# SM DiHiggs precision shapes

## Higgs 2020


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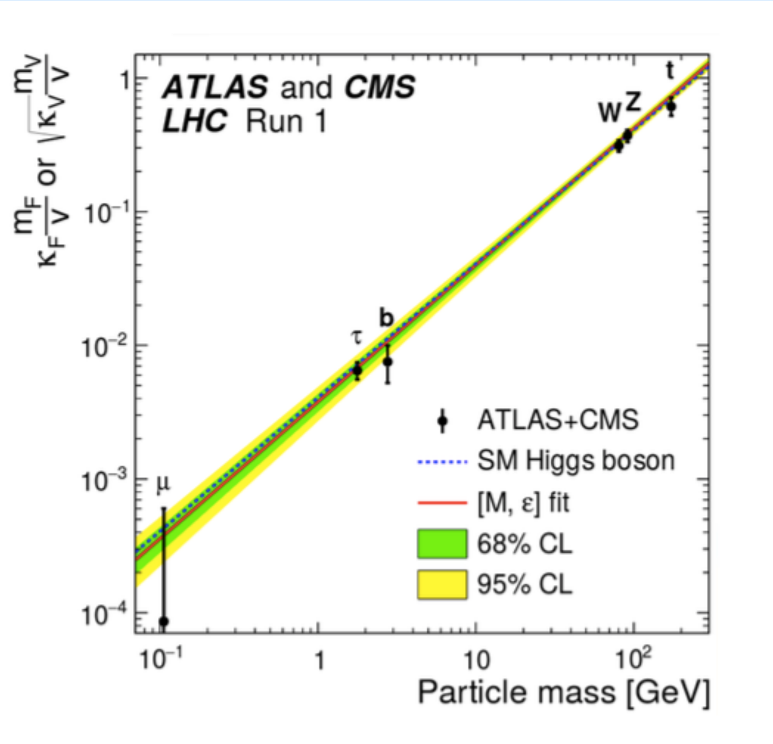
Based on arxiv: 1811.05692, 2003.03227 and 2008.11626v2

# Motivation

- Detection of a Higgs boson with a mass ~ 125 GeV
- Higgs mass, coupling strengths, spin and CP already determined
- *S*elf-coupling strength still unknown



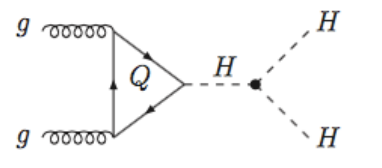

$$V(\phi) = \frac{\lambda}{2}\left\{|\phi|^2 - \frac{v^2}{2}\right\}^2$$

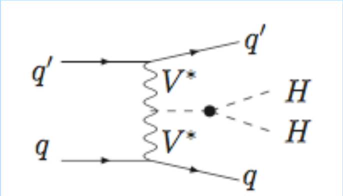
$$\lambda_{h^3} = 3\frac{m_h^2}{v}$$

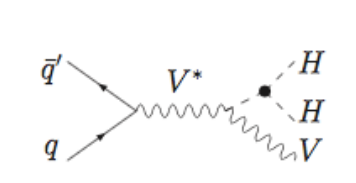
$$\lambda_{h^4} = 3\frac{m_h^2}{v^2}$$

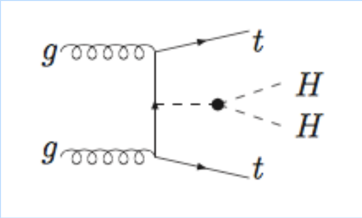
# Higgs boson pair production

## Production channels

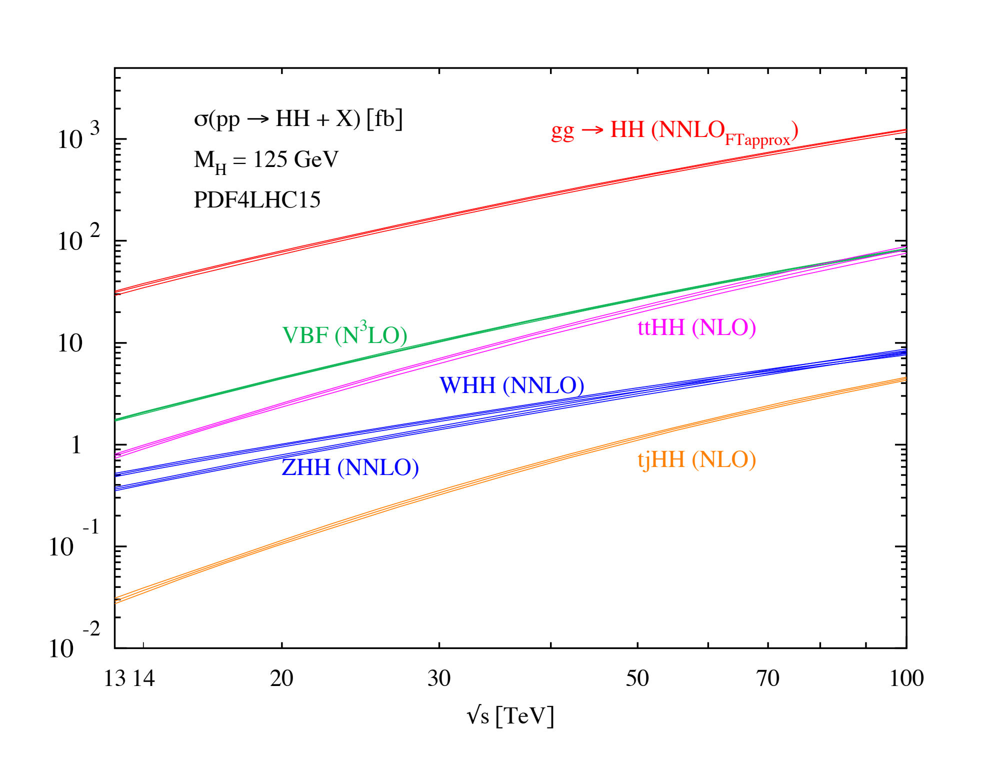


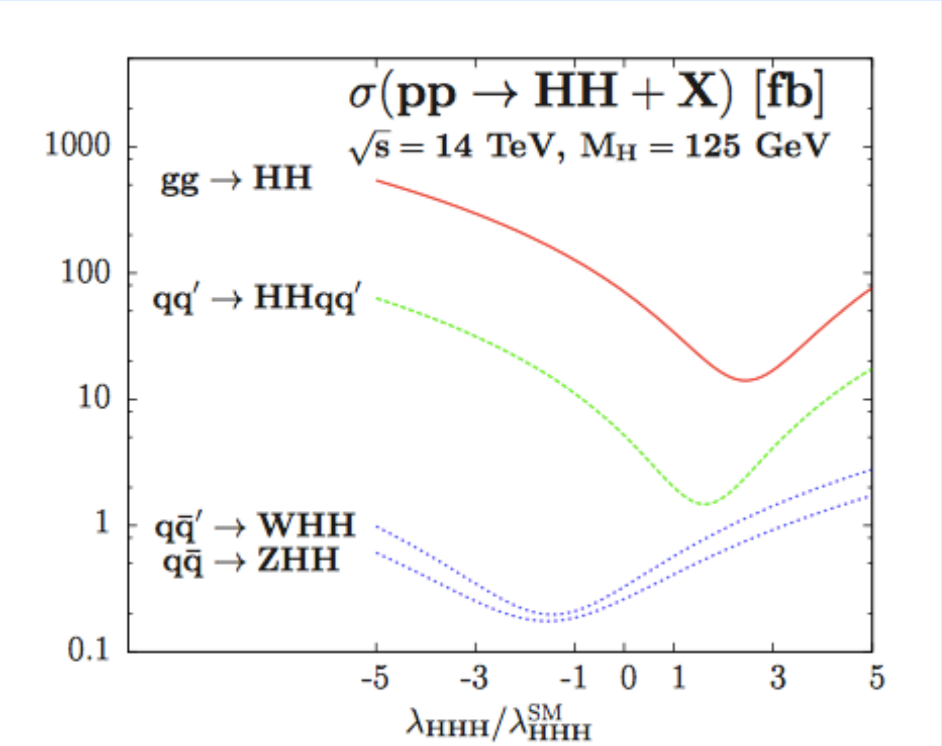

## Cross sections




HH White Paper

# Motivation

## Uncertainties:




$$gg \to HH : \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$$

Baglio, Djouadi, Gröber,
Mühlleitner, Quevillon, Spira

# Current Status

- Virtual & real (N)NLO QCD corrections in large top mass limit (HTL): ~100%

  Dawson,Dittmaier,Spira
  de Florian,Mazzitelli
  Grigo,Melnikov,Steinhauser

- Large top mass expansion: ~ ±10%

  Grigo, Hoff, Melnikov, Steinhauser

- NLO mass effects of the real NLO correction alone ~ -10 %

  Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro

- NLO QCD corrections including the full top mass dependence: - 15 % NLO mass effects

  Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke

  Baglio,Campanario,SG,Mühlleitner, Ronca,Spira,Streicher

- NNLO result in HTL merged with NLO mass effect and supplemented by massive double-real corrections

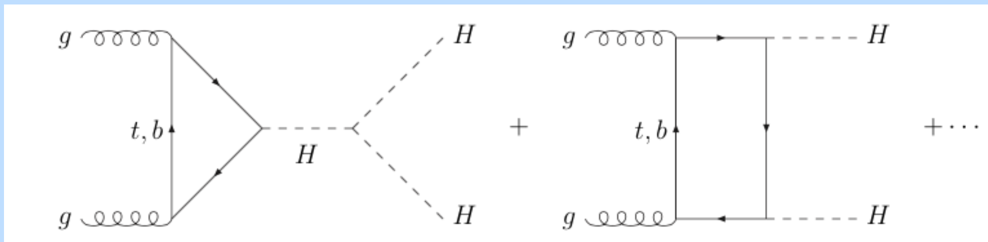
  Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli

- New expansion/extrapolation methods:
  - $1/m_t^2$ expansion & conformal mapping & Padé approximants — Gröber,Maier,Rauh
  - $p_T^2$ expansion — Bonciani,Degrassi,Giardino,Gröber
  - high-energy — Davies,Mishima,Steinhauser,Wellmann

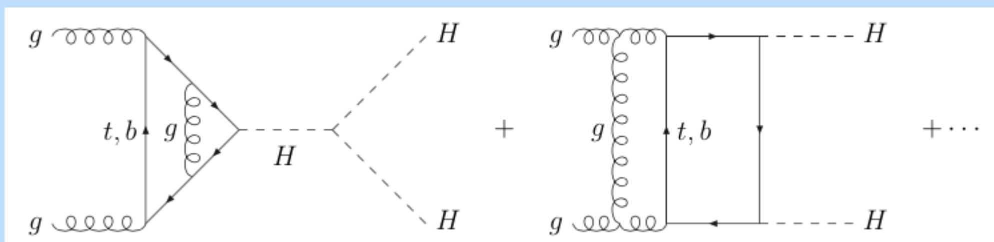
# NLO Corrections

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta\sigma_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

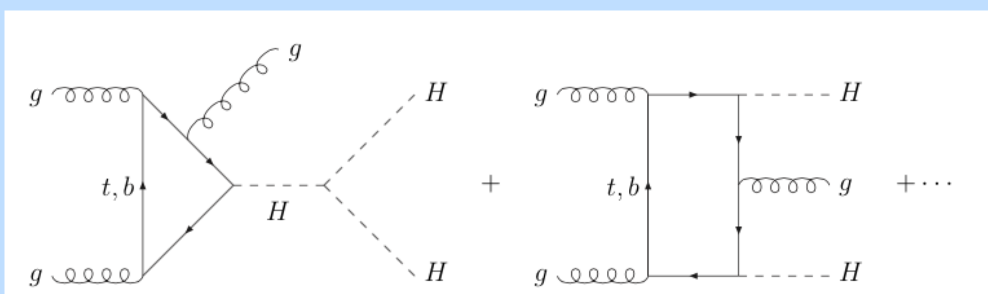
$\sigma_{LO}$:




$\Delta\sigma_{virt}$:



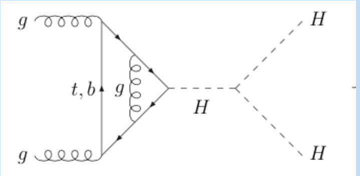

$\Delta\sigma_{ij}$:



# Virtual Corrections

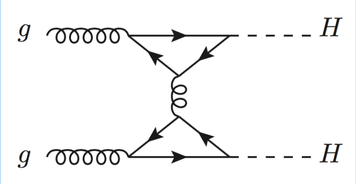
Triangular diagrams

- Use existing results of single Higgs calculation

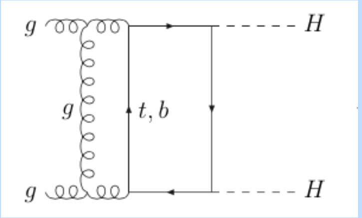



One-particle reducible diagrams

- Use existing results of $H \to Z\gamma$




Box diagrams




- Treat every diagram individually (no reduction to master integrals)
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions
- Extract the infrared and collinear divergences using a proper subtraction of the integrand
- Integration by parts to cope with numerical instabilities above the thresholds where

$$m_{HH}^2 > 0, m_{HH}^2 > 4m_t^2 \Rightarrow m_t^2 \to (1 - i\bar{\epsilon})\, m_t^2 \text{ with } \bar{\epsilon} \ll 1$$

# Virtual Corrections

## Total virtual corrections

- Numerical evaluation using Vegas (P. Lepage)

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

$(Q^2 = m^2_{HH})$

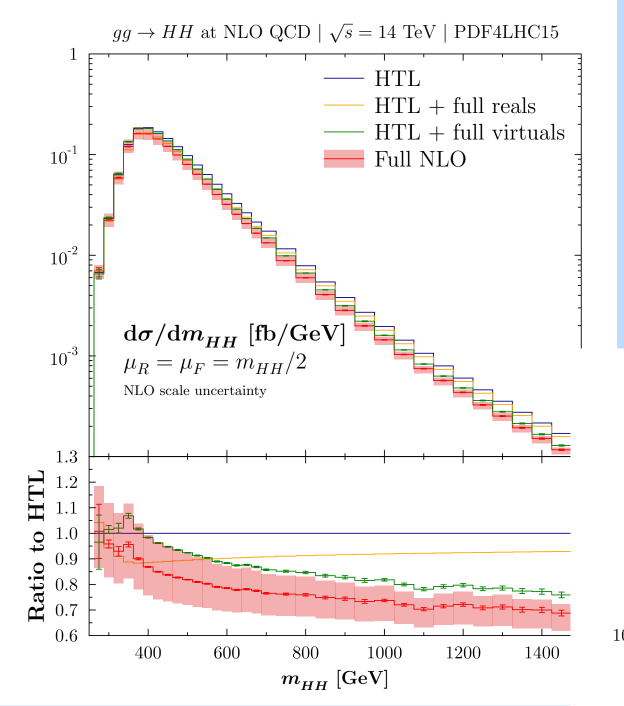
- Renormalization: $\alpha_s$ in $\overline{MS}$ with $N_F = 5$ and $m_t$ on-shell (central value)
- Subtraction of Born-improved HTL $\longrightarrow$ IR finite top mass effects
- Numerical instabilities due to the small imaginary parts of the top mass: *Richardson extrapolation* to derive the narrow-width limit for $m_t$

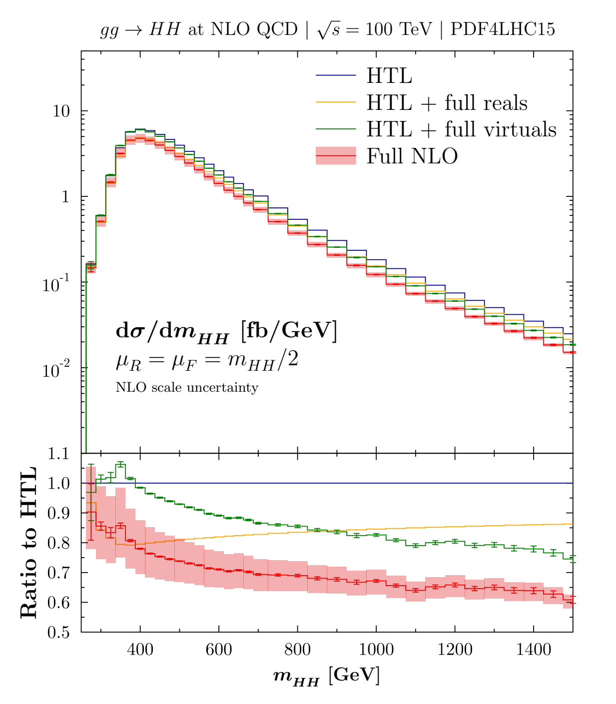
## Real corrections

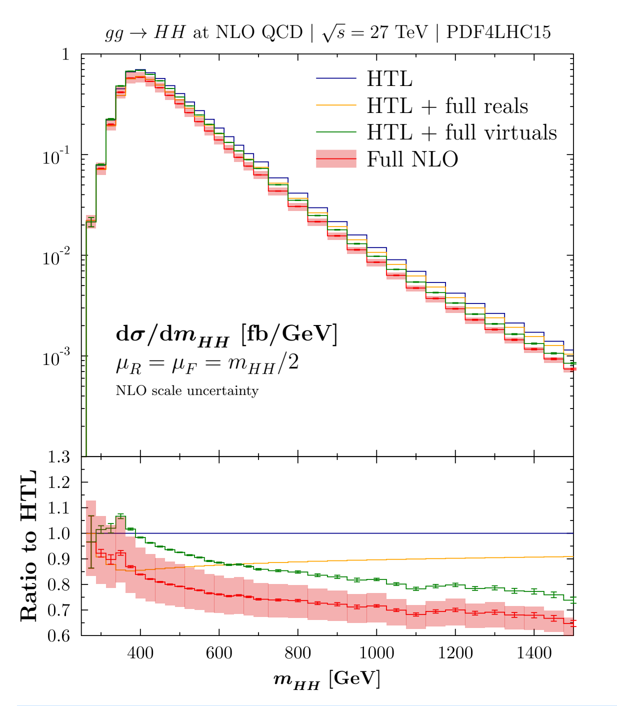
- Full matrix elements generated with FeynArts and FormCalc
- Matrix element in the HTL (massive LO) subtracted $\longrightarrow$ IR finite top mass effects

# Differential cross section

Differential cross section for different centre of mass energies



# Total hadronic cross section

→ integration over differential cross section

using combination of trapezoidal method and Richardson extrapolation

$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 27.73(7)^{+13.8\%}_{-12.8\%} \text{ fb},$$
$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 32.81(7)^{+13.5\%}_{-12.5\%} \text{ fb},$$
$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 127.0(2)^{+11.7\%}_{-10.7\%} \text{ fb},$$
$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1140(2)^{+10.7\%}_{-10.0\%} \text{ fb},$$

number in brackets: numerical error

per-centage entries: renormalization and factorization scale dependence

# Top scale and scheme uncertainty

## Uncertainty due to $m_t$: differential cross section

- → uncertainty related to the scheme and scale choice of the top mass
- → calculated the total NLO results from the differential cross section for the $\overline{MS}$ top mass at different scale choices
- → $\overline{MS}$ top mass scale in the range [*Q*/4, *Q*], $m_t$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=300\,\mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=400\,\mathrm{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \mathrm{fb/GeV}$$

$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=600\,\mathrm{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \mathrm{fb/GeV}$$

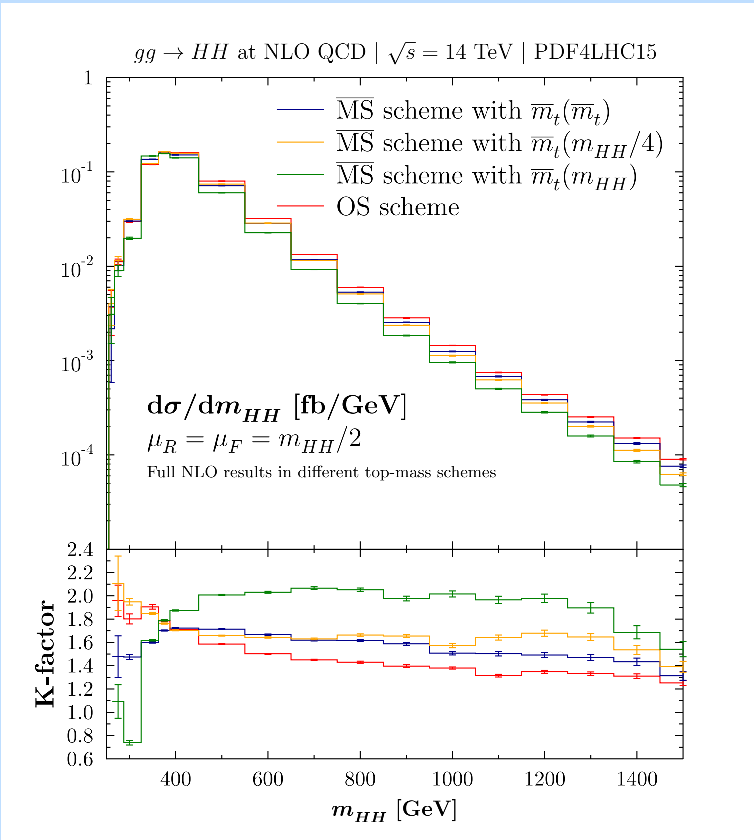
$$\left.\frac{\mathrm{d}\sigma(\mathrm{gg}\to\mathrm{HH})}{\mathrm{dQ}}\right|_{Q=1200\,\mathrm{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \mathrm{fb/GeV}$$

$$(\sqrt{s} = 14\,\mathrm{TeV})$$

(dynamical scale motivated by high energy expansion)

## Uncertainty due to $m_t$: total hadronic cross section

Take for individual Q values the maximum / minimum differential cross section and integrate




$$\sqrt{s} = 13 \text{ TeV}: \quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%} \text{ fb},$$
$$\sqrt{s} = 14 \text{ TeV}: \quad \sigma_{tot} = 32.81(7)^{+4\%}_{-18\%} \text{ fb},$$
$$\sqrt{s} = 27 \text{ TeV}: \quad \sigma_{tot} = 127.8(2)^{+4\%}_{-18\%} \text{ fb},$$
$$\sqrt{s} = 100 \text{ TeV}: \quad \sigma_{tot} = 1140(2)^{+3\%}_{-18\%} \text{ fb}$$

*with PDF4LHC15*

# Combined uncertainties

## Current recommended prediction:

- Central value: NNLO QCD corrections in the HTL combined with LO and NLO mass effects and massive one-loop double-real corrections at NNLO
- Uncertainties: Renormalization and factorization scale uncertainties

$$\begin{aligned}
\sqrt{s} = 13\ \text{TeV}: \quad \sigma_{tot} &= 31.05^{+2.2\%}_{-5.0\%}\ \text{fb}, \\
\sqrt{s} = 14\ \text{TeV}: \quad \sigma_{tot} &= 36.69^{+2.1\%}_{-4.9\%}\ \text{fb}, \\
\sqrt{s} = 27\ \text{TeV}: \quad \sigma_{tot} &= 139.9^{+1.3\%}_{-3.9\%}\ \text{fb}, \\
\sqrt{s} = 100\ \text{TeV}: \quad \sigma_{tot} &= 1224^{+0.9\%}_{-3.2\%}\ \text{fb}.
\end{aligned}$$

*How should we combine the scheme and scale uncertainties from the top-mass to the current recommendation?*

Our proposal: add relative uncertainties **linearly** since

- they are nearly independent of the renormalization and factorization scale uncertainties (agrees with total envelope at NLO)
- scales with dominant part of radiative corrections beyond NLO

$$\begin{aligned}
\sqrt{s} = 13\ \text{TeV}: \quad \sigma_{tot} &= 31.05^{+6\%}_{-23\%}\ \text{fb}, \\
\sqrt{s} = 14\ \text{TeV}: \quad \sigma_{tot} &= 36.69^{+6\%}_{-23\%}\ \text{fb}, \\
\sqrt{s} = 27\ \text{TeV}: \quad \sigma_{tot} &= 139.9^{+5\%}_{-22\%}\ \text{fb}, \\
\sqrt{s} = 100\ \text{TeV}: \quad \sigma_{tot} &= 1224^{+4\%}_{-21\%}\ \text{fb}
\end{aligned}$$

## Conclusion

- Calculation of two-loop integrals with three free parameter ratios
- NLO top mass effects of ~ -15% compared to Born-improved HTL result
- Factorisation / renormalisation scale dependence: ~ 15% uncertainties
- Top mass scheme and scale uncertainties: $\lesssim$ 30%   (differential)

  total cross section at 14 TeV: $\sigma(gg \to HH) = 32.81(7)^{+4\%}_{-18\%}\ \mathrm{fb}$
- Combination of the factorization and renormalization scale uncertainties with the scheme and scale uncertanties of the top mass

## Outlook

- Extension to the 2HDM
- Bottom loops

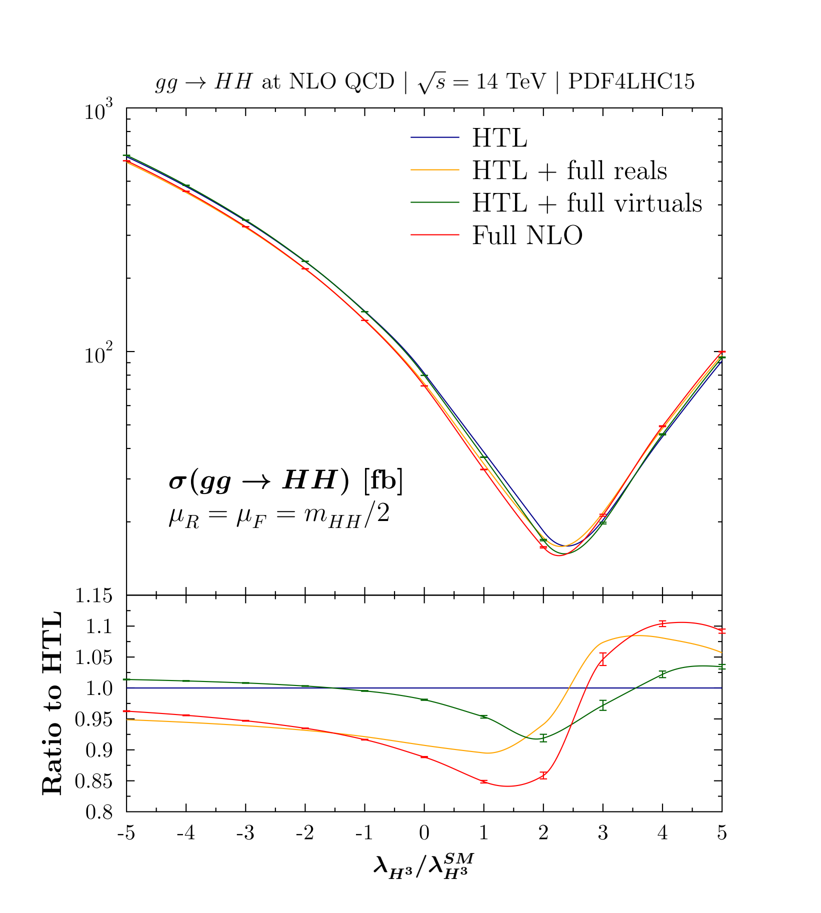
# Zoom Meeting room:

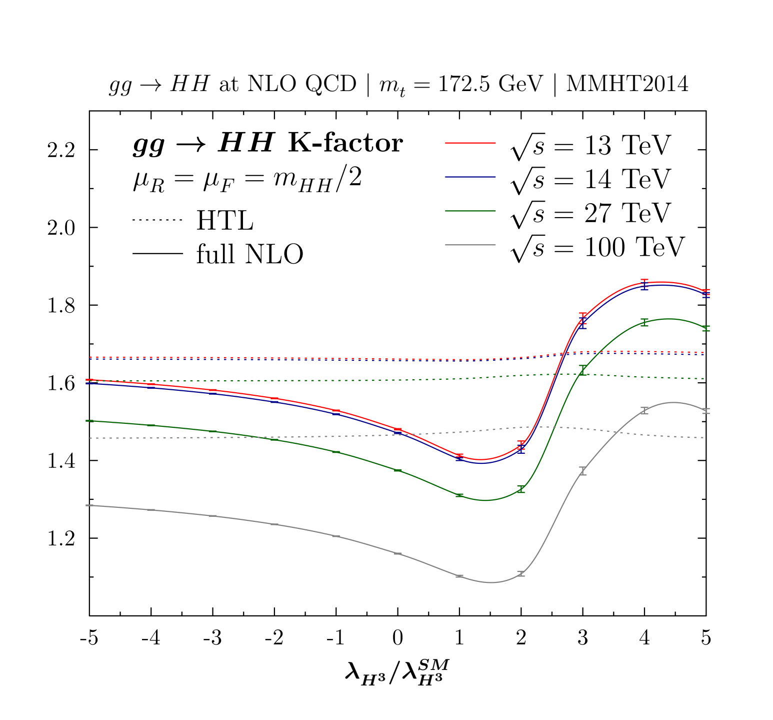
**https://zoom.us/j/5074112410?pwd=NWNFQVRhbnNqb3FKNzZ3VXlvUUYxZz09**

# BACK-UP

# Lambda Variation

## Variation of the cross section with $\lambda_{H^3}$



# Scale dependence

## Factorisation / renormalisation scale dependence

varying both scales by a factor of two around central value of $\mu_F = \mu_R = m_{hh}/2$

Differential cross section:

$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=300\ \mathrm{GeV}} = 0.02978(7)^{+15.3\%}_{-13.0\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=400\ \mathrm{GeV}} = 0.1609(4)^{+14.4\%}_{-12.8\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=600\ \mathrm{GeV}} = 0.03204(9)^{+10.9\%}_{-11.5\%}\ \mathrm{fb/GeV},$$
$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=1200\ \mathrm{GeV}} = 0.000435(4)^{+7.1\%}_{-10.6\%}\ \mathrm{fb/GeV}\,.$$

Total cross section:

$$\sigma(gg \to HH) = 32.81(7)^{+13.5\%}_{-12.5\%}\,\mathrm{fb} \quad (\mathrm{PDF4LHC15})$$

$(\sqrt{s} = 14\,\mathrm{TeV})$

Baglio,Campanario,G,Mühlleitner,Ronca,Spira,Streicher: 2003.03227

# Total hadronic cross section

| Energy | $m_t$ = 173 GeV | $m_t$ = 172.5 GeV |
|---|---|---|
| **13 TeV** | $27.80(9)^{+13.8\%}_{-12.8\%}$ fb | $27.73(7)^{+13.8\%}_{-12.8\%}$ fb |
| **14 TeV** | $32.91(10)^{+13.6\%}_{-12.6\%}$ fb | $32.78(7)^{+13.5\%}_{-12.5\%}$ fb |
| **27 TeV** | $127.7(2)^{+11.5\%}_{-10.4\%}$ fb | $127.0(2)^{+11.7\%}_{-10.7\%}$ fb |
| **100 TeV** | $1149(2)^{+10.8\%}_{-10.0\%}$ fb | $1140(2)^{+10.7\%}_{-10.0\%}$ fb |

HH White Paper

only factorization and renormalization scale uncertainties

# NLO Corrections

$$\sigma_{\rm NLO}(pp \to HH + X) = \sigma_{\rm LO} + \Delta\sigma_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}},$$

$$\begin{aligned}
\sigma_{\rm LO} &= \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\rm LO}(Q^2 = \tau s) \\
\Delta\sigma_{\rm virt} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau}\, \hat{\sigma}_{\rm LO}(Q^2 = \tau s)\; C \\
\Delta\sigma_{gg} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau\, \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\rm LO}(Q^2 = z\tau s) \left\{ -zP_{gg}(z) \log\frac{M^2}{\tau s} \right. \\
&\qquad \left. + d_{gg}(z) + 6[1 + z^4 + (1-z)^4] \left(\frac{\log(1-z)}{1-z}\right)_+ \right\} \\
\Delta\sigma_{gq} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\rm LO}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log\frac{M^2}{\tau s(1-z)^2} \right. \\
&\qquad \left. + d_{gq}(z) \right\} \\
\Delta\sigma_{q\bar{q}} &= \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^{1} \frac{dz}{z}\, \hat{\sigma}_{\rm LO}(Q^2 = z\tau s)\; d_{q\bar{q}}(z)
\end{aligned}$$

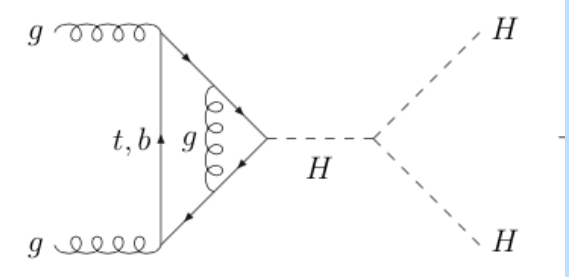
$$C \to \pi^2 + \frac{11}{2} + C_{\Delta\Delta}, \qquad d_{gg} \to -\frac{11}{2}(1-z)^3, \qquad d_{gq} \to \frac{2}{3}z^2 - (1-z)^2, \qquad d_{q\bar{q}} \to \frac{32}{27}(1-z)^3$$

# Virtual Corrections

47 two-loop box diagrams + 8 triangular diagrams + 2 one-particle reducible diagrams

Triangular Diagrams

⟵ single Higgs case

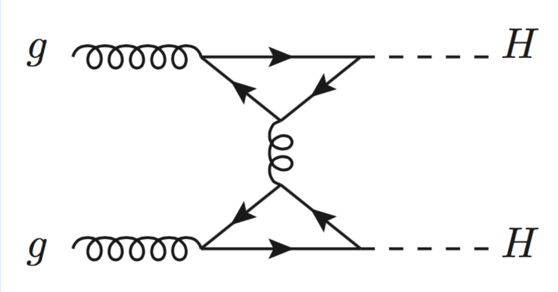



One-particle reducible diagrams

⟶ analytical results for $C_{\triangle\triangle}$

$(H \to Z\gamma)$

see e.g. Degrassi, Giardino, Gröber



# Virtual Corrections

## Box diagrams

- Generate matrix element form factors for all possible diagrams using Feynman rules (by hand → *Reduce, Mathematica, Form*)
- Perform Feynman parametrisation → additional 6-dimensional integrals
- Extract the ultraviolet divergences of the matrix elements using endpoint subtractions of the 6-dimensional Feynman integrals

$$\int_0^1 dx \, \frac{f(x)}{(1-x)^{1-\epsilon}} = \int_0^1 dx \, \frac{f(1)}{(1-x)^{1-\epsilon}} + \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx \, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- Extract the infrared and collinear divergences using a 'proper' subtraction of the integrand (based on HTL calculation)
- Integration by parts due to numerical instabilities at the thresholds

$m_{hh}^2 > 4m_t^2 \Rightarrow m_t^2 \to m_t^2(1-i\bar{\epsilon})$ with $\bar{\epsilon} \ll 1$

$$\int_0^1 \mathrm{dx} \, \frac{\mathrm{f(x)}}{(\mathrm{a}+\mathrm{bx})^3} = \frac{\mathrm{f(0)}}{2\mathrm{a}^2\mathrm{b}} - \frac{\mathrm{f(1)}}{2\mathrm{b}(\mathrm{a}+\mathrm{b})^2} + \int_0^1 \mathrm{dx} \, \frac{\mathrm{f}'(\mathrm{x})}{2\mathrm{b}(\mathrm{a}+\mathrm{bx})^2}$$

# Virtual Corrections

## Differential cross section

$$Q^2 \frac{d\Delta\sigma_{virt}}{dQ^2} = \tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{virt}(Q^2)\Big|_{\tau=\frac{Q^2}{s}} \qquad (Q^2 = m^2_{HH})$$

$$\frac{\mathrm{d}\mathcal{L}^{gg}}{\mathrm{d}\tau} = \text{gluon luminosity}$$

$$\hat{\sigma}_{virt} = \text{virtual part of the partonic cross section}$$

⟶ 7 dimensional integrals (6 Feynman and one phase space integration)

⟶ use Vegas for numerical integration (P. Lepage)

⟶ numerical instabilities due to the small imaginary parts of the top mass above the thresholds: Richardson extrapolation

# Virtual Corrections

## Renormalisation

$\alpha_s$ and $m_t$ need to be renormalised

→ $\alpha_s$ in $\overline{MS}$ with $N_F = 5$

→ $m_t$ on shell (→ central prediction)

$$\delta\sigma = \delta\alpha_s \frac{\delta\sigma_{LO}}{\delta\alpha_s} + \delta m_t \frac{\delta\sigma_{LO}}{\delta m_t}$$

Subtraction of the heavy-top limit → virtual mass effects only (infrared finite)

$$\Delta C_{mass} = C^0 - C^0_{HTL}$$

Adding back the results in the heavy-top limit (HPAIR)

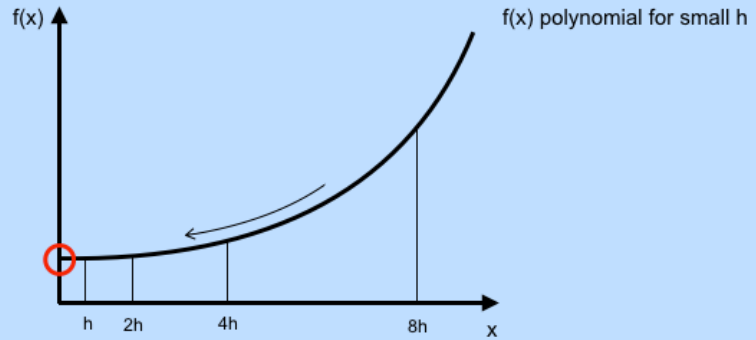
$$C = \underbrace{C_{HTL}}_{\text{HPAIR}} + \Delta C_{mass}$$

## Richardson extrapolation

→ sequence acceleration method to obtain a better convergence behaviour

Approximation polynomial

$$M_{i+1}(h) = \frac{t^{k_i} M_i(\frac{h}{t}) - M_i(h)}{t^{k_i} - 1}$$




*h* and *h/t* the two step sizes and $k_i$ the truncation error

$$M_2[f(h), f(2h)] = 2f(h) - f(2h) = f(0) + \mathcal{O}(h^2)$$

$$M_4[f(h), f(2h), f(4h)] = (8f(h) - 6f(2h) + f(4h))/3 = f(0) + \mathcal{O}(h^3)$$

$$M_8[f(h), f(2h), f(4h), f(8h)] = (64f(h) - 56f(2h) + 14f(4h) - f(8h))/21 = f(0) + \mathcal{O}(h^4)$$

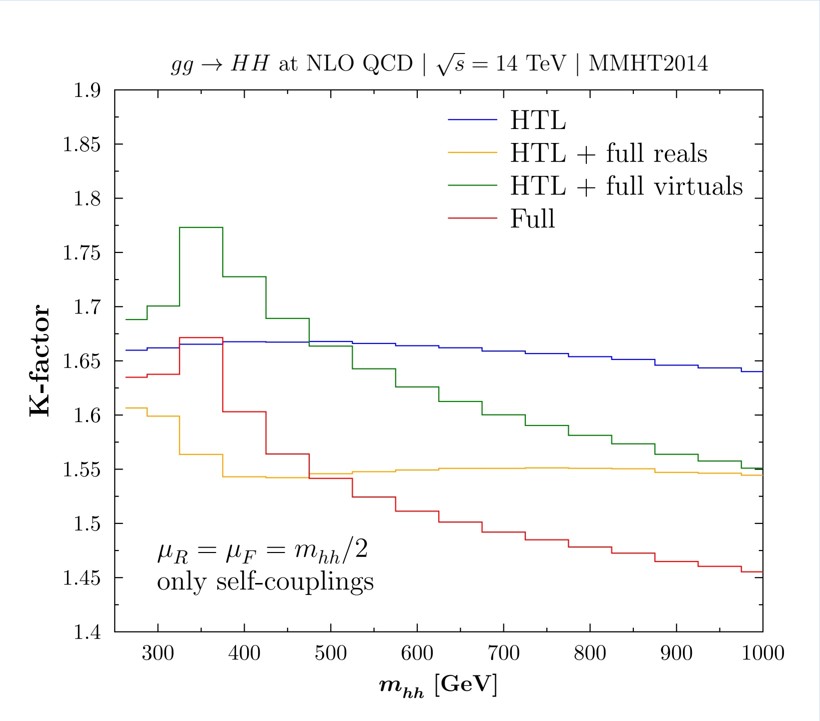
In our case $h = \bar{\epsilon}$ and $\bar{\epsilon}_n = 0.05 \times 2^n \quad n = 0 \ldots 9$

Theoretical error from Richardson extrapolation estimated by the difference of the fifth and the

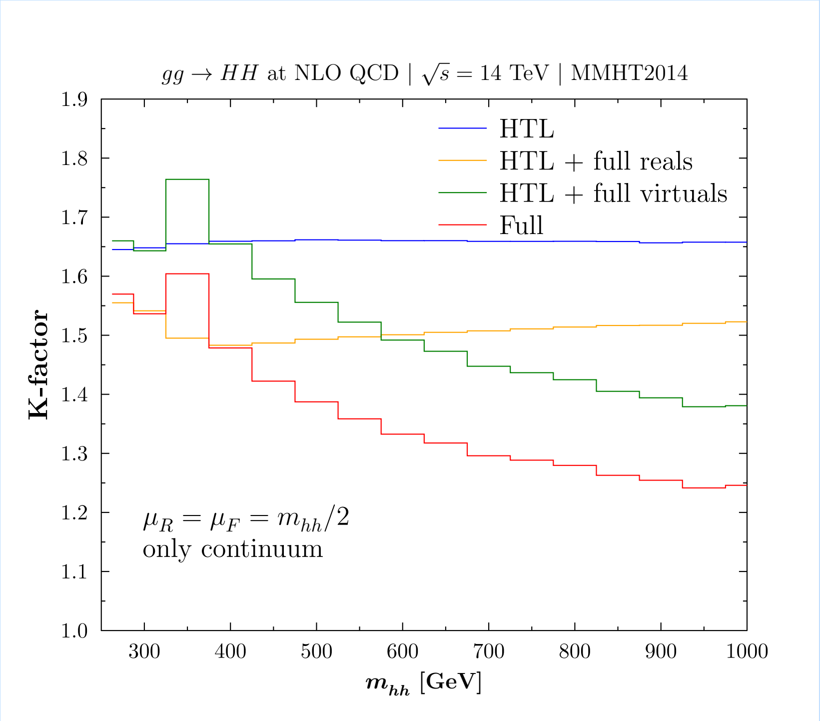
## K-factor distribution

$$K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$$

### Triangular contributions

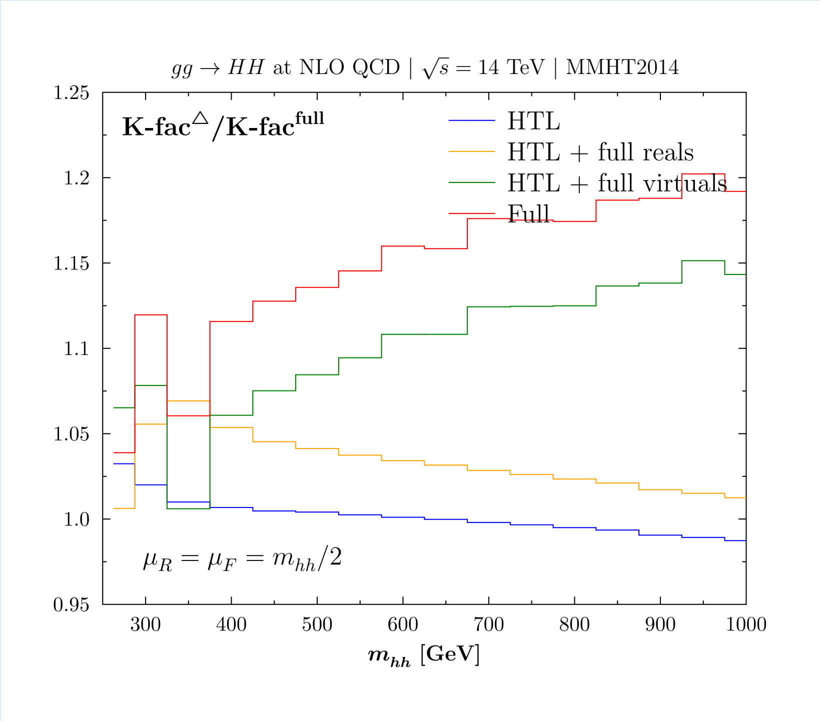



### Box contributions

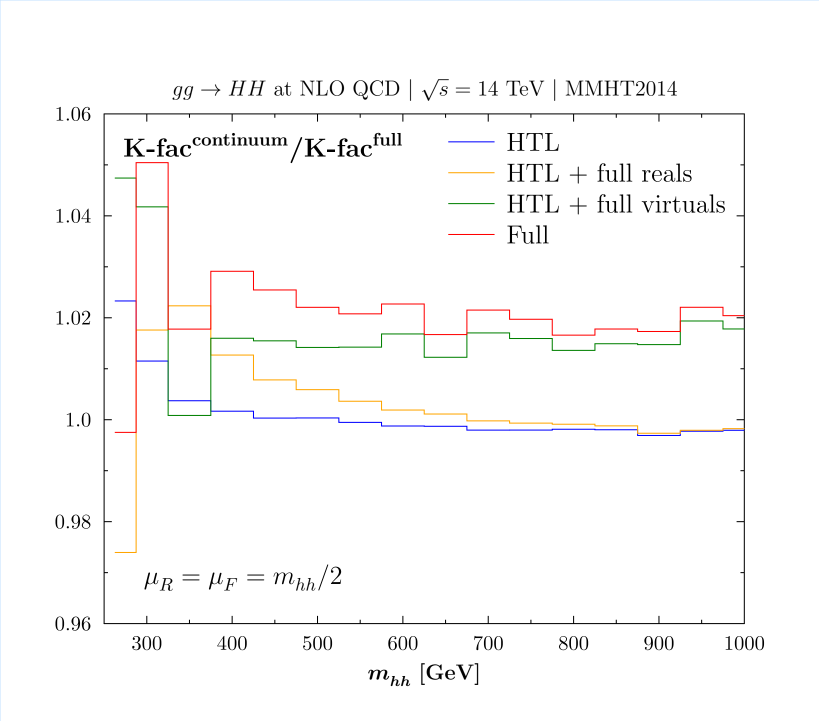


# Results

## Triangular contributions




## Box contributions



## Uncertainty due to $m_t$ for single Higgs

→ $\overline{MS}$ top mass in the range [Q/4,Q]

$$\sigma(gg \to H)\big|_{m_H=125\,\mathrm{GeV}} = 42.17^{+0.4\%}_{-0.5\%}\ \mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=300\,\mathrm{GeV}} = 9.85^{+7.5\%}_{-0.3\%}\ \mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=400\,\mathrm{GeV}} = 9.43^{+0.1\%}_{-0.9\%}\ \mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=600\,\mathrm{GeV}} = 1.97^{+0.0\%}_{-15.9\%}\ \mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=900\,\mathrm{GeV}} = 0.230^{+0.0\%}_{-22.3\%}\ \mathrm{pb}$$
$$\sigma(gg \to H)\big|_{m_H=1200\,\mathrm{GeV}} = 0.0402^{+0.0\%}_{-26.0\%}\ \mathrm{pb}$$