A short note on the MC's for Higgs Physics at the LHC

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Abstract
A short note on the Monte Carlo's at NLO and with ME merging for the Higgs LHC WG

Contents

1. INTRODUCTION

In recent years Monte Carlo event generators have been the subject of great theoretical and practical developments, most significantly in the extension of existing parton shower simulations to consistently include exact next-to-leading order (NLO) corrections [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19] and, separately, in the consistent combination of parton shower simulations and high multiplicity tree-level matrix element generators [20], [21], [22], [23], [24], [25], [26], [27]. The state-of-the-art in fixed order predictions has also undergone major improvement, resulting in fully differential Monte Carlo predictions at next-to-next-to-leading order (NNLO) for inclusive Higgs production and, simultaneously, NLO accuracy for production in association with a hard jet [28], [29], [30], [31].

In this note we aim at giving simple guidelines for experimentalists to have an idea of the tools available and how to perform a meaningful cross validation of the actual tools used in the analysis vis-a-vis the best theoretical predictions available at a given moment.

In the following we shall concisely review the pertinent features of the simulations achievable today, prior to presenting results for a variety of simple observables concerning the gluon fusion production channel. At the end of the article we summarise the results and comment on the readiness of these theoretical tools for much anticipated Higgs analysis.

2. MULTIPURPOSE MONTE CARLO PROGRAMS FOR THE HIGGS

In modern experimental particle physics, shower Monte Carlo (SMC) programs have become an indispensable tool for data analysis. From a user perspective, these programs provide an approximate but extremely detailed description of the final state in a high-energy process involving hadrons. They provide a *fully exclusive* description of the reaction, as opposed to fixed-order QCD calculations, which are only suitable for the computation of *inclusive* quantities.

After a latency period, research in SMC is once again very active, with significant advances being made in the last decade. In general these developments can be grouped into two main classes:

1. The merging of leading-order matrix elements (ME), characterized by a high number of final-state partons, with parton showers (PS). Examples of such methods are the CKKW matching scheme [20], [21], the MLM matching procedure [24] as well as the newer merging schemes based on truncated parton showers [26], [27].
2. The interfacing of NLO calculations (that are typically available only with a small number of legs in the final state) with parton shower simulations (MC@NLO [1] and POWHEG [8]).

All of them have to face the same problems: avoiding overcounting of events as well as the presence of dead regions.

2.1 MadGraph / MadEvent

Besides the possibility of generating processes in a long and extensible list of theoretical models (SM, MSSM, Higgs effective theory ...), the Monte-Carlo generator MADGRAPH/MADEVENT (MG/ME) [32] is also intended to simulate accurately the QCD radiation from initial and final states when coupled to a parton shower simulation. Such an aim can be achieved by using jet matching and a phase-space slicing technique

In MG/ME, three jet matching schemes (using Pythia 6.4 [33]) are implemented, namely the MLM [24], the $k_\perp$-MLM [34], [35] and the shower-$k_\perp$ [35] schemes. While the first two methods work with both virtuality and p_T-ordered showers, the third one only works with the p_T-ordered showers. For each of these methods, no analytic Sudakov reweighting of the events is performed, instead showered events are rejected if they are not matched to the ME-level partons. A detailed comparison of the $k_\perp$-MLM and shower-$k_\perp$ behaviours has shown that their respective outputs are very similar for the production of heavy colored particles in the SM and beyond [35]. In addition, a comparison for the production of W+jets events between the results from $k_\perp$-MLM and other simulation chains with different matching schemes has led to a similar conclusion [34].

Having the computation of the effective coupling between a scalar or pseudo-scalar to the gluons, the accurate simulation of the production of a Higgs boson accompanied by initial and final-state radiation is therefore relatively straightforward. In order to compare the MG/ME production with the results from the other generators considered in this work, we choose to simulate $H + 0, 1, 2$ partons at the ME level and apply the $k_\perp$-MLM scheme with $Q^{\rm ME}_{\rm cut} = 10$ GeV and $Q_{\rm match} = 15$ GeV. These choices were made in agreement with the smoothness of ME→PS transition regions in the $2 \to 1$ and $1 \to 0$ differential jet rates distributions.

2.2 MC@NLO

The MC@NLO method [1] was the first one to give an acceptable solution to the overcounting problem. The generality of the method has been explicitly proven by its application to processes of increasing complexity, such as heavy-flavour-pair [2] and single-top [3] production.[1] The basic idea of MC@NLO is that of avoiding the overcounting by subtracting from the exact NLO cross section its approximation,

[1] A complete list of processes implemented in MC@NLO can be found at `http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO`.

as implemented in the SMC program to which the NLO computation is matched. Such approximated cross section (which is the sum of what have been denoted in [1] as MC subtraction terms) is computed analytically, and is SMC dependent. On the other hand, the MC subtraction terms are process-independent, and thus, for a given SMC, can be computed once and for all. In the current version of the MC@NLO code, the MC subtraction terms have been computed for Herwig 6 [36], but extensions to other SMC are possible. In general, the exact NLO cross section minus the MC subtraction terms does not need to be positive. Therefore MC@NLO can generate events with negative weights. For the processes implemented so far, negative-weighted events are typically about 10–15% of the total. Their presence does not imply a negative cross section, since at the end physical distributions must turn out to be positive, but affects the overall efficiency of the simulation.

The features implemented in MC@NLO can be summarized as follows:

- Infrared-safe observables have NLO accuracy.
- Collinear emissions are summed at the leading-logarithmic level.
- The double logarithmic region (i.e. soft and collinear gluon emission) is treated correctly since Herwig 6 uses an angular-ordered shower.

2.3 POWHEG

The POWHEG (Positive Weight Hardest Emission Generator) method was proposed in ref. [8]. This method overcomes the problem of negative weighted events, and is not SMC specific. In the POWHEG method, the hardest radiation is generated first, with a technique that yields only positive-weighted events, using the exact NLO matrix elements. The POWHEG output can then be interfaced to any SMC program that is either p_T-ordered, or allows the implementation of a p_T veto.[2] However, when interfacing POWHEG to angular-ordered SMC programs, the double-log accuracy of the SMC is not sufficient to guarantee the double-log accuracy of the whole result. Some extra soft radiation (technically called vetoed-truncated shower in ref. [8]) must also be included in order to recover double-log accuracy. In fact, angular ordered SMC programs may generate soft radiation before generating the radiation with the largest p_T, while POWHEG generates it first. When POWHEG is interfaced to shower programs that use transverse-momentum ordering, the double-log accuracy should be correctly retained if the SMC is double-log accurate. The ARIADNE program [38], Pythia 6.4 [33] (when used with the new showering formalism), ADICIC++ [39] and the new parton showers based on the Catani–Seymour dipole formalism [40, 41] adopt transverse-momentum ordering, and aim to have accurate soft resummation approaches, in the limit of large number of colours.

Up to now, it has successfully been applied to Z pair hadroproduction [9], heavy-flavour production [12], e^+e^- annihilation into hadrons [13] and into top pairs [19], Drell-Yan vector boson production [16, 14], W' production [42], Higgs boson production via gluon fusion [17, 15], Higgs boson production associated with a vector boson (Higgs-strahlung) [15], single-top production [18] $Z + 1$ jet production [**?**], and, very recently, Higgs production in vector boson fusion [43].

The POWHEG BOX is an automated package able to construct a POWHEG implementation of a NLO process, given the following ingredients:

1. The list of all flavour structures of the Born processes.
2. The list of all the flavour structures of the real processes.
3. The Born phase space.
4. The Born squared amplitude, the color correlated and spin correlated Born amplitudes. These are common ingredients of NLO calculations regularized with a subtraction method.
5. The real matrix elements squared for all relevant partonic processes.
6. The finite part of the virtual corrections, computed in dimensional regularization or in dimensional reduction.

[2] All SMC programs compatible with the *Les Houches Interface for User Processes* [37] should comply with this requirement.

7. The Born color structure in the large limit of the number of colors.

The plots in this article were obtained using the POWHEG BOX, and the completion of the shower has been done both with Pythia 6 and with Herwig 6.

2.4 Herwig++

Herwig++ builds and improves upon the physics content of the parent Herwig 6 program, particularly in regard to the accurate simulation of QCD radiation. A major success of the original Herwig 6 program was in its modeling the effects of soft gluon interference, specifically the *colour coherence* phenomenon, whereby the intensity of soft gluon radiation, emitted at wide angles with respect to a bunch of collimated colour charges, is found to be proportional to the *coherent* sum of emissions from the constituents i.e. the jet parent [44], [45], [46], [47], [48], [49], [50], [51], [52], [53]. This effect is manifest in the perturbative series as large soft logarithms and is implemented as an angular ordering of successive emissions in the parton shower.

A further significant accomplishment of the POWHEG [8] formalism is in fully catering for such effects through a careful decomposition of the angular-ordered parton shower into a truncated shower, describing soft wide angle radiation, the hardest emission, as described above, and a vetoed shower comprised of increasingly collinear emissions. In doing so the formalism provides a means of distributing the highest p_T emission in an event according to the exact real-emission matrix element, including resummation effects, without degrading or otherwise disturbing the resummation and colour-coherence properties inherent to the parton shower.

The facility to perform truncated showers is absent from the older fortran Herwig 6 program but is implemented in the new Herwig++ program, which also includes its own native, independent, POWHEG simulation for the gluon-fusion process, the results of which are presented in section 3.3

2.5 Pythia

2.6 Sherpa

Sherpa is a multi-purpose Monte-Carlo event generation framework for colliders [54], [55]. The main goal of this project is a proper simulation of the perturbative aspects of the collision, although significant improvements have been made over the past years regarding the simulation of non-perturbative dynamics, like the process of hadronisation. One of the key features of the Sherpa program is a general implementation of a novel technique for combining tree-level matrix elements with parton showers, in arbitrary QCD or QCD-associated processes [26]. To this end, Sherpa makes use of its two internal tree-level matrix element generators AMEGIC++ [56] and Comix [57], which are capable of simulating both Standard Model (AMEGIC++ and Comix) and beyond Standard Model (AMEGIC++) reactions with high-multiplicity final states. Soft and collinear parton radiation is generated in Sherpa by means of a parton shower based on Catani–Seymour dipole factorisation [40]. This formalism has apparent advantages compared to more conventional parton showers, which are based on strict $1 \to 2$ splittings and often lack the notion of a well-defined spectator parton. In the new approach, the recoil partner of a splitting parton is always a single external particle. This turned out to be an important ingredient when combining parton-shower evolution with higher-order tree-level matrix elements.

The technique for merging matrix elements and parton showers, which is employed by Sherpa, is based on phase-space slicing. A detailed description of the corresponding algorithm, and its relation with other tree-level merging techniques, can be found in [26]. The method has recently been applied to mixed QCD and electroweak processes, in particular photon and diphoton production [58] and proves to give a consistent and reliable description of data from deep-inelastic lepton-nucleon scattering [59]. Compared to the CKKW algorithm [20, 21], the new merging scheme of ref. [26] is more sophisticated and improves over CKKW by including truncated vetoed parton showers. The results are more accurate, with respect to

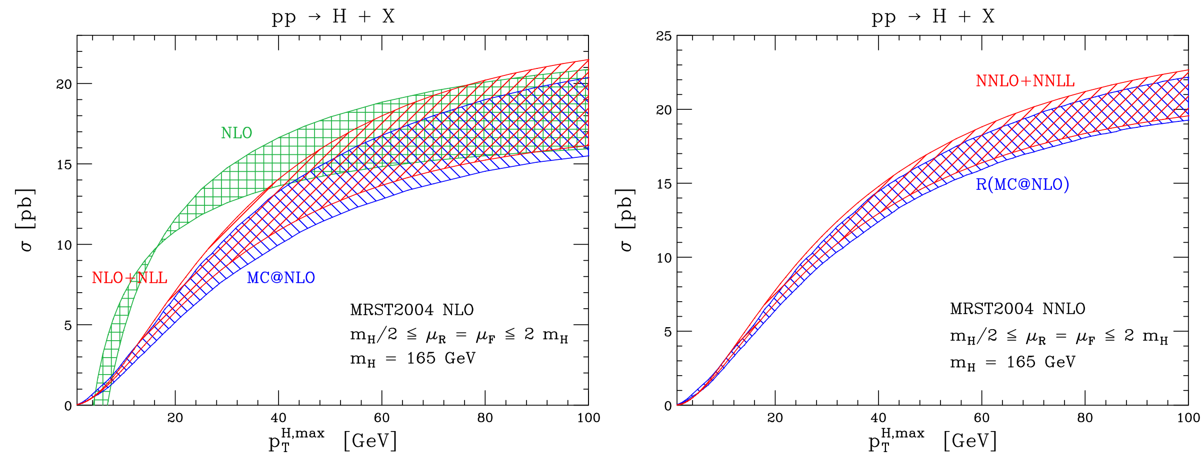

Fig. 1: Integrated Higgs cross section up to a given $p_T^{H,\max}$. On the left plot the NLO and NLO+NLL cross section is compared with MC@NLO. On the right plot the NNLO+NNLL resummed cross section is compared to a rescaled MC@NLO output.

those of the CKKW approach, which has been employed in former versions of the Sherpa event generator with already great success [60], [61], [62], [34].

3. GGH

Several studies are available on comparisons between MC's and fixed order results. See for example Refs [63], [17].

Ref. [63] performs a study on comparison with fixed order and resummed analytic calculations and shower MC (MC@NLO and HERWIG) for Higgs production. The framework is the $H \to WW$ decay mode, where a transverse momentum cut is needed to suppress the $t\bar{t}$ background. The integrated cross section as a function of the cut, computed in the various schemes, is reported in fig. 1 Further details are given , including a study of the impact of different jet clustering methods, hadronization and the underlying event on the signal.

In ref. [17] comparisons of POWHEG and MC@NLO for Higgs production in $gg->H$ are performed. Comparisons are also made with NNLO result and the NNLL resummation. Highlights of this comparison are shown in fig. 2. The rapidity of the hardest jet presents a dip in the central region, for MC@NLO. Other differences have to do with the very high transverse momentum spectrum of the Higgs and of the hardest jet in POWHEG, as shown in fig. 3. These differences are well understood, and discussed at length in several papers. Dips in the jet rapidity are inherited by MC@NLO from HERWIG, and softness in the p_T spectrum is also inherited from HERWIG, via a mechanism illustrated in [17]. A more $gg \to H$ specific issue that affect the shape of the p_T spectrum has to do with scale choice. Higgs production is a α_s^2 process, and Higgs plus one jet is α_s^3. One can argue wether at high p_T one should use $\alpha_s^3(p_T^2)$ or $\alpha_s^2(M_H)\alpha_s(p_T^2)$, for example.

PYTHIA (with ME corrections) is found to agree remarkably with POWHEG, up to a global K-factor [17].

3.1 HNNLO

The `HNNLO` program is based on an extension of the NLO subtraction formalism to NNLO, as described in ref. [30].

The calculation is organized in two parts. In the first part, the contribution of the regularized virtual

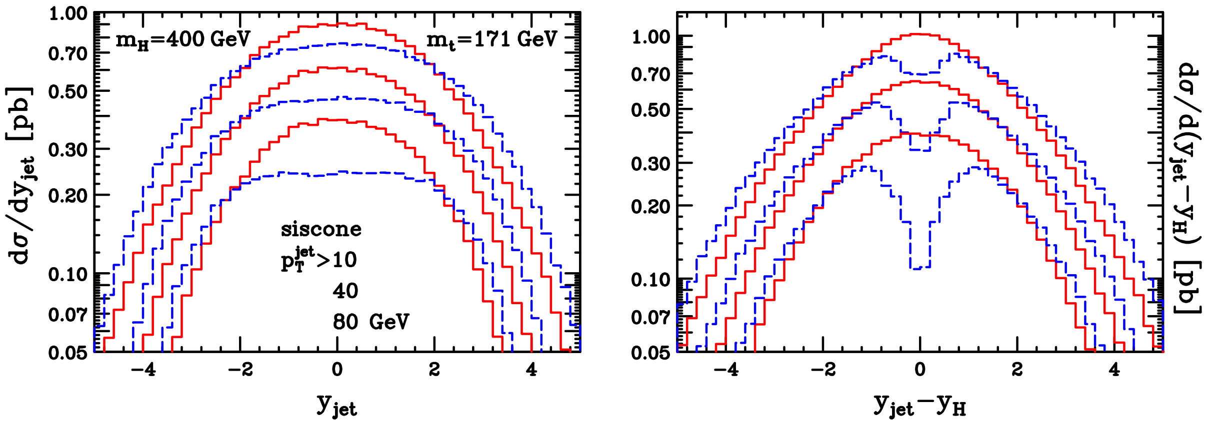

Fig. 2: Dip in rapidity of hardest jet in MC@NLO.

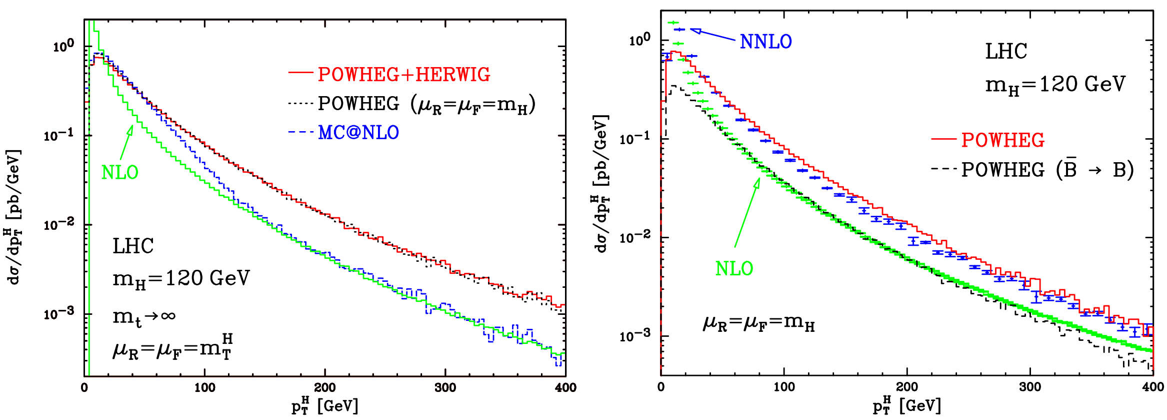

Fig. 3: Dip in rapidity of hardest jet in MC@NLO.

corrections is computed up to two-loop order. In the second part, the cross section for the production of the Higgs boson in association with one jet is first evaluated up to NLO, i.e. up to $\mathcal{O}(\alpha_s^4)$), using conventional NLO subtraction methods. Now, since the $H + \mathrm{jet}$ cross section is divergent when the transverse momentum, q_{T}, of the Higgs boson becomes small, a further counterterm must be subtracted to make the result finite as $q_{\mathrm{T}} \to 0$. To this end the program uses counterterm introduced in ref. [30]. Having regularized the real and virtual parts, the two contributions can be combined to reconstruct the full cross section. Organizing the differential cross section in this way, one can construct a parton-level event generator with which arbitrary infrared safe quantities can be computed. The present version of the program includes the decay modes $H \to \gamma\gamma$ [30], $H \to WW \to l\nu l\nu$ and $H \to ZZ \to 4$ leptons [31].

The calculation is performed in the large top-mass approximation. This is known to be a good approximation provided that the Higgs boson is not too heavy and the transverse momenta of the final state jets are not too large.

3.2 Parameters for the study

In the following, we present results for the pp LHC at a centre-of-mass energy of 10 TeV. The common features of the analysis are the following:

- *Model*: We work in the Standard Model in the large top-mass limit. In this approximation the couplings between the gluons and the Higgs boson are described by a dimension-five effective interaction
$$\mathcal{L}_h = -\frac{1}{4}\, g_h\, G^a_{\mu\nu} G^a_{\mu\nu} H, \tag{1}$$
with $g_h = \alpha_s/(3\pi v)$.
- *Event samples*: The analysis was performed on the generated final state and, with the exception of HNNLO, after parton showering. Hadronization effects were included for the MC@NLO and POWHEG results only.
A Higgs boson mass of $m_H = 120$ GeV was assumed. Tree-level predictions were generated using the leading order CTEQ6l1 PDF set [64], while generators employing the POWHEG method used the next-to-leading order PDF set CTEQ6m [64]. In both cases the parametrisation of the strong coupling was chosen accordingly. All partons (excluding the top quark) were taken to be massless and their Yukawa couplings were neglected.
- *Jet definitions*: Jets were defined using the longitudinally invariant $k_\perp$-algorithm with $D = 0.7$ in the implementation of FastJet [65]. They were required to lie within a rapidity range of $|\eta| < 4.5$ and have transverse momenta of $p_T > 20$ GeV.

3.3 RESULTS

In this section we present and discuss results obtained for some key observables in the analysis of Higgs production via gluon fusion. As already mentioned, we do not expect the LO matched results to provide reliable information on total rates, so we have normalized the corresponding curves, from MADGRAPH/MADEVENT and Sherpa, to the HNNLO result. On the other hand, having NLO accuracy, Herwig++, MC@NLO and POWHEG have not been rescaled.

In fig. 4, we plot the transverse momentum of the Higgs boson and, in fig. 5, its rapidity. The Higgs boson p_T, in particular, is determined by the recoiling QCD radiation, both soft and hard, and, exactly as for Drell-Yan, it is therefore a key observable. The blow up of the small-p_T region (left panel of fig. 4) shows quite good agreement among the various MC approaches with predictions being typically peaked in the range between 5 and 10 GeV. The obvious excess in the HNNLO prediction, at low-p_T, is expected on the grounds that it is based on a fixed order computation, hence, it does not resum the effects of multiple soft emissions, which are essential for a proper description of the $p_T = 0$ region. At higher values of the p_T, the agreement is also excellent, apart from MC@NLO which shows a steeper behaviour

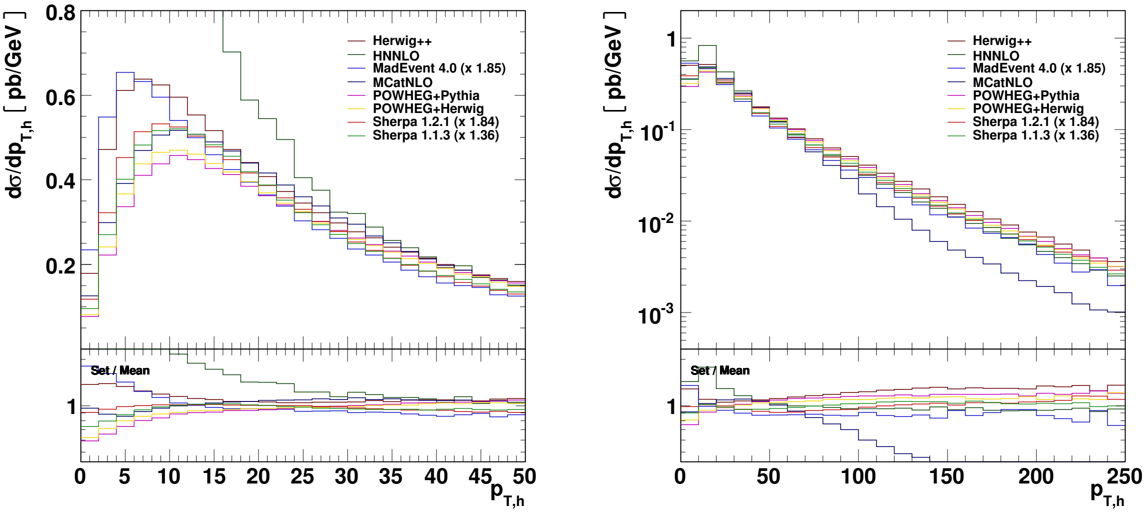

Fig. 4: The transverse momentum spectrum of the Higgs boson. Tree-level predictions have been rescaled by the global factors indicated in the legend. The lower panels display the ratio of individual results and the average of all histograms, excluding the results from HNNLO.

with respect to the results obtained by the POWHEG method and with the matching. As already pointed out in Refs [17], [15], this is due to NNLO terms in the POWHEG formula. It is, however, important to note that, for all the NLO codes, this particular distribution can be predicted only at LO, i.e. no $H + 2$ partons contribution and no $H + 1$ parton one-loop contributions are included. From this point of view, it is reasonable to expect the shape to be sensitive to variations in the renormalization and factorization scales, although, in practice, this sensitivity is much milder due to the resummation of higher order corrections (i.e. the shower). In any case, it is both remarkable to see that the predictions based on the POWHEG method and the ME+PS matching show such good agreement, particularly considering the fundamental differences in their methodology.

In the rapidity distributions of fig. 5, the HNNLO result shows all of its NNLO content: in fact, this is the only plot that receives contributions from the two-loop diagrams.

Figure 6 shows the jet p_T distributions for the four hardest jets (ordered in p_T). Once again the agreement among the various approaches is very good, with MC@NLO leading to significantly less events at very high p_T's; this lower number of events is in exact correspondance with that seen for the Higgs boson transverse-momentum distribution and bears the same explanation. A particularly interesting feature is the agreement found on the 3rd and 4th jet spectra. Only Sherpa has included the corresponding tree-level hard matrix elements, while all other predictions contain only one (NLO codes) or two (HNNLO and MADGRAPH/MADEVENT) hard partons. This good agreement is a mere coincidence, since in POWHEG, MC@NLO and Herwig++ these extra jets come from the shower, and are therefore only corect in the strict collinear limit. Similar comments hold for the pseudorapidity distributions of fig. 7.

Larger discrepancies are instead present for more exclusive quantities, such as the distance in the η-ϕ plane between the two leading jets, ΔR_{12}, shown in fig. 8. Indeed, we start appreciating here some interesting differences in shape: Herwig++, for example, predicts much steeper distribution than Sherpa.

Finally, in fig. 9, we plot the jets rates, which, once again, agree within 50% uncertainty even for higher multiplicities.

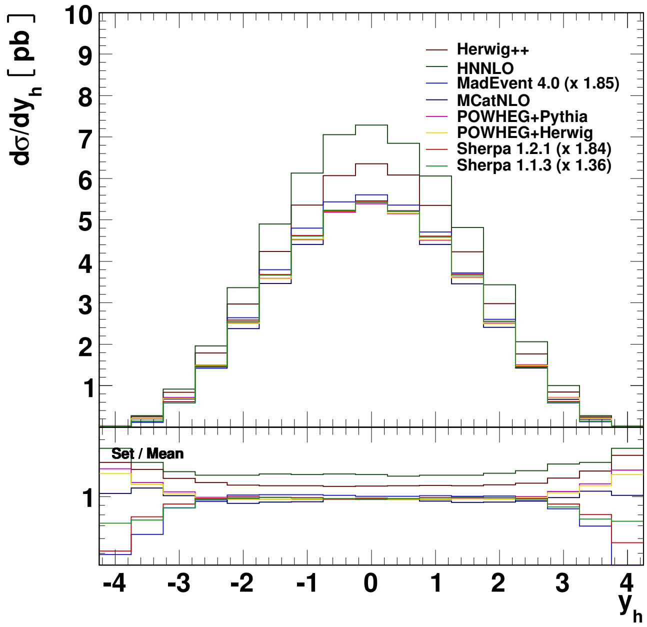

Fig. 5: The rapidity distribution of the Higgs boson. See fig. 4 for details.

4. VBF

Discussion and plots taken from Refs. [66], [67], [43].

5. VH

Discussion and plots taken from Ref. [15]. A fully differential NNLO computation is possible but still not available and therefore no comparison has ever been made.

6. ttH and bbH

No public code is available for Higgs production in association with heavy quarks. It would be interesting to have such a calculation implemented in MC@NLO or via the POWHEG box. This could be done in a very short time.

7. CONCLUSIONS

As we are entering the LHC running phase, we have available several very accurate ”TH”-style predictions in the form of parton-level integrators that can output histograms for any IR-safe observable. On the other hand, MonteCarlo with NLO accuracy are now (or wil be soon) available for all the relevant Higgs production processes. A systematic comparison between various implementations, PS programs and fixed or improved ”TH”-style calculations is now possible. In this brief note we have listed the available tools and also given an example (taken from Ref. [68]) on how such comparisons can be made.

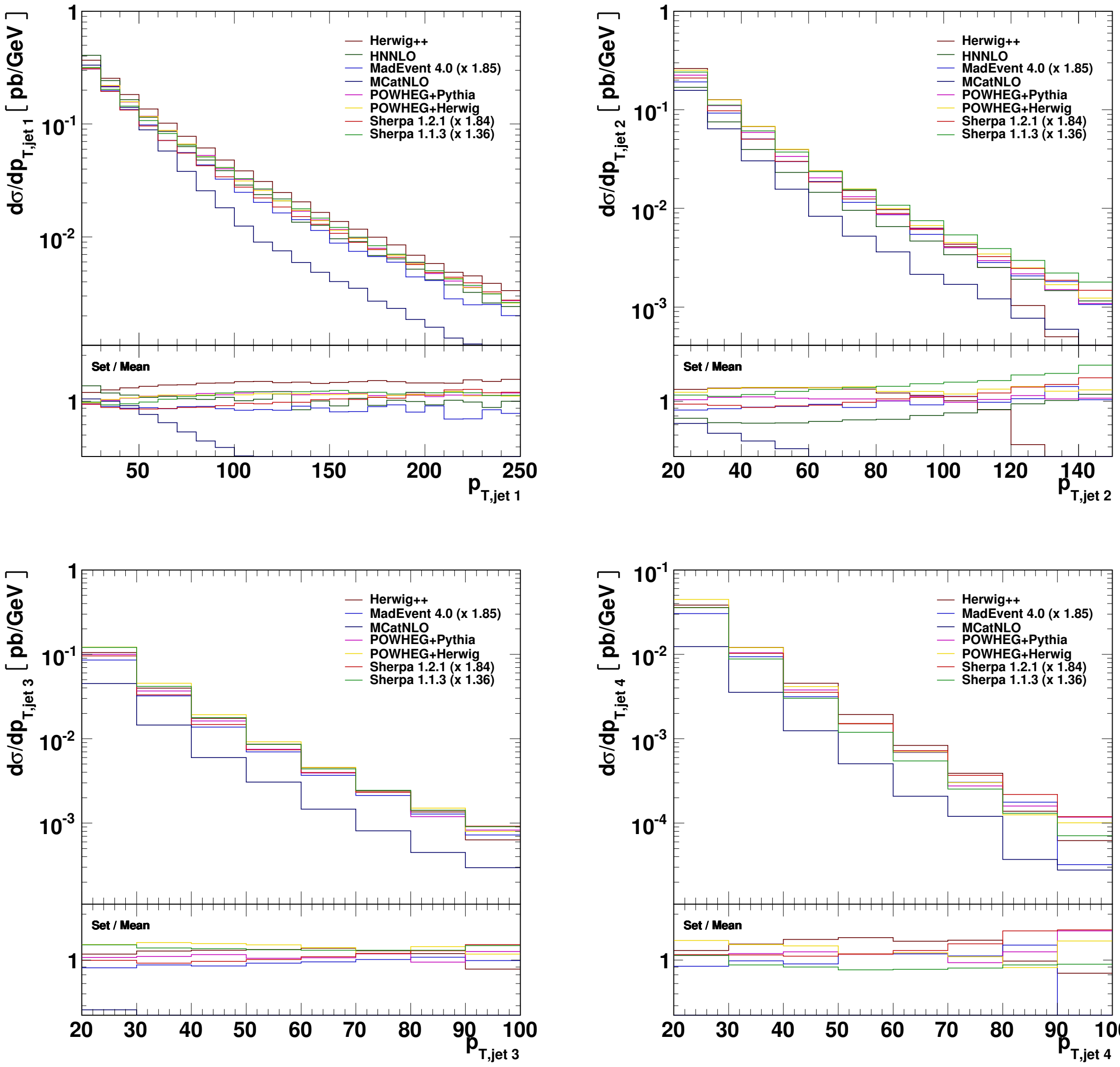

Fig. 6: The transverse momentum spectra of the first four hardest jets, ordered in p_T, accompanying the Higgs boson. See fig. 4 for details.

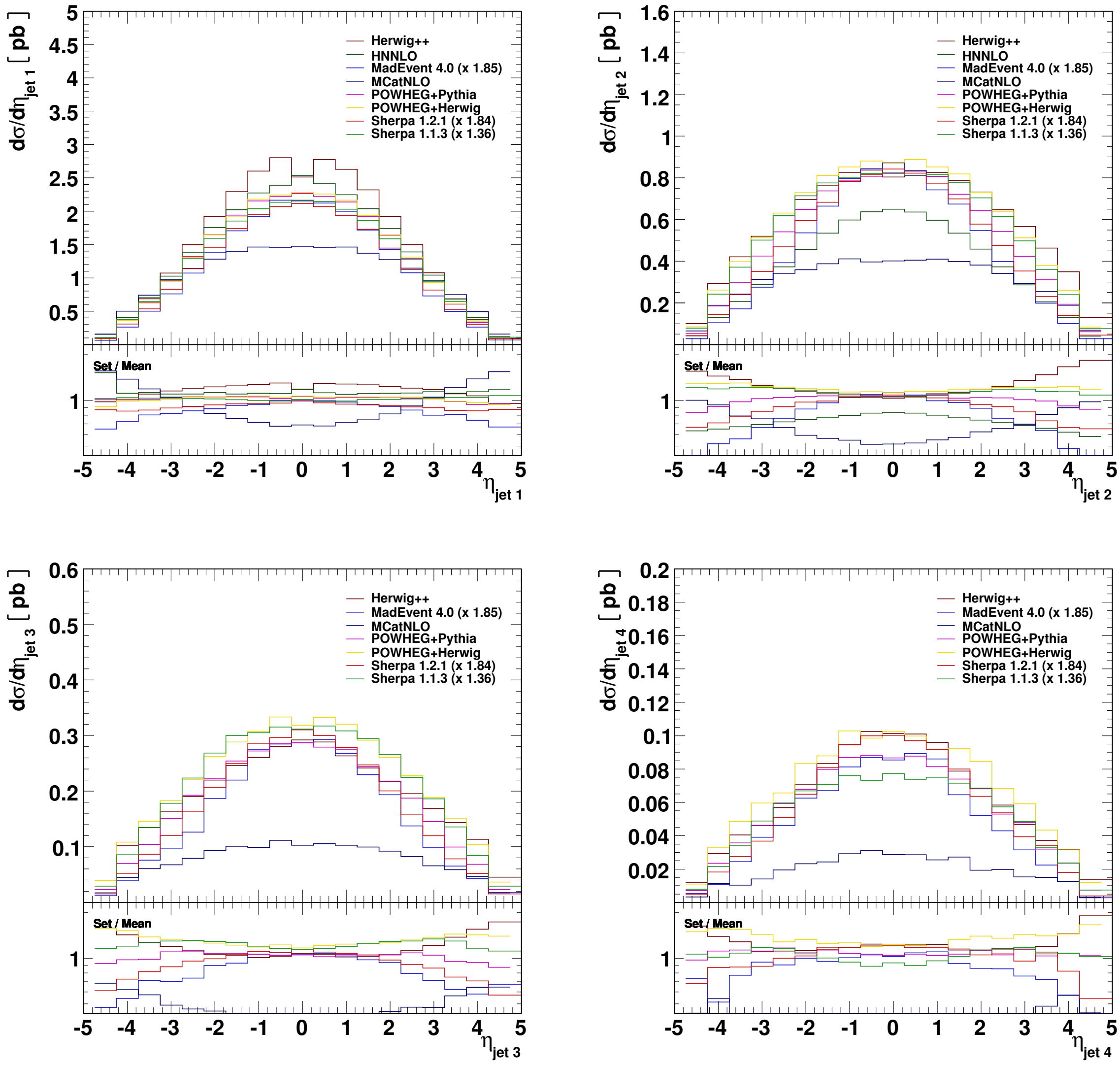

Fig. 7: The pseudorapidity distributions of the first four hardest jets, ordered in p_T, accompanying the Higgs boson. See fig. 4 for details.

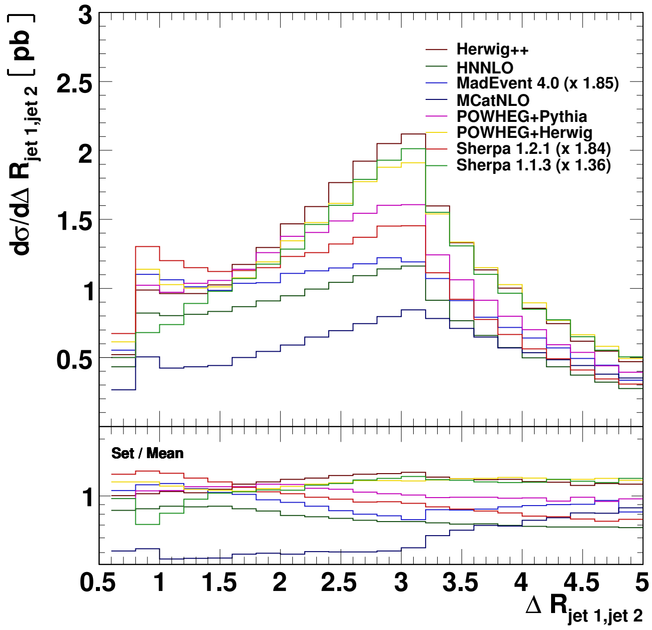

Fig. 8: The separation in the η-ϕ plane of the two leading jets accompanying the Higgs boson. See fig. 4 for details.

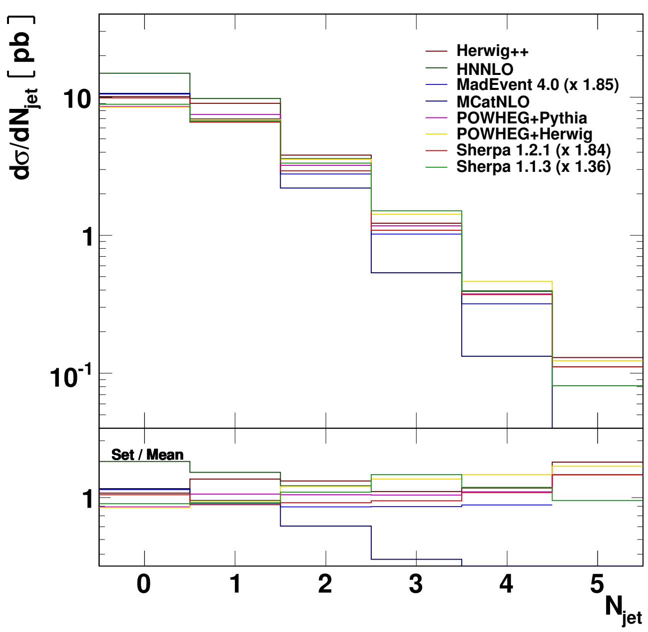

Fig. 9: The production rates for $N_{\rm jet}$ additional jets accompanying the Higgs boson. See fig. 4 for details.

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