

Status of SixTrack Implementation for Hollow Electron Lenses

A. Mereghetti

Acknowledgements: R. Bruce, M. Giovannozzi, D. Mirarchi, S. Redaelli

Material substantially the same as that presented at [ColUSM #122, Informal review of the HL-LHC electron lens design](#)

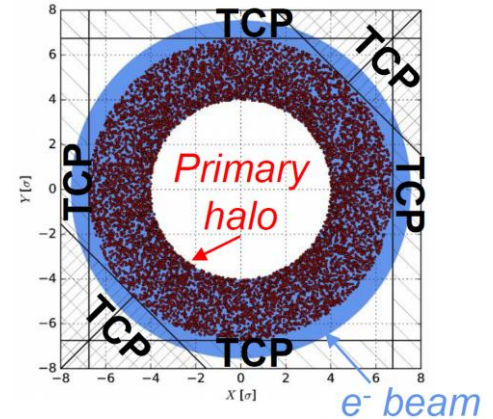


E-Beam – #2 Remote WG Meeting

1st April 2020

Introduction

- Hollow Electron Lenses (HELs) are presently part of the baseline upgrade foreseen by the High Luminosity Large Hadron Collider (HL-LHC) project;
- Their scope is to clean beam tails (e.g. $>3.5\sigma$) to avoid magnet quenches or permanent damage to collimators in case of orbit jitters or crab cavity phase slips;
 - Scaling scraping measurements of beam population at the LHC to HL-LHC beam intensities shows that ~ 35 MJ are expected in the beam tails at flat top ([B. Salvachua Ferrando, International Review of the HL-LHC Collimation System](#));
- Working principle:
 - The electron beam is hollow, covering the amplitude range between the desired cut and the TCP cut;
 - Diffusion speed of tails is enhanced on purpose to dispose them;
 - The beam core should be un-affected;
- Big **simulation campaign**, to **define operational scenarios** and **optimal parameters** of e-Lens;
 - Using **ideal** e-Lens;
 - **Pulsing mode**: most promising, for fast removal of tails;
 - **DC mode**: promising for continuous and less aggressive tail cleaning;
 - [D. Mirarchi, 9th HL-LHC Annual Meeting](#);



Ideal Electron Lens: SixTrack Implementation

- Electron beam: indefinitely long, with cylindrical symmetry, no changes along the lens;
- No longitudinal component of field lines taken into account, no update of energy of beam particles;
 - Kick applied only to transverse momenta, based only on transverse position;
- No fringe fields;
- Thin lens only!
- Can simulate hollow and full lens
 - Foxification (i.e. 6D closed orbit calculation via Taylor maps) of full e-lens completed!
- Can simulate three radial profiles: constant, Gaussian, from ASCII file (e.g. from measurements);
- Compatible with all species tracked by SixTrack;
- Lenses are DYNK-able (i.e. kick can be varied with time);
- Dynamic memory allocation, i.e. no hard-coded limit in number of lenses;
- Flexible user interface;
 - E.g. beam of lens composed by particles other than electrons and geometrical parameters of lens expressed in normalised units;

As done in other tracking codes, like LifeTrack or Merlin

Implied implementing a general module for polynomial interpolation in SixTrack
→ Interpolation of any desired degree!

Recent enhancements with respect to previous implementation by M. Fitterer *et al.*

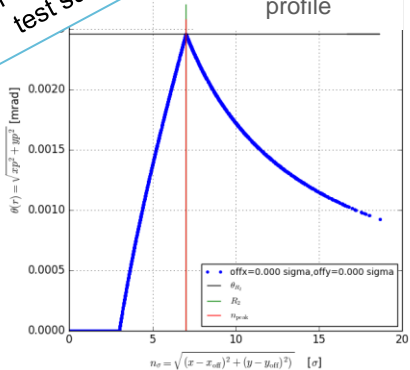
Assumption fine for beams of few GeV (Z);
→ Outlook: considering to extend the implementation, starting from an approximated Hamiltonian;

Ideal Electron Lens: SixTrack Implementation (II)

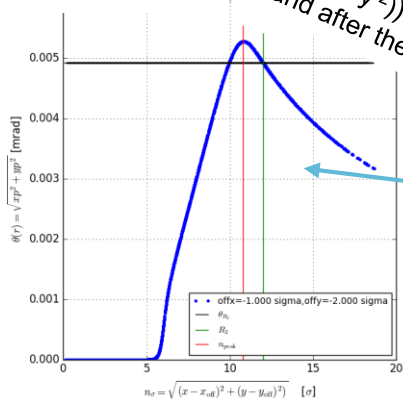
Examples taken from the SixTrack test suite

Hollow e-lenses

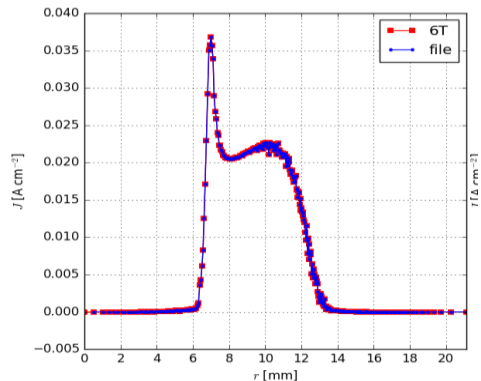
Constant radial profile



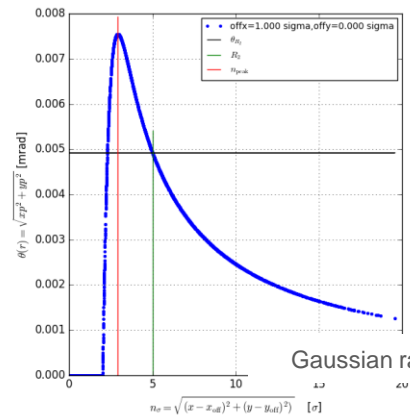
Radial profile from .txt file



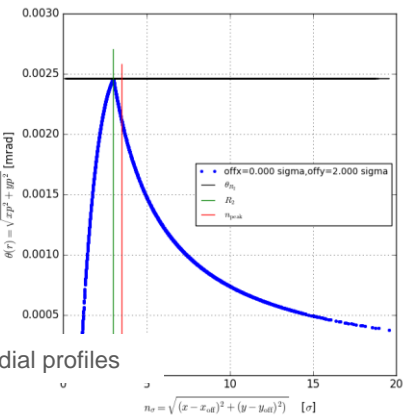
Difference in "radial" angle (i.e. $\sqrt{x^2+y^2}$) between before and after the electron lens



Original radial profile



Gaussian radial profiles



Preliminary draft 09:42 31 March 2020

31 March 2020

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Physics of the Implementation of Electron Lenses in SixTrack v5.xx.yy

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Keywords: Electron lens, SixTrack, symplectic tracking map

Report about implemented physics in preparation!
(including proof of symplecticity)

Towards a More Realistic Description of the Electron Lens

- The description of the ideal electron lens can be deployed to identify key working parameters;
- When evaluating impact of electron lens on beam tails and core, it is important also to take into account other effects, e.g.:
 - Effect of electric field in the **region of the main bends** of the electron lens (injection/extraction of electron beam);
 - **Evolution** of transverse distribution of **electron beam along the lens** (e.g. due to space-charge in electron beam);
- In order to take into account these effects, it is necessary to simulate the actual magnetic configuration of the lens and the electron beam dynamics;
 - Approach can be only numerical!

- Method outlined by G. Stancari in [FERMILAB-FN-0972-APC](#), based on Chebyshev polynomials:
 1. Use numerical simulations to define distribution of electrons and compute the electric potential and field thus generated as 3D maps;
 2. Longitudinally integrate the maps, to get the integrated values – from 3D maps to 2D maps;
 3. Fit the 2D maps by means of Chebyshev polynomials – from 2D maps to fit coefficients;
 4. Deploy the fit coefficients in tracking code, to estimate effects of integrated fields on proton beam;
- The method is effective to simulate heavily (transversely) non-linear electric fields in a CPU-efficient way;
 - Method already implemented in LifeTrack;

We need a simulation set-up that allows to take into account asymmetries in the electron beam

First implementation in SixTrack!

Method

- Method outlined by G. Stancari in [FERMILAB-FN-0972-APC](#), based on Chebyshev polynomials:
 - Use numerical simulations to define distribution of electrons and compute the electric potential and field thus generated as 3D maps;
 - Longitudinally integrate the maps, to get the integrated values – from 3D maps to 2D maps;
 - Fit the 2D maps by means of Chebyshev polynomials – from 2D maps to fit coefficients;
 - Deploy the fit coefficients in SixTrack, to estimate effects of integrated fields on proton beam;

This formalism is symplectic by construction (see [FERMILAB-FN-0972-APC](#))

Electric Potential

$$V(x, y) = \int_{z_1}^{z_2} \phi(x, y, z) dz$$

$$\begin{aligned} V(x, y) &= C_{00} + C_{10} \cdot T_1\left(\frac{x}{a}\right) + C_{01} \cdot T_1\left(\frac{y}{a}\right) + \\ &C_{20} \cdot T_2\left(\frac{x}{a}\right) + C_{11} \cdot T_1\left(\frac{x}{a}\right) \cdot T_1\left(\frac{y}{a}\right) + C_{02} \cdot T_2\left(\frac{y}{a}\right) + \dots \\ &= \sum_{n=0}^N \sum_{j=0}^n C_{j, (n-j)} \cdot T_j\left(\frac{x}{a}\right) \cdot T_{n-j}\left(\frac{y}{a}\right) \end{aligned}$$

$$T_0(u) = 1$$

$$T_1(u) = u$$

$$T_n(u) = 2u \cdot T_{n-1}(u) - T_{n-2}(u)$$

Electric Field

$$k_x(x, y) \equiv \int_{z_1}^{z_2} E_x(x, y, z) dz,$$

$$k_y(x, y) \equiv \int_{z_1}^{z_2} E_y(x, y, z) dz,$$

$$k_x(x, y) = -\frac{\partial V}{\partial x} = -\frac{1}{a} \sum_{n=0}^N \sum_{j=0}^n C_{j, (n-j)} \cdot T_j'\left(\frac{x}{a}\right) \cdot T_{n-j}\left(\frac{y}{a}\right)$$

$$k_y(x, y) = -\frac{\partial V}{\partial y} = -\frac{1}{a} \sum_{n=0}^N \sum_{j=0}^n C_{j, (n-j)} \cdot T_j\left(\frac{x}{a}\right) \cdot T_{n-j}'\left(\frac{y}{a}\right)$$

$$(1 - u^2) \cdot T_n'(u) = n \cdot [T_{n-1}(u) - u \cdot T_n(u)]$$

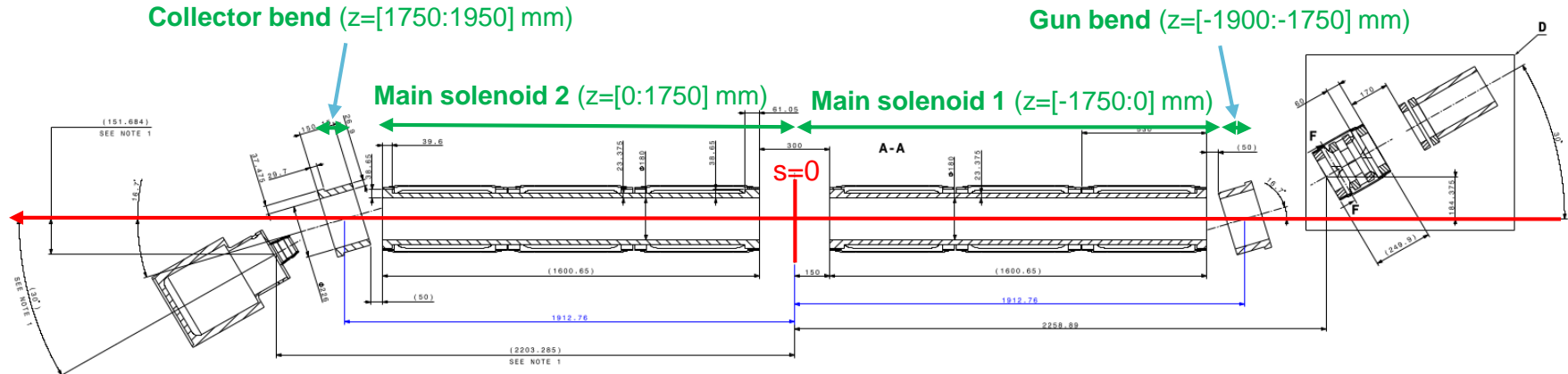
Longitudinal integration

Definition based on Chebyshev polynomials

Definition of Chebyshev polynomials

Applying the Method to the HL-LHC HEL

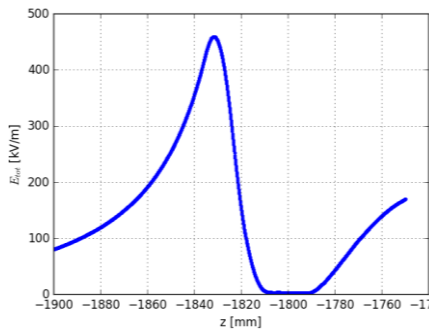
1. To get 3D maps of electric field and electric potential – D.Nikiforov, with CST;
 2. To integrate them longitudinally and fit them with Chebyshev polynomials – A.Mereghetti, numpy;
 3. To plug Chebyshev polynomials into SixTrack and see the effect in tracking simulations;
- Maps generated by D.Nikiforov:
 - $x=[-5:5:0.1]$ mm, $y=[-5:5:0.1]$ mm, $z=[-1900:1950:0.1]$ mm;
 - Electron current: 5A, beam potential in main solenoid: 11.2kV (electron beam compression);
 - ASCII files very large: ~35 GB for E field, ~10 GB for V → split in 4 pieces:



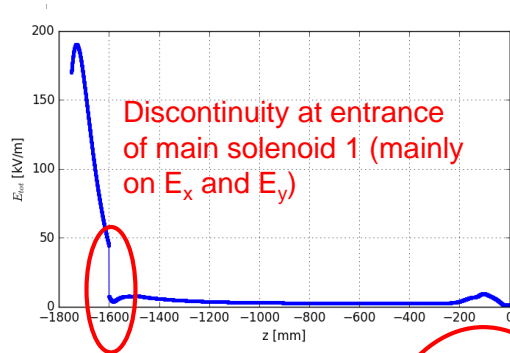
Longitudinal Profile (1D) at x=0, y=0

$$E_{tot}^2 = E_x^2 + E_y^2 + E_z^2$$

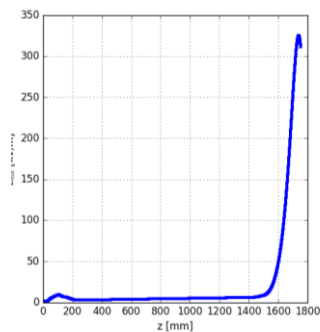
Gun bend



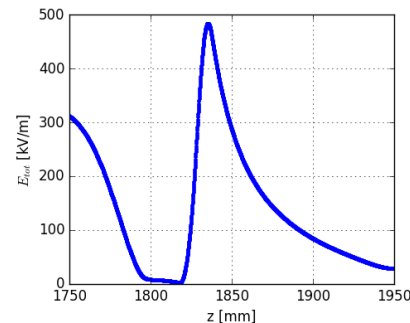
Main solenoid 1



Main solenoid 2

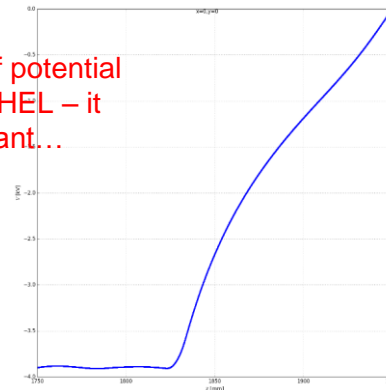
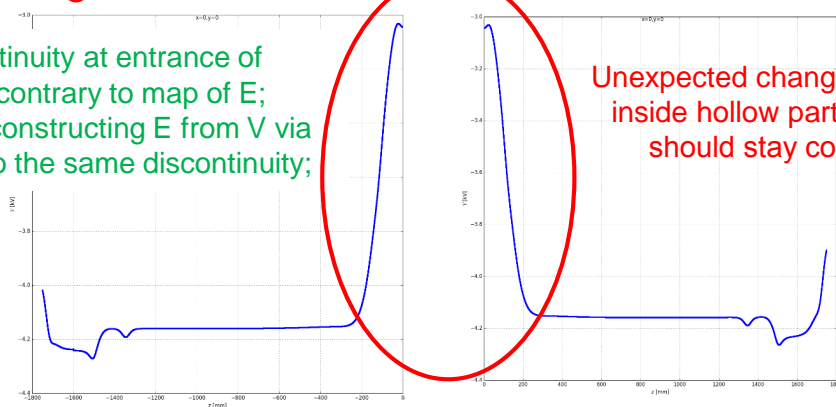
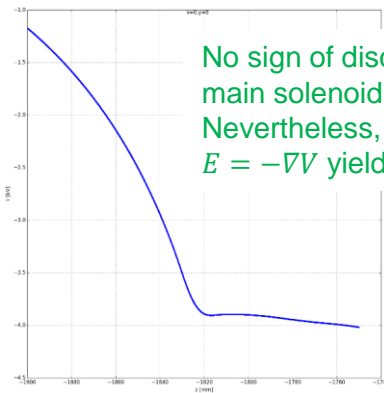


Collector bend



Electric Potential

No sign of discontinuity at entrance of main solenoid 1, contrary to map of E; Nevertheless, reconstructing E from V via $E = -\nabla V$ yields to the same discontinuity;



Unclear origin of discontinuity in electric field at entrance of main solenoid 1;
 → cannot proceed with these maps for production of results;
 In contact with D. Nikiforov to check origin of issue (mostly caused by matching boundary conditions);



Longitudinal Profile (2D) at x=0

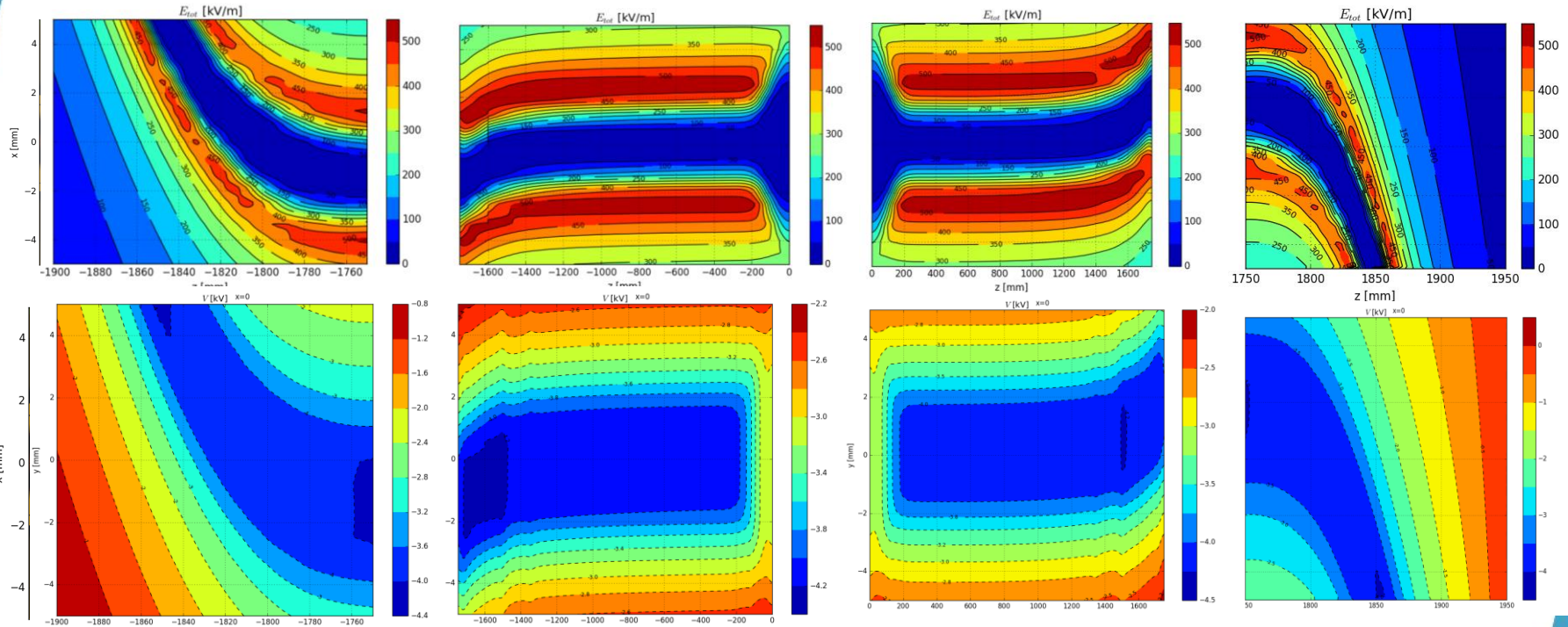
$$E_{tot}^2 = E_x^2 + E_y^2 + E_z^2$$

Gun bend

Main solenoid 1

Main solenoid 2

Collector bend



Shift at entrance and exit of main solenoid clearly visible (also presented by [A.Rossi, E-beams, #1 remote WG meeting](#));
 → Proposal of mitigation presented by D.Nikiforov ([CoIUSM #122](#));

Integrated Kicks

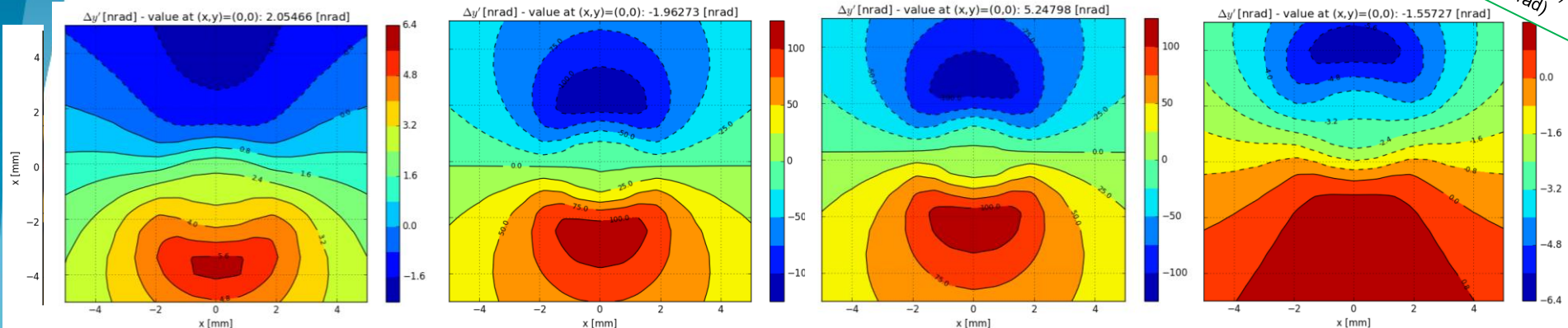
All maps: (integrated) vertical kick (nrad)

Gun bend

Main solenoid 1

Main solenoid 2

Collector bend



x=y=0, 7TeV	$\Delta x'$ [nrad]	$\Delta y'$ [nrad]	ΔE [keV]
Gun Bend	0.052	2.05	2.85
Main 1 solenoid	0.4	-1.96	-0.98
Main 2 solenoid	-0.4	5.25	0.85
Collector Bend	0.03	-1.56	-3.9

(V) Values at gun/collector bends are comparable to those computed by G. Stancari in [FERMILAB-FN-0972-APC](https://arxiv.org/abs/1707.0972)

NB: Ref sys of e-beam \rightarrow proton beam has a local ref sys rotate by 180° around the y-axis

Fitting the Maps – Example: Gun Bend

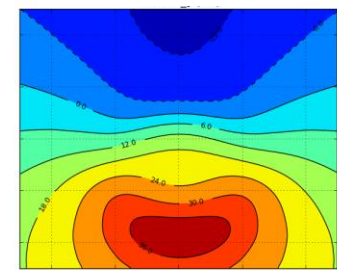
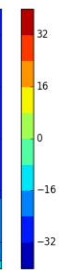
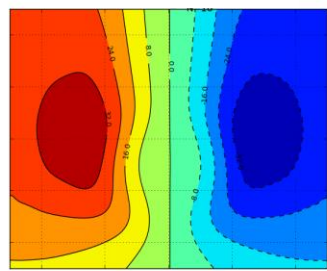
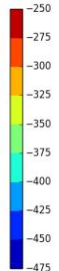
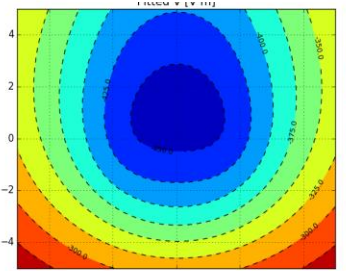
$N=18$

Fitted

V [V m]

E_x [kV]

E_y [kV]

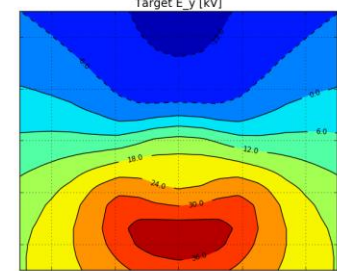
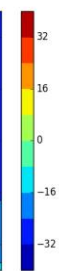
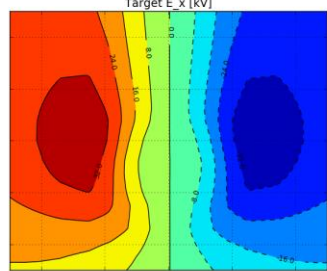
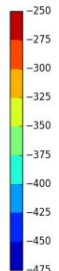
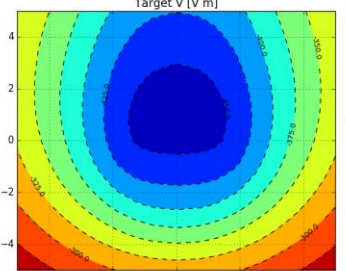


Target

Target V [V m]

Target E_x [kV]

Target E_y [kV]

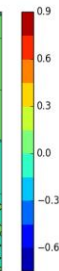
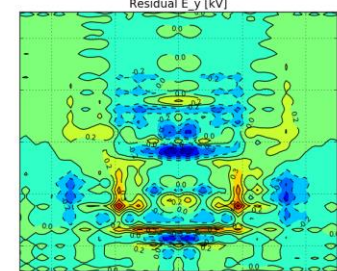
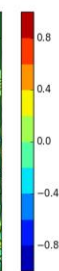
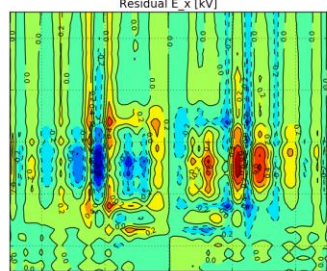
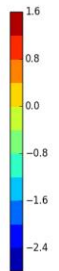
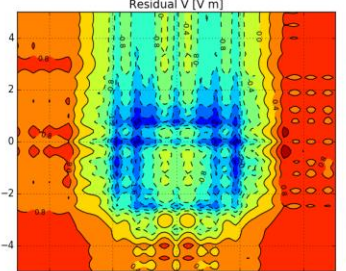


Residuals

Residual V [V m]

Residual E_x [kV]

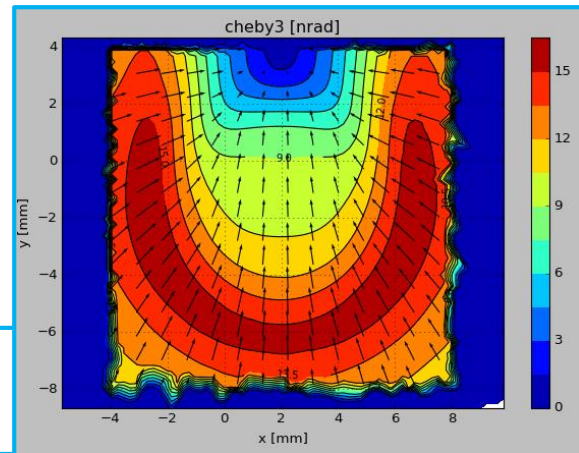
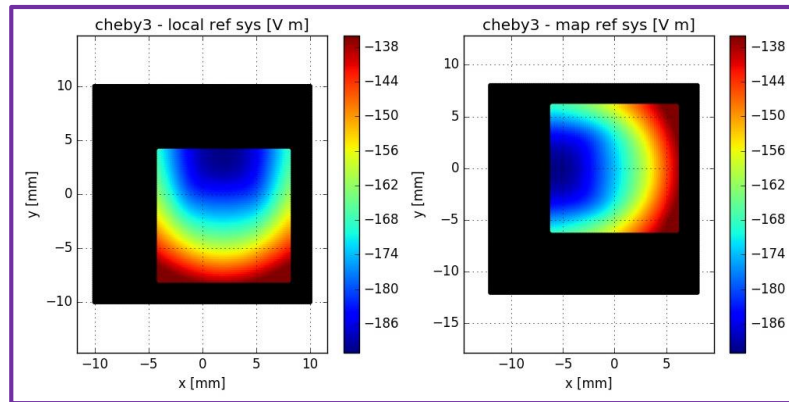
Residual E_y [kV]



Chebyshev Lens: SixTrack Implementation

- Chebyshev polynomials implemented;
 - Module separate from that of ideal electron lens, so that it can be used for other purposes – e.g. e-cloud?
 - Electric field only, for the time being;
 - Outlook: considering magnetic fields as well, and their superposition;
- Compatible with all species tracked by SixTrack;
- Echo of integrated potential map as from read Chebyshev coefficients;
- Possibility to rotate and offset original maps;
- Dynamic allocation of memory, i.e. no hard-coded limit in number of elements or dimension of map;
- Lenses are DYNK-able (i.e. kick can be varied with time);
- Foxification (i.e. 6D closed orbit calculation via Taylor maps) almost done;

Echo of potential map



Kick applied to 450 GeV
protons (test with 60
protons, 400 turns)

Conclusions / Remarks

- Solid implementation in SixTrack of a module for simulating ideal electron lenses;
 - Module significantly expanded wrt original implementation (e.g. full lens, Gaussian electron beam, beam from measured radial profile, etc...);
- General module for maps with Chebyshev polynomials for simulating pure electric fields;
 - User can define as many maps as necessary for their application (e.g. several longitudinal slices);
- Modules are pretty flexible and general, with no hard-coded assumptions targeting HL-LHC HELs;
 - Even though HL-LHC HELs are the main study case;
- This is the current framework used for SixTrack simulations for the HL-LHC HELs:
 - Optimization of working point of HEL: ideal electron lens;
 - Effects on proton beam core: module for Chebyshev maps;
 - Issues with 3D maps for Chebyshev polynomials being discussed with our Russian collaborators...
- Outlook:
 - Look into describing the kick by ideal electron lens starting from the Hamiltonian;
 - Look into extending Chebyshev formalism to magnetic fields and mixed fields;



Thanks a lot!



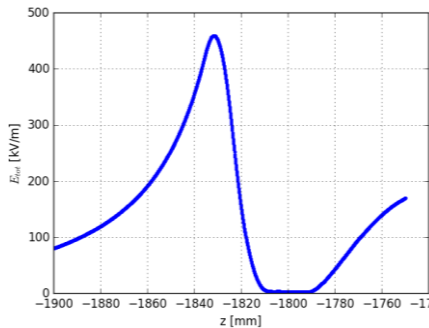
Longitudinal Profile (1D) of E_{tot} at $x=0, y=0$

$$E_{tot}^2 = E_x^2 + E_y^2 + E_z^2$$

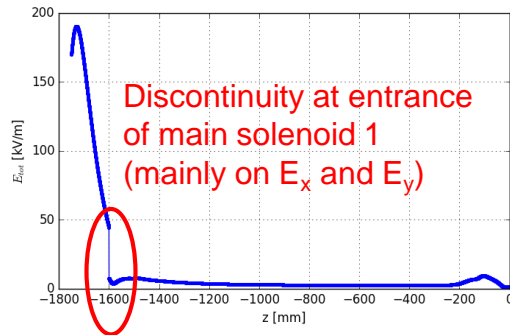
$$E_{tot}^2 = E_x^2 + E_y^2 + E_z^2$$

$$E = -\nabla V$$

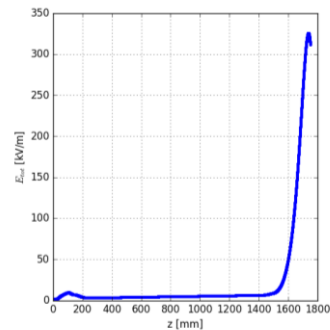
Gun bend



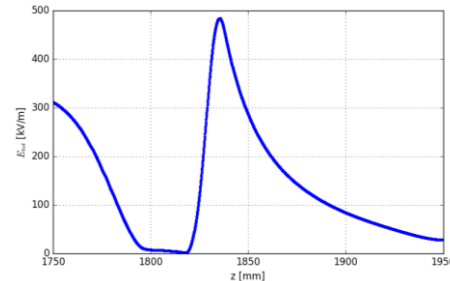
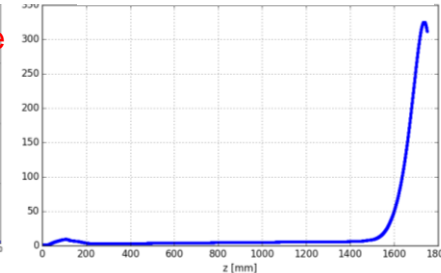
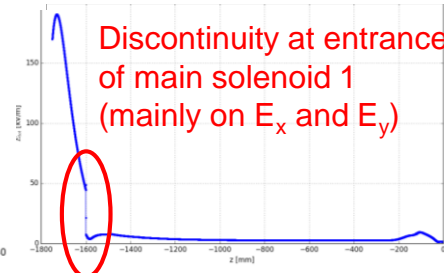
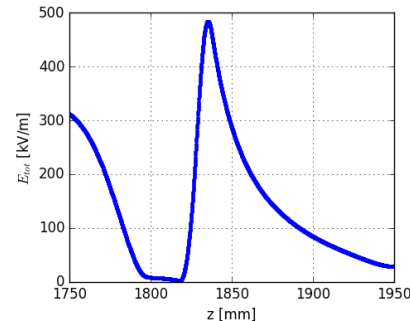
Main solenoid 1



Main solenoid 2



Collector bend



In the effort of by-passing the issue on the electric field, I computed it on my own from $E = -\nabla V$, but got into the same result;
 → Something might be wrong with Opera calculation itself...

Transverse Profiles (2D)

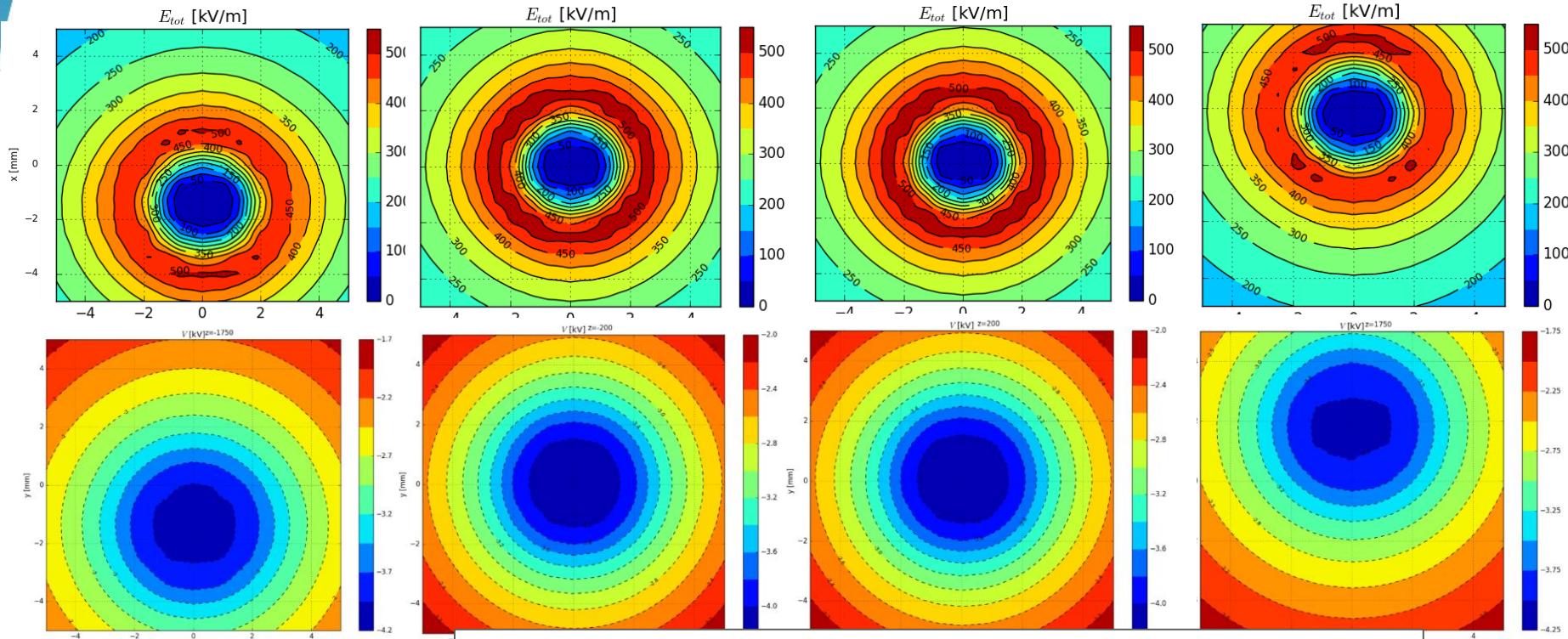
$$E_{tot}^2 = E_x^2 + E_y^2 + E_z^2$$

Gun bend

Main solenoid 1

Main solenoid 2

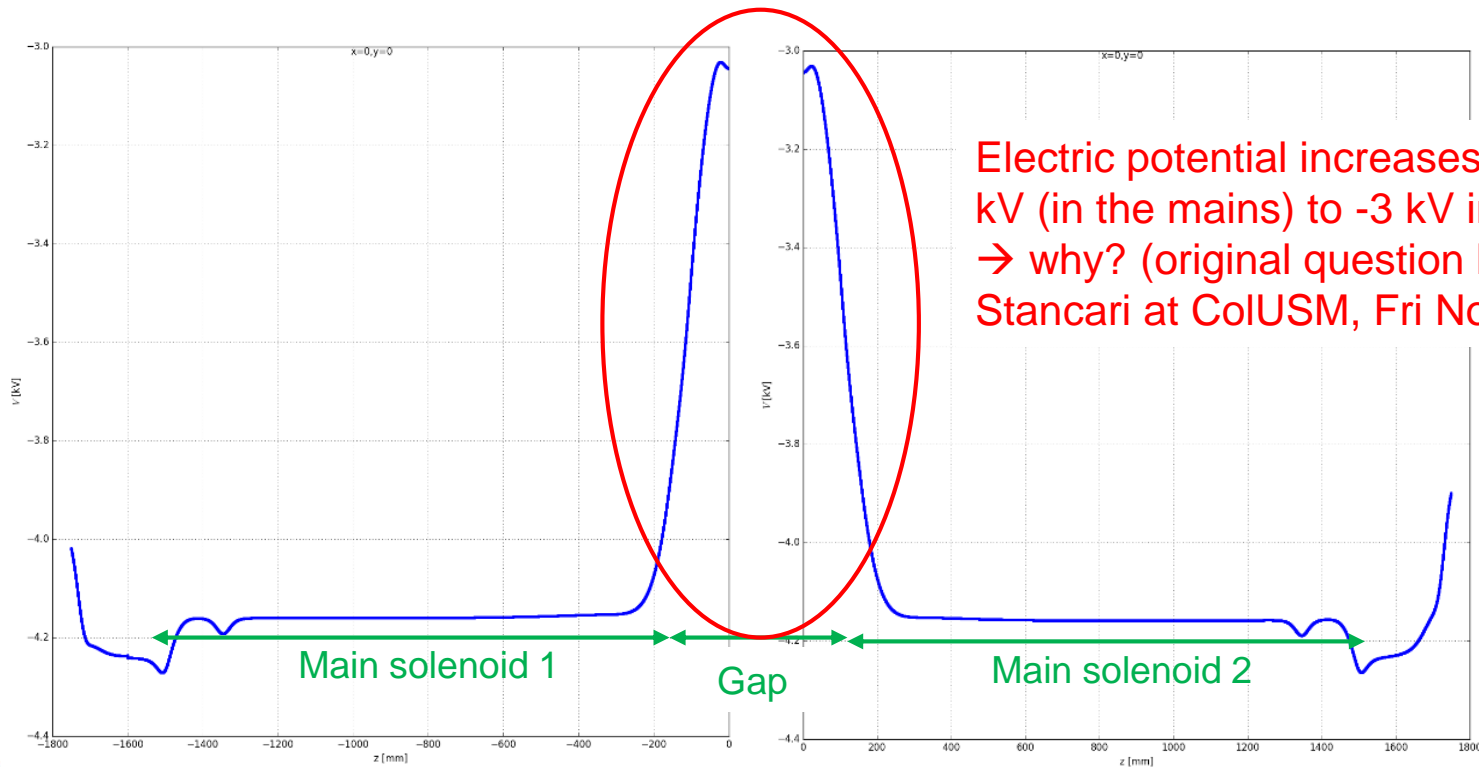
Collector bend



Shift at entrance and exit of main solenoid clearly visible (also presented by [A.Rossi, E-beams, #1 remote WG meeting](#));
 → Proposal of mitigation presented by D.Nikiforov ([CoIUSM #122](#));

Electric Potential in Gap between Main Solenoids

Electric potential: longitudinal profile at $x=0, y=0$ (i.e. main solenoids longitudinal axis);



Electric potential increases from -4.2 kV (in the mains) to -3 kV in the gap;
→ why? (original question by G. Stancari at CoLUSM, Fri Nov 22nd)

Electric Potential in Ideal HEL

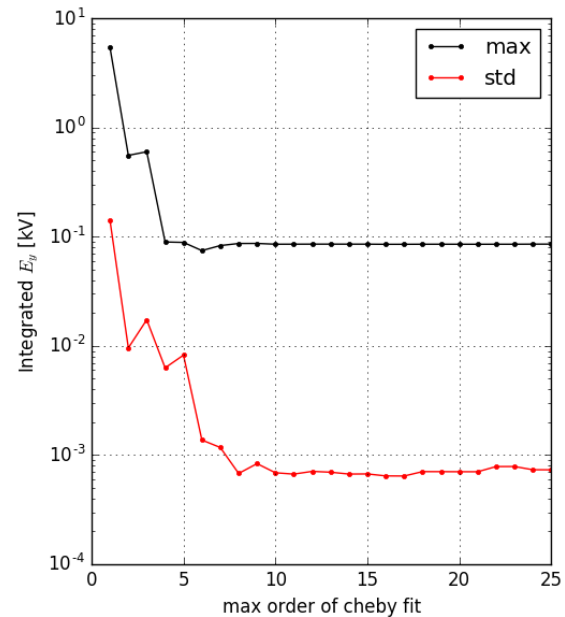
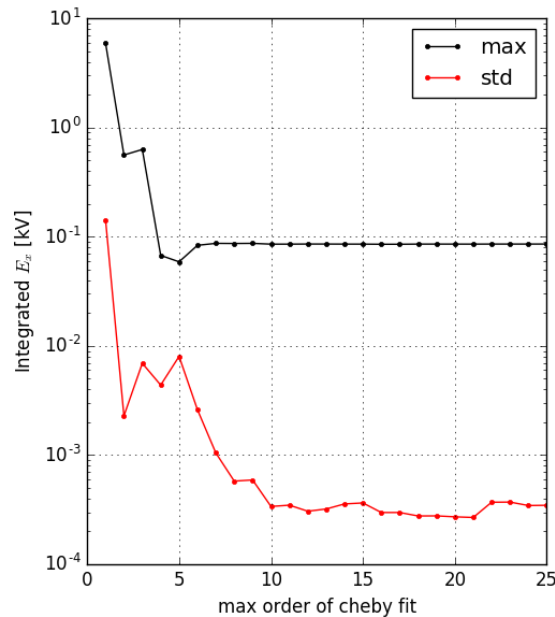
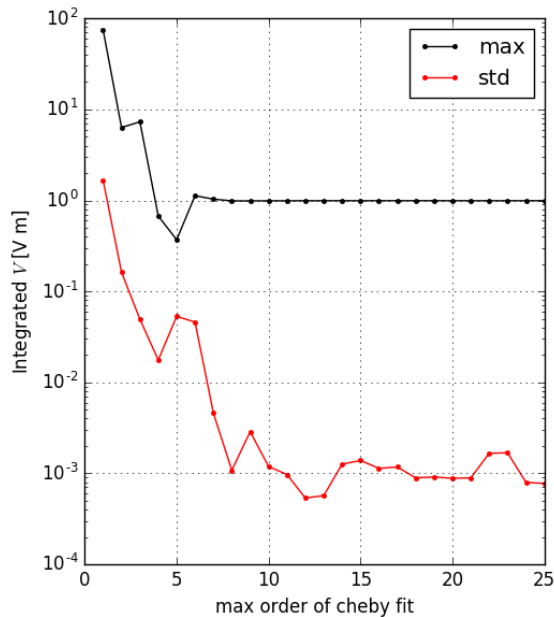
- In an ideal HEL (i.e. e-beam perfectly cylindrical and co-axial with solenoid, infinite solenoid and e-beam, no e-beam injection/extraction, no beam pipes), the electric potential in the hollow part of the HEL is constant and given by the analytical formula (if $V(R_2) = 0$):

$$V_{(r < R_1)} \div \frac{1}{(F^2 - 1)} \log\left(\frac{1}{F}\right)$$

where R_1 and R_2 are the inner and outer radius of e-beam, respectively, and $F = R_2/R_1$;

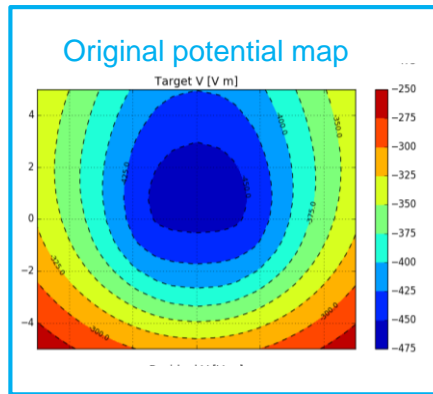
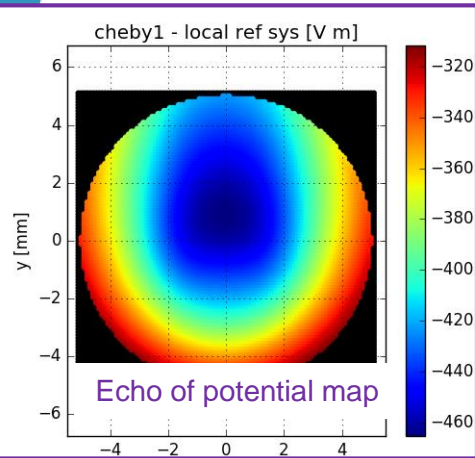
- Why do we see the change in electric potential in the gap between the two main solenoids? Possible answers:
 1. F changes inside the gap;
 2. Asymmetry in e-beam distribution in the gap;

Fitting the Maps – Example: Gun Bend (II)



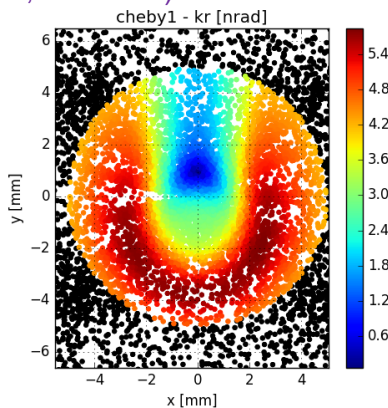
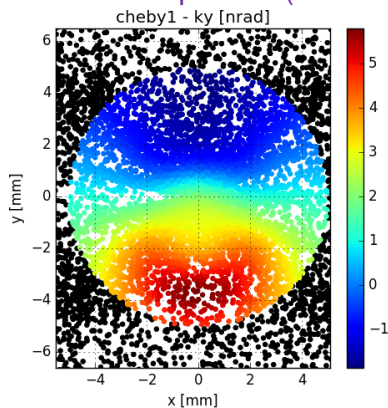
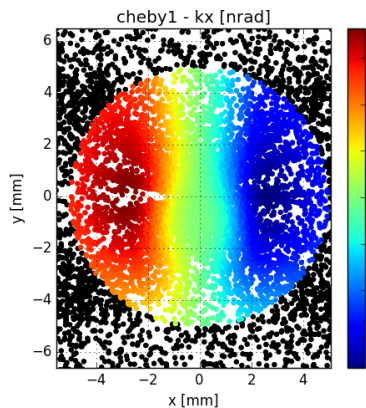
Residuals vs fitting order do not show big changes for $N > 10$;
→ The same applies to the fitting of the other maps;

Chebyshev Lens: SixTrack Implementation

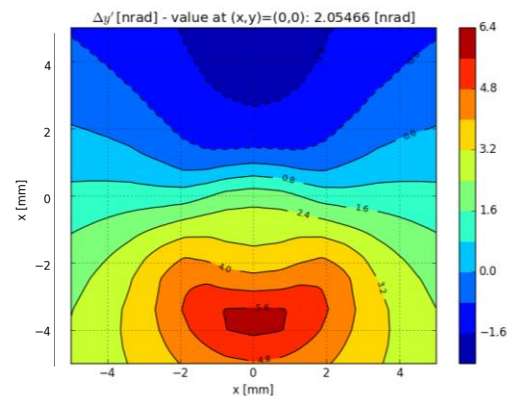


- Chebyshev polynomials implemented;
- Compatible with all species tracked by SixTrack;
- Echo of integrated potential map as from read Chebyshev coefficients;
- Possibility to rotate and offset original maps;
- Dynamic allocation of memory, i.e. no hard-coded limit in number of elements or dimension of map;
- Lenses are DYNK-able (i.e. kick can be varied with time);
- Foxification (i.e. 6D closed orbit calculation via Taylor maps) almost done;

Kick applied to 7 TeV protons (test with 60, 400 turns)



Original integrated map of vertical field



Examples of Tracking Studies

Parameters explored

- Effect of several parameters studied:

- ✓ Inner radius ($r1$): 3, 5, 7, 9 σ
- ✓ Pulsing pattern: Continuous (DC), Random ON-OFF (RND), Continuous with random current between 0 A and 5 A (RNDI), pulsed every 1, 2, 3, ..., 10 turns
- ✓ e-beam current: 1 A, 2 A, 3 A, 4 A, 5 A
- ✓ Octupole current (MO): -600 A, -450 A, -300 A, -150 A, 0 A, 150 A, 300 A
- ✓ Chromaticity (Q'): 0, 2, 5, 10, 15

- Machine optics:

- ✓ HL-LHC v1.3, 7 TeV, $\beta^* = 15$ cm, separated beams, multipolar errors included (completer list of machine and e-lens settings reported in backup as reference)

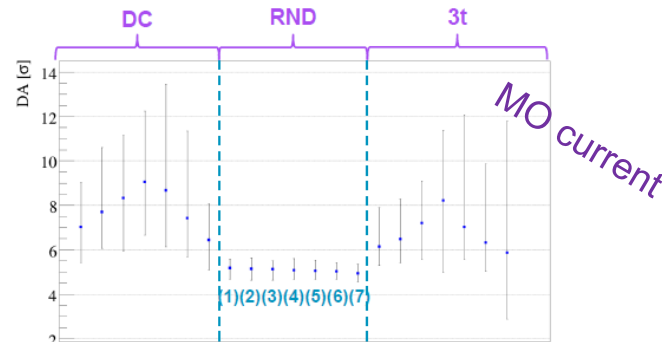


D. Mirarchi, 9th HL-LHC Collaboration Meeting 6

Example for **random ON-OFF** excitation with $r1 = 5 \sigma$, $MO = 0$ A and $Q' = 2$



Example for **DC, RND and 3t** excitations with $r1 = 5 \sigma$ and $Q' = 2$



(1) MO = -600 A, (2) MO = -450 A, (3) MO = -300 A, (4) MO = -150 A, (5) MO = 0 A, (6) MO = 150 A, (7) MO = 300 A



Courtesy of D. Mirarchi, 9th HL-LHC Annual Meeting;