Resummation benchmark: Scale variations in NangaParbat

Valerio Bertone and Giuseppe Bozzi

INFN and Università di Pavia

LHC EW WG Subgroup Meeting

March 26, 2020

TMD evolution equations

- TMD factorisation allows one to obtain the **evolution equations**:

$$\begin{cases} \dfrac{d\ln F}{d\ln\mu} = \gamma(\mu,\zeta) \\[2ex] \dfrac{d\ln F}{d\ln\sqrt{\zeta}} = K(\mu) \end{cases}, \qquad \frac{d^2\ln F}{d\ln\mu\, d\ln\sqrt{\zeta}} = \begin{cases} \dfrac{d\gamma}{d\ln\sqrt{\zeta}} \\[2ex] \dfrac{dK}{d\ln\mu} \end{cases} = \gamma_K(\alpha_s(\mu))$$

- To solve these equations we need to fix **two pairs of (*i.e.* four) scales**:
  - **initial** scales: (μ_0, ζ_0)
  - **final** scales: (μ, ζ)
- The solution is **unique** and reads:

$$F(\mu,\zeta) = R\left[(\mu,\zeta)\leftarrow(\mu_0,\zeta_0)\right] F(\mu_0,\zeta_0)$$

$$R\left[(\mu,\zeta)\leftarrow(\mu_0,\zeta_0)\right] = \exp\left\{K(\mu_0)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_F(\alpha_s(\mu')) - \gamma_K(\alpha_s(\mu'))\ln\frac{\sqrt{\zeta}}{\mu'}\right]\right\}$$

- The question is: **how do we choose these four scales?**

Scale variations

- A sensible choice of the scales is important to **allow perturbation theory to be reliable**:
  - **no large unresummed logarithms** should be introduced,
  - each scale has to be set in the **vicinity of its natural (central) value**,
  - scale variations (within a reasonable range) give an estimate of HO corrs.
- In TMD factorisation ($q_T \ll Q$) for DY the relevant scales are $\boldsymbol{q_T}$ and $\boldsymbol{Q}$:
  - natural to expect $\mu_0 \sim \sqrt{\zeta_0} \sim q_T \sim b_T^{-1}$ and $\mu \sim \sqrt{\zeta} \sim Q$
- In fact, it turns out that (in the $\overline{\text{MS}}$ scheme) the **central scales** are:

$$\mu_0 = \sqrt{\zeta_0} = \frac{2e^{-\gamma_E}}{b_T} \equiv \mu_b \quad \text{and} \quad \mu = \sqrt{\zeta} = Q$$

- This choice **nullifies** all unresummed logs. One should thus consider:

$$\mu_0 = C_i^{(1)} \mu_b, \quad \sqrt{\zeta_0} = C_i^{(2)} \mu_b, \quad \mu = C_f^{(1)} Q, \quad \sqrt{\zeta} = C_f^{(2)} Q,$$

Scale variations

- A sensible choice of the scales is important to **allow perturbation theory to be reliable**:
  - **no large unresummed logarithms** should be introduced,
  - each scale has to be set in the **vicinity of its natural (central) value**,
  - scale variations (within a reasonable range) give an estimate of HO corrs.
- In TMD factorisation ($q_T \ll Q$) for DY the relevant scales are $\boldsymbol{q_T}$ and $\boldsymbol{Q}$:
  - natural to expect $\mu_0 \sim \sqrt{\zeta_0} \sim q_T \sim b_T^{-1}$ and $\mu \sim \sqrt{\zeta} \sim Q$
- In fact, it turns out that (in the $\overline{\text{MS}}$ scheme) the **central scales** are:

$$\mu_0 = \sqrt{\zeta_0} = \frac{2e^{-\gamma_E}}{b_T} \equiv \mu_b \quad \text{and} \quad \mu = \sqrt{\zeta} = Q$$

- This choice **nullifies** all unresummed logs. One should thus consider:

$$\mu_0 = C_i^{(1)} \mu_b, \quad \sqrt{\zeta_0} = C_i^{(2)} \mu_b, \quad \mu = C_f^{(1)} Q, \quad \sqrt{\zeta} = C_f^{(2)} Q,$$

Scale variations

- To reason why variations of ζ have **no effect** is that:

$$\frac{d\sigma}{dq_T} \propto H\left(\frac{\mu}{Q}\right) F_1(\mu, \zeta_1) F_2(\mu, \zeta_2) \quad \text{with} \quad \boxed{\zeta_1 \zeta_2 \stackrel{!}{=} Q^4}$$

- It is easy to see that:

$$F_1(\mu, \zeta_1) F_2(\mu, \zeta_2) = \underbrace{R\left[(\mu, \zeta_1) \leftarrow (\mu_0, \zeta_0)\right] R\left[(\mu, \zeta_2) \leftarrow (\mu_0, \zeta_0)\right]}_{f(\zeta_1 \zeta_2) = f(Q^4)} F_1(\mu_0, \zeta_0) F_2(\mu_0, \zeta_0)$$

- The single dependence on ζ_1 and ζ_2 **drops** in the combination:
  - we choose $\zeta_1 = \zeta_2 = Q^2$ but any other choice such that $\zeta_1 \zeta_2 = Q^4$ is **identical**.
- In addition, in NangaParbat we have chosen to set $\mu_0 = \sqrt{\zeta_0}$:
  - not strictly necessary but **probably a conservative choice**.
- At the end of the day, we have **two scales** to be varied:

$$\boxed{\mu_0 = \sqrt{\zeta_0} = C_i \mu_b \quad \text{and} \quad \mu = C_f Q}$$

Comparison to q_T resummation

- In q_T resummation, the **resummation scale** M is introduced as:

$$L = \ln\left(\frac{Q}{\mu_b}\right) = \ln\left(\frac{M}{\mu_b}\right) + \ln\left(\frac{Q}{M}\right)$$

- These logs are **exposed** by expressing integral representations of the argument of the Sudakov in terms of the **functions** g_n:

$$\begin{aligned}\int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[A(\alpha_s(\mu'))\ln\left(\frac{Q}{\mu'}\right) + B(\alpha_s(\mu'))\right] &= Lg_0(\alpha_s L) + \sum_{n=1}^{\infty} \alpha_s^{n-1} g_n(\alpha_s L) \\ &= Lg_0(\alpha_s L) + \sum_{n=1}^{k} \alpha_s^{n-1} g_n(\alpha_s L) + \mathcal{O}(\alpha_s^{k+n} L^n)\end{aligned}$$

- The series in the r.h.s. is **truncated** according to the log accuracy:
  - the truncation is responsible for the **explicit dependence on** M.
- If the l.h.s. integral is computed exactly, no dependence on M appears:
  - this is what we do in NangaParbat by computing the integral numerically,
  - therefore, we have **no resummation scale dependence**.

Comparison to q_T resummation

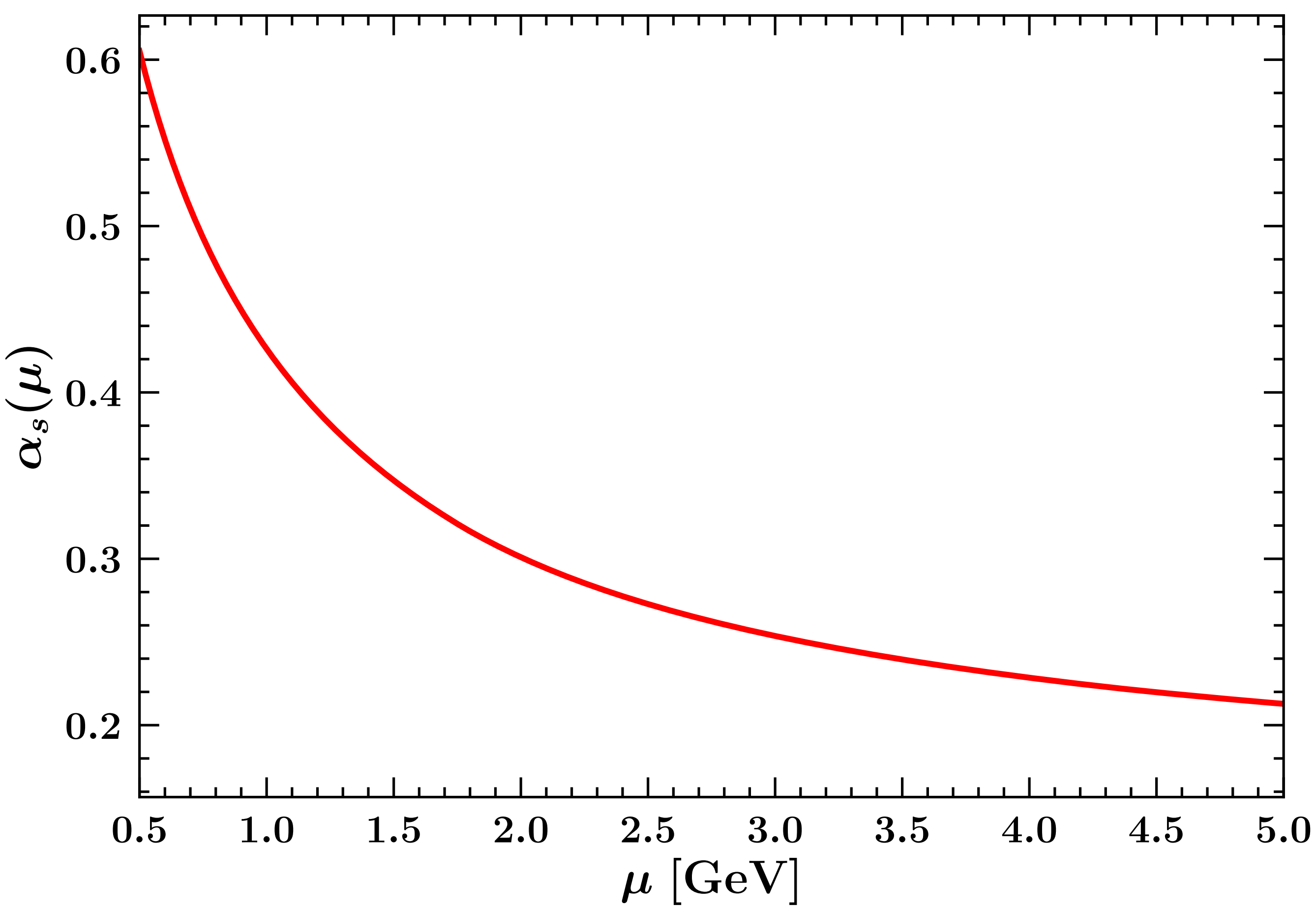
- The **renormalisation scale** μ_R in q_T resummation is probably to be (partly) identified with the scale μ in the TMD formalism:
  - this is the large scale at which the strong coupling α_s is computed.
- The **factorisation scale** μ_F present in q_T resummation is absent in the TMD formalism:
  - in the TMD approach, PDFs are computed at the low scale μ_0:
    - μ_0 is varied around μ_b,
  - in q_T resummation, PDFs are evolved from *exactly* μ_b up to μ_F:
    - μ_F is varied around Q.
  - variations of μ_0 are typically much larger than variations of μ_F because at the energies relevant to the benchmark $\alpha_s(\mu_0) \gg \alpha_s(\mu_F)$:
    - problems with NangaParbat in using a b_{max} too large with scale variations.

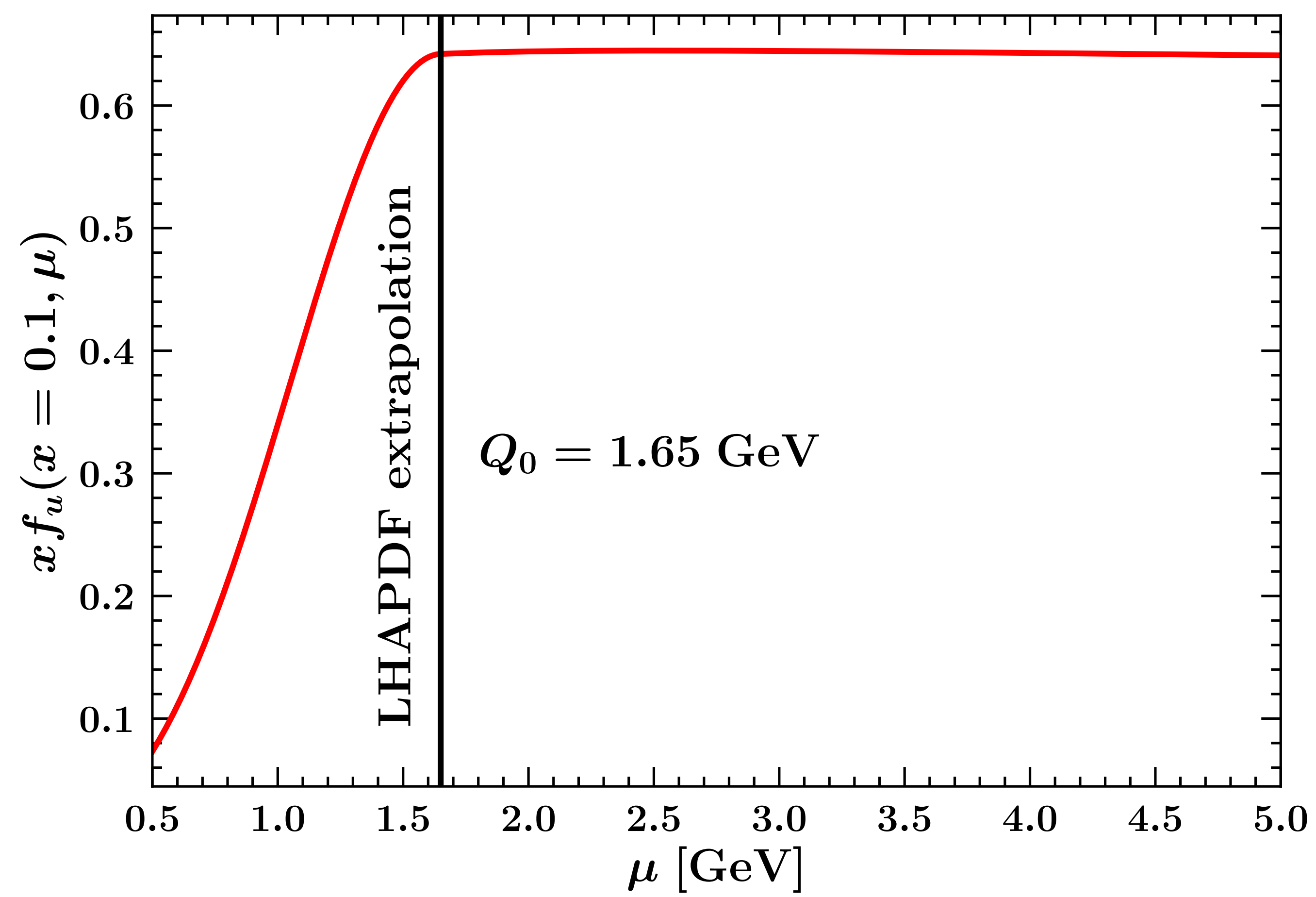
Problems with b_{max}

- In the benchmark settings $b_{\max}$ is set such that:

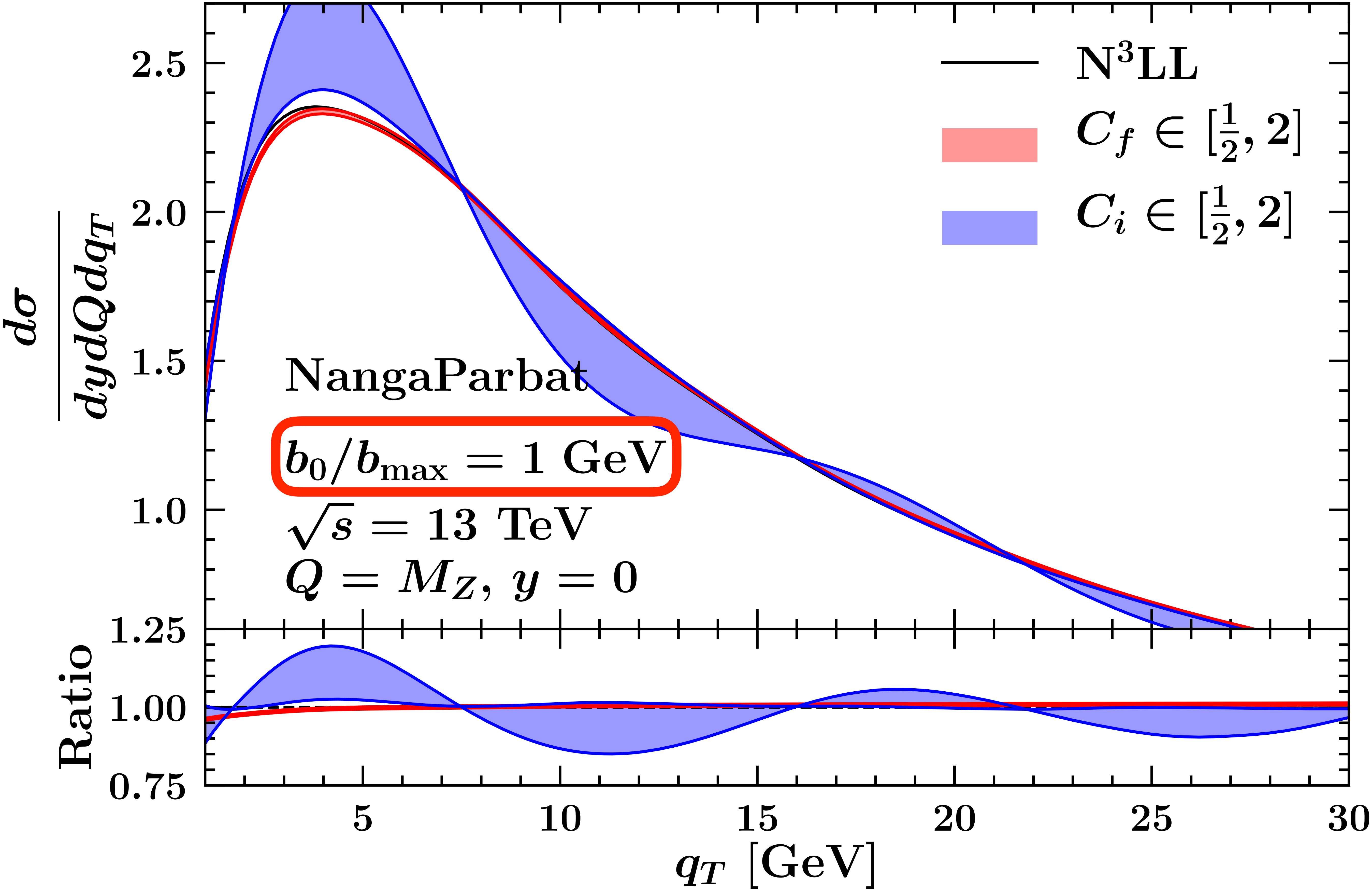
$$\mu_{\min} = \frac{b_0}{b_{\max}} = \frac{2e^{-\gamma_E}}{b_{\max}} = 1\ \text{GeV}$$

- This is the **minimum** value of the scale at α_s which and PDFs are called.
- Scales variations in NangaParbat cause $\mu_{\min}$ to be scaled by some factor:
  - if the scale variation is a factor 1/2, then $\mu_{\min} = 0.5$ GeV α_s where is very large and **PDFs are crazy**.

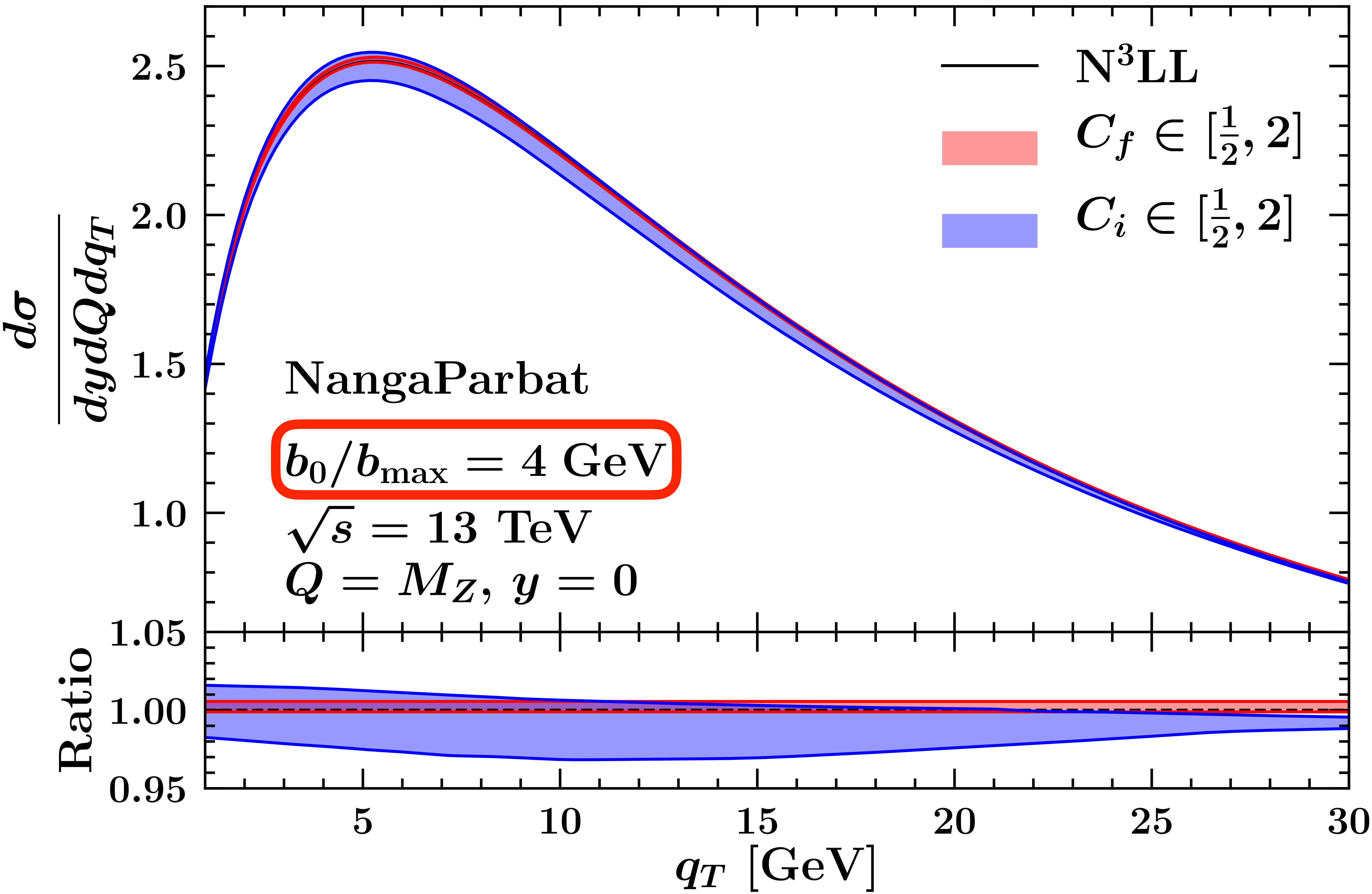




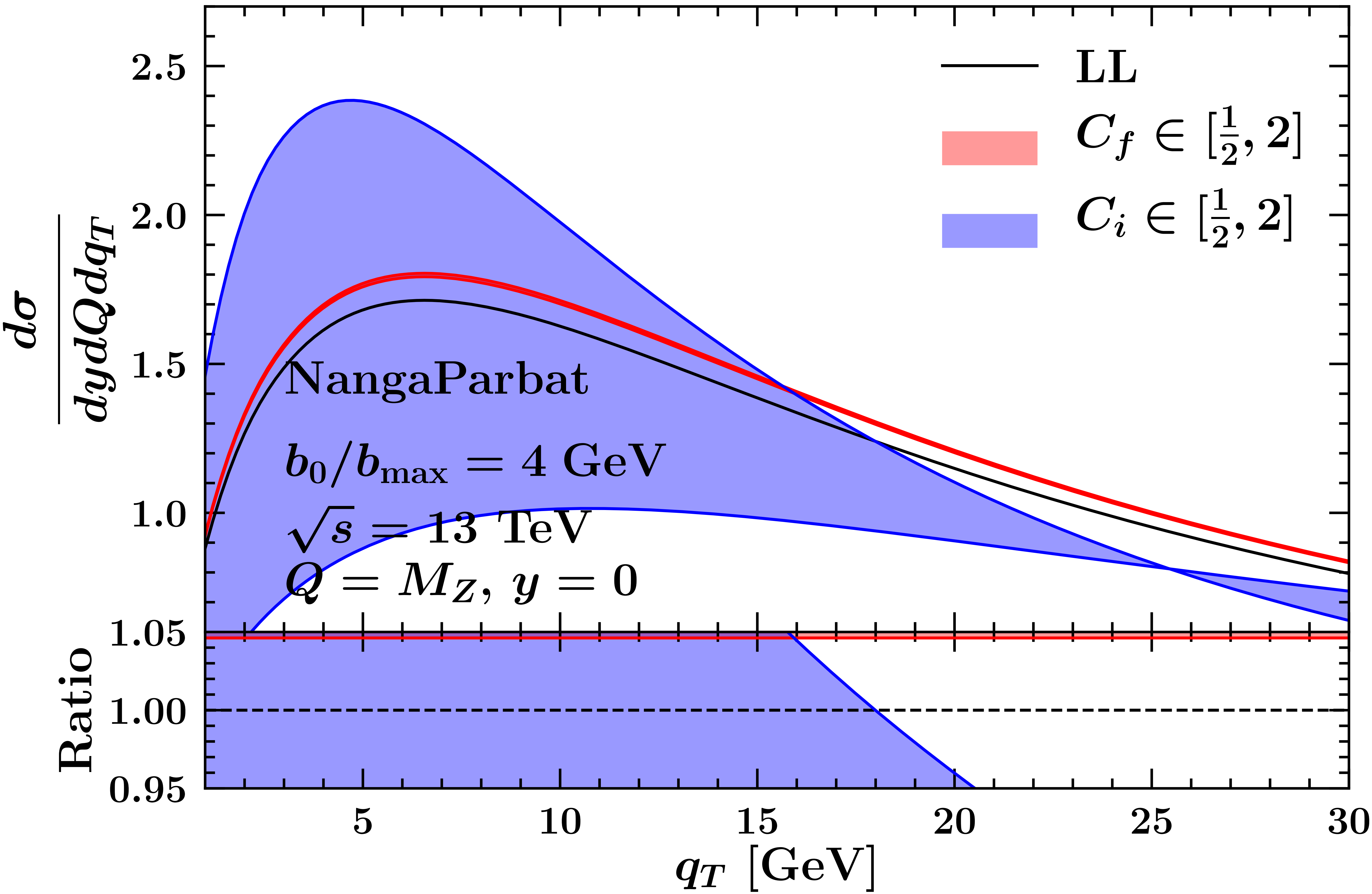
Numerical effect



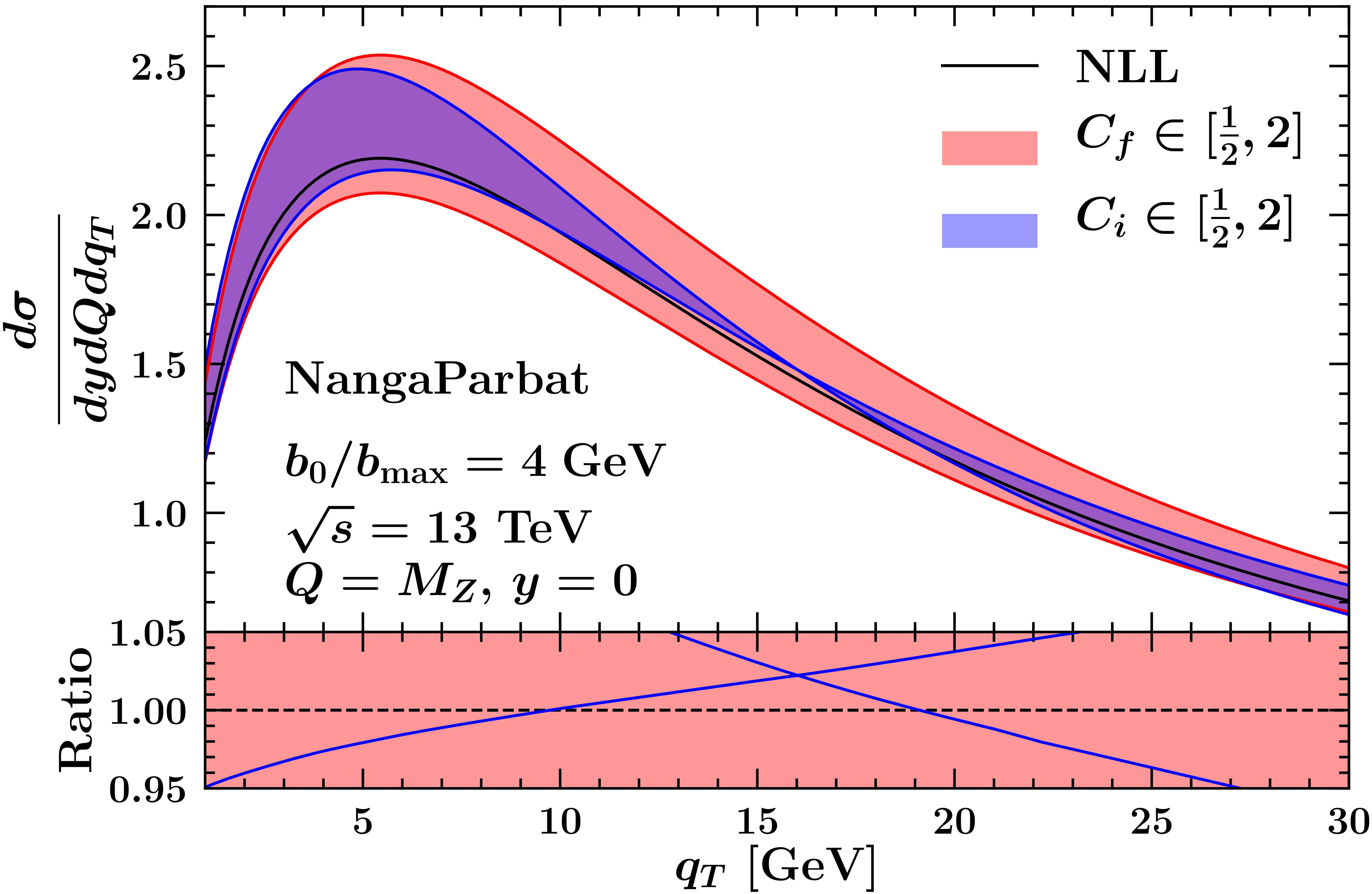
Numerical effect



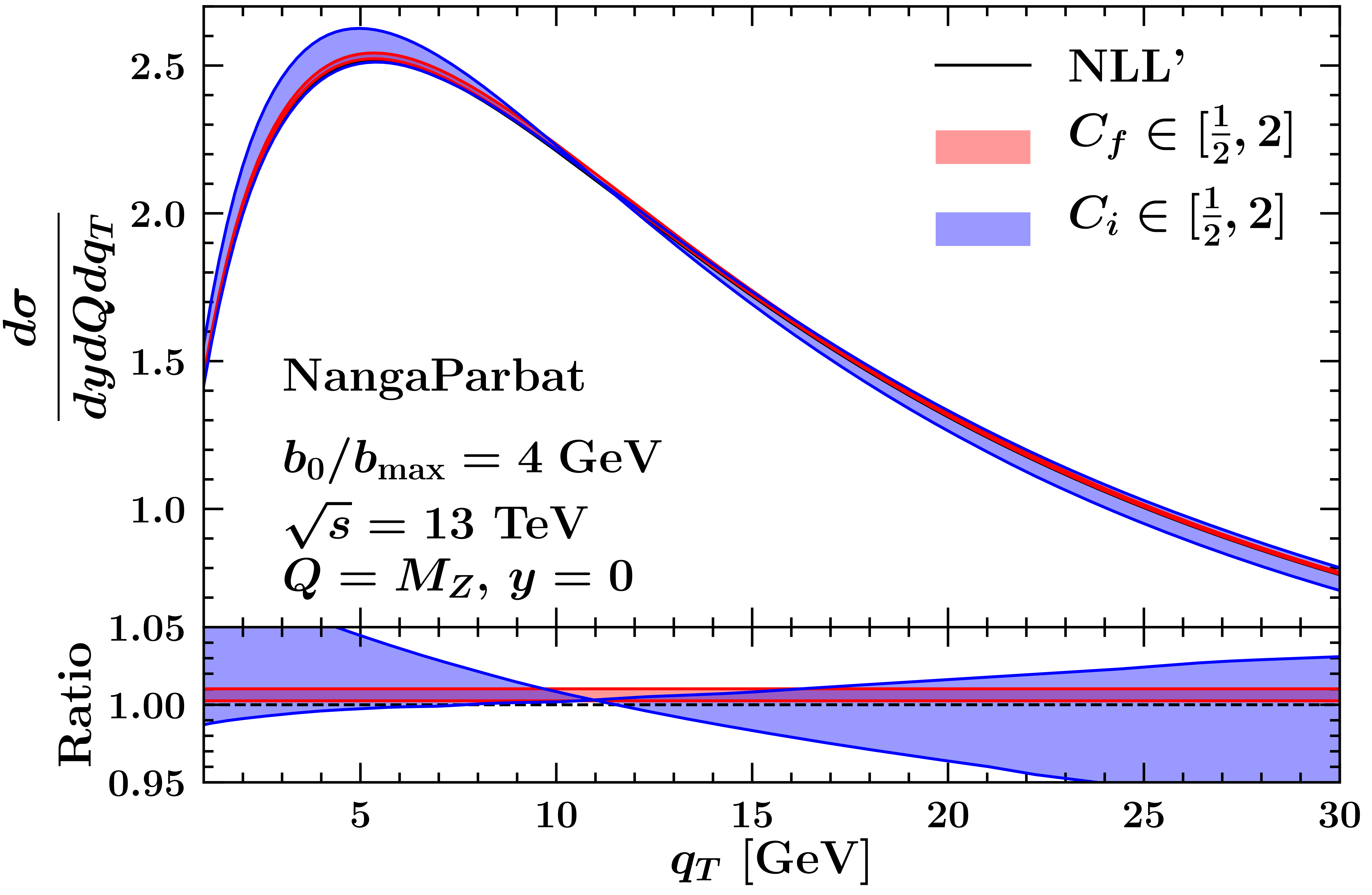
Perturbative convergence



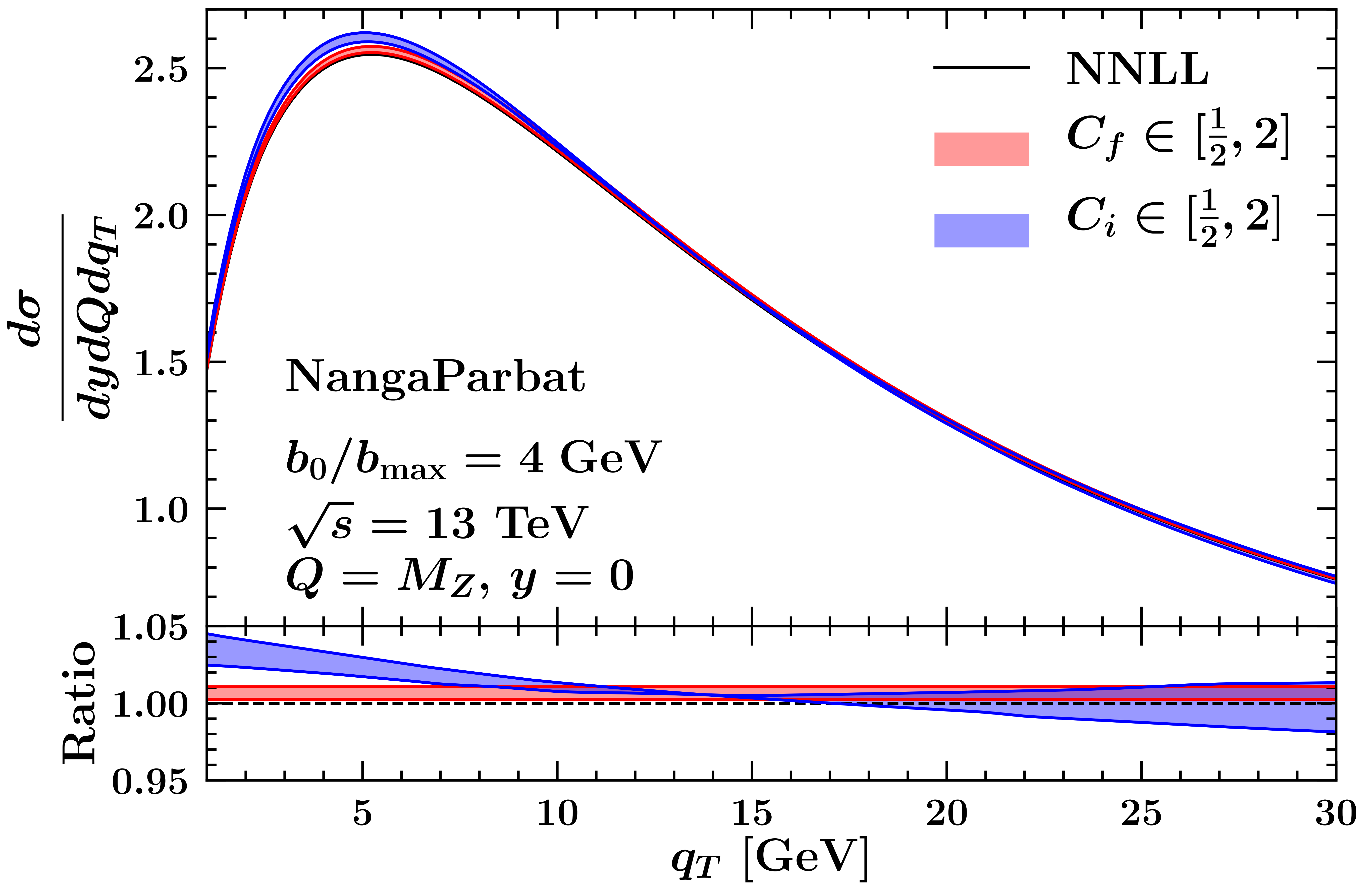
Perturbative convergence



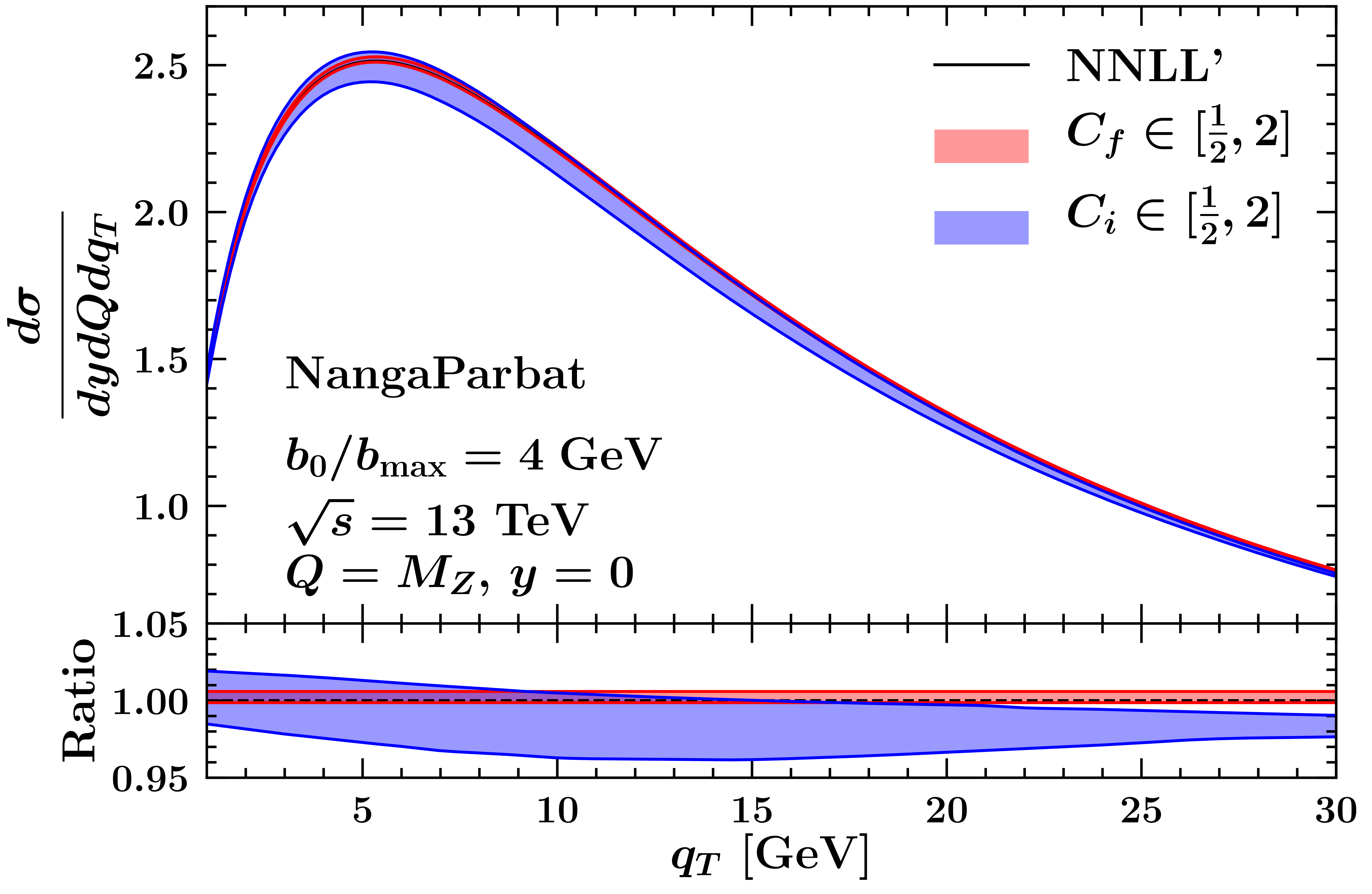
Perturbative convergence



Perturbative convergence



Perturbative convergence



Perturbative convergence