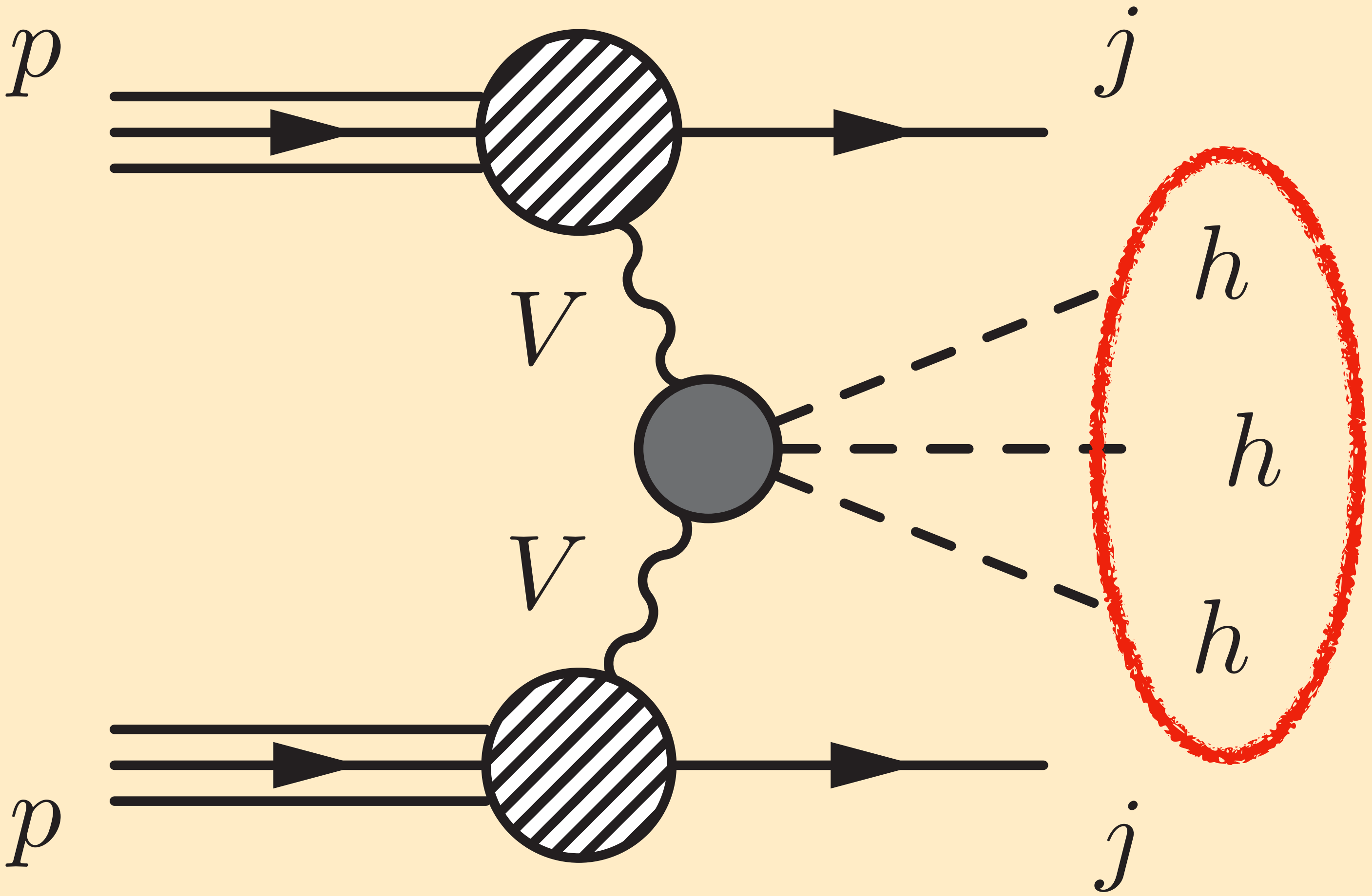
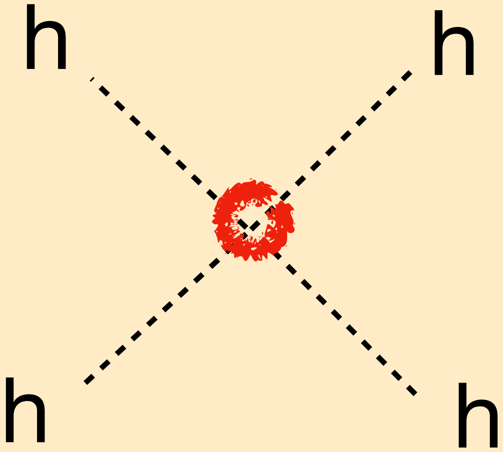




Triple Higgs and quartic interactions at future colliders

[pp + μμ *collisions*]

INFN Roma *Barbara Mele*

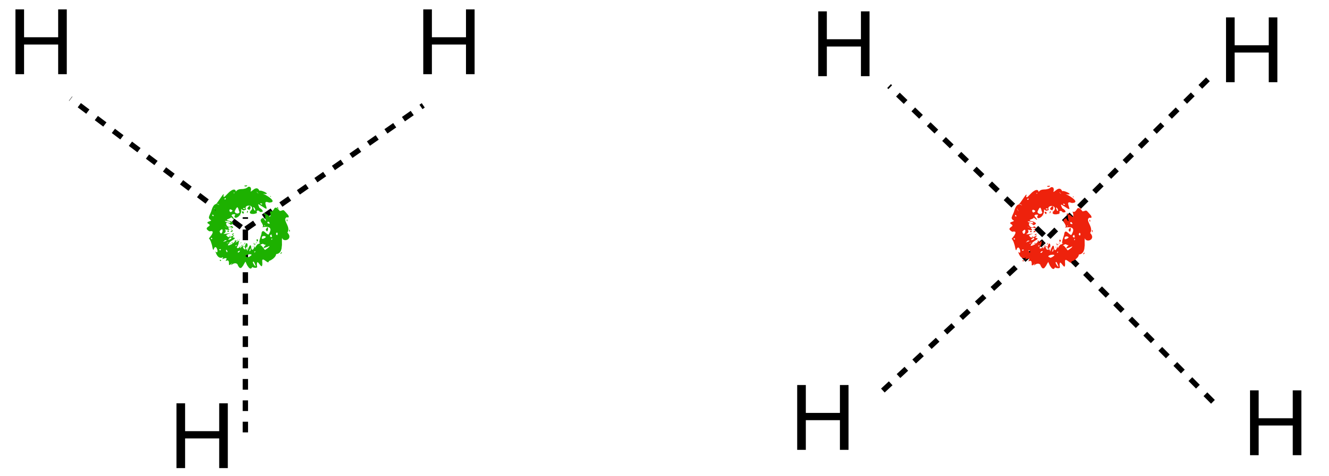
Higgs self-interaction couplings

- the "tough topic" even at "most-future" colliders
- most interesting to measure from theory side....

Higgs potential :

$$\mathcal{L} = -\frac{1}{2} m_h^2 h^2 - \boxed{\lambda_3} \frac{m_h^2}{2v} h^3 - \boxed{\lambda_4} \frac{m_h^2}{8v^2} h^4$$

$$\lambda_3^{SM} = \lambda_4^{SM} = 1$$



previous talks

this talk

- HHH projected reach on λ_4 (λ_3) at 100-TeV pp collider [$gg \rightarrow HHH$]

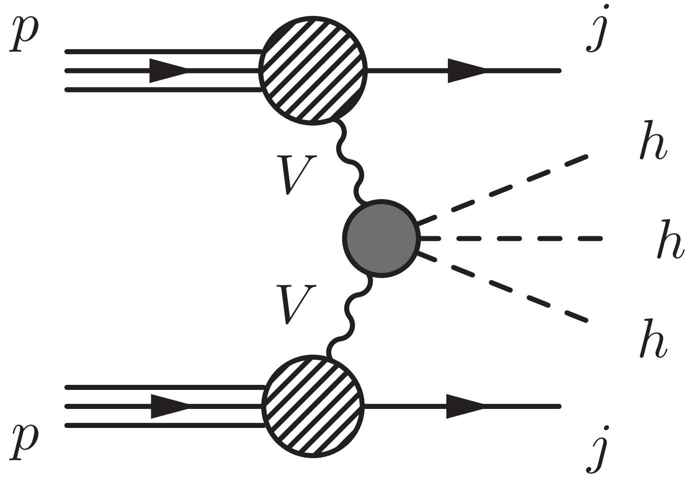
- multi-TeV $\mu\mu$ collider → tentative parameters and timescale after EPPSU

- HHH projected reach on λ_4 (λ_3) at multi-TeV $\mu\mu$ colliders [VBF → HHH]

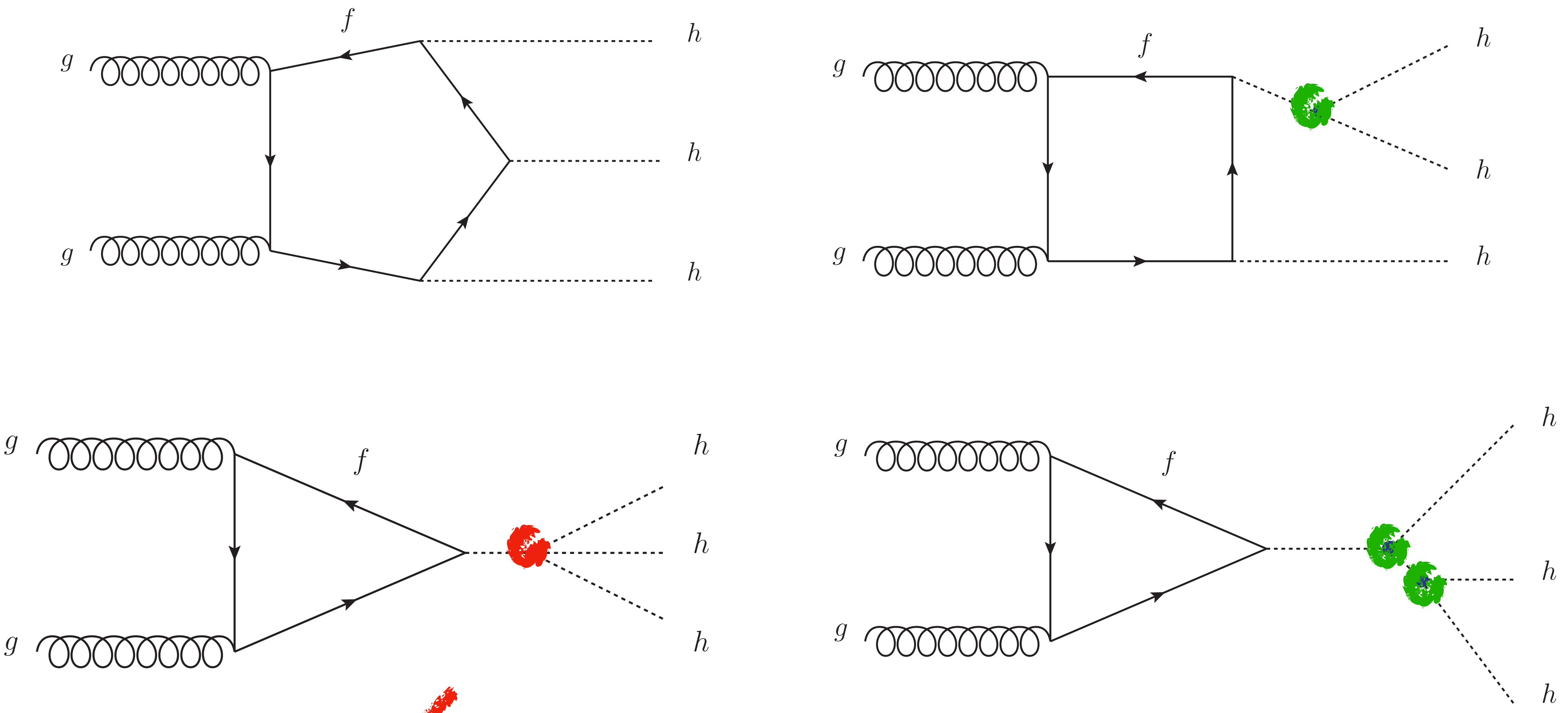
recent study by Chiesa et al.
JHEP 09 (2020) 098

not covered here: indirect λ_4 bounds from
H and HH production [see 1810.04665,1811.12366]

pp collisions

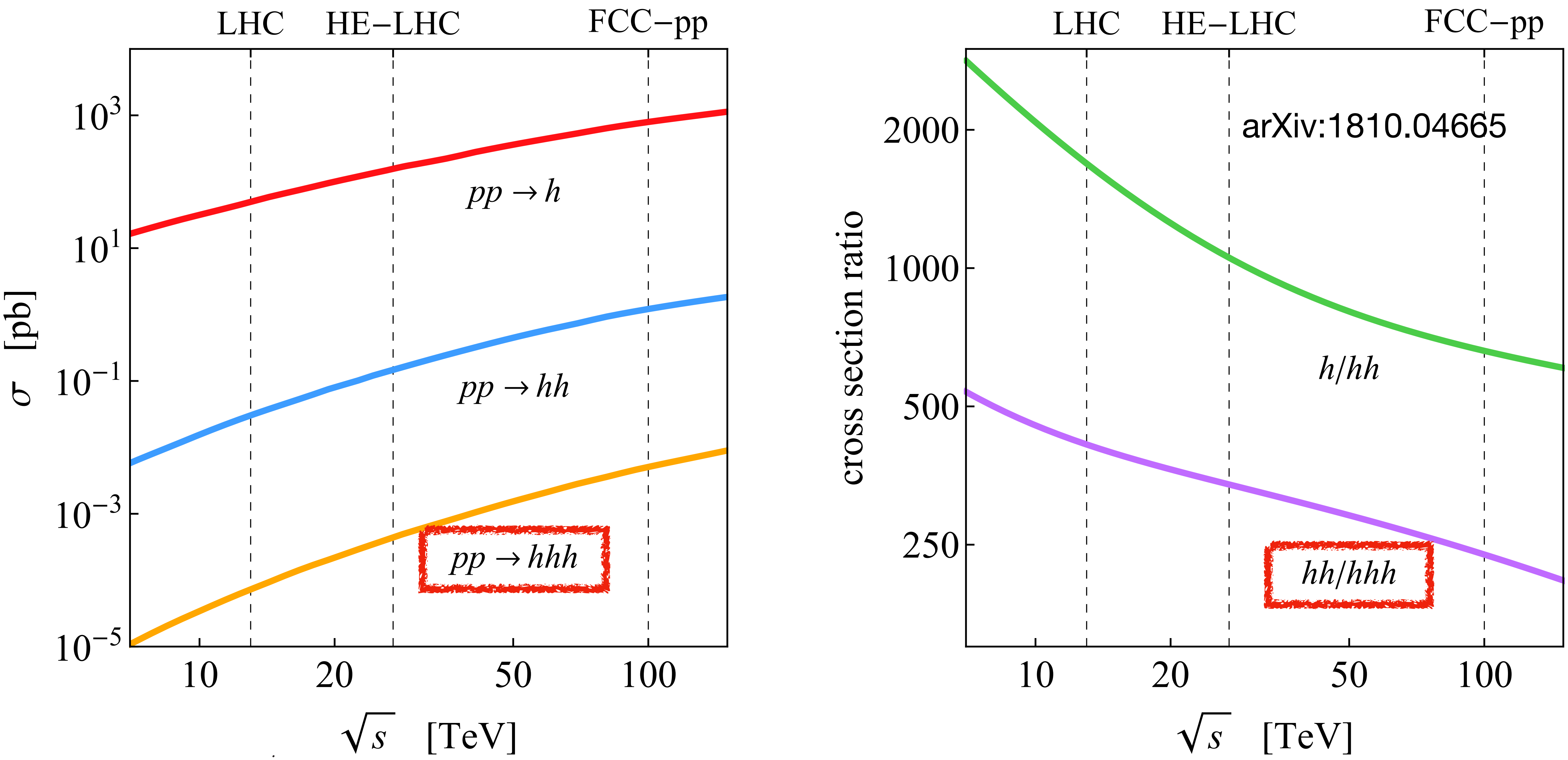


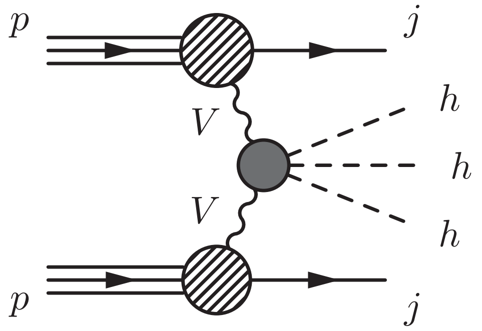
✱ gg → HHH



$$\sigma(\lambda_3, \lambda_4) = A\lambda_4^2 + (B\lambda_3^2 + C\lambda_3 + D)\,\lambda_4 + E\lambda_3^4 + F\lambda_3^3 + G\lambda_3^2 + H\lambda_3 + I$$

(SM) $\sigma_{(HHH)}$ vs $\sigma_{(HH, H)}$ [pp collisions]





$\sigma_{(HHH)} < \sigma_{(HH)}/100 < \sim$ fb

at $\sqrt{S} <$ 100 TeV

arXiv:1508.06524

- $hhh \to (b\bar{b})(b\bar{b})(b\bar{b})$
- $hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$,
- $hhh \to (b\bar{b})(b\bar{b})(\tau^+\tau^-)$,
- $hhh \to (b\bar{b})(\tau^+\tau^-)(\tau^+\tau^-)$,
- $hhh \to (b\bar{b})\ (W^+W^+)(W^+W^-)$

$hhh \to$ final state	BR (%)	σ (ab)	$N_{30\mathrm{ab}^{-1}}$
$(b\bar{b})(b\bar{b})(b\bar{b})$	19.21	1110.338	33310
$(b\bar{b})(b\bar{b})(WW_{1\ell})$	7.204	416.41	12492
$(b\bar{b})(b\bar{b})(\tau\bar{\tau})$	6.312	364.853	10945
$(b\bar{b})(\tau\bar{\tau})(WW_{1\ell})$	1.578	91.22	2736
$(b\bar{b})(b\bar{b})(WW_{2\ell})$	0.976	56.417	1692
$(b\bar{b})(WW_{1\ell})(WW_{1\ell})$	0.901	52.055	1561
$(b\bar{b})(\tau\bar{\tau})(\tau\bar{\tau})$	0.691	39.963	1198
$(b\bar{b})(b\bar{b})(ZZ_{2\ell})$	0.331	19.131	573
$(b\bar{b})(WW_{2\ell})(WW_{1\ell})$	0.244	14.105	423
$(b\bar{b})(b\bar{b})(\gamma\gamma)$	0.228	13.162	394
$(b\bar{b})(\tau\bar{\tau})(WW_{2\ell})$	0.214	12.359	370
$(\tau\bar{\tau})(WW_{1\ell})(WW_{1\ell})$	0.099	5.702	171
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{1\ell})$	0.086	4.996	149
$(b\bar{b})(ZZ_{2\ell})(WW_{1\ell})$	0.083	4.783	143
$(b\bar{b})(\tau\bar{\tau})(ZZ_{2\ell})$	0.073	4.191	125
$(b\bar{b})(\gamma\gamma)(WW_{1\ell})$	0.057	3.291	98
$(b\bar{b})(\tau\bar{\tau})(\gamma\gamma)$	0.05	2.883	86
$(WW_{1\ell})(WW_{1\ell})(WW_{1\ell})$	0.038	2.169	65
$(\tau\bar{\tau})(WW_{2\ell})(WW_{1\ell})$	0.027	1.545	46
$(\tau\bar{\tau})(\tau\bar{\tau})(\tau\bar{\tau})$	0.025	1.459	43
$(b\bar{b})(WW_{2\ell})(WW_{2\ell})$	0.017	0.956	28
$(WW_{2\ell})(WW_{1\ell})(WW_{1\ell})$	0.015	0.882	26
$(b\bar{b})(b\bar{b})(ZZ_{4\ell})$	0.012	0.69	20
$(\tau\bar{\tau})(\tau\bar{\tau})(WW_{2\ell})$	0.012	0.677	20
$(b\bar{b})(ZZ_{2\ell})(WW_{2\ell})$	0.011	0.648	19
$(\tau\bar{\tau})(ZZ_{2\ell})(WW_{1\ell})$	0.009	0.524	15
$(b\bar{b})(\gamma\gamma)(WW_{2\ell})$	0.008	0.446	13
$(\tau\bar{\tau})(\gamma\gamma)(WW_{1\ell})$	0.006	0.36	10

many many different HHH final states with N_{ev} > 10 at 100 TeV (30 ab^{-1})

quite a few studies of gg → HHH at pp colliders :

hep-ph/0507321, arXiv:1508.06524
arXiv:1510.04013, arXiv:1602.05849
arXiv:1606.09408, arXiv:1702.03554
arXiv:1704.04298, arXiv:1708.03580
arXiv:1810.04665, arXiv:1811.12366
arXiv:1909.09166 . . .

anomalous Higgs self-coupling parametrization

$$\lambda^{\rm SM}_{hhh} = \lambda^{\rm SM}_{hhhh} = \frac{m_h^2}{2v^2}$$

$$V_{\rm h} = \frac{m_h^2}{2} h^2 + (1+\delta_3)\lambda^{\rm SM}_{hhh} v h^3 + \frac{1}{4}(1+\delta_4)\lambda^{\rm SM}_{hhhh} h^4$$

typical of well-behaved EFTs → → $\delta_3 = \bar{c}_6$ $\qquad$ $\delta_4 = 6\,\bar{c}_6 + \bar{c}_8$

$$V^{\rm NP}(\Phi) \equiv \sum_{n=3}^{\infty} \frac{c_{2n}}{\Lambda^{2n-4}} \left(\Phi^\dagger\Phi - \frac{1}{2}v^2\right)^n$$

$$\bar{c}_6 \equiv \frac{c_6 v^2}{\lambda^{SM}\Lambda^2} = \delta_3$$

$$\bar{c}_8 \equiv \frac{4 c_8 v^4}{\lambda^{SM}\Lambda^4} = \delta_4 - 6\delta_3$$

3 interesting benchmarks :

- $\delta_3 = 0\,;$ *free* δ_4
- $\delta_4 = 6\,\delta_3$ ***(well-behaved SMEFT)***
- *free* (δ_3, δ_4)

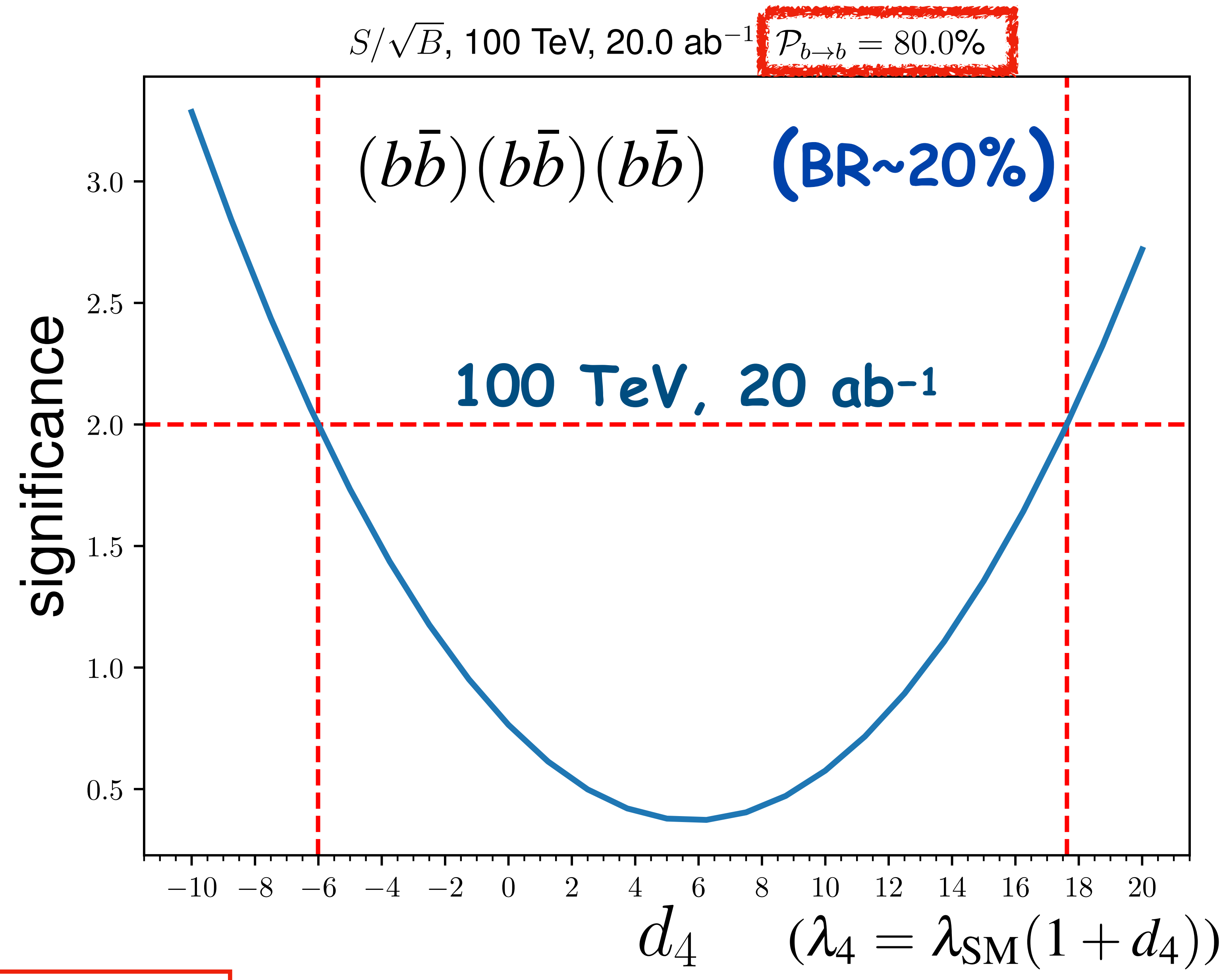
be agnostic about how UV dynamics modifies Higgs self-interactions

→ no assumption about the actual size of (δ_3, δ_4)

$$\lambda_3 = \lambda_{SM}(1+\delta_3)$$

$$\lambda_4 = \lambda_{SM}(1+\delta_4)$$

σ(HHH → bbbbbb) [pp , 100 TeV]



arXiv:1909.09166

$$\lambda_3 = \lambda_{SM}(1+\delta_3)$$

$$\lambda_4 = \lambda_{SM}(1+\delta_4)$$

[δ_3=0] $-6 < \delta_4 < 18$ (95%CL)

"typical" constraining power of HHH in pp

$hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$ **pp, 100 TeV, 30 ab-1**

(BR~0.2%) $S/B \sim 0.5$ $S/\sqrt{B} \sim 2.1$

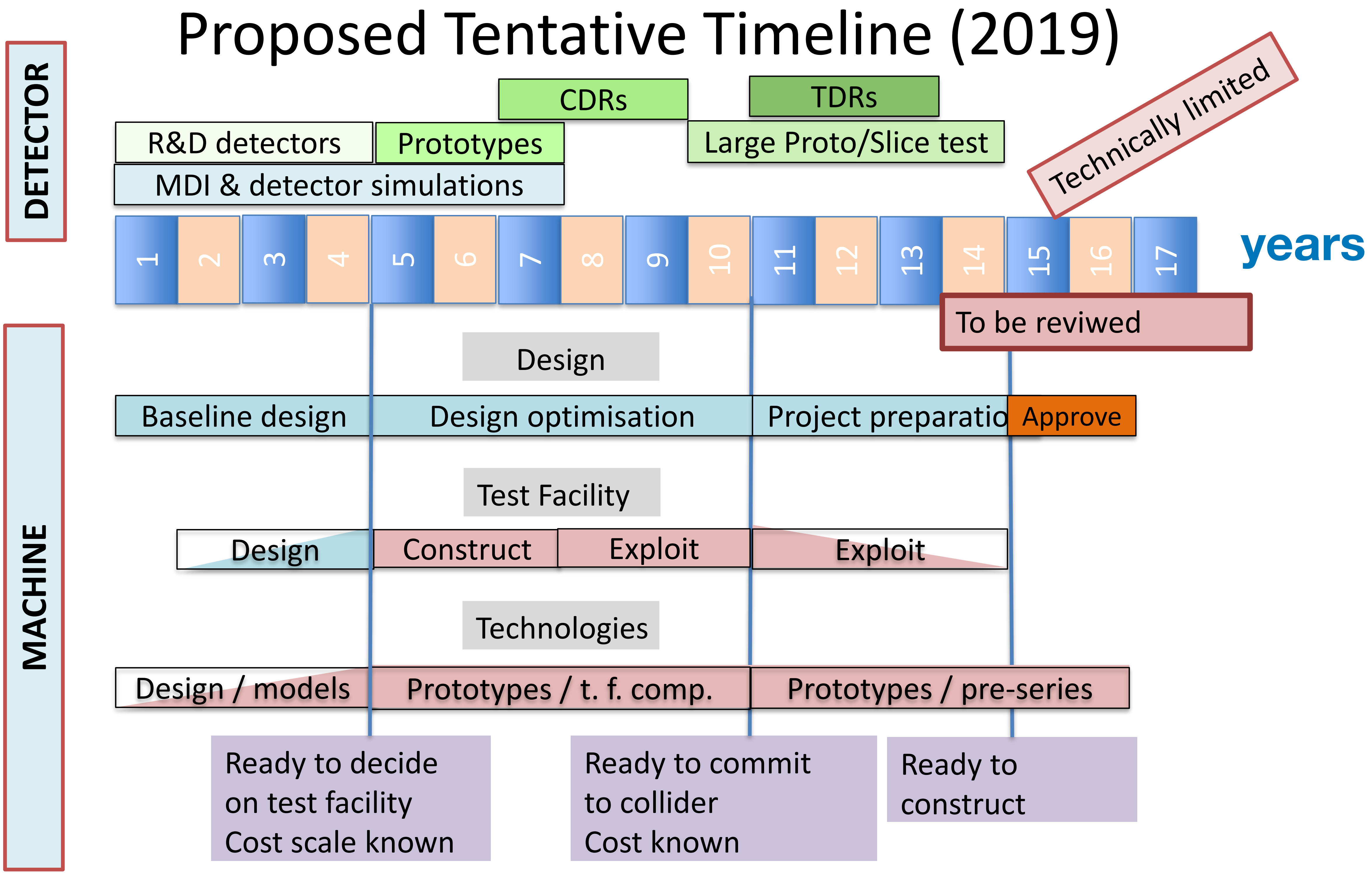
process	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO} \times {\rm BR} \times \mathcal{P}_{\rm tag}$ (ab)	$\epsilon_{\rm analysis}$	$N^{\rm cuts}_{30\ {\rm ab}^{-1}}$
$hhh \to (b\bar{b})(b\bar{b})(\gamma\gamma)$, SM	2.89	5.4	0.06	9.7
$b\bar{b}b\bar{b}\gamma\gamma$	1.28	1050	2.6×10^{-4}	8.2
hZZ, (NLO) $(ZZ \to (b\bar{b})(b\bar{b}))$	0.817	0.8	0.002	$\ll 1$
hhZ, (NLO)$(Z \to (b\bar{b}))$	0.754	0.8	0.007	$\ll 1$
hZ, (NLO) $(Z \to (b\bar{b}))$	8.02×10^3	1130	$\mathcal{O}(10^{-5})$	$\ll 1$
$b\bar{b}b\bar{b}\gamma$ + jets	2.95×10^3	2420	$\mathcal{O}(10^{-5})$	$\mathcal{O}(1)$
$b\bar{b}b\bar{b}$ + jets	5.45×10^3	4460	$\mathcal{O}(10^{-6})$	$\ll 1$
$b\bar{b}\gamma\gamma$ + jets	98.7	4.0	$\mathcal{O}(10^{-5})$	$\ll 1$
hh + jets, SM	275	593	7×10^{-4}	12.4

[δ_3=0]   -5 < δ_4 < 15   (95%CL)

in "optimistic" scenario !!!

arXiv:1508.06524
arXiv:1606.09408

Muon-Collider possible timescale



Proposed Tentative Timeline (2019)

DETECTOR

CDRs

TDRs

R&D detectors

Prototypes

Large Proto/Slice test

MDI & detector simulations

Technically limited

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 **years**

To be reviwed

MACHINE

Design

Baseline design | Design optimisation | Project preparatio | Approve

Test Facility

Design | Construct | Exploit | Exploit

Technologies

Design / models | Prototypes / t. f. comp. | Prototypes / pre-series

Ready to decide
on test facility
Cost scale known

Ready to commit
to collider
Cost known

Ready to
construct

D. Schulte

Muon collider, AF-EF, July 2020

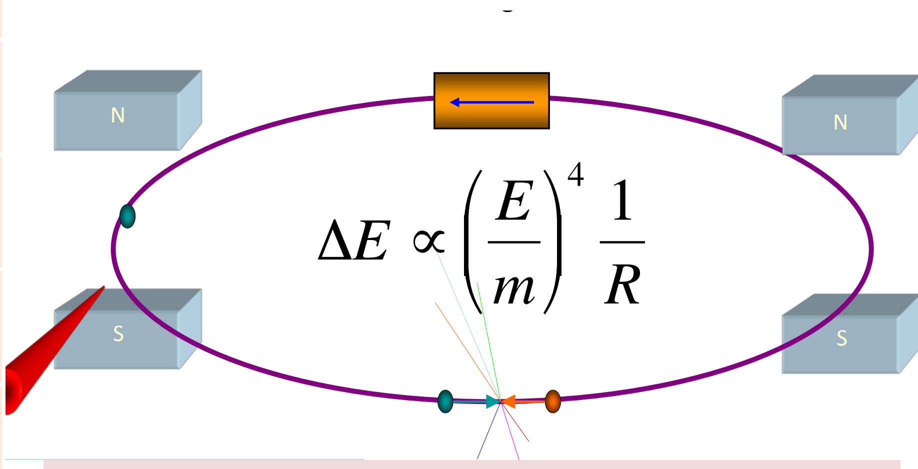
Physics Briefing Book
arXiv:1910.11775

Tentative Target Parameters

Parameter	Unit	3 TeV	10 TeV	14 TeV
L	10^{34} cm^{-2}s^{-1}	1.8	20	40
N	10^{12}	2.2	1.8	1.8
f_r	Hz	5	5	5
P_{beam}	MW	5.3	14.4	20
C	km	4.5	10	14
<B>	T	7	10.5	10.5
ε_L	MeV m	7.5	7.5	7.5
σ_E / E	%	0.1	0.1	0.1
σ_z	mm	5	1.5	1.07
β	mm	5	1.5	1.07
ε	µm	25	25	25
$\sigma_{x,y}$	µm	3.0	0.9	0.63

Schulte, July 2020

Based on extrapolation of MAP parameters



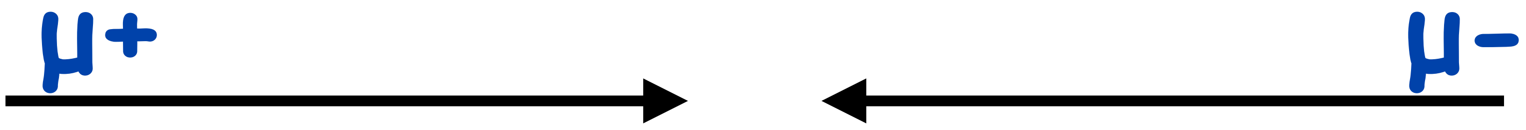
cf. CLIC_3TeV requiring a 50 Km tunnel !

$$L \sim 10^{35} cm^{-2} s^{-1} \sim 1\, ab^{-1}/y$$

integrated lumi for 5 years (10^7 s) run

$$\mathcal{L} = \boxed{(E_{\rm CM}/10\ \ {\rm TeV})^2 \times 10\,{\rm ab}^{-1}}$$

our "reference frame" :



$\sqrt{S}_{\mu\mu} \sim$ 3, 6, 10, 14, 30 TeV

$$\mathcal{L} = (E_{\rm CM}/10\ {\rm TeV})^2 \times 10\,{\rm ab}^{-1}$$

$\sim 1\,fb\,(\frac{10\,TeV}{\sqrt{S}})^2$

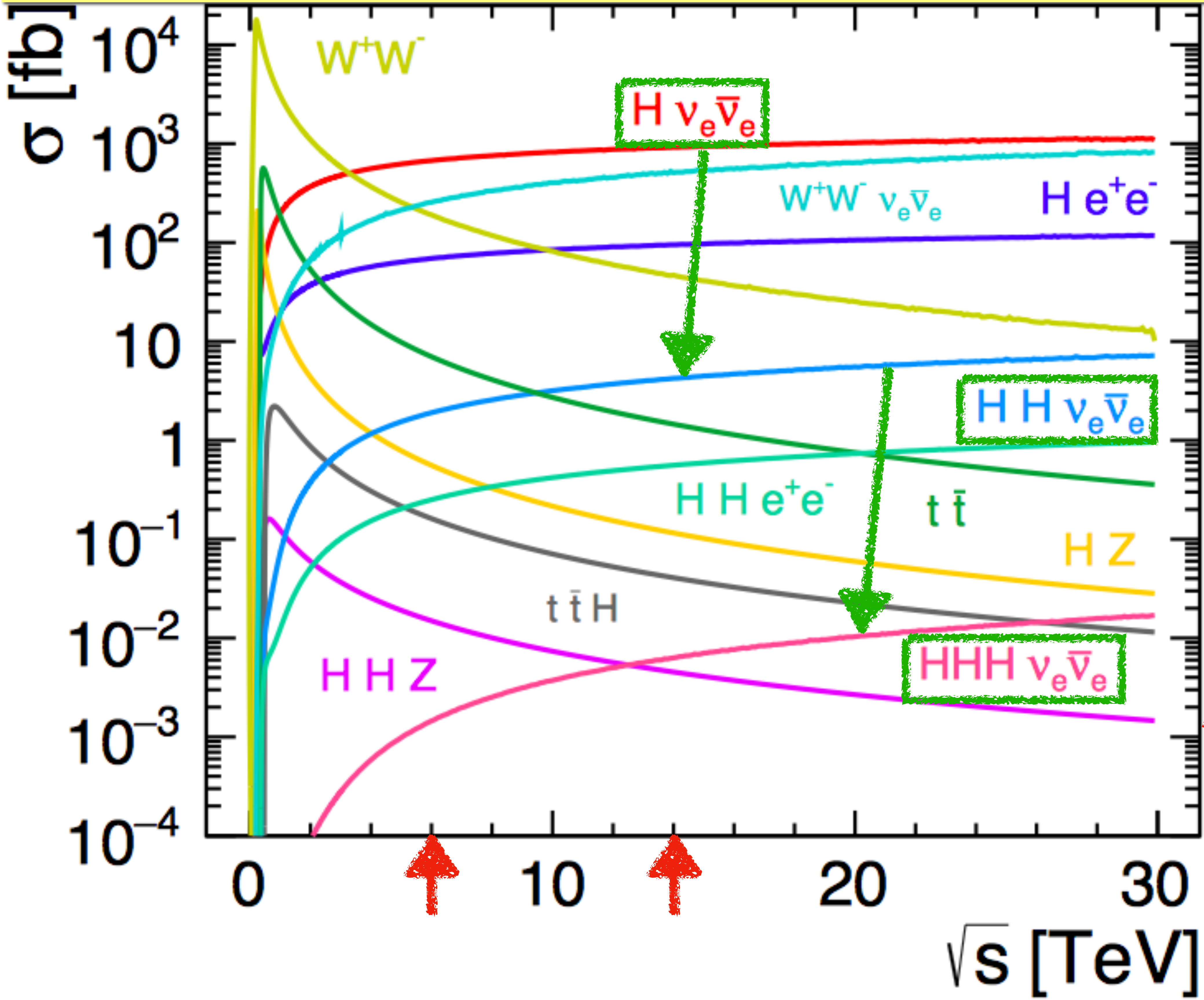
$\sigma_{point} \times \int L \sim 10^4$ evts

$\delta_{stat} \sim 1\%$

allows precision on whatever is pair-produced in s-channel !

not yet systematic Physics studies, just a few preliminary projections !

at [$\sqrt{S_{\mu\mu}}$ > a few TeV's], point $\sigma_{\mu\mu\to X}$ (~1/s) superseeded by $\sigma_{WW\to X}$ (~log^n s) !



[10^2 ab^{-1}]

10^8ev

10^6ev

10^4ev

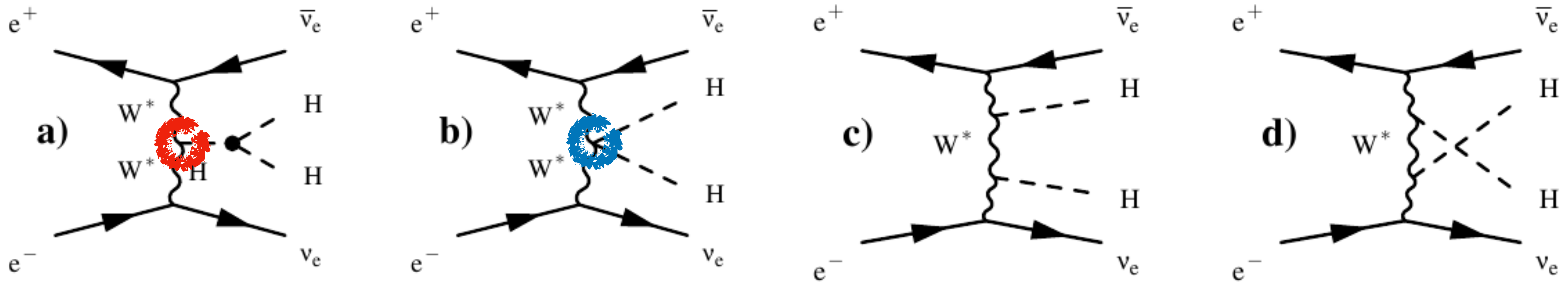
plenty of Higgs's !

in a clean environment !

trilinear Higgs coupling at Muon Colliders

$$\mathcal{L} = -\frac{1}{2}m_h^2 h^2 - \boxed{\lambda_3}\frac{m_h^2}{2v}h^3 - \lambda_4\frac{m_h^2}{8v^2}h^4$$

✱ **40.000 HH pairs at 14 TeV !**



$HH \to 4b$

$$p_T(b) > 30\ \text{GeV}, \quad 10^\circ < \theta_b < 170^\circ, \quad \Delta R_{bb} > 0.4. \quad |m_{jj} - m_H| < 15\ \text{GeV}$$

$\sqrt{s}$ (TeV) benchmark lumi (ab^{-1})	3 1	6 4	10 10	14 20	30 90
HHWW $(\Delta\kappa_{W_2})_{\text{in}}$	5.3%	1.3%	0.62%	0.41%	0.20%
HHH $(\Delta\kappa_3)_{\text{in}}$	25%	10%	5.6%	3.9%	2.0%

(other projects)

5% CLIC

5% FCC-hh 68%CL

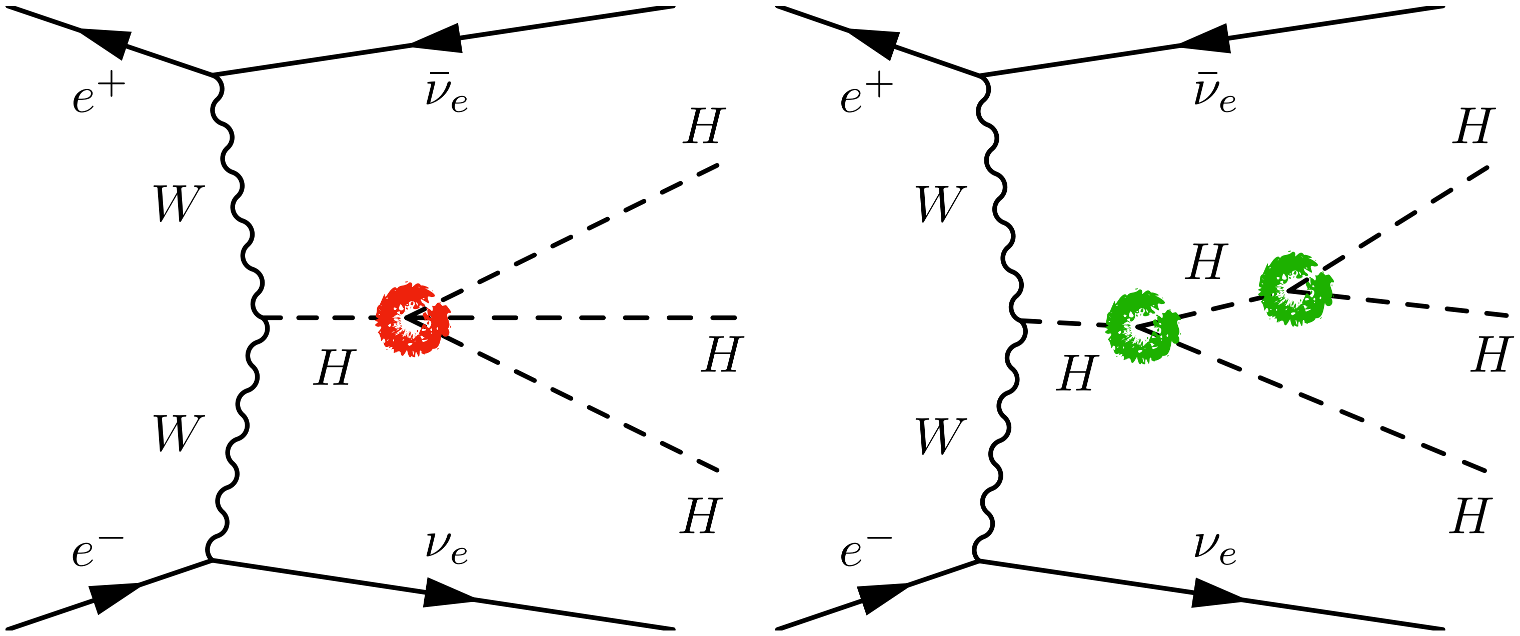
(95% CL, single-parameter fit)

T. Han et al. arXiv:2008.12204

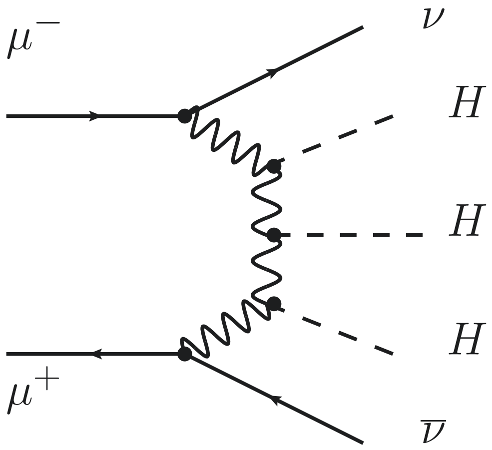
talk by Xing Wang in "Precision Higgs IV"

$\mu^+\mu^- \to HHH\nu\bar{\nu},\ (\nu = \nu_e, \nu_\mu, \nu_\tau)$

$$V_{\rm h} = \frac{m_h^2}{2}h^2 + (1+\delta_3)\lambda_{hhh}^{\rm SM} v h^3 + \frac{1}{4}(1+\delta_4)\lambda_{hhhh}^{\rm SM} h^4$$

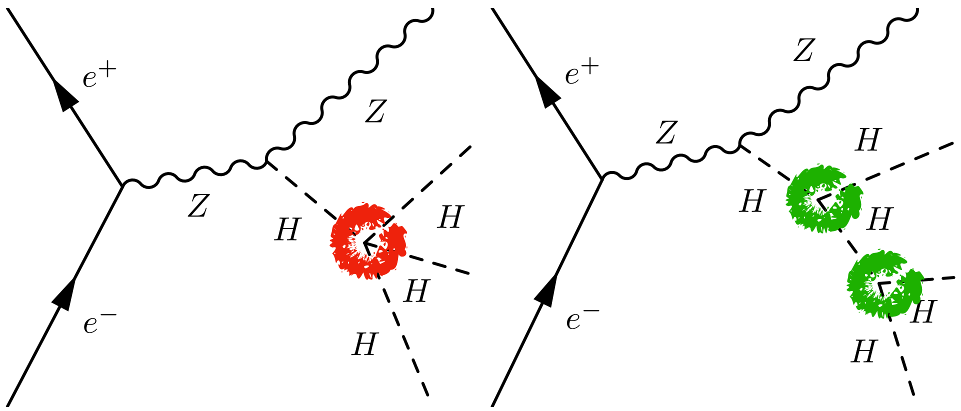


$$\mu^+\mu^- \to W^*W^*\nu_\mu\bar{\nu}_\mu$$
$$\to HHH\nu_\mu\bar{\nu}_\mu$$



+ . . .

$$\sigma = c_1 + c_2\delta_3 + c_3\delta_4 + c_4\delta_3\delta_4 + c_5\delta_3^2 + c_6\delta_4^2 + c_7\delta_3^3 + c_8\delta_3^2\delta_4 + c_9\delta_3^4$$



HHHZ subdominant !

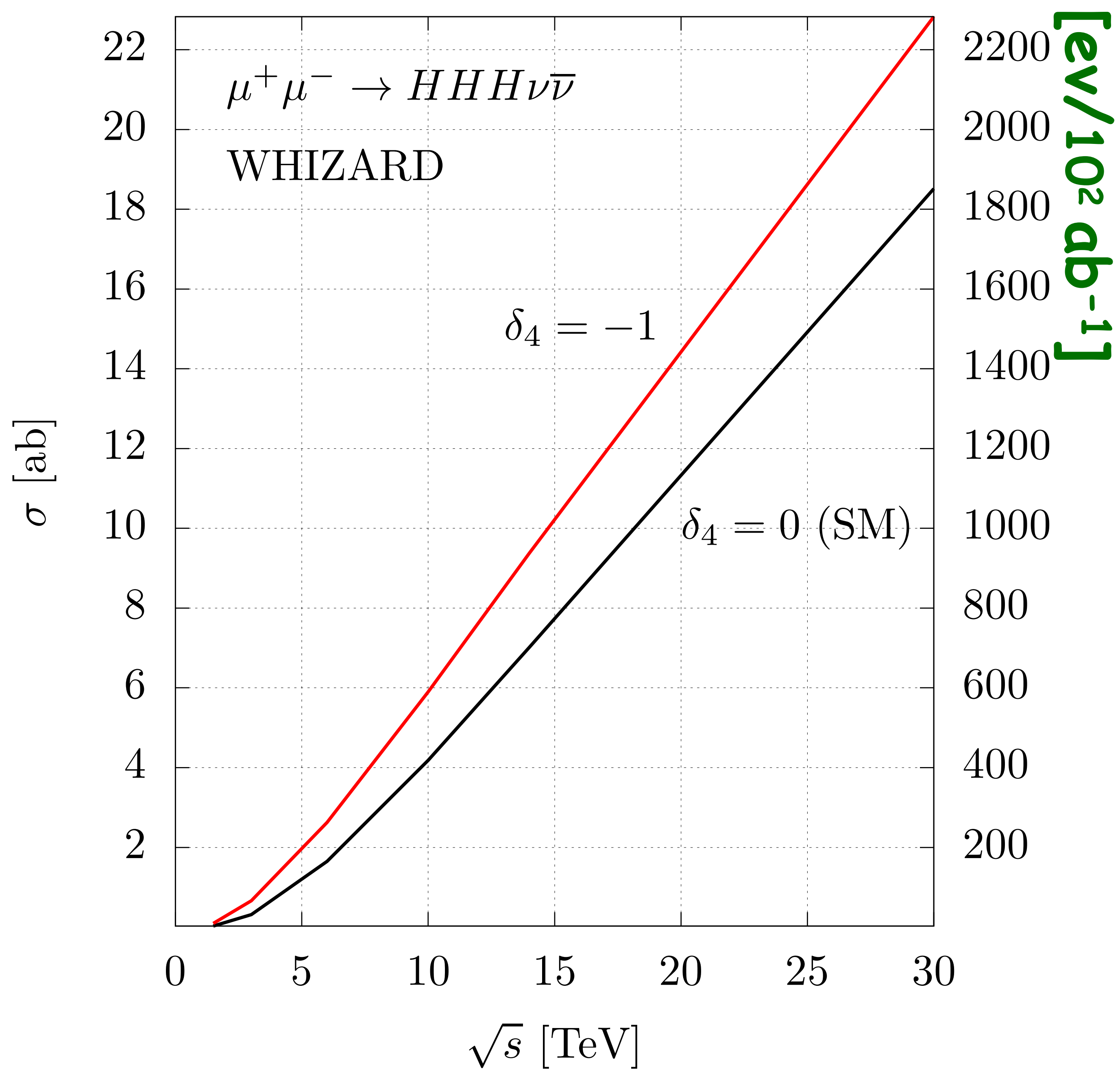
$\sigma_{\rm HHHZ}$ ~ 1/2 $\sigma_{\rm HHH\nu\nu}$ @ 3TeV

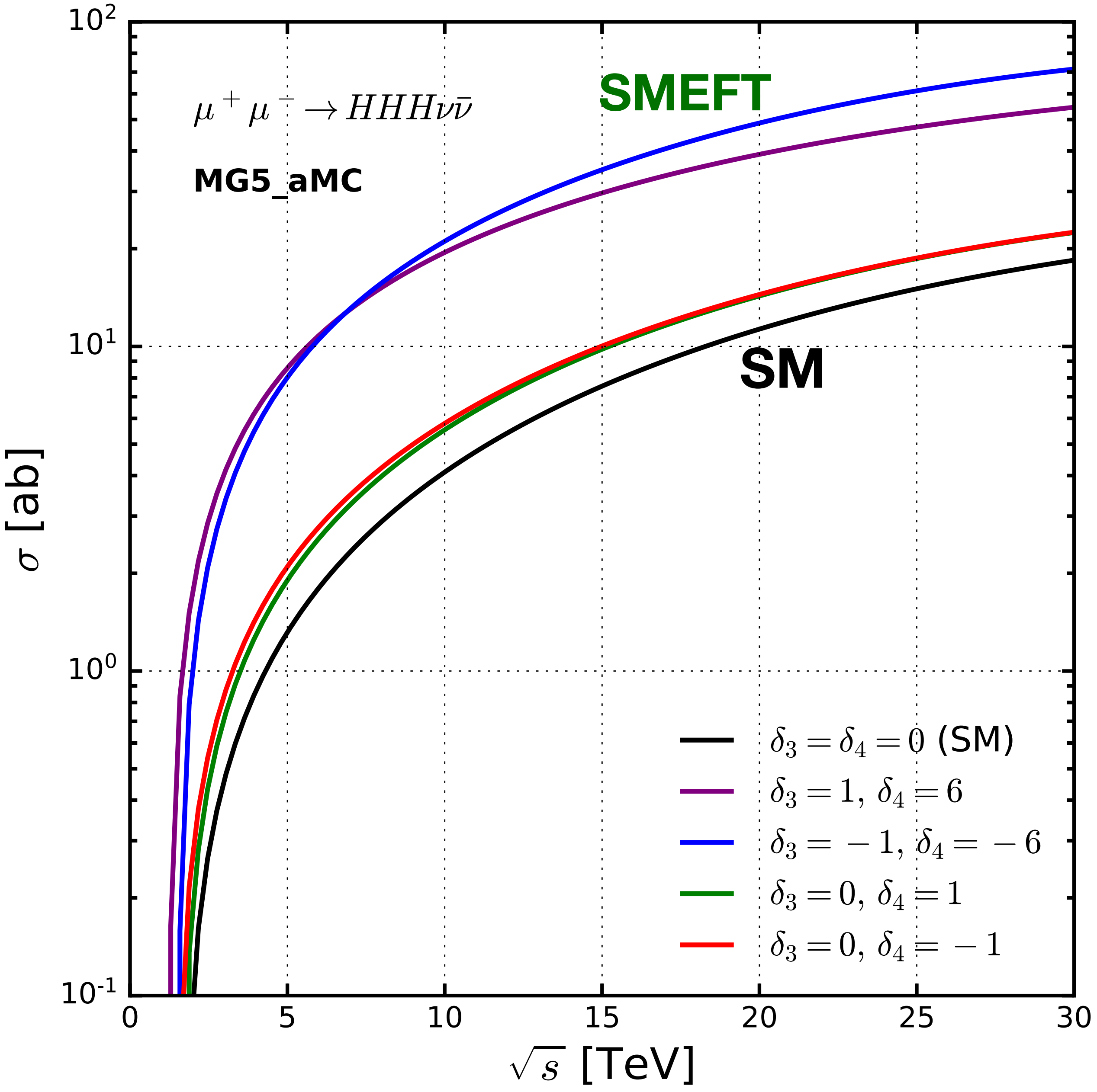
~ 1/50 $\sigma_{\rm HHH\nu\nu}$ @ 30TeV

$\sigma_{HHH\nu\nu}\ (\delta_3\ ,\ \delta_4)$ [μμ collisions]

Chiesa, Maltoni, Mantani, BM, Piccinini, Zhao,
arXiv:2003.13628, JHEP 09 (2020) 098

$$\lambda_3 = \lambda_{SM}(1+\delta_3)$$
$$\lambda_4 = \lambda_{SM}(1+\delta_4)$$

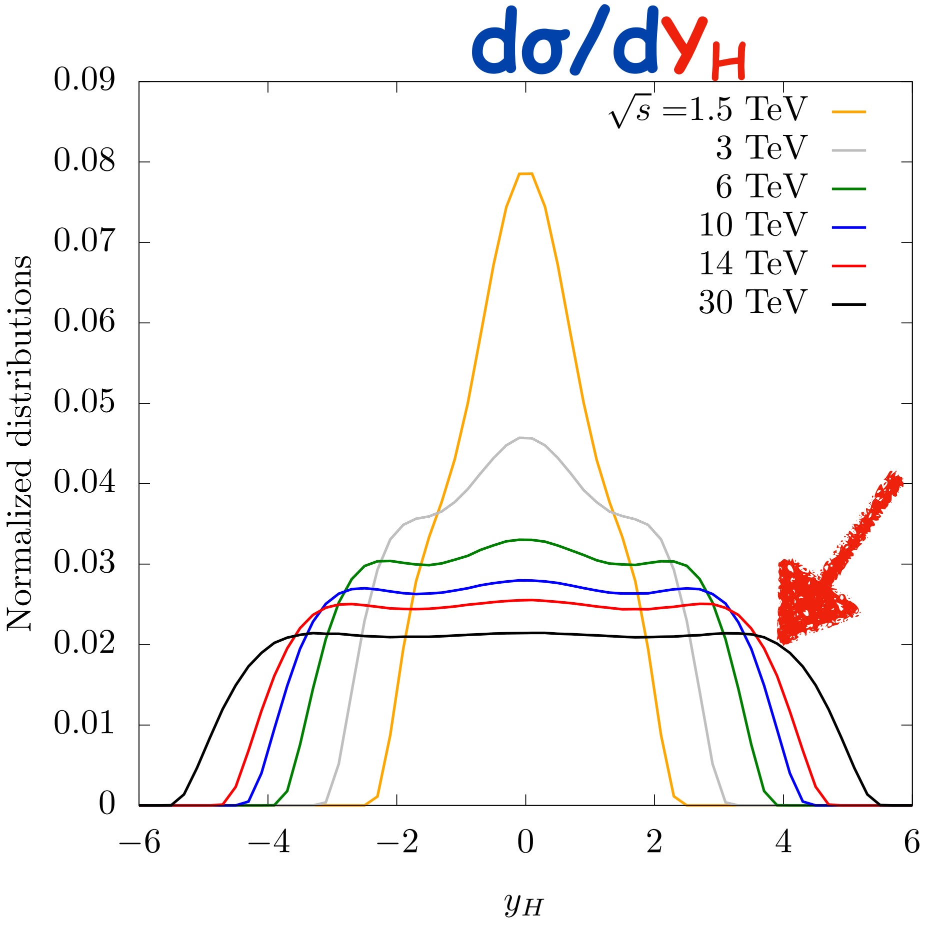


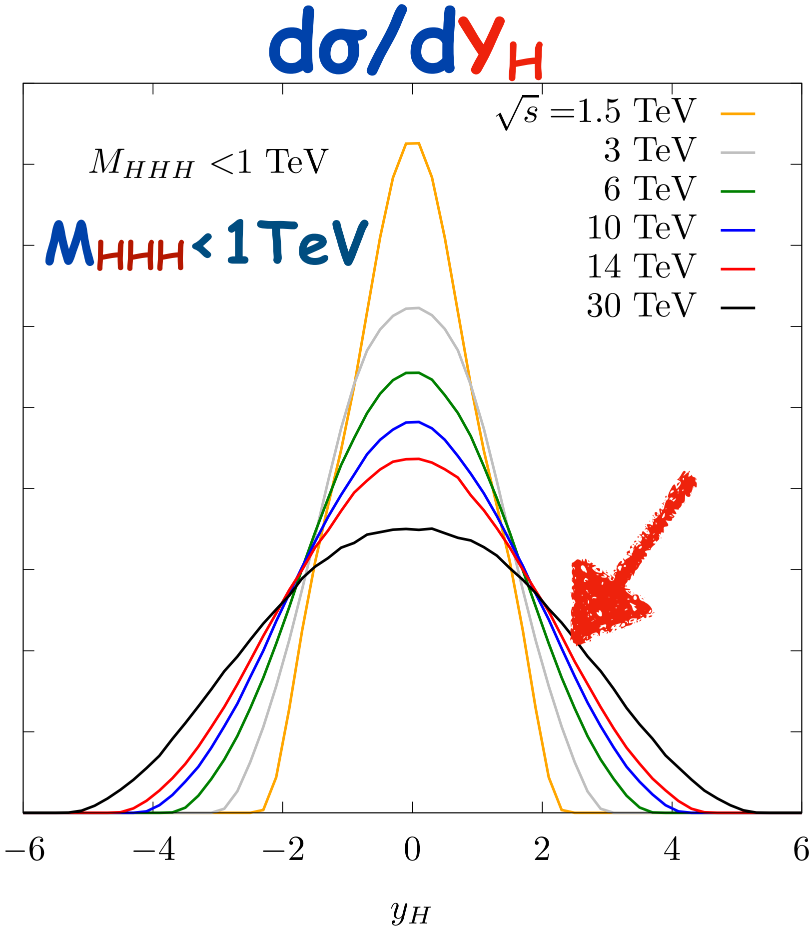


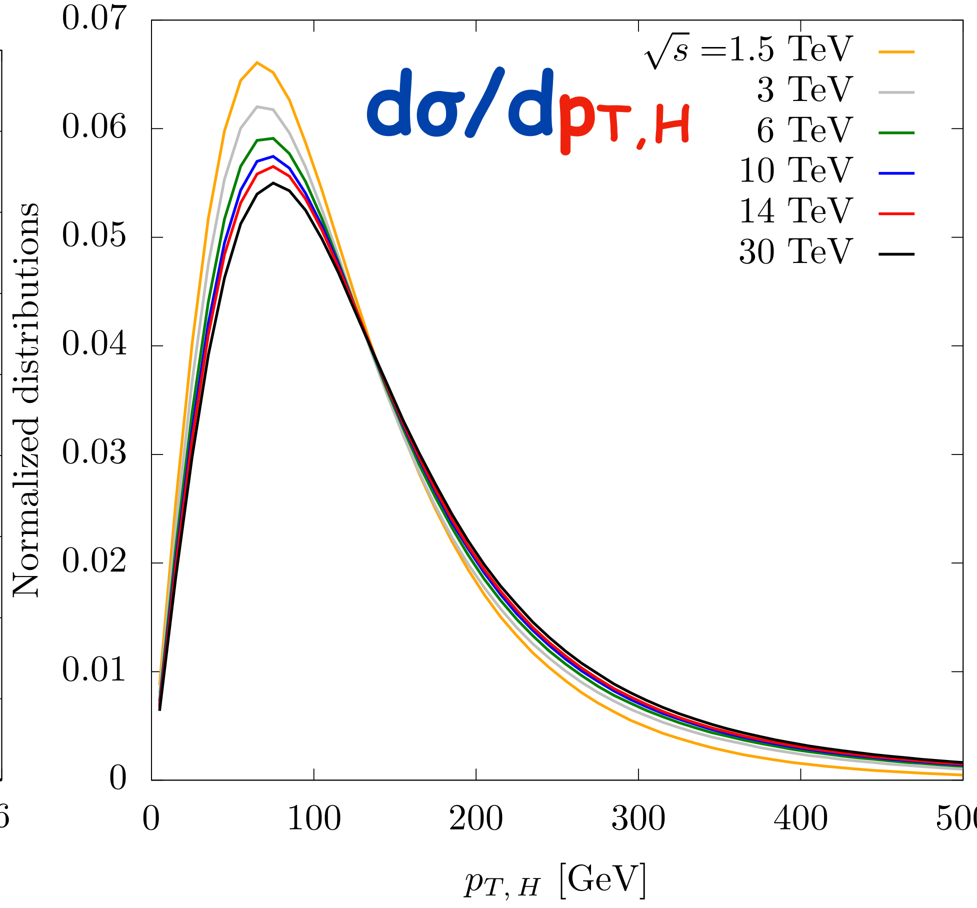
$\sigma_{HHH\nu\nu}$ (SM) [$\mu\mu$ collisions]

$\sqrt{s}$ (TeV) / L (ab^{-1})	10 / 20	14 / 33	30 / 100
σ_{SM} (ab) $[N_{\mathrm{ev}}]$			
σ^{tot}	4.18 [84]	7.02 [232]	18.51 [1851]
$\sigma(M_{HHH} < 3\mathrm{TeV})$	2.89 [58]	3.98 [131]	6.69 [669]
$\sigma(M_{HHH} < 1\mathrm{TeV})$	0.37 [7]	0.45 [15]	0.64 [64]

$\boxed{M_{\bar{\nu}\nu} \gtrsim 150\mathrm{GeV}}$ *applied everywhere (selects VBF contribution)*

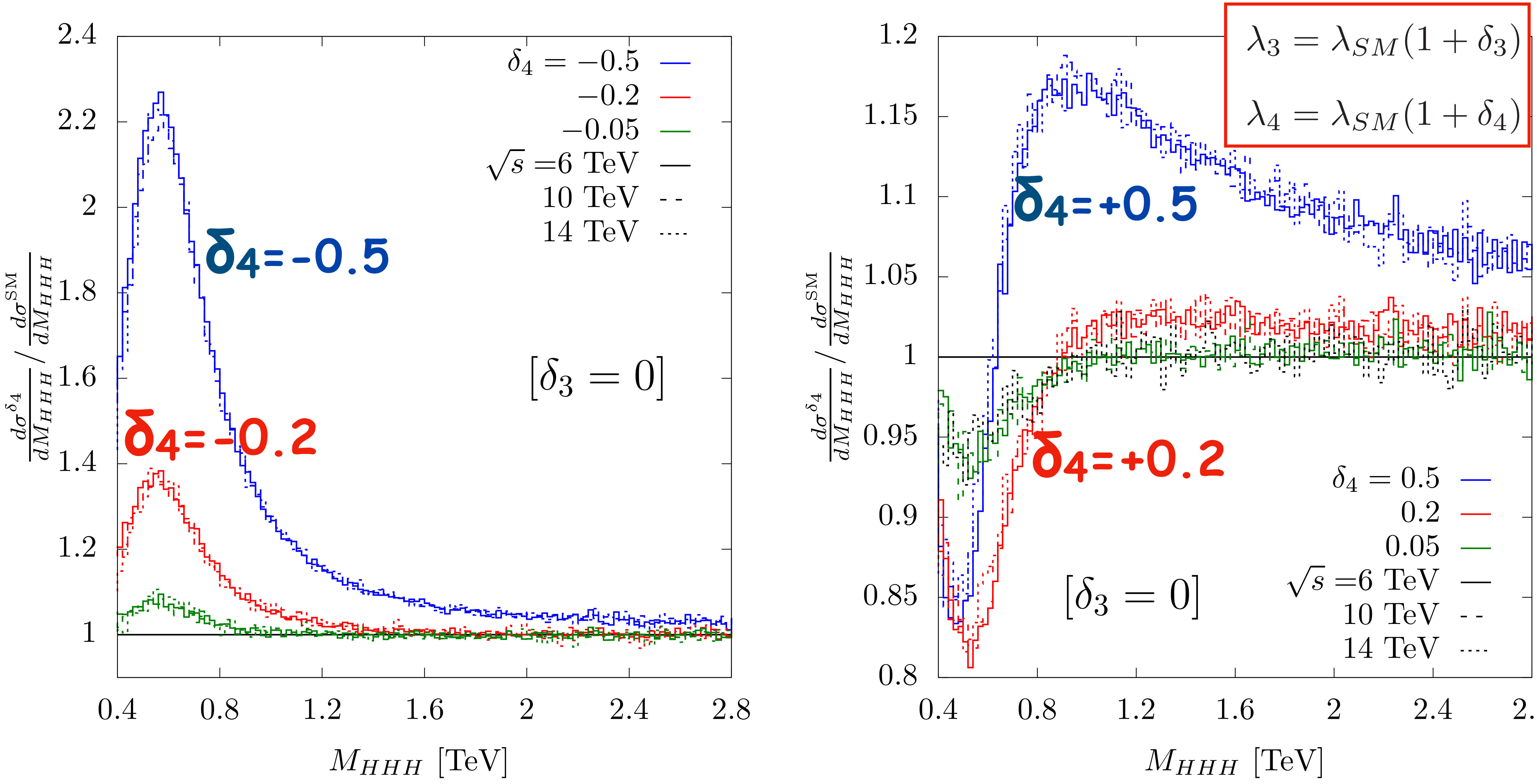






arXiv:2003.13628

m_{HHH} distribution vs δ_4 (δ_3 =0)



$$\lambda_3 = \lambda_{SM}(1 + \delta_3)$$

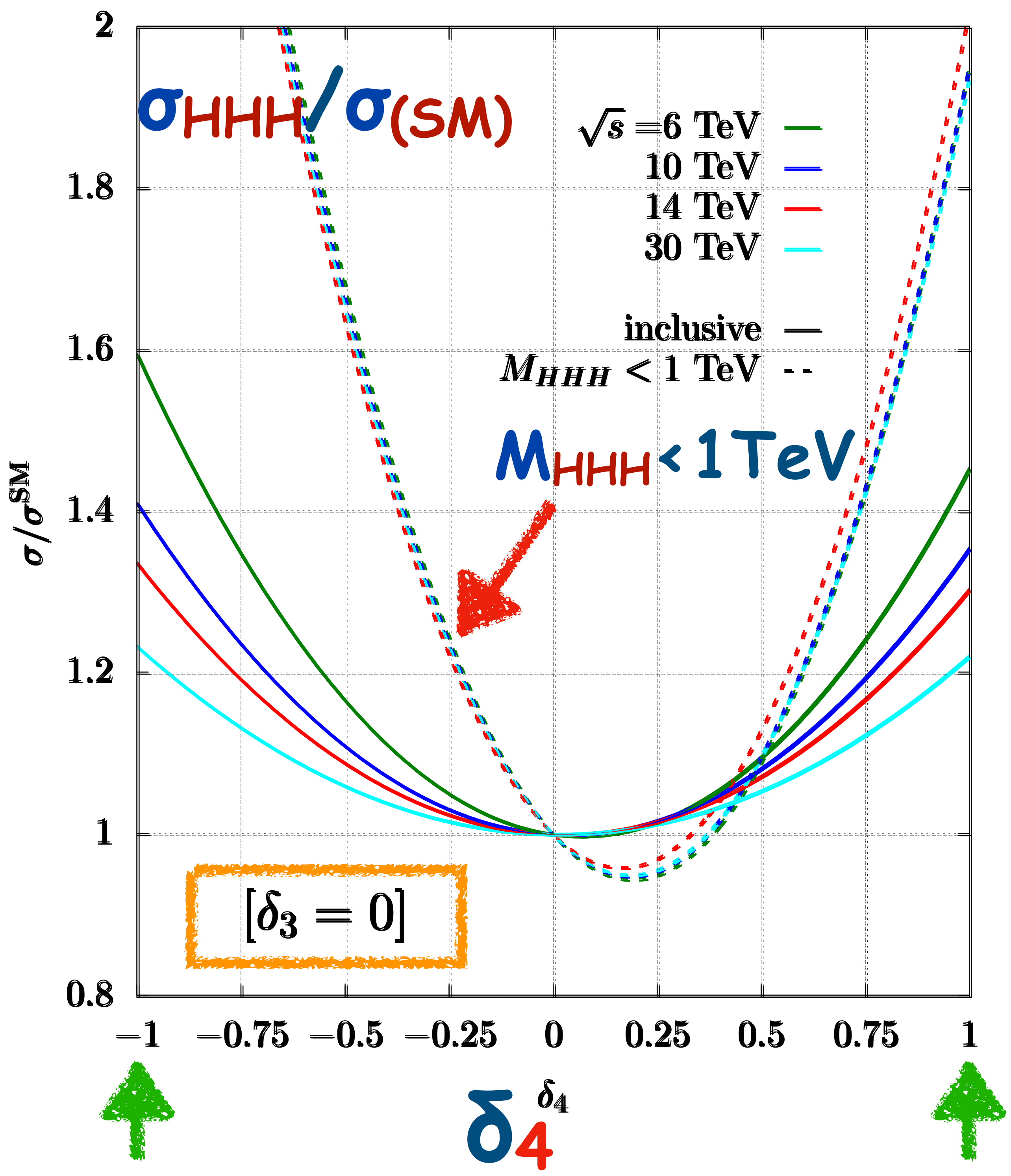
$$\lambda_4 = \lambda_{SM}(1 + \delta_4)$$

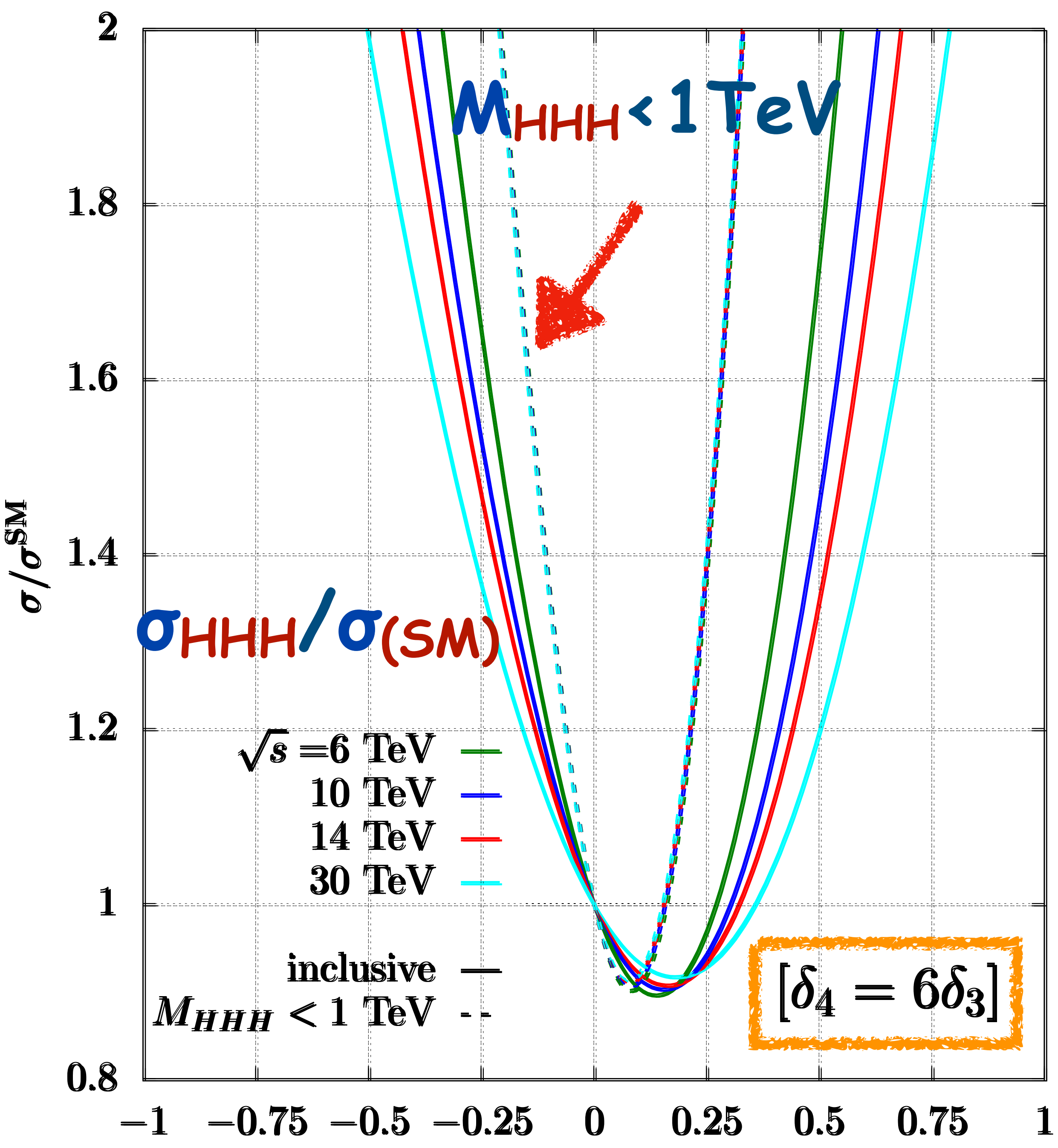
arXiv:2003.13628

- maximal λ_4 (λ_3) sensitivity for M_{HHH} close to threshold [independent of $\sqrt{S_{\mu\mu}}$]

$\sigma_{HHH}/\sigma_{(SM)}$ versus (δ_3, δ_4)

arXiv:2003.13628





$$\lambda_3 = \lambda_{SM}(1+\delta_3)$$

$$\lambda_4 = \lambda_{SM}(1+\delta_4)$$

$(N-N_{SM})/\sqrt{N_{SM}} \sim 1$ vs (δ_3, δ_4)

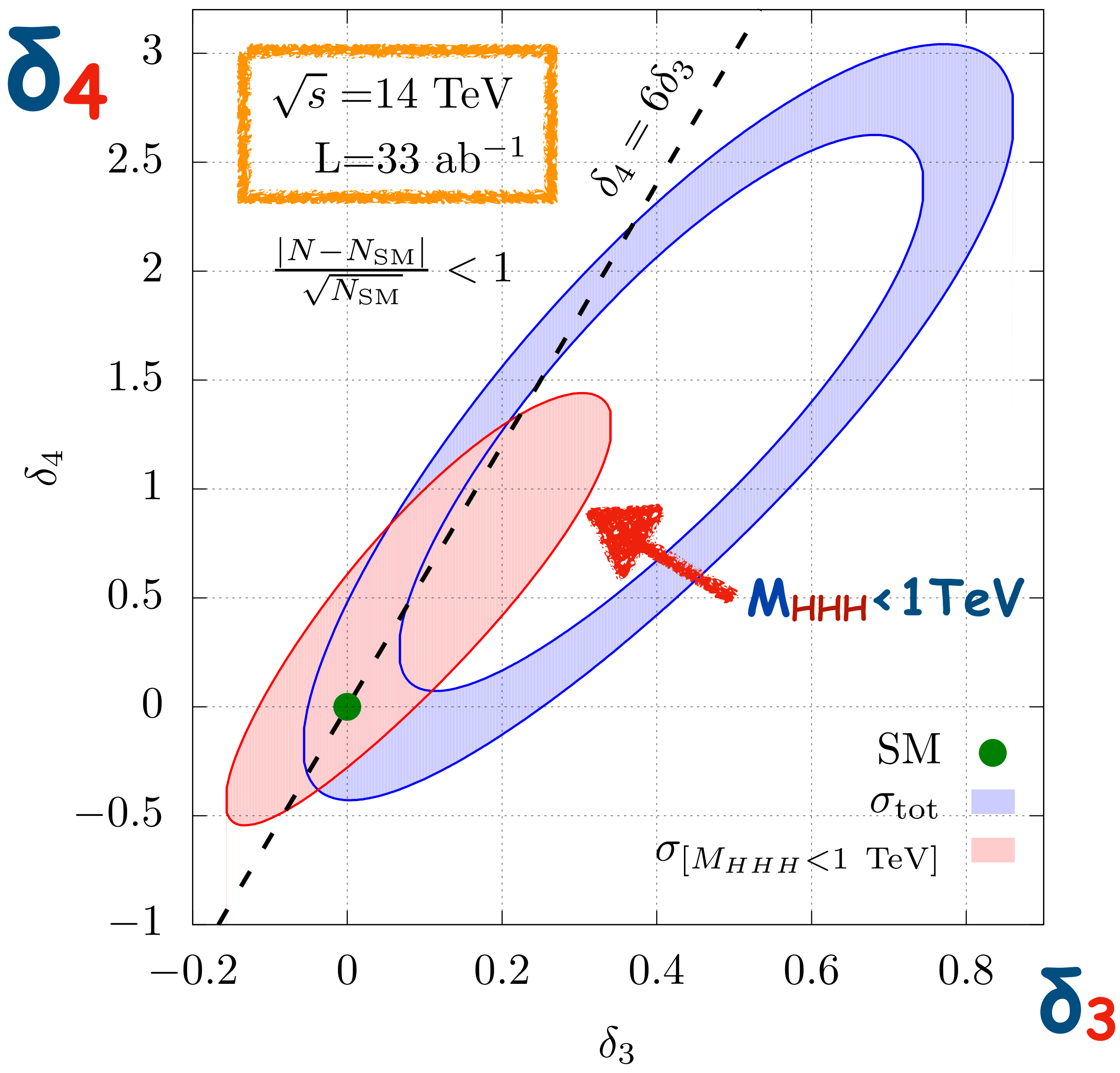
VBF→HHH

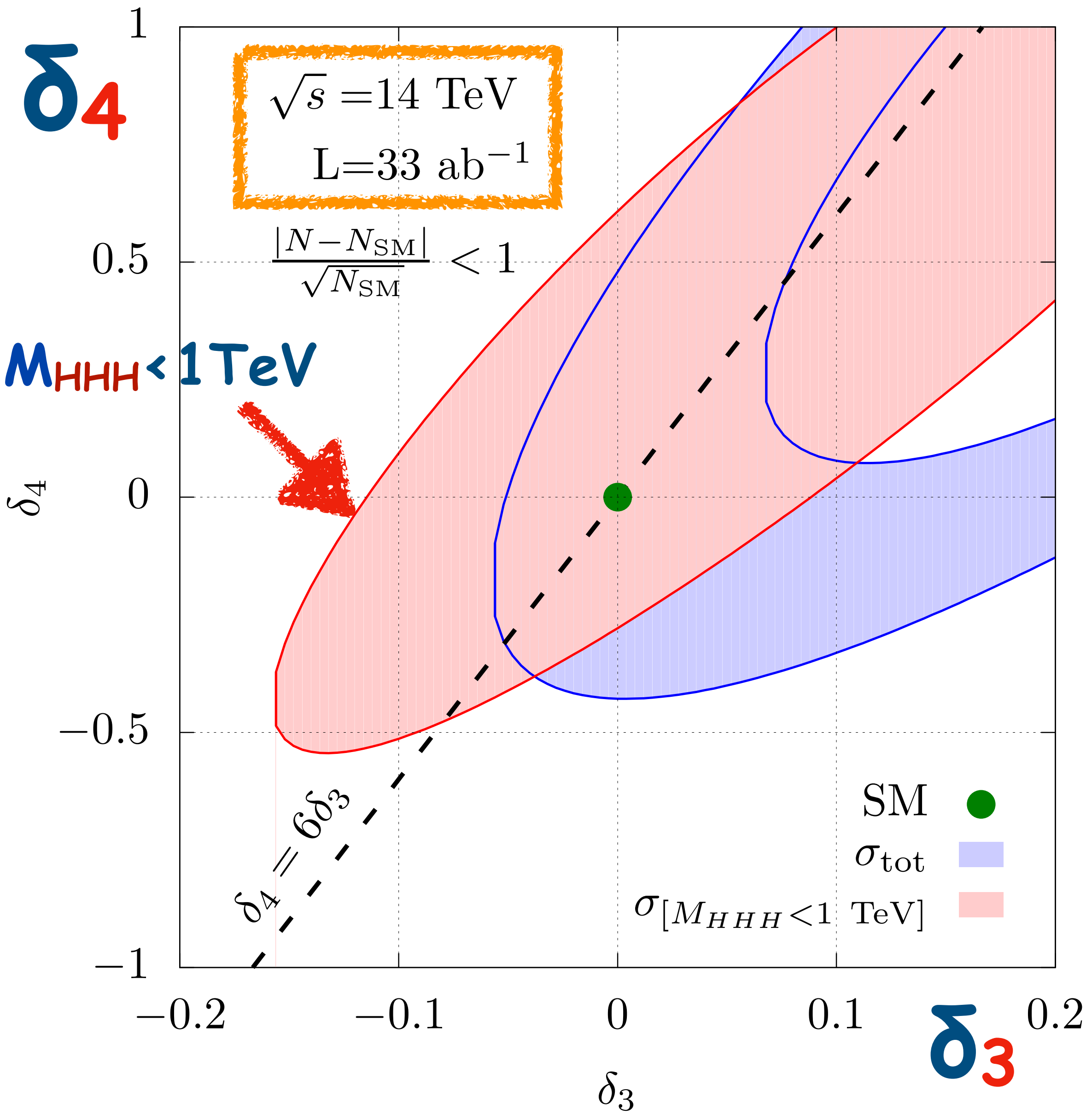
arXiv:2003.13628

$$\lambda_3 = \lambda_{SM}(1+\delta_3)$$

$$\lambda_4 = \lambda_{SM}(1+\delta_4)$$

$\sqrt{S_{\mu\mu}}$ = 14 TeV





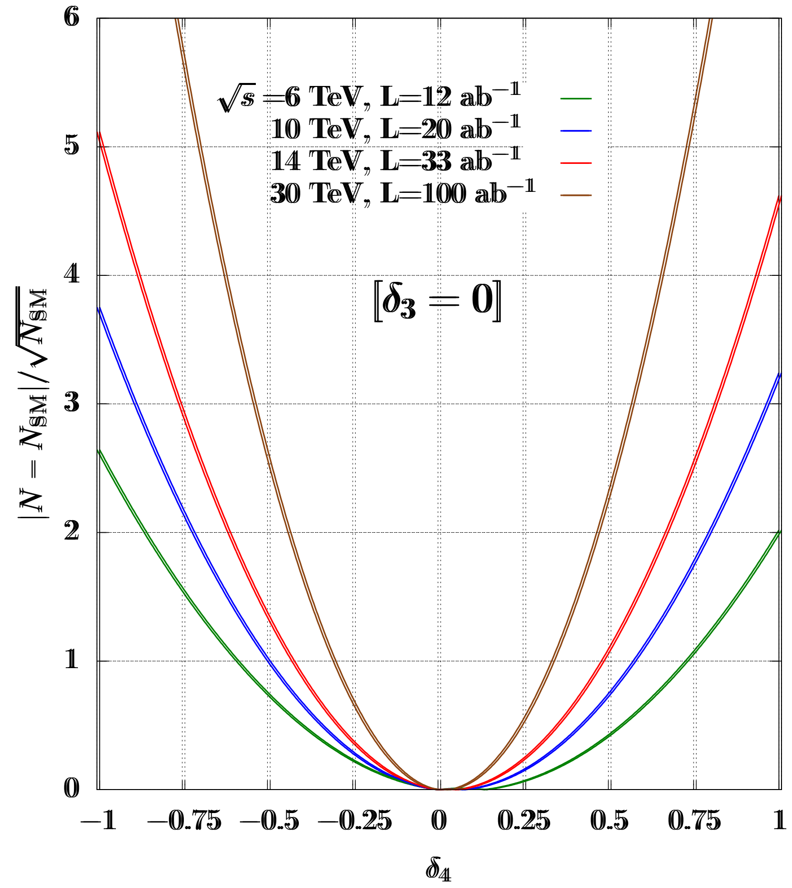
[δ_3=0] -0.3 < δ_4 <0.5 (68%CL) !!!

δ_4 bounds from $\sigma_{HHH(tot)}$ [δ_3=0] vs $\sqrt{S_{\mu\mu}}$

$\sqrt{s}$ (TeV)	Lumi (ab^{-1})	x-sec only 1 σ	x-sec only 2 σ
6	12	$[-0.60, 0.75]$	$[-0.90, 1.00]$
10	20	$[-0.50, 0.55]$	$[-0.70, 0.80]$
14	33	$[-0.45, 0.50]$	$[-0.60, 0.65]$
30	100	$[-0.30, 0.35]$	$[-0.45, 0.45]$
3	100	$[-0.35, 0.60]$	$[-0.50, 0.80]$

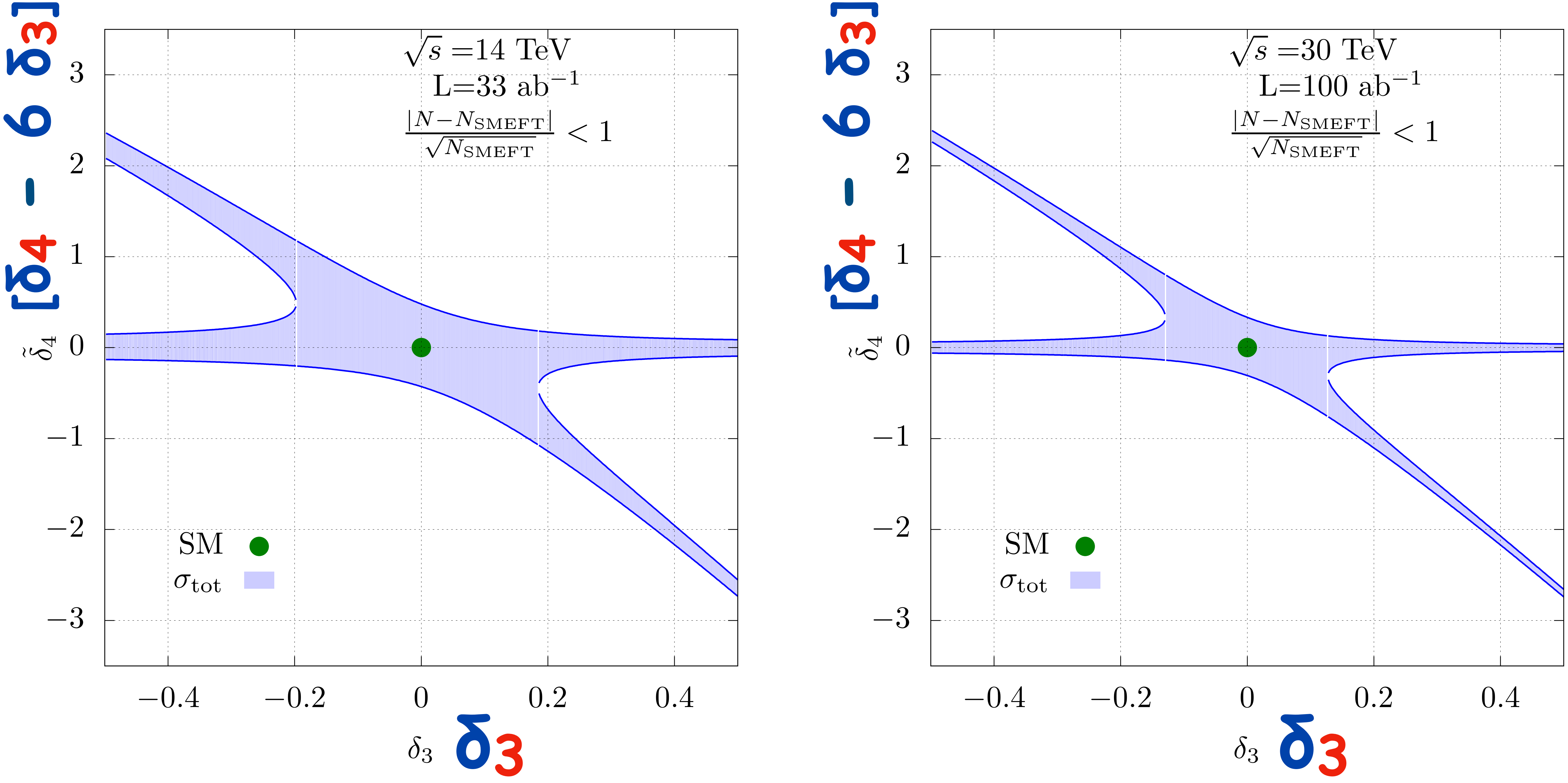
~20 x L$_{(CLIC)}$!!

- full HHH statistics
- no background
- no optimization from kinem.features of (δ_3,δ_4)-depending sub-amplitudes



for $\delta_3 \neq 0$, can constrain deviations from SMEFT configuration $[\delta_4 \sim 6\ \delta_3]$

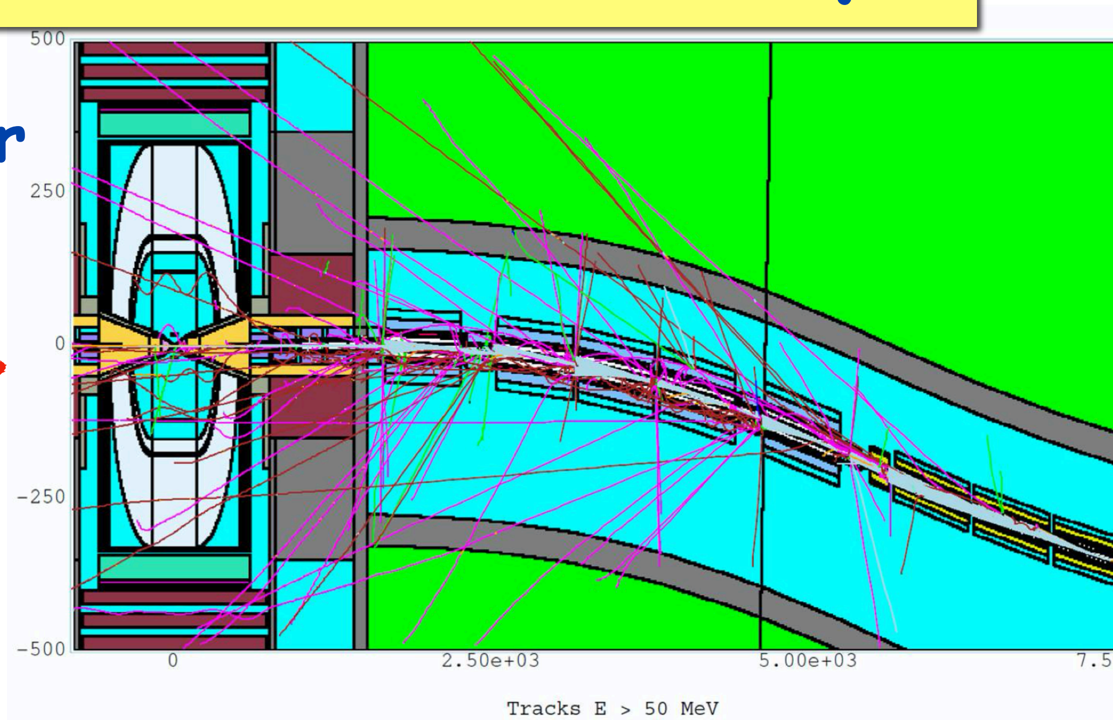
$$|N(\delta_3, \tilde{\delta}_4 + 6\delta_3) - N(\delta_3, 6\delta_3)| / \sqrt{N(\delta_3, 6\delta_3)} < 1$$



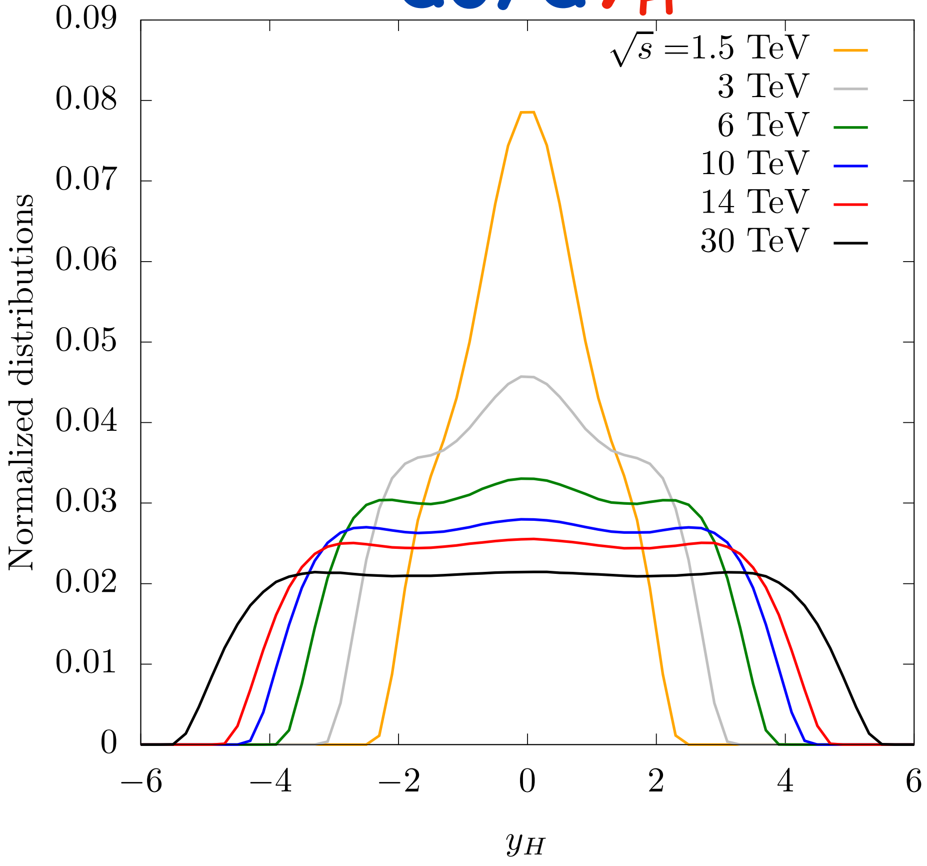
Beam Induced Bckgr (BIB) from muons' decay

two tungsten nozzles can mitigate BIB in detector
→ → reduced acceptance in forward regions

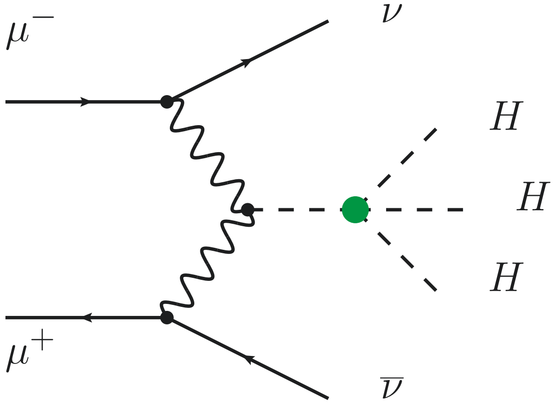
(→ MAP studies at $\sqrt{S_{\mu\mu}}$~1.5 TeV)



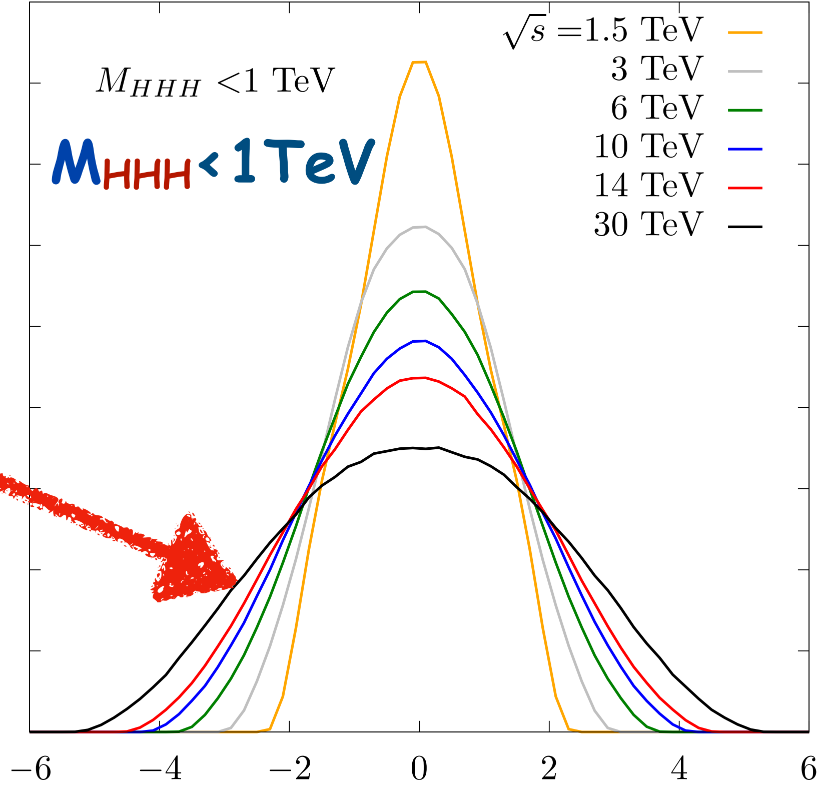
dσ/dY_H



δ_4 sensitivity enhanced for M_{HHH}<1TeV where final p.les are more centrally produced



dσ/dY_H



→→cutting off small angles can increase sensitivity !

δ_4 bounds [δ_3=0] : σ_{tot} vs $\sigma_{[reduced\ accept.]}$

$\sqrt{s}$ (TeV)	Lumi (ab^{-1})	x-sec [tot] $1\ \sigma$	p_T >20 GeV $\lvert\eta\rvert < 3$ $1\ \sigma$
6	12	$[-0.60, 0.75]$	$[-0.50, 0.70]$
10	20	$[-0.50, 0.55]$	$[-0.37, 0.54]$
14	33	$[-0.45, 0.50]$	$[-0.28, 0.43]$
30	100	$[-0.30, 0.35]$	$[-0.15, 0.30]$
3	100	$[-0.35, 0.60]$	$[-0.34, 0.64]$

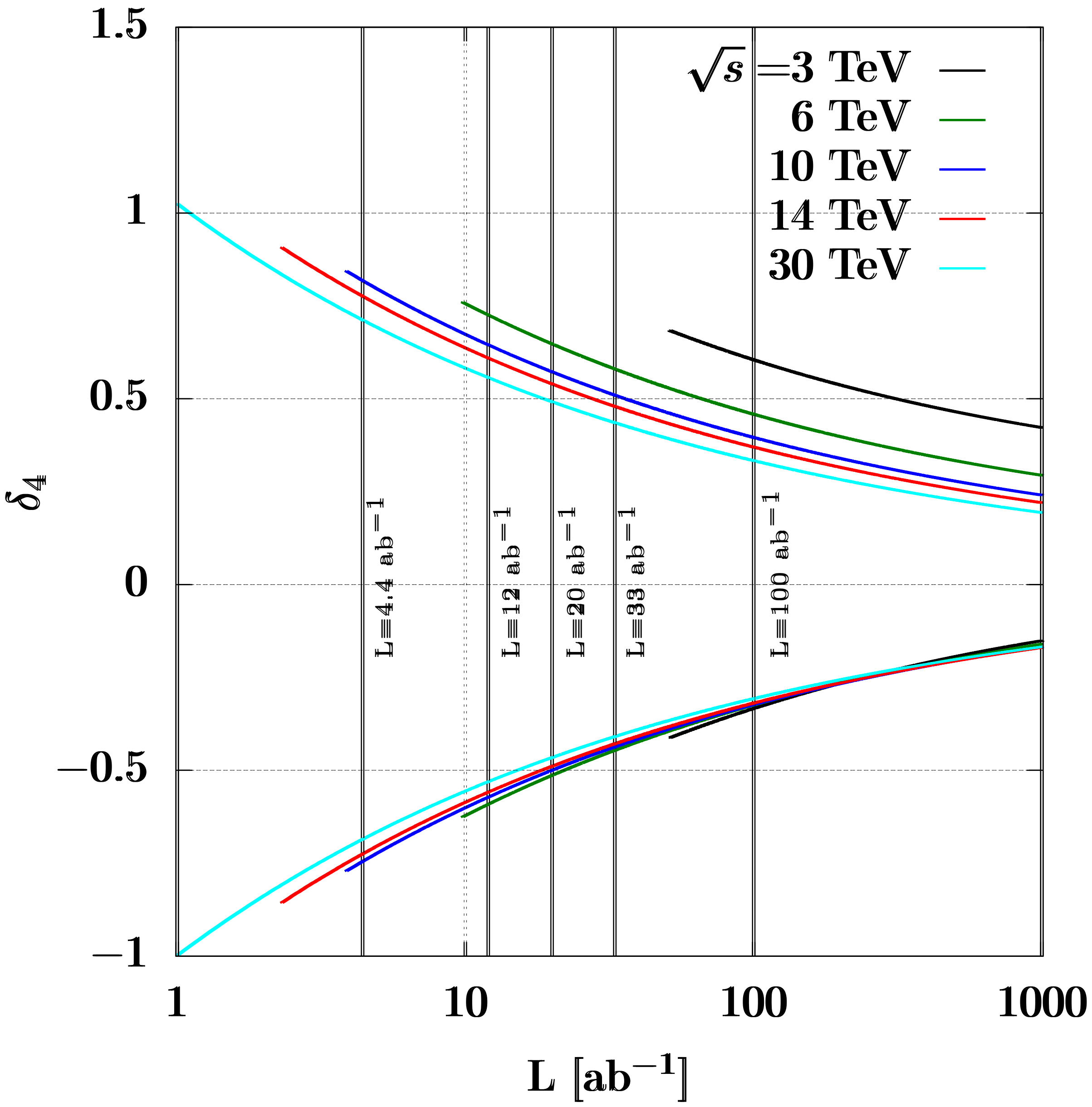
δ_4

- geometrical selection on H → bb decay products

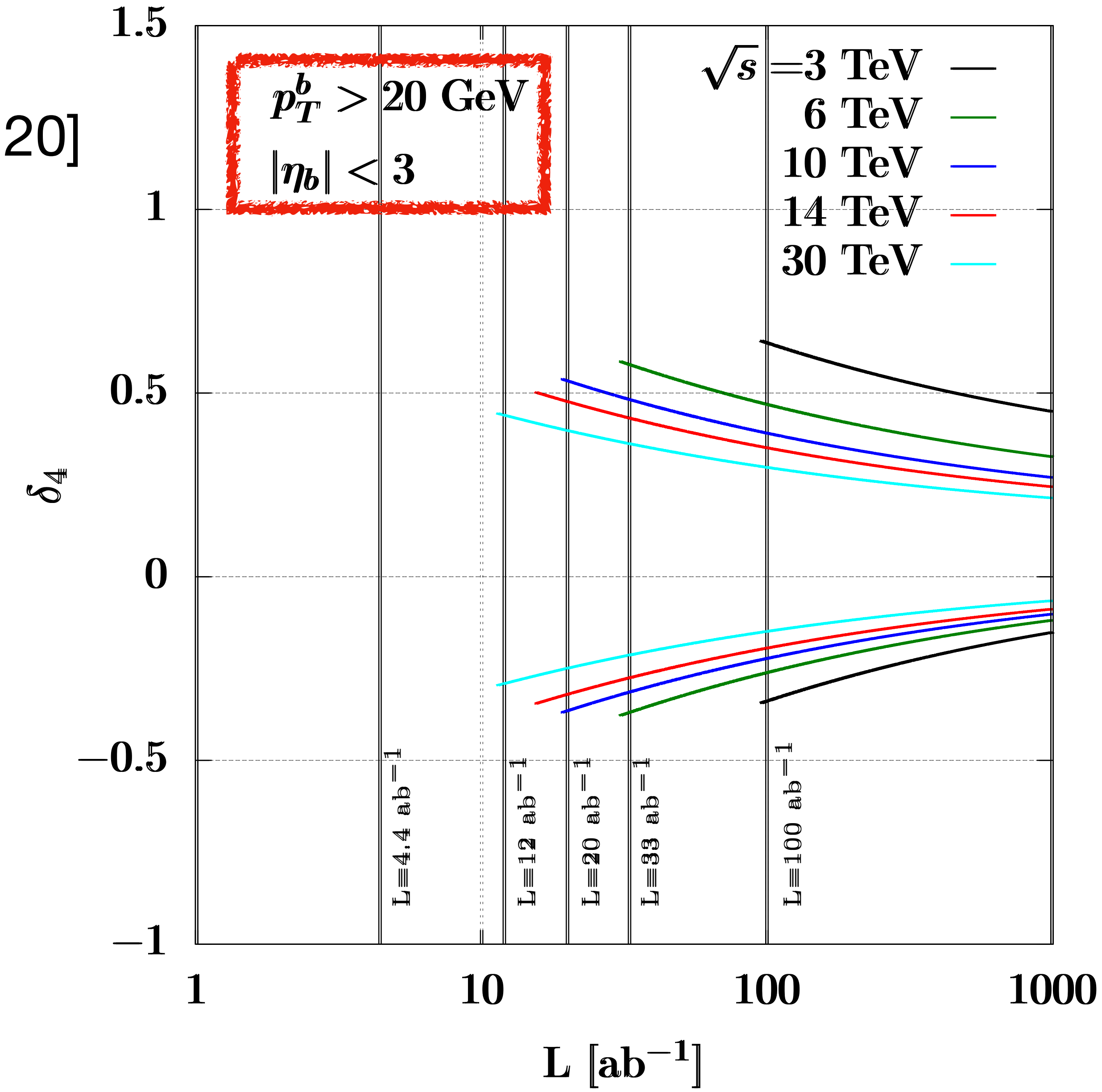
  (in principle inclusive on H → bb,cc,gg,ττ...→ no BR applied)

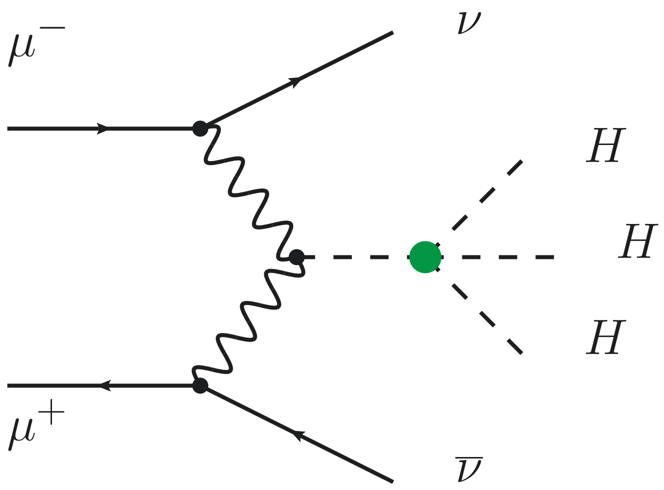
- clear improvement in sensitivity !

killing acceptance in forward regions increases sensitivity



[N$_{\mathrm{ev}}$ > 20]

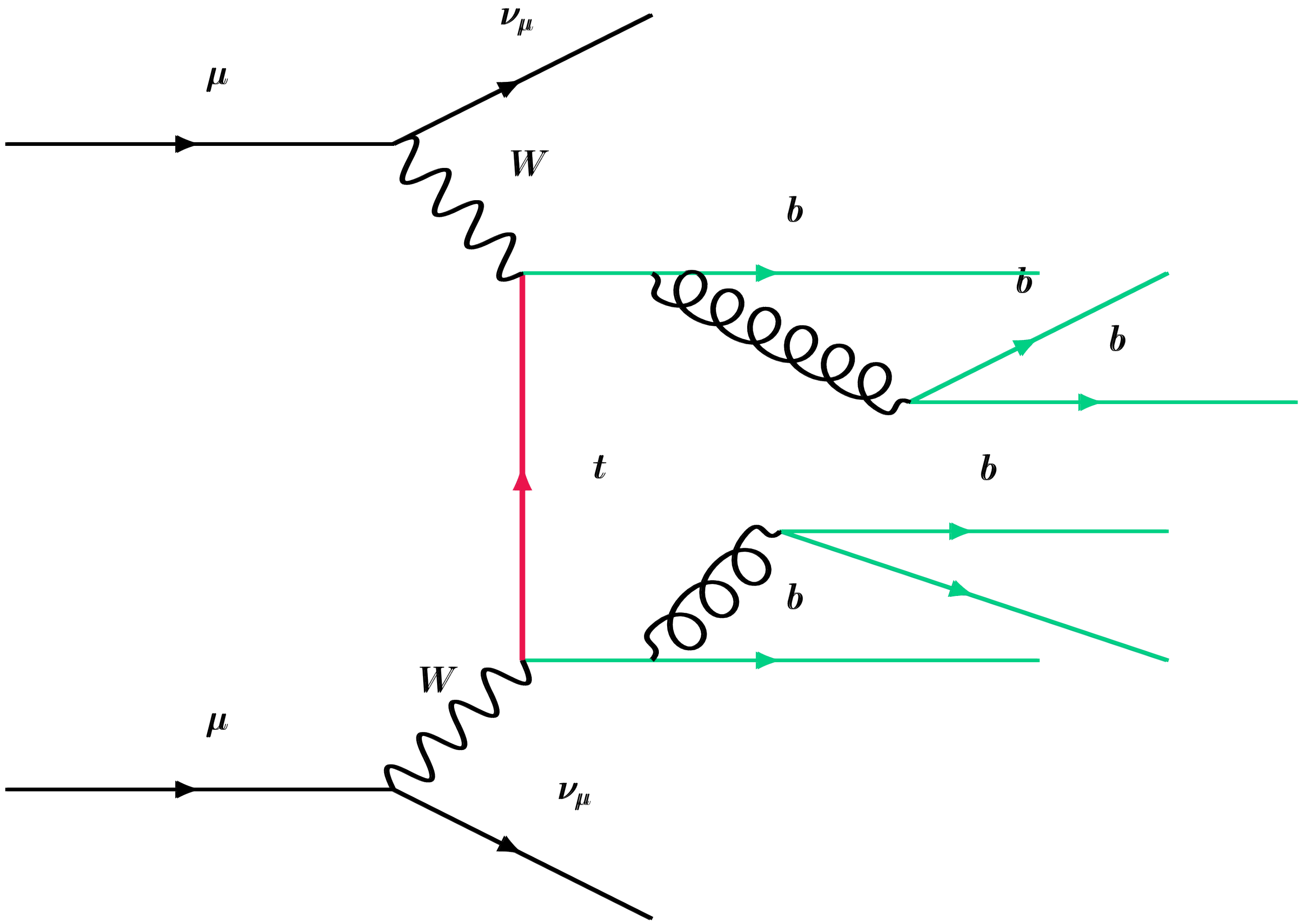




self-coupling measurement robust against beam-induced bckgr effects !!!

"Physics" bckgds to VBF → HHH

- all HHH decay modes with sizeble BR's are relevant !
- 8-body final states (at least !)
  → hard to evaluate via MC's
- 6b-jet bckgr moderate at FCC-hh [arXiv:1801.10157]
- might be S/B >> 1 at multi-TeV muon colliders... →



Chiesa et al., in progress

cf. bckgds to VBF → HH at CLIC_3TeV

$\sqrt{s} = 3\,\mathrm{TeV} \quad \mathcal{L} = 5\,\mathrm{ab}^{-1}$

Process	σ/fb	$\epsilon_{\mathrm{tightBDT}}$	N_{tightBDT}
$e^+e^- \to HH\nu\bar{\nu}$	0.59	8.43 %	367
only $HH \to b\bar{b}b\bar{b}$	0.19	26.3 %	361
only $HH \to$ other	0.40	0.2 %	6
$e^+e^- \to q\bar{q}q\bar{q}$	547	0.00033 %	13
$e^+e^- \to q\bar{q}q\bar{q}\nu\bar{\nu}$	72	0.017 %	90
$e^+e^- \to q\bar{q}q\bar{q}l\bar{\nu}$	107	0.0029 %	23
$e^+e^- \to q\bar{q}H\nu\bar{\nu}$	4.7	0.56 %	174
$e^\pm\gamma \to \nu q\bar{q}q\bar{q}$	523	0.0014 %	52
$e^\pm\gamma \to q\bar{q}H\nu$	116	0.0026 %	21

Roloff at al, arXiv:1901.05897

S/B ~ 1

outlook

- testing Higgs potential via Higgs self-coupling measurement of paramount importance !
- triple Higgs production only direct access to quartic self-coupling
- projections at FCC-hh can give few-% accuracy on λ_3 but only mild bounds on λ_4 ($\delta\lambda_4/\lambda_4 \sim 10$) at present
- first indications that μ colliders @10+TeV with $L \sim 10^{35} cm^{-2}s^{-1}$ might provide a λ_4 determination with few-10% accuracy ($\delta\lambda_4/\lambda_4 \sim 1$), i.e. significantly better that other future colliders !
- physics bckgds expected mild (also for hadronic final states) ➜ detailed simulations needed (challenging ➜ many particles in phase-space)