{c}} l v$ decays:

$$
\frac{\Gamma(b \rightarrow u \ell \nu)}{\Gamma(b \rightarrow c \ell \nu)} \approx \frac{\left|V_{u b}\right|}{\left|V_{c b}\right|} \approx \frac{1}{50}
$$

- convergence of Heavy Quark Expansion spoiled in partial rate calculations, but kinematic selection required to suppress backgrounds:


$$
\begin{aligned}
& p_{\ell}{ }^{*} \text { lepton momentum } \\
& \mathrm{M}_{X} \text { hadronic invariant mass } \\
& q^{2} \quad \text { squared momentum transfer } \\
& P^{+}=E_{X}-\left|p_{X}\right| \text { light-cone momentum }
\end{aligned}
$$

- introduces dependencies on non-perturbative shape functions to account for efficiency loss in inaccessible regions of phase space)
$\Rightarrow$ Tradeoff between extending measurements into higher backgrounds regions and increased theory uncertainties on $\left|\mathrm{V}_{\mathrm{ub}}\right|$ extraction


## 

Belle $\left|\mathrm{V}_{\mathrm{ub}}\right|$ measurement based on inclusive $B \rightarrow X_{u} l v$ with hadronic tag reconstruction

- $\mathrm{p}_{\ell}{ }^{*}>1.0 \mathrm{GeV} / \mathrm{c}$
- Suppression of $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{c}} l v$ background via 20-input Boosted Decision Tree


Yield extracted from 2D fit to $M_{x}, q^{2}$ with background floated:



Analysis accesses
~90\% of available
$\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v$ phase space

## Inclusive $\left|\mathrm{V}_{\mathrm{tb}}\right|$

New BABAR measurement of $B \rightarrow X_{u} l v$ using hadronic tag reconstruction ( $468 \times 10^{6} \mathrm{BB}$ pairs)

- $\mathrm{p}_{\ell}{ }^{*}>1.0 \mathrm{GeV} / \mathrm{c}$
- Cut-based selection using similar variables as Belle multivariate analysis

- Measure partial branching fractions in six regions of phase space which have limited charm background



- $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v$
$-\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{C}} l v$ background - other background

Background subtracted distributions

## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from $\mathrm{B} \rightarrow \mathbf{X}_{\mathrm{u}} l v$

Most precise BABAR value obtained for full $M_{x}, q^{2}$ determination with $p_{l}{ }^{*}>1.0 \mathrm{GeV} / \mathrm{c}$

- Equivalent phase space coverage to Belle analysis
- Significantly reduced theory uncertainties compared with other methods

Partial branching fraction measurements translated into values of $\left|\mathrm{V}_{\mathrm{ub}}\right|$ using theoretical models (BLNP, GGOU, DGE ADFR

$$
\begin{aligned}
& \Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu ; p_{\ell}^{*}>1.0 \mathrm{GeV} / c\right)= \\
& \quad(1.80 \pm 0.13 \pm 0.15) \times 10^{-3} \\
& (1.96 \pm 0.17 \pm 0.16) \times 10^{-3}
\end{aligned}
$$



## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from $\mathrm{B} \rightarrow(\pi, \rho) l v$

$\left|\mathrm{V}_{\mathrm{ub}}\right|$ can be extracted from measurements of exclusive $\mathrm{B} \rightarrow \pi l v$ and $\mathrm{B} \rightarrow \rho l v$ differential branching fractions

$$
\frac{\mathrm{d} \Gamma(B \rightarrow \pi \ell \nu)}{\mathrm{d} q^{2}}=\frac{G_{F}^{2}\left|V_{u b}\right|^{2}}{24 \pi^{3}}\left|p_{\pi}\right|^{3}\left|f_{+}\left(q^{2}\right)\right|^{2}
$$

- theory input needed for form factor $f_{+}\left(\mathrm{q}^{2}\right)$ determination

New preliminary Belle measurement based on $605 \mathrm{fb}^{-1}$ of data:

$$
\begin{aligned}
& \mathrm{B}\left(\mathrm{~B}^{0} \rightarrow \pi^{-} l^{+} v\right)= \\
& \quad(1.49 \pm 0.04 \text { stat } \pm 0.07 \text { syst }) \times 10^{4}
\end{aligned}
$$



## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from $\mathrm{B} \rightarrow(\pi, \rho) l v$

BABAR measurement of $\mathrm{B} \rightarrow\left(\pi^{ \pm}, \pi^{0}, \rho^{ \pm}, \rho^{0}\right) l v$ based on $337 \times 10^{6}$ BB pairs

- Neutrino inferred from total event missing momentum vector
- Multivariate (NN) selection to suppress large $B \rightarrow X_{c} l v$ background as well as continuum and other $\mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v$ backgrounds
- Branching fractions extracted from simultaneous fit with isospin constraint in $m_{E S}, \Delta E$ and $q^{2}$ :

$$
\begin{aligned}
& \mathrm{B}\left(\mathrm{~B}^{0} \rightarrow \pi^{-} l^{+} v\right)=(1.41 \pm 0.05 \pm 0.07) \times 10^{4} \\
& \mathrm{~B}\left(\mathrm{~B}^{0} \rightarrow \rho^{-} l^{+} v\right)=(1.75 \pm 0.15 \pm 0.27) \times 10^{-4}
\end{aligned}
$$

BABAR
preliminary

arXiv:1005.3288[hep-ex]

## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from $\mathrm{B} \rightarrow(\pi, \rho) l v$

Extract shape of the $\mathrm{B} \rightarrow \pi l v$ form factor $f_{+}\left(\mathrm{q}^{2}\right)$ from differential branching fraction spectrum:

Extract $\left|\mathrm{V}_{\mathrm{ub}}\right|$ by integrating form factor predictions over relevant $q^{2}$ range


|  | $q^{2}$ Range $\left(G e V^{2}\right)$ | $\begin{gathered} \Delta \mathcal{B} \\ \left(10^{-4}\right) \end{gathered}$ | $\begin{gathered} \Delta \zeta \\ \left(\mathrm{ps}^{-1}\right) \end{gathered}$ | $\begin{gathered} \left\|V_{u b}\right\| \\ \left(10^{-3}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| LCSR 1 | 0-16 | $1.10 \pm 0.07$ | $5.44 \pm 1.43$ | $3.63 \pm 0.12_{-0.40}^{+0.59}$ |
| LCSR 2 | $0-12$ | $0.88 \pm 0.06$ | $4.00_{-0.95}^{+1.01}$ | $3.78 \pm 0.13_{-0.40}^{+0.55}$ |
| HPQCD | $16-26.4$ | $0.32 \pm 0.03$ | $2.02 \pm 0.55$ | $3.21 \pm 0.17_{-0.36}^{+0.55}$ |



## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from $\mathrm{B} \rightarrow(\pi, \rho) l v$

$\left|\mathrm{V}_{\mathrm{ub}}\right|$ from alternatively be extracted from a simultaneous fit to data and lattice (FNAL/MILC):


$$
\left|\mathrm{V}_{\mathrm{ub}}\right|=(2.95 \pm 0.31) \times 10^{-3}
$$

$$
\left|\mathrm{V}_{\mathrm{ub}}\right|=(3.43 \pm 0.33) \times 10^{-3}
$$


$f_{+}\left(q^{2}\right)$ expressed in terms of $z$ to to remove known QCD effects

## $\left|\mathrm{V}_{\mathrm{bb}}\right|$ summary

$$
\begin{aligned}
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(4.46 \pm 0.27 \pm 0.24) \times 10^{-3} \\
& \mathcal{B} \text { Inclusive } \mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v\left(p_{\ell}^{*}>1.0 \mathrm{GeV} / \mathrm{c}, M_{x}, q^{2} \text { fit }\right) \\
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(4.32 \pm 0.16 \pm 0.23) \times 10^{-3} \\
& \text { Inclusive } \mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v \text { HFAG average } \\
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(4.27 \pm 0.23 \pm 0.26) \times 10^{-3} \quad \sum \mathrm{New} \\
& \text { 5. Inclusive } \mathrm{B} \rightarrow \mathrm{X}_{\mathrm{u}} l v\left(p_{\ell}{ }^{*}>1.0 \mathrm{GeV} / \mathrm{c}, M_{x}, q^{2} \text { fit }\right) \\
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(2.95 \pm 0.31) \times 10^{-3} \\
& \text { Exclusive } \mathrm{B} \rightarrow \pi l v \text { (fit with lattice) } \\
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(3.40 \pm 0.20) \times 10^{-3} \\
& \text { Exclusive } \mathrm{B} \rightarrow \pi l v \text { HFAG average (HPQCD) } \\
& \left|\mathrm{V}_{\mathrm{ub}}\right|=(3.43 \pm 0.33) \times 10^{-3} \quad \sum \mathrm{New} \\
& \mathcal{B} \text { Exclusive } \mathrm{B} \rightarrow \pi l v \text { (fit with lattice) }
\end{aligned}
$$

- Significant improvements in techniques for $\left|\mathrm{V}_{\mathrm{ub}}\right|$ extraction in recent years, but long-standing discrepancy between inclusive and exclusive determinations persists


## $\left|\mathbf{V}_{\mathbf{u b}}\right|$ and $\mathbf{B}^{+} \rightarrow \tau^{+} \mathbf{v}$

Theoretically clean determination of $\left|\mathrm{V}_{\mathrm{ub}}\right|$ from helicity suppressed leptonic modes

$$
\mathcal{B} r\left(B^{+} \rightarrow \ell^{+} \nu_{\ell}\right)=\frac{G^{2}\left(\frac{\pi}{8 \pi}\left|V_{u b}\right|^{2} f_{B}^{2} \eta_{B} m_{\ell}^{2} \tau_{B}\left(1-\frac{m_{\ell}^{2}}{m_{B}^{2}}\right)^{2}, ~\right.}{}
$$



Experimental challenge due to small branching fractions and limited kinematic information

- $\mathrm{B} \rightarrow \tau v$ most accessible mode at B factories
- Possible to use both hadronic and semileptonic reconstruction of tag B:

$$
\begin{aligned}
m_{\mathrm{ES}} & \equiv \sqrt{E_{\text {beam }}^{* 2}-p_{B}^{* 2}} \\
\Delta E^{*} & \equiv E_{B}^{*}-E_{\text {beam }}^{*}
\end{aligned}
$$



## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ and $\mathrm{B}^{+} \rightarrow \tau^{+} \mathbf{v}$

Topological selection of $\tau$ decay candidates in e, $\mu, \pi$ and $\rho$ modes from particles not associated with the tag B candidate

- Signal $\mathrm{B}^{+} \rightarrow \tau^{+} v$ events expected to have little or no other activity in the detector, while backgrounds have higher multiplicity

Characterize additional activity by $\mathrm{E}_{\text {extra }} \quad$ (summed energy of all remaining calorimeter activity)

- Validate $\mathrm{E}_{\text {extra }}$ shape using samples in which the second $B$ is exclusively reconstructed



## $\left|\mathbf{V}_{\mathbf{u b}}\right|$ and $\mathbf{B}^{+} \rightarrow \tau^{+} \mathbf{v}$



arXiv:1008.0104 [hep-ex]

| Decay Mode |  | $\epsilon \times 10^{-4}$ |
| :--- | :---: | :---: |
| $\tau^{+} \rightarrow e^{+} \nu \bar{\nu}$ | Branching Fraction $\left(\times 10^{-4}\right)$ |  |
| $\tau^{+} \rightarrow \mu^{+} \nu \bar{\nu}$ | 2.92 | $0.39_{-0.79}^{+0.89}$ |
| $\tau^{+} \rightarrow \pi^{+} \nu$ | 1.55 | $1.23_{-0.90}^{+0.89}$ |
| $\tau^{+} \rightarrow \rho^{+} \nu$ | 0.85 | $4.0_{-1.3}^{+1.5}$ |
| combined | 8.05 | $4.3_{-1.9}^{+2.2}$ |

- Both results consistent with previously published SL tag analysis and Belle hadronic tag analyses*

| Decay Mode | Signal Yield | $\varepsilon, 10^{-4}$ | $\mathcal{B}, 10^{-4}$ |
| :---: | :---: | :---: | :---: |
| $\tau^{-} \rightarrow e^{-} \bar{\nu}_{e} \nu_{\tau}$ | $73_{-22}^{+23}$ | 5.9 | $1.90{ }_{-0.57}^{+0.59}{ }_{-0.35}^{+0.33}$ |
| $\tau^{-} \rightarrow \mu^{-} \bar{\nu}_{\mu} \nu_{\tau}$ | $12_{-17}^{+18}$ | 3.7 | $0.50_{-0.72}^{+0.76+0.21}{ }^{+0.18}$ |
| $\tau^{-} \rightarrow \pi^{-} \nu_{\tau}$ | $55_{-20}^{+21}$ | 4.7 | $1.80{ }_{-0.66}^{+0.69}{ }_{-0.37}^{+0.36}$ |
| Combined | $143_{-35}^{+36}$ | 14.3 | $1.54{ }_{-0.37}^{+0.38}{ }_{-0.31}^{+0.29}$ |

* K. Ikado et al., Phys. Rev. Lett. 97, 251802 (2006)
B. Aubert et al., Phys. Rev. D 81, 051101(R) (2010)


## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ and $\mathrm{B}^{+} \rightarrow \tau^{+} \mathbf{v}$

- Combination of all four (statistically independent) measurements yields

$$
\mathrm{B}\left(\mathrm{~B}^{+} \rightarrow \tau^{+} v\right)=(1.68 \pm 0.31) \times 10^{4}
$$



- Comparison with B mixing measurements permits cancelation of parametric uncertainty from $f_{B}$


$$
\begin{aligned}
& \Delta m_{d}=\frac{G_{F}^{2}}{6 \pi^{2}} \eta_{B} m_{L_{d}} f_{B_{d}}^{2} \beta_{d} m_{W}^{2} S\left(x_{t}\right)\left|V_{t d} V_{t b}^{*}\right|^{2} \\
& \eta_{B}=0.551 \pm 0.007 \quad S\left(x_{t}\right) \approx 0.784 x_{t}^{0.76} \quad x_{t}=\bar{m}_{t}^{2} / m_{W}^{2}
\end{aligned}
$$

- "Tension" with respect to indirect $\left|\mathrm{V}_{\mathrm{cb}}\right|$ determination from $\sin 2 \beta$ at the level of ~2.5 $\sigma$

CKMfitter Group (J. Charles et al.),
Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184],

## Conclusions

Measurements of CKM element magnitudes provide an important counterpoint to CP violation studies at B factories and hadron colliders

- Substantial improvements in experimental techniques and theoretical input in recent years have resulted in significantly decreased uncertainties on $\left|\mathrm{V}_{\mathrm{c}}\right|$ and $\left|\mathrm{V}_{\mathrm{bb}}\right|$ determinations
- Discrepancies persist between inclusive and exclusive determinations, as well as between $\left|\mathrm{V}_{\mathrm{cb}}\right|$ measurements and the unitarity triangle fit driven by $\sin 2 \beta$
- Recent (and internally consistent) $\mathrm{B}^{+} \rightarrow \tau^{+} v$ measurements also favouring
 large $\left|\mathrm{V}_{\mathrm{ub}}\right|$

