HiLumi
LARGE HADRON COLLIDER

Research supported by the High Luminosity LHC project

HiLumi LHC
Effects of Feed-Down from b_3

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Outline

Aim

- Beam quality strongly influenced by errors in D1, D2/MCBRD, and MCBXF magnets
  - simulations show this is mainly due to b_3 component
  - not clear however if this is due to pure b_3 or feed-down
- Create script to cancel feed-down from higher-order errors
- Use script to distinguish pure effects from feed-down effects, investigating impact on
  - β-beating
  - tune shift
  - β^*-beating

Outline

Cancelling Feed-Down

- **Calculate reference orbit** (before errors are assigned):

  But feed-down from other magnets' errors changes orbit..

  ⇒ Temporarily remove errors of which feed-down will be cancelled

- **Add feed-down correction** to the correct multipoles:

  Using existing error routines proved to be nearly impossible

  ⇒ Write all errors to file and adapt by python script

  ⇒ Hence use k_n, k_n^s rather than b_n, a_n

Review: Feed-Down

Displacement from reference orbit creates extra contributions to the multipoles coming from higher orders:

$$\tilde{k}_0 = k_0 + \Delta x\, k_1 - \Delta y\, k_1^s + \frac{\Delta x^2 - \Delta y^2}{2} k_2 - \Delta x \Delta y\, k_2^s + \mathcal{O}(n{=}3)$$

$$\tilde{k}_0^s = k_0^s + \Delta x\, k_1^s + \Delta y\, k_1 + \frac{\Delta x^2 - \Delta y^2}{2} k_2^s + \Delta x \Delta y\, k_2 + \mathcal{O}(n{=}3)$$

$$\tilde{k}_1 = k_1 + \Delta x\, k_2 - \Delta y\, k_2^s + \mathcal{O}(n{=}3)$$

$$\tilde{k}_1^s = k_1^s + \Delta x\, k_2^s + \Delta y\, k_2 + \mathcal{O}(n{=}3)$$

$\dots$

(see derivation in backup slides)

Cancelling Feed-Down in b_2 from b_3

Cancelling feed-down is as easy as subtracting it from the multipole where it is fed into. E.g. to cancel the feed-down in b_2 from b_3, we have to make the substitution

$$k_1 \to k_1 - \Delta x\, k_2$$

However, this creates an extra spurious feed-down to b_1 and a_1 which is unwanted. Hence, besides the above substituton we also have to substitute

$$k_0 \to k_0 + \Delta x^2 k_2$$
$$k_0^s \to k_0^s + \Delta x \Delta y\, k_2$$

Cancelling Feed-Down

in b_2 from b_3

$$k_1 \to k_1 - \Delta x\, k_2$$
$$k_0 \to k_0 + \Delta x^2 k_2$$
$$k_0^s \to k_0^s + \Delta x \Delta y\, k_2$$

in a_2 from b_3

$$k_1^s \to k_1^s - \Delta y\, k_2$$
$$k_0 \to k_0 - \Delta y^2 k_2$$
$$k_0^s \to k_0^s + \Delta x \Delta y\, k_2$$

in b_2 from a_3

$$k_1 \to k_1 + \Delta y\, k_2^s$$
$$k_0 \to k_0 - \Delta x \Delta y\, k_2^s$$
$$k_0^s \to k_0^s - \Delta y^2 k_2^s$$

in a_2 from a_3

$$k_1^s \to k_1^s - \Delta x\, k_2^s$$
$$k_0 \to k_0 - \Delta x \Delta y\, k_2^s$$
$$k_0^s \to k_0^s + \Delta x^2 k_2^s$$

Cancelling Feed-Down

in b_2 from b_3

$k_1 \to k_1 - \Delta x\, k_2$

$k_0 \to k_0 + \Delta x^2 k_2$

$k_0^s \to k_0^s + \Delta x \Delta y\, k_2$

BETA-BEATING

in b_2 from a_3

$k_1 \to k_1 + \Delta y\, k_2^s$

$k_0 \to k_0 - \Delta x \Delta y\, k_2^s$

$k_0^s \to k_0^s - \Delta y^2 k_2^s$

in a_2 from b_3

$k_1^s \to k_1^s - \Delta y\, k_2$

$k_0 \to k_0 - \Delta y^2 k_2$

$k_0^s \to k_0^s + \Delta x \Delta y\, k_2$

COUPLING

in a_2 from a_3

$k_1^s \to k_1^s - \Delta x\, k_2^s$

$k_0 \to k_0 - \Delta x \Delta y\, k_2^s$

$k_0^s \to k_0^s + \Delta x^2 k_2^s$

Testing: β-Beating

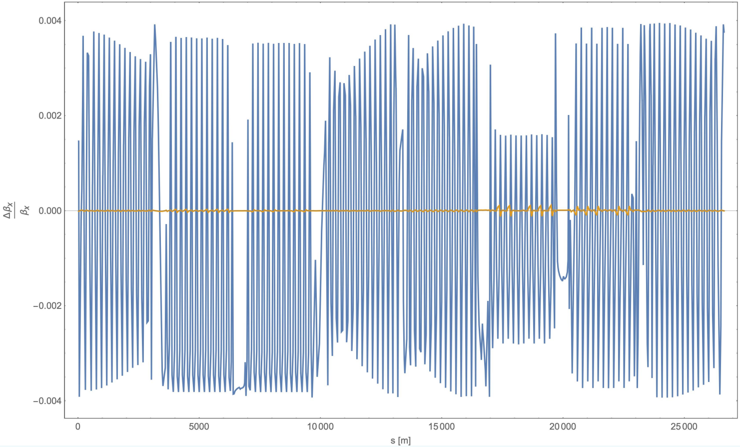
Any b_2 component, whether it is pure or feed-down, will generate β-beating following:

$$\frac{\Delta\beta}{\beta} = \frac{1}{2\sin 2\pi(Q - \lfloor Q \rfloor)} \int \mathrm{d}s\, k_1(s)\beta(s)\cos(\ldots)$$

To test our script, we assign a $b_3 = 1$ error to 1 slice of 1 magnet only (MBXF.4R1..1). With 1.4 optics, this generates a feed-down of $k_1 = -3.92 \cdot 10^{-7}$, giving a maximum β-beating of around 0.004.

Testing: β-Beating

Indeed, cancelling this feed-down reduces the β-beating almost completely:



Testing: β-Beating

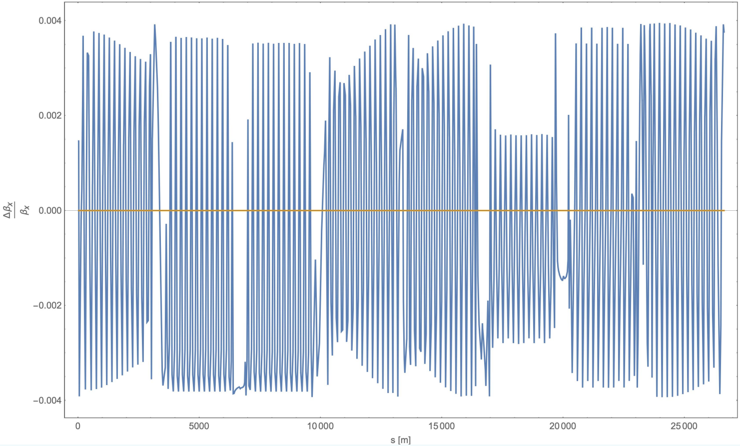
The remnant beta beating can be explained by the small second-order feed-down from b_3 to b_1 and a_1, as the latter will change the orbit slightly and hence change our feed-down correction in b_2 as well. To correct for these, we additionally make the substitutions

$$k_0 \to k_0 - \frac{\Delta x^2 - \Delta y^2}{2} k_2$$

$$k_0^s \to k_0^s - \Delta x \Delta y \, k_2$$

Testing: β-Beating

And indeed, cancelling this extra feed-down removes virtually all β-beating:

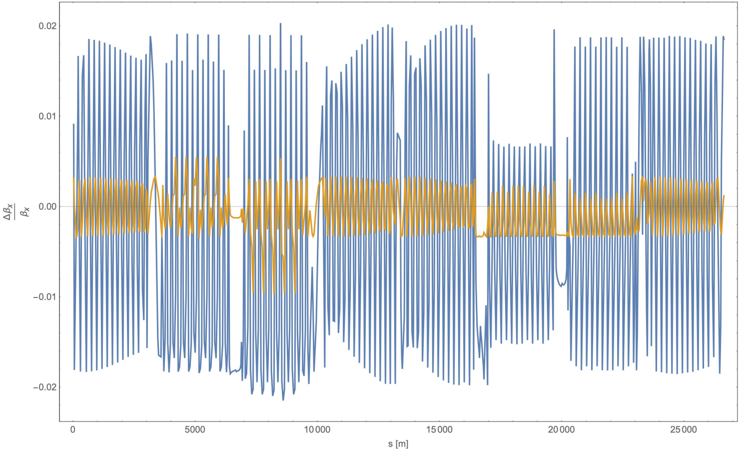


Testing: β-Beating

Next we assign a $b_3 = 1$ error to all D1 magnets, still keeping all other errors at zero. With 1.4 optics, this generates a maximum β-beating of around 0.02.

Testing: β-Beating

Cancelling the feed-down is now much less efficient:



Testing: β-Beating

In theory all β-beating should disappear completely, because:

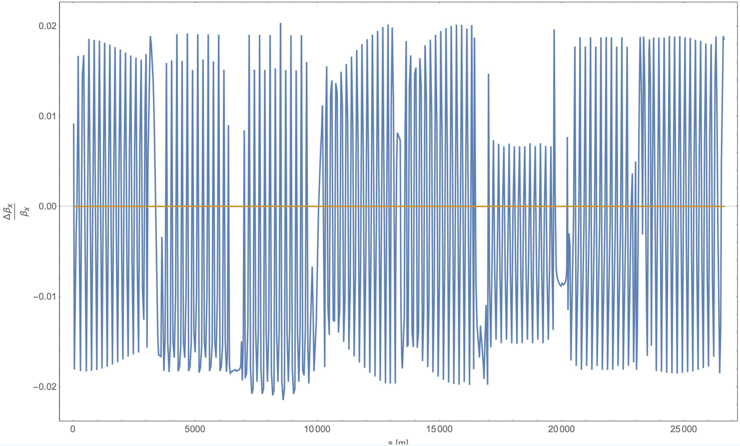
- no other errors assigned in the machine
- β-beating calculated before/after error assignment

One possible explanation is that the b_3 also feeds into a_2, which creates coupling, which influences the orbit, which distorts our correction.

To test this, we cancel the feed-down to a_2 as well (and again also correct the spurious new feed-downs to b_1 and a_1).

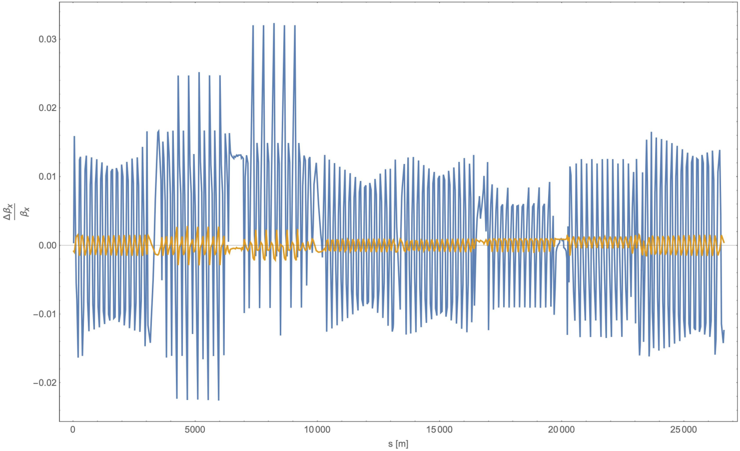
Testing: β-Beating

And indeed, cancelling now **all** feed-down from b_3 again removes all β-beating:



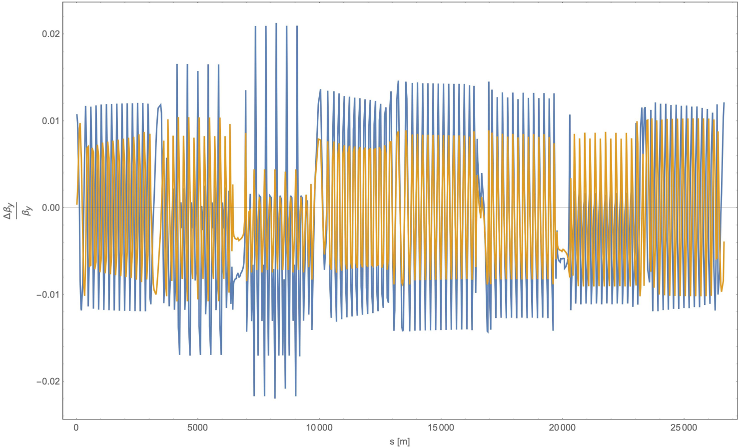
Testing: β-Beating

As a final test we assign all nominal errors to D1:



Testing: β-Beating

This is not a good result, even disastrous in the vertical:



Testing: β-Beating

A logical explanation is that we only cancelled feed-down from b_3, while other orders might of course contribute as well!

The biggest displacements are in MBXF.4R1..1 ($x = 0.013$) and MBXF.4L5..4 ($y = -0.013$). Let's investigate the error strengths and corresponding feed-downs at these locations.

Testing: β-Beating

$k_2 = 2.71 \cdot 10^{-5}$ → $(k_1)_{\text{FD}} \sim 3.5 \cdot 10^{-7}$ (in MBXF.4R1..1)

$k_2^s = 2.01 \cdot 10^{-5}$ → $(k_1)_{\text{FD}} \sim 2.6 \cdot 10^{-7}$ (in MBXF.4L5..4)

$k_3 = -2.23 \cdot 10^{-4}$ → $(k_1)_{\text{FD}} \sim 1.9 \cdot 10^{-8}$ (in MBXF.4L5..4)

$k_3^s = -4.92 \cdot 10^{-4}$ → $(k_1)_{\text{FD}} \sim 1.2 \cdot 10^{-10}$ (in MBXF.4R1..1)

$k_4 = -5.14 \cdot 10^{-2}$ → $(k_1)_{\text{FD}} \sim -1.9 \cdot 10^{-8}$ (in MBXF.4R1..1)

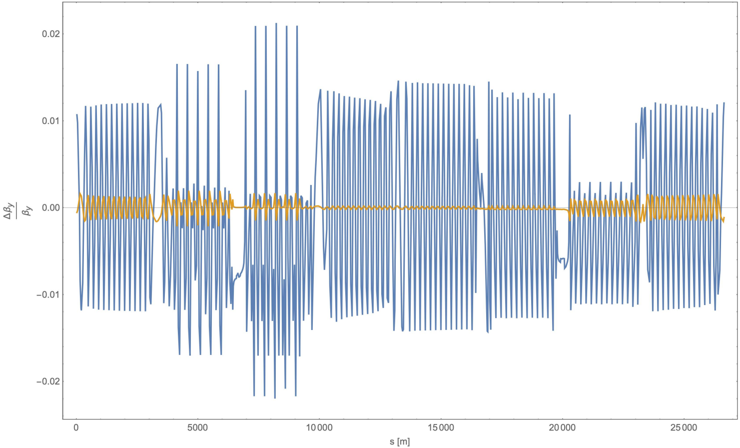
$k_4^s = -1.69 \cdot 10^{-2}$ → $(k_1)_{\text{FD}} \sim 6.2 \cdot 10^{-9}$ (in MBXF.4L5..4)

$k_5 = -2.81$ → $(k_1)_{\text{FD}} \sim -3.3 \cdot 10^{-9}$ (in MBXF.4R1..1)

$k_5^s = 3.20$ → $(k_1)_{\text{FD}} \sim -2.2 \cdot 10^{-11}$ (in MBXF.4R1.).1

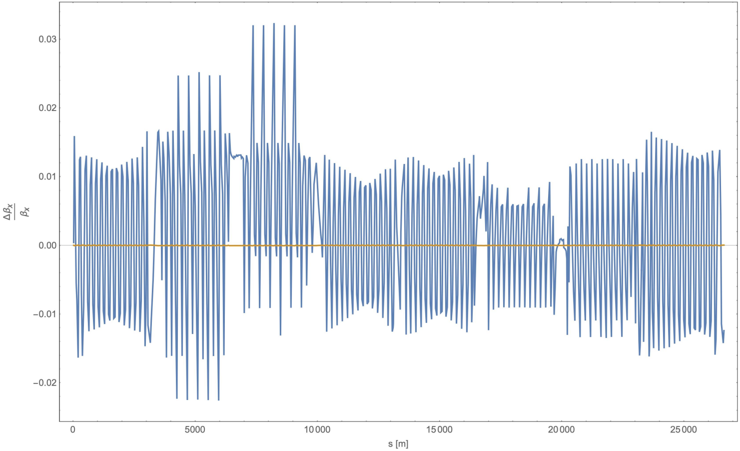
Testing: β-Beating

Indeed, adding the cancellation of feed-down from a_3 saves the vertical:



Testing: β-Beating

And cancelling feed-down up to b_6 removes all β-beating:

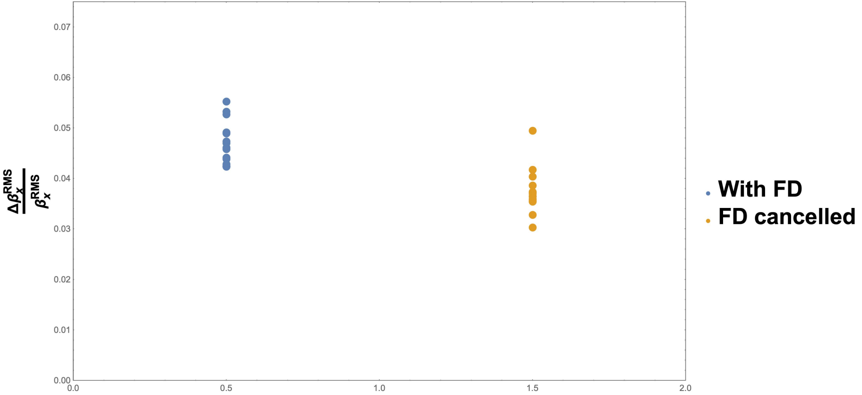


Success

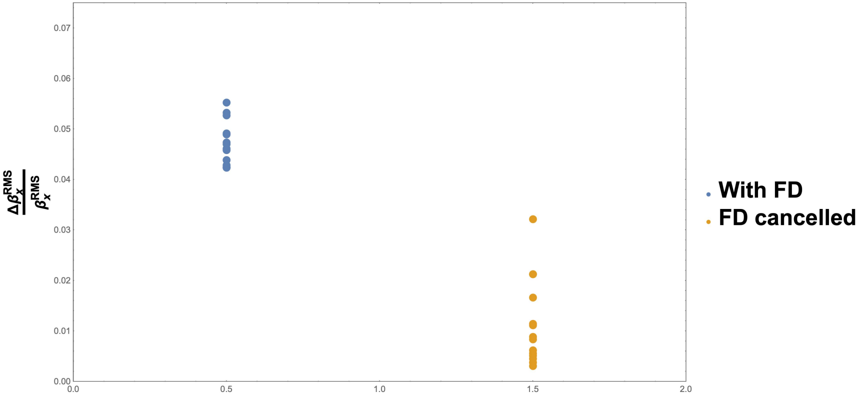
Now that we have shown that the script does exactly what it should, we can apply it to a realistic case, in which the machine has had error correction routines, coupling correction, and orbit matching

Outline

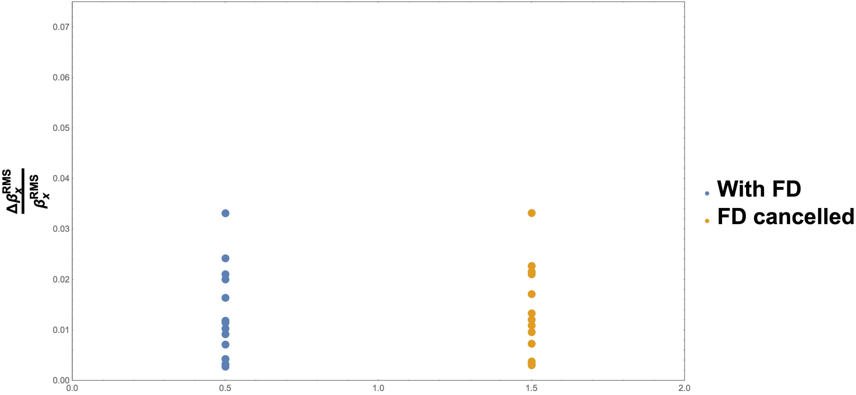
β-Beating from b_3 FD in MCBXF



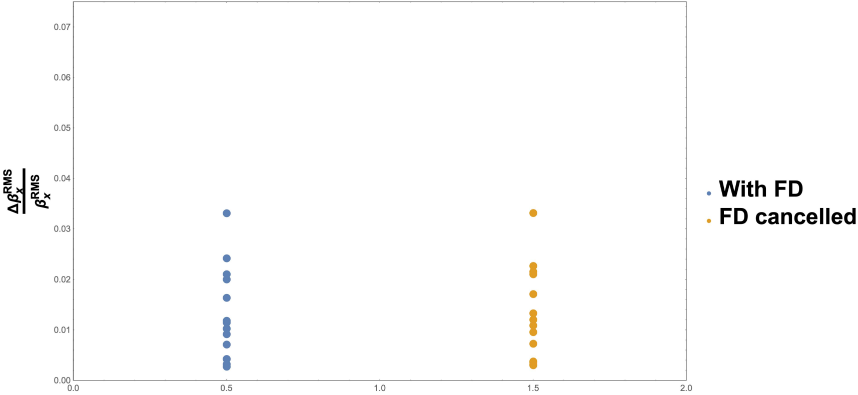
β-Beating from b_3 and a_3 FD in MCBXF



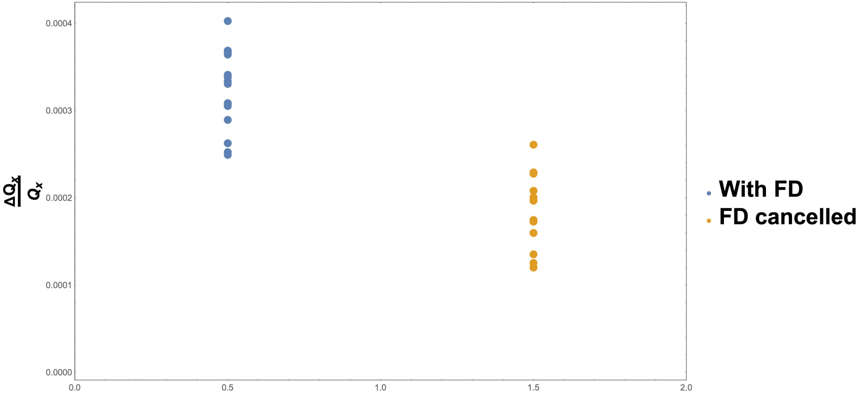
β-Beating from b_3 and a_3 FD in MBRD



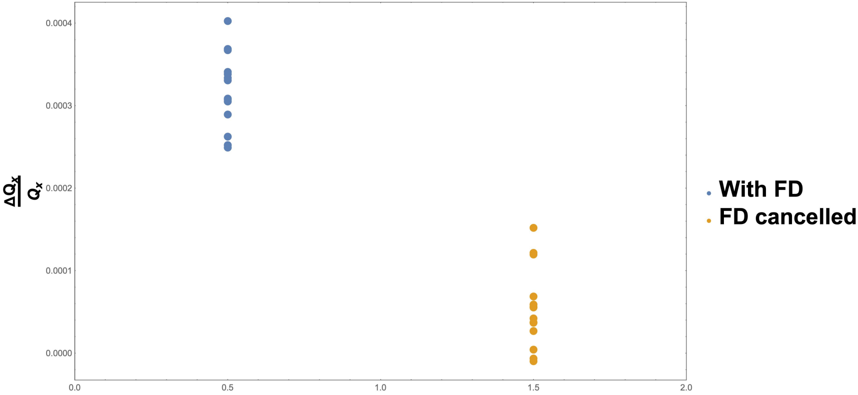
β-Beating from b_3 and a_3 FD in MCBRD



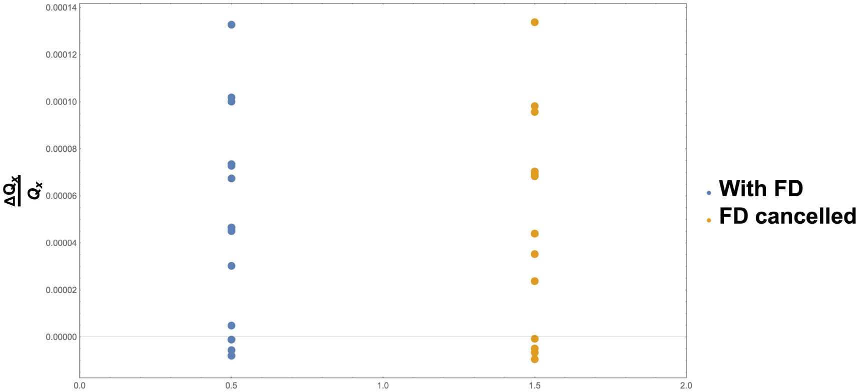
Tune shift from b_3 FD in MCBXF



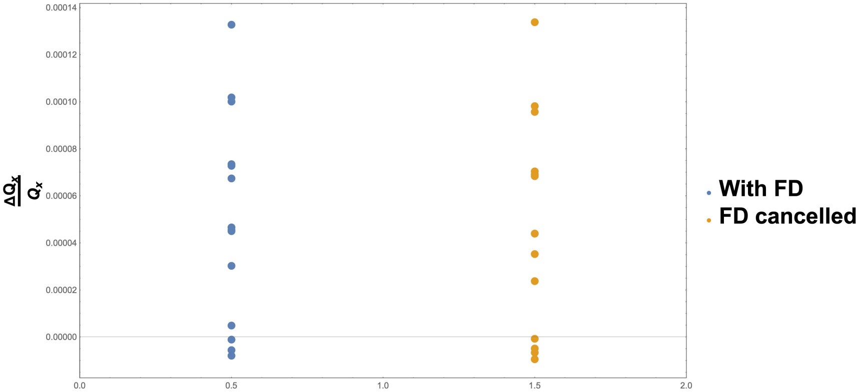
Tune shift from b_3 and a_3 FD in MCBXF



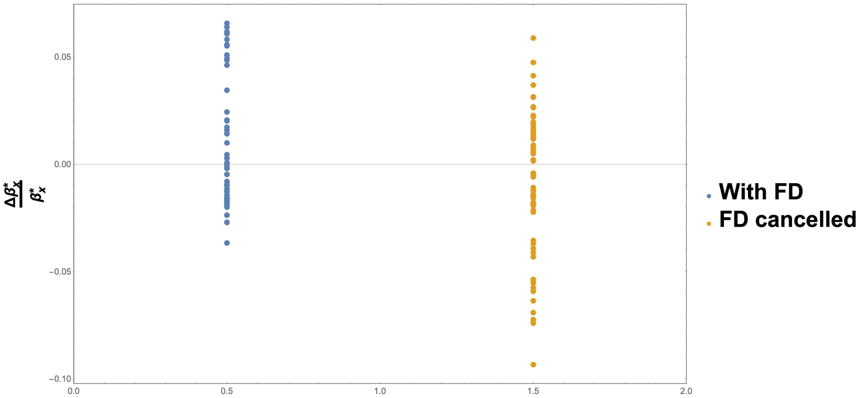
Tune shift from b_3 and a_3 FD in MBRD



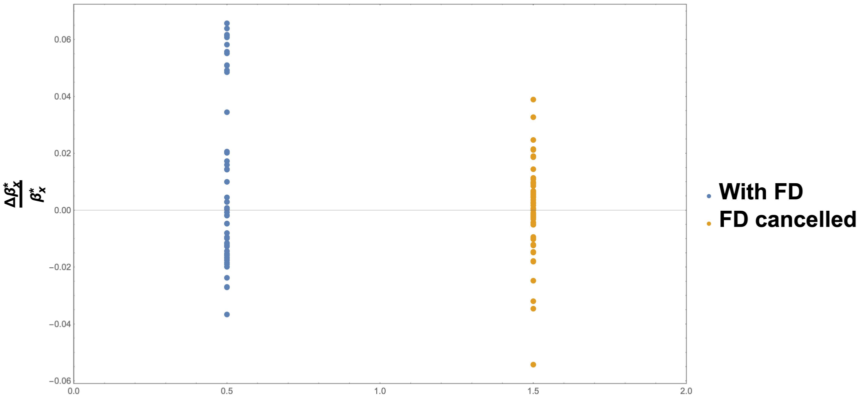
Tune shift from b_3 and a_3 FD in MCBRD



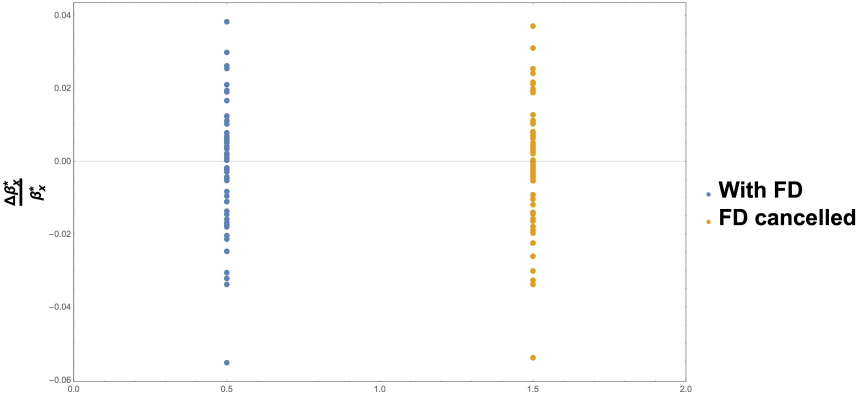
β^*-beating from b_3 FD in MCBXF



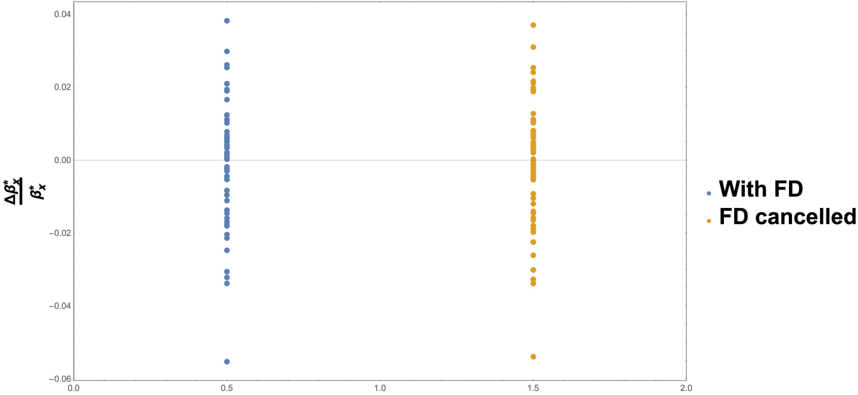
β^*-beating from b_3 and a_3 FD in MCBXF



β^*-beating from b_3 and a_3 FD in MBRD



β^*-beating from b_3 and a_3 FD in MCBRD



Outline

Conclusions and Outlook

- feed-down from b_3 and a_3 in MCBXF is substantial
- feed-down from b_3 and a_3 in D2 and MCBRD are neglible
- feed-down from b_3 and a_3 in D1 not well understood

- look at impact on DA
- how to realise this feed-down cancellation in reality (Q4?)

Backup Slides

Calculation of Feed-Down

$$B_y + iB_x = \sum_{n=0}^{\infty} (b_{n+1} + ia_{n+1}) \frac{(x + \Delta x + iy + i\Delta y)^n}{R^n}$$

$$= \sum_{n=0}^{\infty} (b_{n+1} + ia_{n+1}) \sum_{k=0}^{n} \binom{n}{k} \frac{(x+iy)^k}{R^k} \frac{(\Delta x + i\Delta y)^{n-k}}{R^{n-k}}$$

using $\sum_{n=0}^{\infty} \sum_{k=0}^{n} = \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} n \leftrightarrow k$

$$= \sum_{n=0}^{\infty} \left(\tilde{b}_{n+1} + i\tilde{a}_{n+1}\right) \frac{(x+iy)^n}{R^n}$$

Calculation of Feed-Down

$$\tilde{b}_{n+1} = \sum_{k=n}^{\infty} \sum_{m=0}^{\lfloor \frac{k-n}{2} \rfloor} \frac{(-)^m}{R^{k-n}} \binom{k}{n} \left[\binom{k-n}{2m} b_{k+1}\, \Delta x^{k-n-2m} \Delta y^{2m} - \binom{k-n}{2m+1} a_{k+1}\, \Delta x^{k-n-2m-1} \Delta y^{2m+1} \right]$$

$$\tilde{a}_{n+1} = \sum_{k=n}^{\infty} \sum_{m=0}^{\lfloor \frac{k-n}{2} \rfloor} \frac{(-)^m}{R^{k-n}} \binom{k}{n} \left[\binom{k-n}{2m+1} b_{k+1}\, \Delta x^{k-n-2m-1} \Delta y^{2m+1} + \binom{k-n}{2m} a_{k+1}\, \Delta x^{k-n-2m} \Delta y^{2m} \right]$$

Calculation of Feed-Down

$$\tilde{b}_1 = b_1 + \frac{\Delta x}{R} b_2 - \frac{\Delta y}{R} a_2 + \frac{\Delta x^2 - \Delta y^2}{R^2} b_3 - 2\frac{\Delta x \Delta y}{R^2} a_3 + \mathcal{O}\left(n=4\right)$$

$$\tilde{a}_1 = a_1 + \frac{\Delta x}{R} a_2 + \frac{\Delta y}{R} b_2 + \frac{\Delta x^2 - \Delta y^2}{R^2} a_3 + 2\frac{\Delta x \Delta y}{R^2} b_3 + \mathcal{O}\left(n=4\right)$$

$$\tilde{b}_2 = b_2 + 2\frac{\Delta x}{R} b_3 - 2\frac{\Delta y}{R} a_3 + \mathcal{O}\left(n=4\right)$$

$$\tilde{a}_2 = a_2 + 2\frac{\Delta x}{R} a_3 + 2\frac{\Delta y}{R} b_3 + \mathcal{O}\left(n=4\right)$$

$\ldots$

Calculation of Feed-Down

$$-\Delta p_x + i\Delta p_y = \sum_{n=0}^{\infty} (k_n + i k_n^s)\,\frac{(x+iy)^n}{n!}$$

$$\tilde{k}_n = \sum_{l=n}^{\infty} \sum_{m=0}^{\lfloor \frac{l-n}{2} \rfloor} \frac{(-)^m}{(l-n)!} \Bigg[\binom{l-n}{2m} k_l \,\Delta x^{l-n-2m} \Delta y^{2m} - \binom{l-n}{2m+1} k_l^s \,\Delta x^{l-n-2m-1} \Delta y^{2m+1} \Bigg]$$

$$\tilde{k}_n^s = \sum_{l=n}^{\infty} \sum_{m=0}^{\lfloor \frac{l-n}{2} \rfloor} \frac{(-)^m}{(l-n)!} \Bigg[\binom{l-n}{2m+1} k_l \,\Delta x^{l-n-2m-1} \Delta y^{2m+1} + \binom{l-n}{2m} k_l^s \,\Delta x^{l-n-2m} \Delta y^{2m} \Bigg]$$