Bunch by bunch tune shifts for HL-LHC: tune separation along the bunch train

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HL-LHC baseline scheme (assumed in TDR)

Resistive wall (collimators and all resistive elements) 2019 HL-LHC impedance model

Flat top
N=2.3e11 ppb
Q'=15
σ= 0.08994
Qx=62.31
Qy=60.32
HL-LHC: PyHEADTAIL multi-bunch simulation

Full tune shift over the bunch train is in the order of $10^{-3}$
### Maximum tune shift along the train due to impedance

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Cycle state</td>
<td>Injection</td>
<td>Flat-top</td>
<td>Injection</td>
<td>Flat-top</td>
<td>Injection</td>
<td>Flat-top</td>
</tr>
<tr>
<td>Energy, GeV</td>
<td>450</td>
<td>6500</td>
<td>450</td>
<td>7000</td>
<td>1300</td>
<td>13500</td>
</tr>
<tr>
<td>Intensity, ppb</td>
<td>1.05x10^{11}</td>
<td>1.05x10^{11}</td>
<td>2.3x10^{11}</td>
<td>2.3x10^{11}</td>
<td>2.2x10^{11}</td>
<td>2.2x10^{11}</td>
</tr>
<tr>
<td>Current full, A</td>
<td>0.67</td>
<td>0.67</td>
<td>1.47</td>
<td>1.47</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Imp, MΩ/m</td>
<td>100</td>
<td>900</td>
<td>100</td>
<td>800</td>
<td>250</td>
<td>2000</td>
</tr>
<tr>
<td>Tune shift full</td>
<td>8.8x10^{-4}</td>
<td>5.5x10^{-4}</td>
<td>1.9x10^{-3}</td>
<td>1.0x10^{-3}</td>
<td>1.6x10^{-3}</td>
<td>1.2x10^{-3}</td>
</tr>
</tbody>
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\[
\Delta Q_{full} = \frac{1}{4\pi Q}\frac{R}{E/e}Z_{\perp}I_b
\]

**Approximations:**
- Uniform bunch train of length \( \tau \)
- Resistive wall impedance \( Z \sim (j/\omega)^{1/2} \)

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**L. Vos, EPAC 2000**
HL-LHC: PyHEADTAIL multi-bunch simulation

What about the tune separation along the bunch train?

Full tune shift over the bunch train is in the order of $10^{-3}$

Requires absolute accuracy of the simulated tunes
Single bunch, 1 turn wake

Single bunch, multi-turn wake

Negligible effect of the multi-turn wake

Single bunch tune shift significantly smaller compared to the tune shift experienced by the first bunch of the train
HL-LHC: PyHEADTAIL multi-bunch simulation

Multi-bunch, 1 turn wake

With single turn wake bunches only see the effect of the previous bunches over 1 turn

Multi-bunch, multi-turn wake

With multi-turn wake all bunches see the effect of all other bunches over the decay time of the wake in #turns

The bunch by bunch tune shift along the train is a single-turn wake effect

The additional tune shift in the order of $-10^{-3}$ in $y$ and $10^{-3}$ in $x$ appears only in the case of multi-bunch (multi-turn wake)
The multi-bunch multi-turn effect is associated with detuning impedance.

HL-LHC: PyHEADTAIL multi-bunch simulation

Full tune shift over the bunch train is in the order of $10^{-3}$

Additional tune shift in the order of $-10^{-3}$ in $y$ and $10^{-3}$ in $x$ due to detuning impedance (multi-bunch multi-turn wake effect)
Tune separation along the bunch train

Reduction in tune separation dominated by the additional tune shift in the order of $-10^{-3}$ in $y$ and $10^{-3}$ in $x$
Summary

• PyHEADTAIL multi-bunch simulations predict the maximum tune shift of the full train of bunches for HL-LHC beam at flat top to be in the order of $10^{-3}$
  – In good agreement with the numbers obtained from L. Vos formula

• In the case under study ($Q_x=62.31$, $Q_y=60.32$) the tune separation between horizontal and vertical plane is in the order of $8 \times 10^{-3}$
  – Without correction the tune separation is reduced by about $2 \times 10^{-3}$ due to the additional tune shift of all bunches from detuning impedance.

• Along the train the tune separation is reduced by about $2 \times 10^{-4}$. 
Thank you for your attention