Theory Challenges in exclusive rare B decays

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Anomalies in $b \to s\ell^+\ell^-$

- Anomalies in $b \to s\mu^+\mu^-$: (LHCb, Belle, ATLAS, CMS)

$$P_5'(B \to K^*\mu^+\mu^-) \sim 3\sigma\ ; \quad \mathcal{B}(B_s \to \phi\mu^+\mu^-) \sim 2\sigma$$

$$\mathcal{B}(B \to K^{(*)}\mu^+\mu^-) \sim 1\sigma\ ; \quad \text{many others} \sim 0\sigma$$

Combined (180 Observables) $\sim 5\sigma$

Significantly alleviated if

Descotes, Matias, Virto 2013

$$\boxed{\mathcal{L}_{NP} \simeq (35\,\mathrm{TeV})^{-2}\ [\bar{s}\gamma_\nu P_L b][\bar{\mu}\gamma^\nu\mu]}$$

- LFNU: (LHCb, Belle)

$$R_K,\, R_{K^*} \gtrsim 2\sigma\ ; \quad Q_5 \equiv P_{5\mu}' - P_{5e}' \gtrsim 1\sigma$$

Combined $\sim 4\sigma$

Consistent(*) with $b \to s\mu^+\mu^-$

Alonso, Camalich, Grinstein 2014

NP interpretation requires accurate TH predictions of $B \to M\ell^+\ell^-$ obs

Theory predictions

- TH predictions require very involved and nontrivial perturbative and non-perturbative calculations.

- The fact that $\Lambda_{QCD} \ll m_b \ll \Lambda_{EW}, \Lambda_{BSM}$ helps a lot

- Weak Effective Theory:

$$\mathcal{L}_{WET} = \mathcal{L}_{QCD+QED} + \sum_i C_i \, \mathcal{O}_i$$

Short distance C_i known in SM to NNLL Bobeth, Misiak, Urban, Gorbahn Haisch,...

BSM part of C_i is the target

- This leaves non-perturbative MEs as the **leading challenge**

Theory predictions

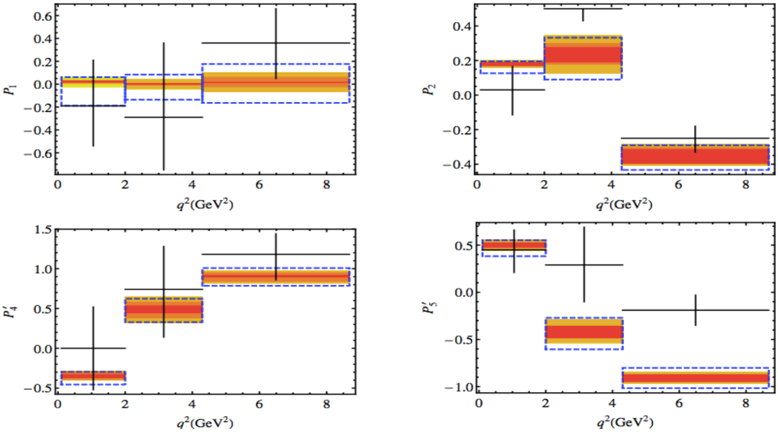
What we need depends on **mode** and **observable**. E.g.:

- $B_q \to \ell^+\ell^-$ needs f_{B_q} (up to QED...)
- $B \to D^{(*)}\ell\nu$ needs **local** form factors (3 for D, 7 for D^*)
- $\mathcal{R}_{D^{(*)}}$ needs local form factor ratios
- $b \to s\ell\ell$ needs both **local** and **non-local** form factors
  - $\mathcal{B}$ needs all (3 for K, 7 for K^*, ϕ)
  - Ratios such as P_5' need only **ratios** (SCET & HQET relations)
  - $\mathcal{R}_{K,K^*,...}$ (LFNU) almost exact cancellation of FFs (both local and non-local) within SM, but NOT in LFNU BSM in general
  - QED effects relevant in LFNU ratios and angular distributions

Theory predictions

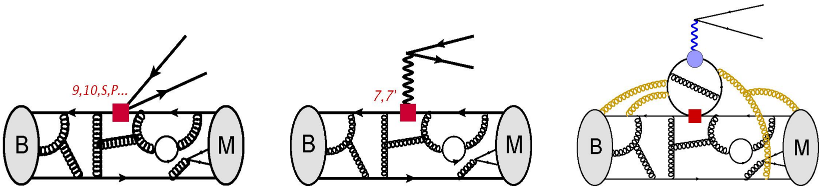
Example of error breakdown (2014)

Descotes-Genon, Hofer, Matias, Virto 2014



E.g., $\langle P_5' \rangle_{[2,4.3]} = -0.411^{+0.050+0.017+0.109+0.016}_{-0.072-0.015-0.101-0.020}$

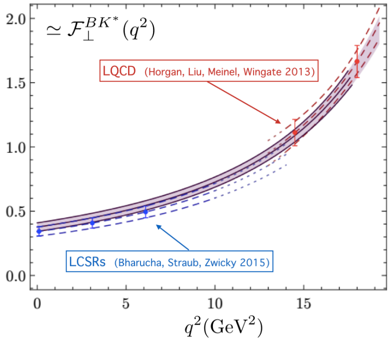
Anatomy of $B \to M_\lambda \ell^+ \ell^-$ EFT Amplitudes

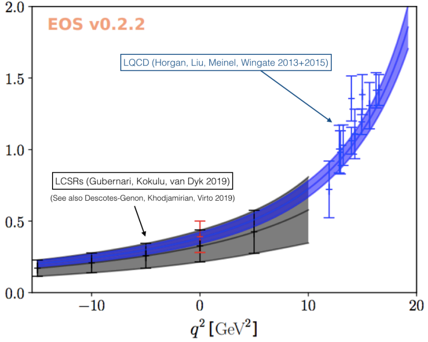


$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s} \, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \, \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Local Form Factors

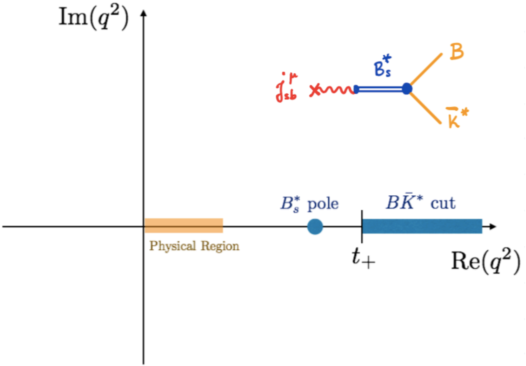




- Two main approaches: (1) Lattice QCD (large q^2) (2) LCSRs (low q^2)
- Two approaches to LCSRs, in terms of (1) K^* LCDAs (2) B LCDAs
- q^2 dependence can be parametrized model-independently

Local Form Factors : q^2-dependence from analyticity

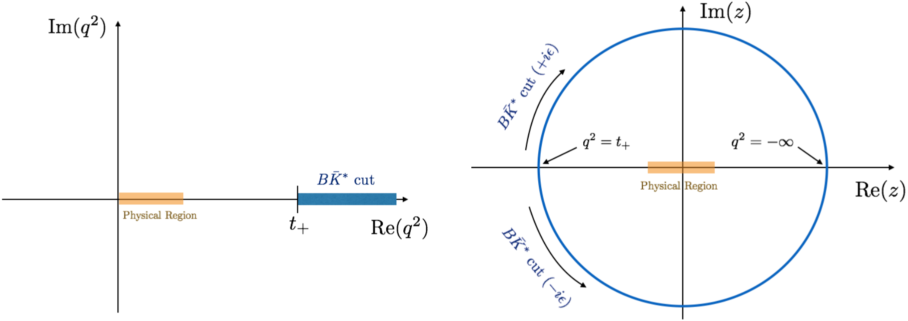
$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k)|\, \bar{s}\, \Gamma_\lambda^{(T)}\, b\, |\bar{B}(k+q)\rangle$: Analytic structure in q^2 :



$\widehat{\mathcal{F}}_\lambda^{(T)}(q^2) \equiv (q^2 - m_{B_S^*}^2)\, \mathcal{F}_\lambda^{(T)}(q^2)$ has no pole, only cut.

Local Form Factors : q^2-dependence from analyticity

- Conformal mapping : $$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

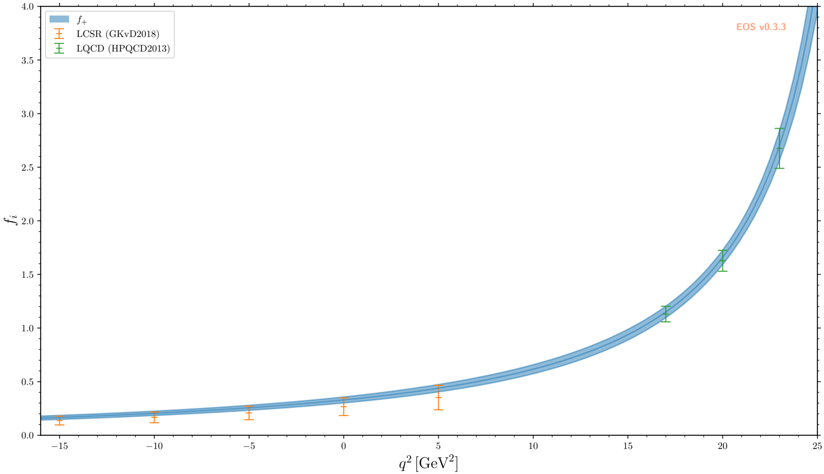


- "z-parametrization" : $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$ is **analytic in** $|z| < 1$ $\quad (|z_{\rm phys}| < 0.15)$

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m_{B_s^*}^2)} \sum_k \alpha_k \, z(q^2)^k$$

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

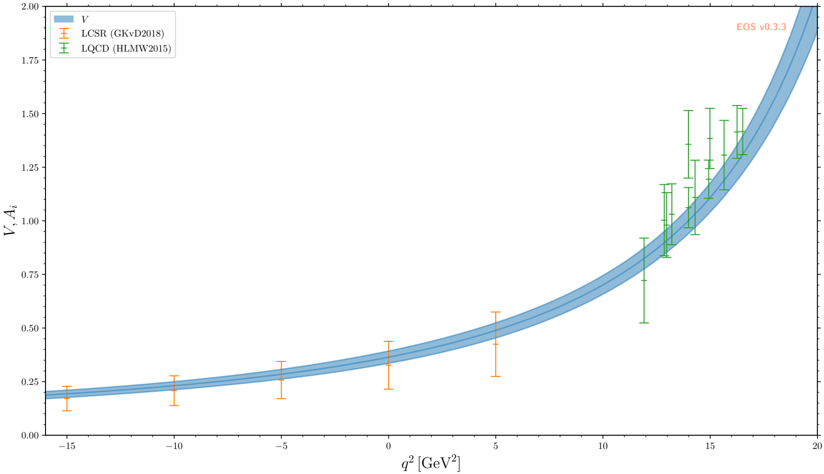
- $B \to K\ell\ell$



Thanks to Méril Reboud for producing these plots

Local Form Factors : New Fits to (B-DAs) LCSRs + LQCD

- $B \to K^* \ell\ell$



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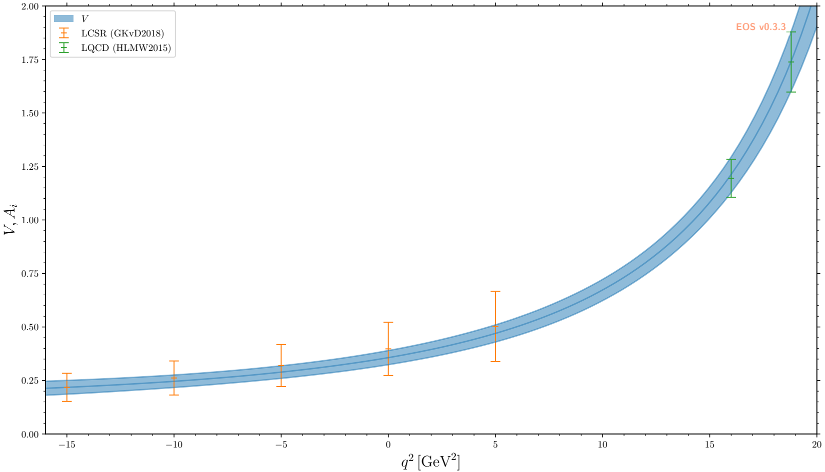
Local Form Factors : New Fits to (B-DAs) LCSRs + LQCD

- $B_s \to \phi\ell\ell$



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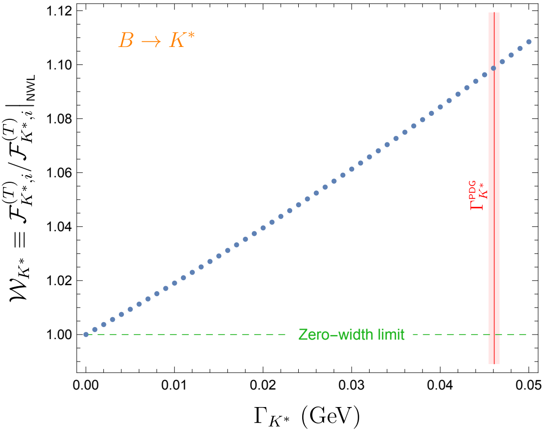
Descotes-Genon, Khodjamirian, Virto, 1908.02267



$$\mathcal{W}_{K^*} \simeq 1 + 1.9\,\frac{\Gamma_{K^*}}{m_{K^*}}$$

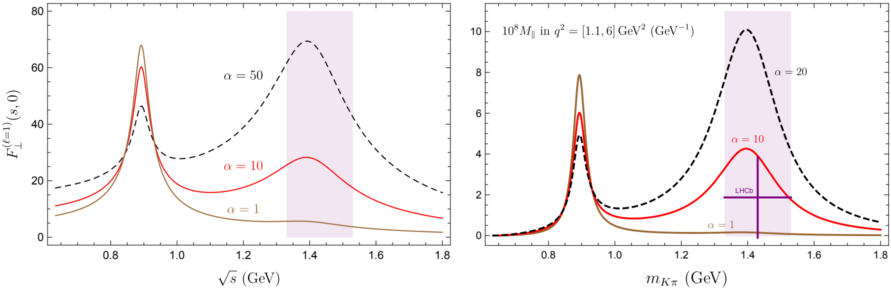
$$\mathcal{W}_{K^*} = 1.09 \pm 0.01$$

- $\mathcal{W}_{K^*}$ is independent of the form factor type
- $\mathcal{W}_{K^*}$ is indep. of q^2

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K^*}|^2 \simeq 1.2$. Ratios unaffected.

Descotes-Genon, Khodjamirian, Virto, 1908.02267

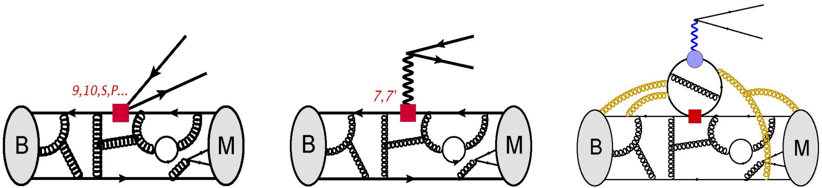
Set $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$ with α a floating parameter



$\alpha = 1:\ \mathcal{F}_{K^*,\perp}(0) = 0.28\ ;\quad \alpha = 10:\ \mathcal{F}_{K^*,\perp}(0) = 0.22\ ;\quad \alpha = 50:\ \mathcal{F}_{K^*,\perp}(0) = 0.11\ .$

Constrained by angular measurements on 1430 region (LHCb)

Non-Local Form Factors



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2}\left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b}\mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) |\, \bar{s}\, \Gamma_\lambda^{(T)}\, b \,| \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i\,\mathcal{P}_\mu^\lambda \int d^4x\; e^{iq\cdot x}\, \langle \bar{M}_\lambda(k) | \mathcal{T}\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Non-local form factors: Operator Product Expansion

$$\mathcal{H}^\mu(q,k) = i\int d^4x\, e^{iq\cdot x}\, \langle \bar{M}_\lambda(k)|\mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\, \mathcal{O}_i(0)\}|\bar{B}(q+k)\rangle$$

- Large-q^2: Dominated by $x \sim 0$ (short-distance dominance - OPE)

  Grinstein, Pirjol; Beylich, Buchalla, Feldmann

- Low-q^2: Dominated by $x^2 \sim 0$ (light-cone dominance - LCOPE)

  Khodjamirian, Mannel, Pivovarov, Wang

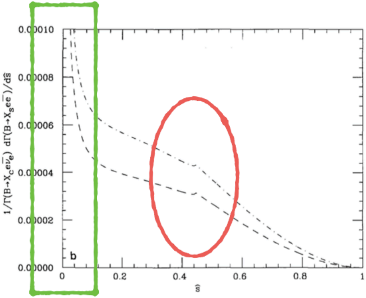
+ Must analytically-continue from OPE region to physical region

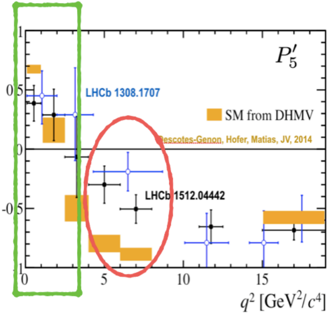
Non-local form factors: Importance of on-shell cuts

- QCD Factorization   Beneke, Feldmann, Seidel 2001

$$\mathcal{H}_\lambda(q^2) \sim \Delta C_9^\lambda(q^2)\,\mathcal{F}_\lambda(q^2) + \frac{1}{q^2}\Delta C_7^\lambda(q^2)\,\mathcal{F}_\lambda^T(q^2) + HSS + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

- It is assumed that the charm loop is dominated by short distances

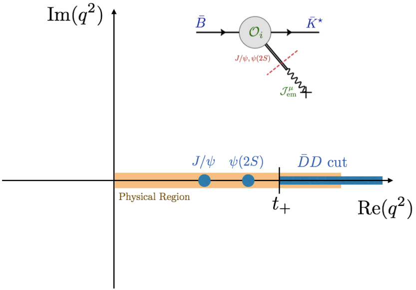


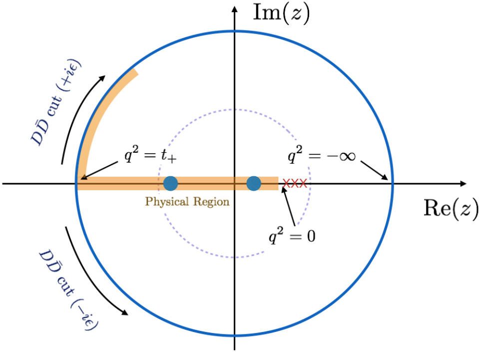


- Kink at $q^2 = 4m_c^2$ symptom of breaking of perturbativity

z-Parametrization for Non-Local FFs

Bobeth, Chrzaszcz, van Dyk, Virto 2017

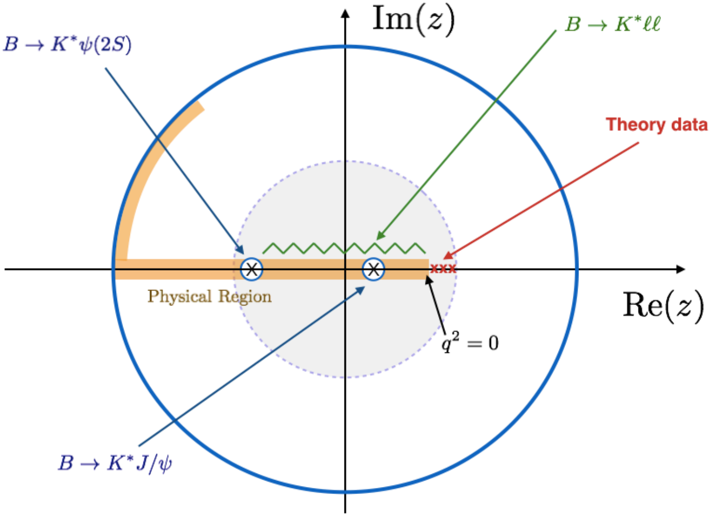




- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\, \mathcal{H}_\lambda(q^2)$ is **analytic in** $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{F}_\lambda(z)$
- Expansion needed for $|z| < 0.52$ ($-7\,\mathrm{GeV}^2 \leq q^2 \leq 14\mathrm{GeV}^2$)

Fit to z-parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017



Fit to z-parametrisation

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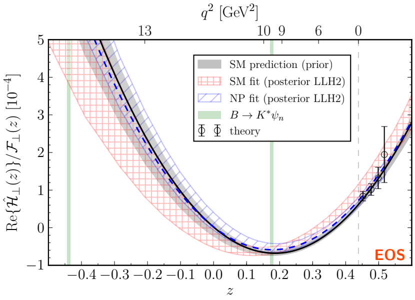
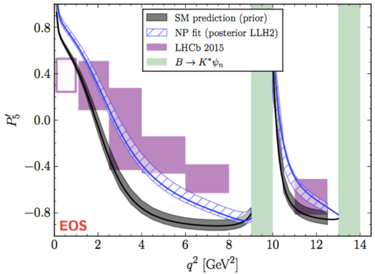




Fit includes $B \to K^* \psi$, theory at $q^2 < 0$ (Khojamirian et al 2010) for SM prior and $B \to K^* \mu\mu$ with $\mathcal{C}_9^{\mathrm{NP}} = -1$ for NP posterior.

A few new theory improvements

- Analytic calculation of leading order OPE matching at NLO

Asatrian, Greub, Virto 2019

- Recalculation of subleading LCOPE contributions $(q^2 \lesssim 0)$

Gubernari, van Dyk, Virto 2020

$$\mathcal{H}_\lambda(q^2) = \Delta C_9(q^2)\,\mathcal{F}_\lambda(q^2) + \Delta C_7(q^2)\,\mathcal{F}^T_\lambda(q^2) + \tilde{\mathcal{V}}_\lambda(q^2) + \cdots$$

- Dispersive bound for z-expansion

Gubernari, van Dyk, Virto 2020

$$\hat{\mathcal{H}}^{B\to M}_\lambda(z) = \sum_{n=0}^{\infty} a^{B\to M}_{\lambda,n}\, p^{B\to M}_n(z)$$

$$\sum_{n=0}^{\infty} \left\{ 2\left|a^{B\to K}_{0,n}\right|^2 + \sum_{\lambda=\perp,\|,0} \left[2\left|a^{B\to K^*}_{\lambda,n}\right|^2 + \left|a^{B_s\to\phi}_{\lambda,n}\right|^2 \right] \right\} < 1\,.$$

Recalculation of subleading LCOPE contributions

Recalculation of charm-loop effect Gubernari, van Dyk, Virto, 2011.09813

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$	This work	Ref. [11]
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\text{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\text{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\text{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\text{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\text{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\text{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\text{GeV}$	—

- We reproduce the result of KMPW'2010
- We incude complete set of 3-particle LCDAs Braun,Li,Manashov 2017
- Cancellations + Parametric lead to a reduction of the effect of **two orders of magnitude**
- Local matrix elements $\lambda_{E,H}$ crucial in this cancellation. Revisit (see e.g. Rahimi, Wald 2020)

Summary

- Theory predictions in B decays require calculation of **local** and **non-local** form factors
- Local form factors are fitted to a z-expansion using LCSRs and LQCD, with good and consistent results with 10% uncertainties. We mostly rely on LQCD for improvements. Finite width and non-resonant effects must be controlled from the continnum/experimental sides.
- Non-local form factors are the bottleneck, but understanding and strategy has improved in recent years. We should reconfirm rapid convergence of OPE and test convergence of z-expansion.
- New Theory Predictions for exclusive $b \to s\ell\ell$ observables will require data and a "global" approach (simultaneous fits to LQCD+LCSRs+data). Still work in progress (with consistent LCSRs all with B-DAs).
Current global fits still use 2015 technology.

Extra

Non-local form factors: Operator Product Expansion

We write

$$\mathcal{H}^\mu(q,k) = \langle \bar{M}_\lambda(k)|\mathcal{K}^\mu(q)|\bar{B}(q+k)\rangle$$

With the operator $\mathcal{K}^\mu(q)$ given by

$$\mathcal{K}^\mu(q) = i\int d^4x\, e^{iq\cdot x}\,\mathcal{T}\big\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\,\mathcal{O}_i(0)\big\}$$

It turns out that: **Leading-order OPE = Leading order LCOPE**

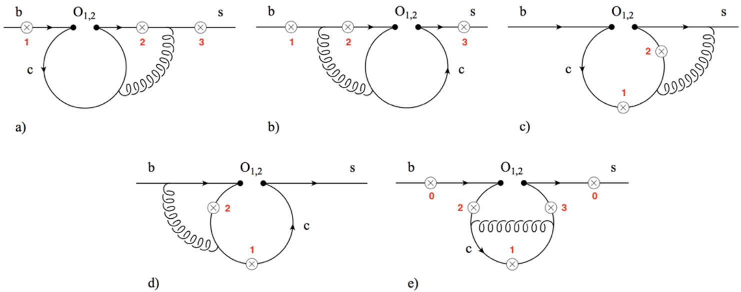
$$\mathcal{K}^\mu_{\rm OPE}(q) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\,\bar{s}\gamma_\nu P_L b + \Delta C_7(q^2)\,2im_b\,\bar{s}\sigma^{\mu\nu}q_\nu P_R b + \cdots$$

With this we have:

$$\mathcal{H}^\mu_{\rm OPE}(q,k) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\mathcal{F}_\nu + 2im_b\,\Delta C_7(q^2)\mathcal{F}^{T\mu} + \cdots$$

Asatrian, Greub, Virto 2019

Objective: Fully analytical calculation in two variables: q^2 and m_c.



Two-loop Master Integrals

$$J_i(q^2, m_c) = (2\pi)^{-2d} \int \frac{(m_b^2)^{N_i-4} (\tilde{\mu}^2)^{2\epsilon}\, d^d\ell\, d^d r}{P_{i_1}^{n_{i_1}} P_{i_2}^{n_{i_2}} P_{i_3}^{n_{i_3}} P_{i_4}^{n_{i_4}} P_{i_5}^{n_{i_5}} P_{i_6}^{n_{i_6}} P_{i_7}^{n_{i_7}}}$$

$$\begin{aligned}
P_1 &= (\ell + q)^2 - m_c^2 & P_5 &= (r + p - q)^2 & P_9 &= \ell \cdot q \\
P_2 &= \ell^2 - m_c^2 & P_6 &= r \cdot q & P_{10} &= (r + p - q)^2 - m_b^2 \\
P_3 &= (\ell + r)^2 - m_c^2 & P_7 &= \ell \cdot (p - q) & P_{11} &= (r + p)^2 - m_b^2 \\
P_4 &= r^2 & P_8 &= (r + p)^2 & P_{12} &= (\ell + r + q)^2 - m_c^2 \\
P_{13} &= r \cdot (p - q) & & & &
\end{aligned}$$

Differential Equations in Canonical Form

Henn 2013

$$J_i(q^2, m_c) = (2\pi)^{-2d} \int \frac{(m_b^2)^{N_i-4} (\tilde{\mu}^2)^{2\epsilon}\ d^d\ell\ d^d r}{P_{i_1}^{n_{i_1}} P_{i_2}^{n_{i_2}} P_{i_3}^{n_{i_3}} P_{i_4}^{n_{i_4}} P_{i_5}^{n_{i_5}} P_{i_6}^{n_{i_6}} P_{i_7}^{n_{i_7}}}$$

$$\partial_x J_{i,k}(\epsilon, x, y) = a_{i,x}^{k\ell}(\epsilon, x, y)\, J_{i,\ell}(\epsilon, x, y)\ , \quad \partial_y J_{i,k}(\epsilon, x, y) = a_{i,y}^{k\ell}(\epsilon, x, y)\, J_{i,\ell}(\epsilon, x, y)\ ,$$

$\rightarrow$ Transformation to "Canonical" Basis: $\vec{M}(x,y) = T(\epsilon, x, y) \cdot \vec{J}(x,y)$

$$\partial_x \vec{M}(\epsilon, x, y) = \epsilon\, A_x(x,y)\, \vec{M}(\epsilon, x, y) \quad ; \quad \partial_y \vec{M}(\epsilon, x, y) = \epsilon\, A_y(x,y)\, \vec{M}(\epsilon, x, y)$$

Matching calculation at NLO

Asatrian, Greub, Virto 2019

Iterative solution of DEs

$$\partial_x \vec{M}(\epsilon, x, y) = \epsilon\, A_x(x, y)\, \vec{M}(\epsilon, x, y) \quad ; \quad \partial_y \vec{M}(\epsilon, x, y) = \epsilon\, A_y(x, y)\, \vec{M}(\epsilon, x, y)$$

$$\vec{M}(\epsilon, x, y) = \sum_{n=0}^{\infty} \epsilon^n\, \vec{M}_n(x, y)$$

$$\partial_{x,y} \vec{M}_n(x, y) = A_{x,y}(x, y) \vec{M}_{n-1}(x, y)$$

Iterative solution of DEs First y dependence, then x:

$$
\begin{aligned}
\vec{M}_0(x,y) &= \vec{C}_0(x)\ , \\
\vec{M}_1(x,y) &= \sum_{j_1} \left[A_y^{j_1}\, G(w_{j_1}(x);y)\right] \vec{C}_0(x) + \vec{C}_1(x)\ , \\
\vec{M}_2(x,y) &= \sum_{j_2,j_1} \left[A_y^{j_2}\, A_y^{j_1}\, G(w_{j_2}(x), w_{j_1}(x);y)\right] \vec{C}_0(x) \\
&\quad + \sum_{j_2} \left[A_y^{j_2}\, G(w_{j_2}(x);y)\right] \vec{C}_1(x) + \vec{C}_2(x)\ , \\
\vec{M}_3(x,y) &= \cdots
\end{aligned} \tag{1}
$$

 Asatrian, Greub, Virto 2019

Iterative solution of DEs

Solutions in terms of **Generalized Polylogarithms (GPLs)** Goncharov 1998

$$G(w_1,\dots,w_n;y) = \int_0^y \frac{dt}{t-w_1} G(w_2,\dots,w_n;t)\,; \quad G(;y)=1\,; \quad G(\vec{0}_n;x) = \frac{\log^n x}{n!}$$

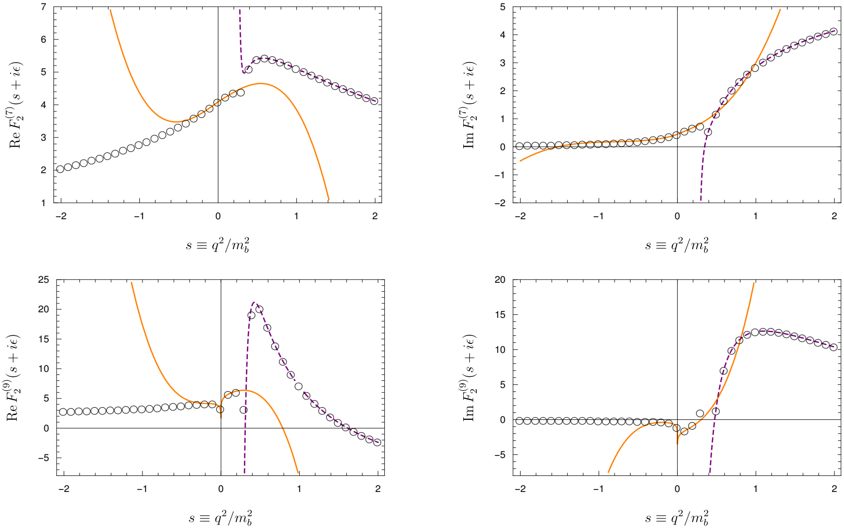
i.e

$$G(1;x) = \log(1-x)\ , \quad G(0,1;x) = -Li_2(x)\ , \quad G(0,0,1;x) = -Li_3(x)\dots$$

Fast numerical evaluation of general GPLs in the complex plane available (C++, python, matlab, ...)

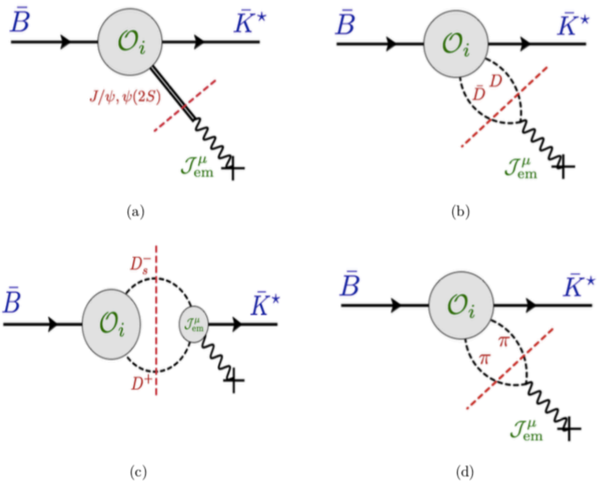
Asatrian, Greub, Virto 2019

Results: Comparison to previous calculations:



Asatrian, Greub, Virto 2019

Preparing the analytic continuation to physical q^2



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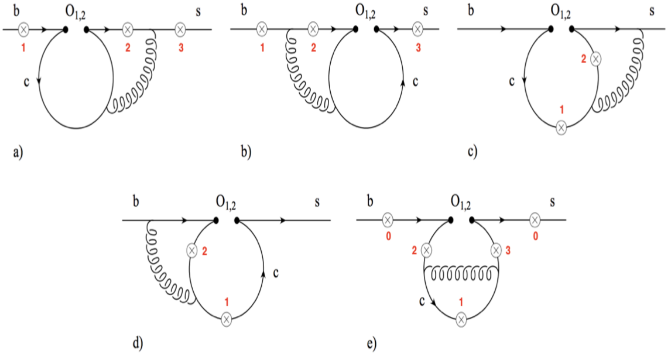

Matching calculation at NLO

Asatrian, Greub, Virto 2019

Checking analytic structure of $\mathcal{H}(q^2)$

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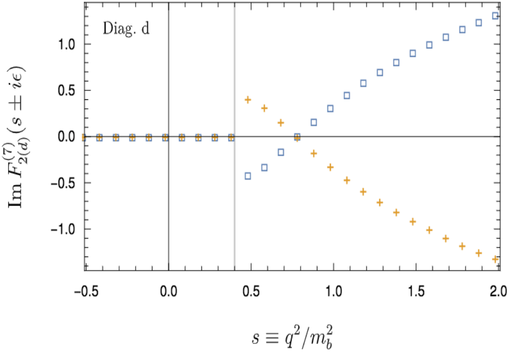

Matching calculation at NLO

Asatrian, Greub, Virto 2019

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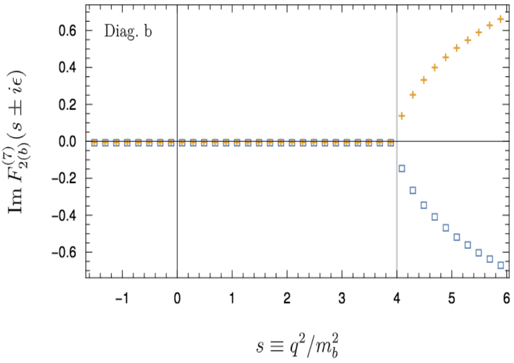
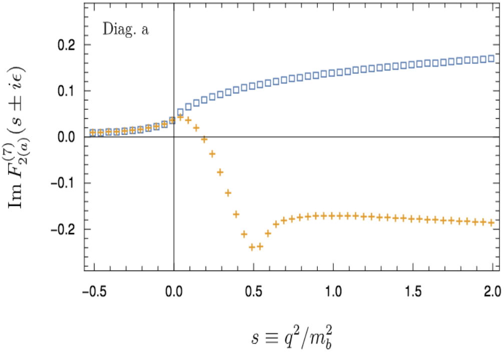
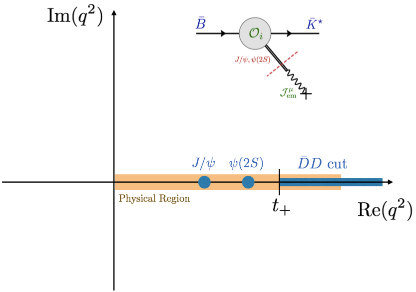


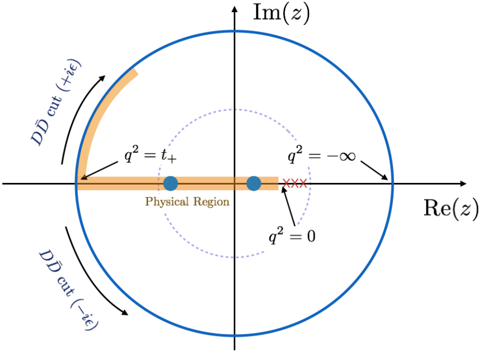
Asatrian, Greub, Virto 2019

Checking analytic structure of $\mathcal{H}(q^2)$



z-parametrisation for $\mathcal{H}_\lambda(q^2)$





- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$ is analytic in $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{F}_\lambda(z)$
- Expansion needed for $|z| < 0.52$ $\left(-7\,\mathrm{GeV}^2 \leq q^2 \leq 14\mathrm{GeV}^2 \right)$

Experimental constraints on z parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017

Experimental constraints :

- The residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

- Angular analyses Belle, Babar, LHCb determine :

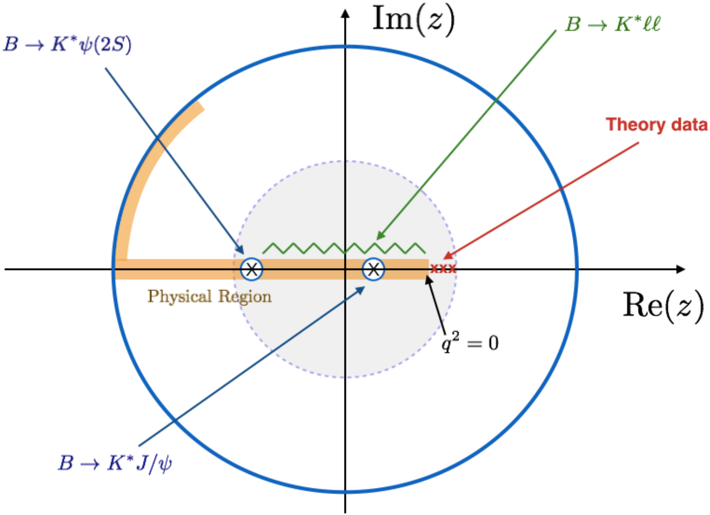
$$|r_\perp^{\psi_n}|,\ |r_\|^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_\|^{\psi_n} r_0^{\psi_n *}\},$$

where $\quad r_\lambda^{\psi_n} \equiv \underset{q^2 \to M_{\psi_n}^2}{\mathrm{Res}} \dfrac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \dfrac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2\, \mathcal{F}_\lambda(M_{\psi_n}^2)}$

- We produce correlated pseudo-observables from a fit (5+5).

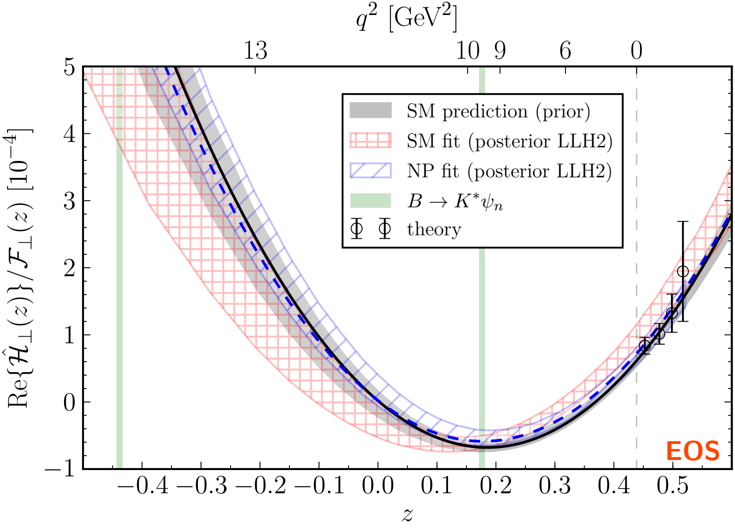
Fit to z-parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017

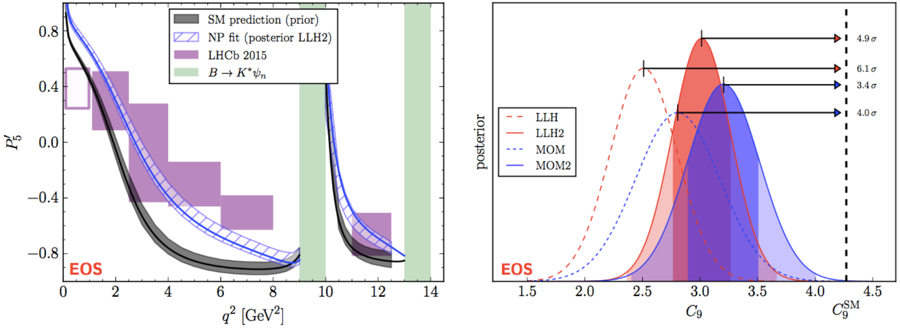


Fit to z-parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017



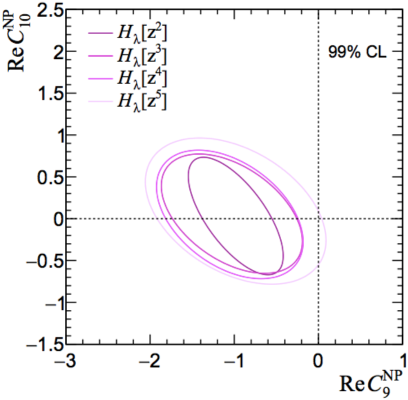
SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $\mathcal{C}_9^{\rm NP}$:

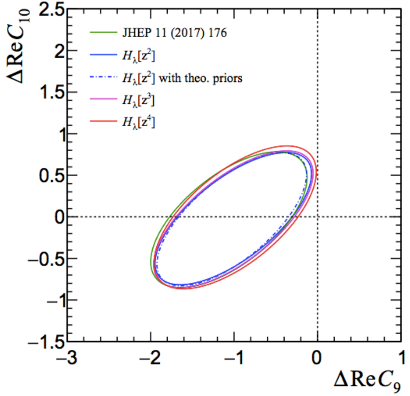


The NP hypothesis with $\mathcal{C}_9^{\rm NP} \sim -1$ is favored strongly in the global fit

Chrzaszcz, Mauri, Serra, Coutinho, van Dyk 1805.06378

Mauri, Serra, Coutinho 1805.06401





Unbinned fits to $B \to K^*\mu\mu$ (Left) and $B \to K^*\ell\ell$ (Right)

Dispersive bound

Gubernari, van Dyk, Virto 2020

$$\Pi^{\mu\nu}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0| T\left\{ O^\mu(q;x), O^{\nu,\dagger}(q;0)\right\} |0\rangle = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) \Pi(q^2)$$

Here, the operators $O^\mu(q;x)$ and $O^{\dagger,\nu}(q;0)$ are defined as

$$O^\mu(q;x) = \left(\frac{-16\pi^2 i}{q^2}\right) \int d^4y\, e^{+iq\cdot y}\; T\{j^\mu_{\rm em}(x+y), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)(x)\}\,,$$

$$O^{\nu,\dagger}(q;0) = \left(\frac{+16\pi^2 i}{q^2}\right) \int d^4z\, e^{-iq\cdot z}\; T\{j^\nu_{\rm em}(z), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)^\dagger(0)\}\,.$$

Dispersive bound

Gubernari, van Dyk, Virto 2020

Twice-subtracted dispersion relation:

$$\chi^{\mathrm{OPE}}(Q^2) \equiv \frac{1}{2i\pi}\int_0^\infty ds\, \frac{\mathrm{Disc}_{b\bar{s}}\Pi^{\mathrm{had}}(s)}{(s-Q^2)^3}$$

$$\begin{aligned}
\frac{3}{32i\pi^3}\,\mathrm{Disc}_{b\bar{s}}\Pi^{\mathrm{had}}(s) = {} & \frac{2M_B^4\,\lambda^{3/2}(M_B^2, M_K^2, s)}{s^4}\left|\mathcal{H}_0^{B\to K}(s)\right|^2 \theta(s-s_{BK}) \\
& + \frac{2M_B^6\sqrt{\lambda(M_B^2, M_{K^*}^2, s)}}{s^3}\left(\left|\mathcal{H}_\perp^{B\to K^*}(s)\right|^2 + \left|\mathcal{H}_\parallel^{B\to K^*}(s)\right|^2 + \frac{M_B^2}{s}\left|\mathcal{H}_0^{B\to K^*}(s)\right|^2\right)\theta(s-s_{BK^*}) \\
& + \frac{M_B^6\sqrt{\lambda(M_{B_s}^2, M_\phi^2, s)}}{s^3}\left(\left|\mathcal{H}_\perp^{B_s\to\phi}(s)\right|^2 + \left|\mathcal{H}_\parallel^{B_s\to\phi}(s)\right|^2 + \frac{M_{B_s}^2}{s}\left|\mathcal{H}_0^{B_s\to\phi}(s)\right|^2\right)\theta(s-s_{B_s\phi}) \\
& + \text{further positive terms}
\end{aligned}$$

Dispersive bound

Gubernari, van Dyk, Virto 2020

Redefine $\mathcal{H}_i$ as before:

$$\begin{aligned}
\hat{\mathcal{H}}_0^{B\to P}(z) &\equiv \phi_0^{B\to P}(z)\,\mathcal{P}(z)\,\mathcal{H}_0^{B\to P}(z)\,, \\
\hat{\mathcal{H}}_\lambda^{B\to V}(z) &\equiv \phi_\lambda^{B\to P}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to P}(z)\,,
\end{aligned}$$

Expand in ortogonal polynomials in unit circle:

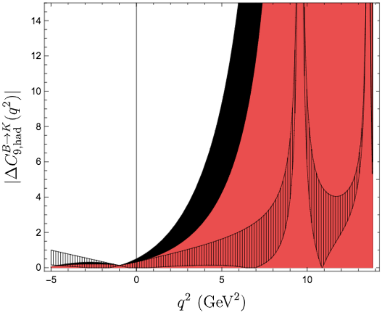
$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} p_n^{B\to M}(z)$$

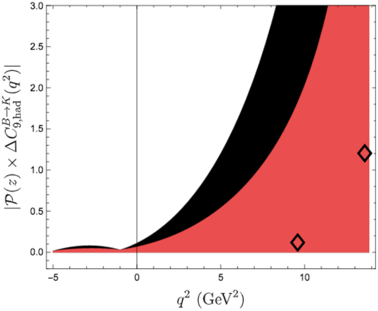
The dispersive bound then takes the simple form

$$\boxed{\sum_{n=0}^{\infty}\left\{2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0}\left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right]\right\} < 1\,.}$$

Example:

$$\Delta C_{9,\mathrm{had}}^{B\to K}(q^2) = \frac{32\pi^2 M_B^2}{q^2}\frac{\mathcal{H}_0^{B\to K}(q^2)}{\mathcal{F}_0^{B\to K}(q^2)}$$

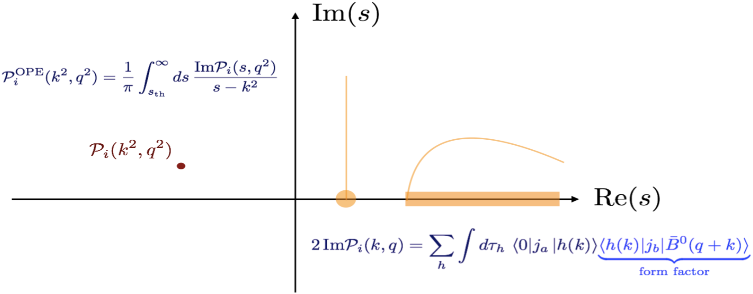




Light-Cone Sum Rules with B-meson LCDAs

Khodjamirian, Mannel, Offen 2006

Consider a correlation function: $\mathcal{P}_{ab}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_a(x), j_b(0)\}|\bar{B}^0(q+k)\rangle$



- Traditionally, $h(k) = K^* + \mathit{continuum} \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^*\, F^{BK*}\,\delta(k^2 - m_{K^*}) + \cdots$

Light-Cone Sum Rules with B-meson LCDAs

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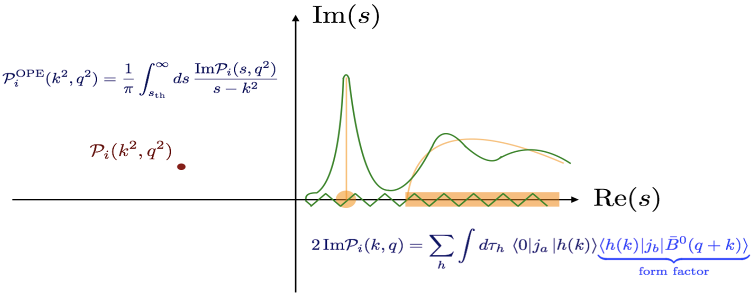
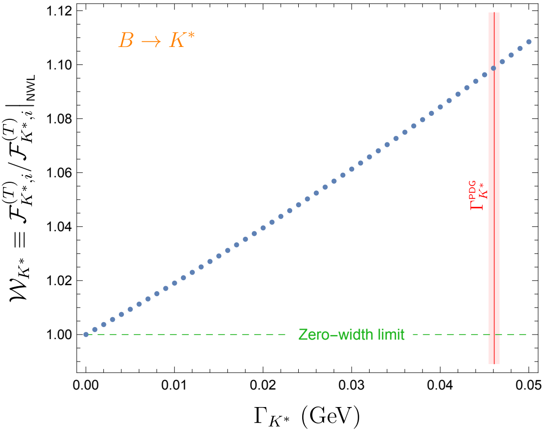


- Traditionally, $h(k) = K^* + continuum \;\Rightarrow\; 2\,\mathrm{Im}\mathcal{P}_{ab}(k,q) \sim f_K^* F^{BK*}\delta(k^2 - m_{K^*}) + \cdots$
- Generalization for unstable mesons Cheng, Khodjamirian, Virto 2017 : $h(k) = K\pi + \cdots$

LCSRs with B-meson DAs, natural for this generalization.

Finite-width effects

Descotes-Genon, Khodjamirian, Virto, 1908.02267



$$\mathcal{W}_{K*} \simeq 1 + 1.9\,\frac{\Gamma_{K*}}{m_{K*}}$$

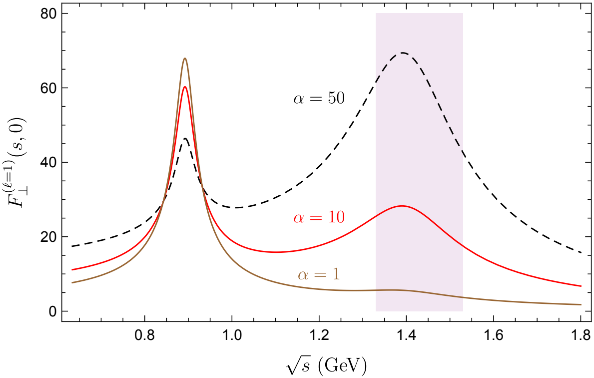
$$\mathcal{W}_{K*} = 1.09 \pm 0.01$$

- $\mathcal{W}_{K*}$ is independent of the form factor type
- $\mathcal{W}_{K*}$ is indep. of q^2

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K*}|^2 \simeq 1.2$. Ratios unaffected.

Beyond the $K^*(892)$

Set $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$ with α a floating parameter



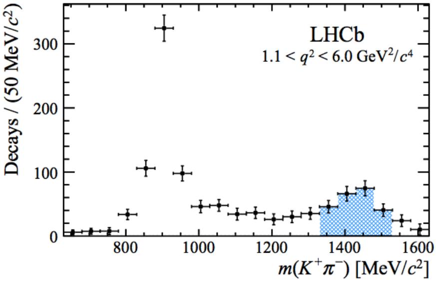
$\alpha = 1 : \ \mathcal{F}_{K^*,\perp}(0) = 0.28\ ; \quad \alpha = 10 : \ \mathcal{F}_{K^*,\perp}(0) = 0.22\ ; \quad \alpha = 50 : \ \mathcal{F}_{K^*,\perp}(0) = 0.11\ .$

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell\, d\cos\theta_K\, d\phi$]

$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega)\, \tilde{\Gamma}_i(q^2, k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ have been measured by LHCb (arXiv:1609.04736) in the bins

$$\sqrt{k^2} \in [1.33, 1.53]\text{GeV}\ , \quad q^2 \in [1.1, 6]\text{GeV}^2$$



High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Differential decay rate including S,P,D waves – – [$d\Omega = d\cos\theta_\ell \, d\cos\theta_K \, d\phi$]

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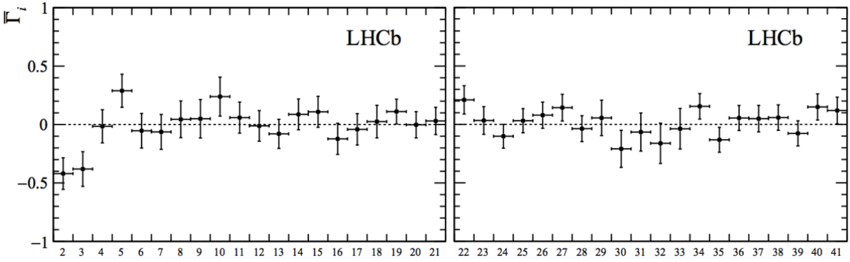

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Descotes-Genon, Khodjamirian, Virto, 1908.02267

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The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ depend on S, P, D-wave amplitudes:

i	$f_i(\Omega)$	$\Gamma_i^{L,\mathrm{tr}}(q^2)/\mathbf{k}q^2$	$\eta_i^{L\to R}$
1	$P_0^0 Y_0^0$	$\left[\lvert H_0^L\rvert^2 + \lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2 + \lvert S^L\rvert^2 + \lvert D_0^L\rvert^2 + \lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2\right]$	$+1$
2	$P_1^0 Y_0^0$	$2\left[\frac{2}{\sqrt{5}} Re(H_0^L D_0^{L*}) + Re(S^L H_0^{L*}) + \sqrt{\frac{3}{5}}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*})\right]$	$+1$
3	$P_2^0 Y_0^0$	$\frac{\sqrt{5}}{7}\,(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2)$ - $\frac{1}{\sqrt{5}}\,(\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) + \frac{2}{\sqrt{5}}\,\lvert H_0^L\rvert^2 + \frac{10}{7\sqrt{5}}\,\lvert D_0^L\rvert^2 + 2\, Re(S^L D_0^{L*})$	$+1$
4	$P_3^0 Y_0^0$	$\frac{6}{\sqrt{35}}\left[- Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + \sqrt{3} Re(H_0^L D_0^{L*})\right]$	$+1$
5	$P_4^0 Y_0^0$	$\frac{2}{7}\left[-2(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + 3\lvert D_0^L\rvert^2\right]$	$+1$
6	$P_0^0 Y_2^0$	$\frac{1}{2\sqrt{5}}\left[(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) + (\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - 2\lvert S^L\rvert^2 - 2\lvert D_0^L\rvert^2 - 2\lvert H_0^L\rvert^2\right]$	$+1$
7	$P_1^0 Y_2^0$	$\left[\frac{\sqrt{3}}{5}\, Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) - \frac{2}{\sqrt{5}}\, Re(S^L H_0^{L*}) - \frac{4}{5}\, Re(H_0^L D_0^{L*})\right]$	$+1$
8	$P_2^0 Y_2^0$	$\left[\frac{1}{14}(\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2) - \frac{2}{7}\lvert D_0^L\rvert^2 - \frac{1}{10}(\lvert H_\parallel^L\rvert^2 + \lvert H_\perp^L\rvert^2) - \frac{2}{5}\lvert H_0^L\rvert^2 - \frac{2}{\sqrt{5}} Re(S^L D_0^{L*})\right]$	$+1$
9	$P_3^0 Y_2^0$	$-\frac{3}{5\sqrt{7}}\left[Re(H_\parallel^L D_\parallel^{L*} + H_\perp^L D_\perp^{L*}) + 2\sqrt{3}\, Re(H_0^L D_0^{L*})\right]$	$+1$
10	$P_4^0 Y_2^0$	$-\frac{2}{7\sqrt{5}}\left[\lvert D_\parallel^L\rvert^2 + \lvert D_\perp^L\rvert^2 + 3\lvert D_0^L\rvert^2\right]$	$+1$
11	$P_1^1 \sqrt{2}\, Re(Y_2^1)$	$-\frac{3}{\sqrt{10}}\left[\sqrt{\frac{2}{3}}\, Re(H_\parallel^L S^{L*}) - \sqrt{\frac{2}{15}}\, Re(H_\parallel^L D_0^{L*}) + \sqrt{\frac{2}{5}}\, Re(D_\parallel^L H_0^{L*})\right]$	$+1$
12	$P_2^1 \sqrt{2}\, Re(Y_2^1)$	$-\frac{3}{5}\left[Re(H_\parallel^L H_0^{L*}) + \sqrt{\frac{5}{3}}\, Re(D_\parallel^L S^{L*}) + \frac{5}{\cdots}\, Re(D_\parallel^L D_0^{L*})\right]$ [illegible]	$+1$

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Descotes-Genon, Khodjamirian, Virto, 1908.02267

Combinations of moments depending only on P-wave:

$$
\begin{aligned}
|\widehat{A}^L_\parallel|^2 + |\widehat{A}^R_\parallel|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 - 5\sqrt{15}\tilde{\Gamma}_{19} + 35\sqrt{3}\tilde{\Gamma}_{21}) \\
|\widehat{A}^L_\perp|^2 + |\widehat{A}^R_\perp|^2 &= \frac{1}{36}(5\tilde{\Gamma}_1 - 7\sqrt{5}\tilde{\Gamma}_3 + 5\sqrt{5}\tilde{\Gamma}_6 - 35\tilde{\Gamma}_8 + 5\sqrt{15}\tilde{\Gamma}_{19} - 35\sqrt{3}\tilde{\Gamma}_{21}) \\
\mathrm{Im}(\widehat{A}^L_\perp \widehat{A}^{L*}_\parallel + \widehat{A}^R_\perp \widehat{A}^{R*}_\parallel) &= \frac{5}{36}(\sqrt{15}\tilde{\Gamma}_{24} - 7\sqrt{3}\tilde{\Gamma}_{26}) \\
\mathrm{Re}(\widehat{A}^L_\perp \widehat{A}^{L*}_\parallel - \widehat{A}^R_\perp \widehat{A}^{R*}_\parallel) &= \frac{1}{36}(-5\sqrt{3}\tilde{\Gamma}_{29} + 7\sqrt{15}\tilde{\Gamma}_{31})
\end{aligned}
$$

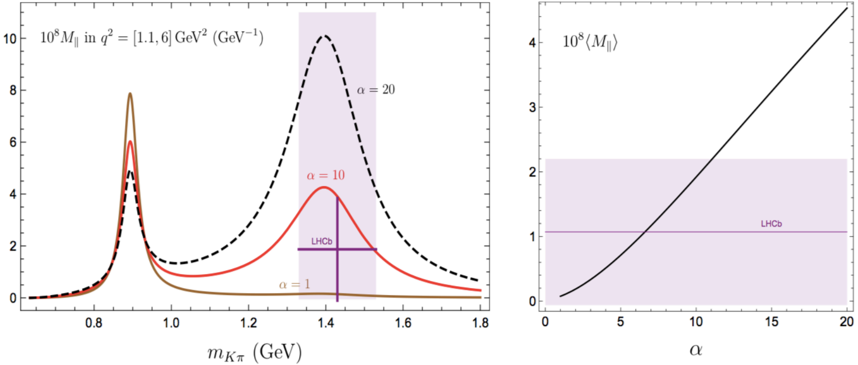
Binned LHCb results (arXiv: 1609.04736) imply:

$$
\begin{aligned}
\tau_B \,\langle|\widehat{A}^L_\parallel|^2 + |\widehat{A}^R_\parallel|^2\rangle &\equiv \langle M_\parallel\rangle = (1.07 \pm 1.13) \times 10^{-8} \\
\tau_B \,\langle|\widehat{A}^L_\perp|^2 + |\widehat{A}^R_\perp|^2\rangle &\equiv \langle M_\perp\rangle = (0.94 \pm 1.06) \times 10^{-8} \\
\tau_B \,\langle\mathrm{Im}(\widehat{A}^L_\perp \widehat{A}^{L*}_\parallel + \widehat{A}^R_\perp \widehat{A}^{R*}_\parallel)\rangle &\equiv \langle M_{\mathrm{im}}\rangle = (-0.75 \pm 0.79) \times 10^{-8} \\
\tau_B \,\langle\mathrm{Re}(\widehat{A}^L_\perp \widehat{A}^{L*}_\parallel - \widehat{A}^R_\perp \widehat{A}^{R*}_\parallel)\rangle &\equiv \langle M_{\mathrm{re}}\rangle = (0.27 \pm 0.50) \times 10^{-8}
\end{aligned}
$$

High $K\pi$-Mass Moments in $B \to K\pi\ell\ell$

Descotes-Genon, Khodjamirian, Virto, 1908.02267

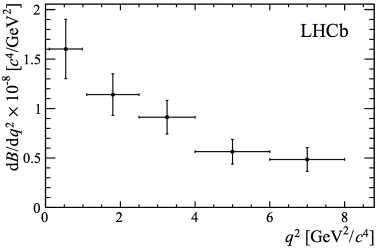
Example: $\langle M_{\|} \rangle$:



Bounds: From $\langle M_{\|} \rangle$: $\alpha \lesssim 11$; From $\langle M_{\perp} \rangle$: $\alpha \lesssim 17$; From $\langle M_{\rm re} \rangle$: $\alpha \lesssim 18$.

Upper bounds on P-wave from differential BR:

$$\frac{d\Gamma}{dq^2 dk^2} = \tilde{\Gamma}_1 = |\widehat{A}^L_\parallel|^2 + |\widehat{A}^R_\parallel|^2 + |\widehat{A}^L_\perp|^2 + |\widehat{A}^R_\perp|^2 + |\widehat{A}^L_0|^2 + |\widehat{A}^R_0|^2 + \ldots$$



$$
\begin{aligned}
10^8 \cdot \langle \mathcal{B} \rangle_{[0.10,0.98]} &= 1.41 \pm 0.27 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[1.10,2.50]} &= 1.60 \pm 0.29 \to \alpha \lesssim 6 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[2.50,4.00]} &= 1.37 \pm 0.26 \to \alpha \lesssim 5 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[4.00,6.00]} &= 1.12 \pm 0.26 \to \alpha \lesssim 4 \\
10^8 \cdot \langle \mathcal{B} \rangle_{[6.00,8.00]} &= 0.98 \pm 0.23 \to \alpha \lesssim 3
\end{aligned}
$$

Bounds are easily improved with some info on S-wave form factors.

Non-local form factors: Strategy

Similar to local form factors:

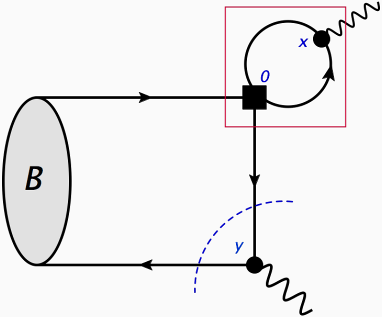
- Calculation at low-q^2 (LCSRs) and high-q^2 (LQCD)
- Interpolation with analytic expansion (z-expansion)

We can apply the same approach to Non-Local form factors:

- Calculate non-local ME at very low q^2
- Access to $q^2 > 0$ via analytic continuation + data

Charm-loop at very low q^2

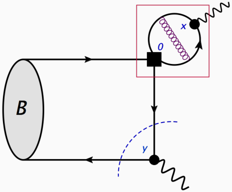
- LCSRs with B-meson DAs Khodjamirian, Mannel, Pivovarov, Wang

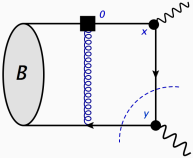


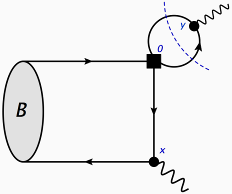
LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$

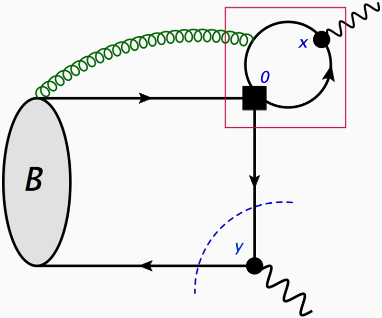






Charm-loop at very low q^2

- LCSRs with B-meson DAs   Khodjamirian, Mannel, Pivovarov, Wang

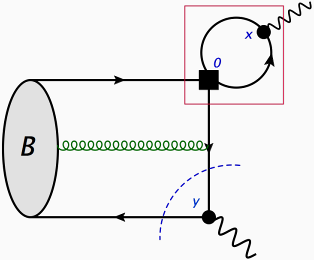


LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} \, [\bar{s}\, \Gamma\, b] +$$

$$+ \,(\text{coeff}) \times [\bar{s}_L \gamma^\alpha (i n_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,] \; + \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



Charm-loop at very low q^2

Recalculation of charm-loop effect Gubernari, van Dyk, Virto, 2011.09813

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\mathrm{GeV}^2)$	This work	Ref. [11]
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\mathrm{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\mathrm{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\mathrm{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\mathrm{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\mathrm{GeV}$	—

- We reproduce the result of KMPW'2010
- We incude complete set of 3-particle LCDAs Braun,Li,Manashov 2017
- Cancellations + Parametric lead to a reduction of the effect of **two orders of magnitude**

B mesons mix and decay due to $\mathcal{L}_{Weak} + \mathcal{L}_{BSM?}$

For $m_B \ll M_W, M_{BSM}$ we use an EFT : $\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_i C_i\, \mathcal{O}_i$

Class	Flavour structure	Number of Ops.	Other flavours	ADM	Example process
Class I	$\bar{s}b\,\bar{s}b$	5+3	$\bar{d}b\,\bar{d}b$	$\hat{\gamma}_{\mathrm{I}}$	$B_q - \overline{B}_q$ mixing
Class II	$\bar{u}b\,\bar{\ell}\nu_{\ell'}$	$(2+3)\times 9$	$\bar{c}b\,\bar{\ell}\nu_{\ell'}$	$\hat{\gamma}_{\mathrm{II}}$	$\overline{B}_d \to \pi^+\mu^-\bar{\nu}$
Class III	$\bar{s}b\,\bar{u}c$	10+10	$\bar{s}b\,\bar{c}u$ $\bar{d}b\,\bar{u}c$ $\bar{d}b\,\bar{c}u$	$\hat{\gamma}_{\mathrm{III}}$	$B^- \to \overline{D}^0 K^-$
Class IV	$\bar{s}b\,\bar{s}d$	5+5	$\bar{d}b\,\bar{d}s$ $\bar{b}s\,\bar{b}d$	$\hat{\gamma}_{\mathrm{IV}}$	$B^- \to \overline{K}^0 K^-$
Class V	$\bar{s}b\,\bar{q}q$ $\bar{s}b\,F$, $\bar{s}b\,G$ $\bar{s}b\,\bar{\ell}\ell$	57+57	$\bar{d}b\,\bar{q}q$ $\bar{d}b\,F$, $\bar{d}b\,G$ $\bar{d}b\,\bar{\ell}\ell$	$\hat{\gamma}_{\mathrm{V}}$	$\overline{B}_d \to D^+D_s^-$ $\overline{B}_d \to X_s\gamma$ $B^- \to K^-\mu^+\mu^-$
Class Vb	$\bar{s}b\,\bar{\ell}\ell'$, $\ell \neq \ell'$	$(5+5)\times 6$	$\bar{d}b\,\bar{\ell}\ell'$	$\hat{\gamma}_{\mathrm{Vb}}$	$\overline{B}_s \to \mu^-\tau^+$
Class Vν	$\bar{s}b\,\bar{\nu}_\ell\nu_{\ell'}$	$(1+1)\times 9$	$\bar{d}b\,\bar{\nu}_\ell\nu_{\ell'}$	zero	$B^- \to K^-\bar{\nu}\nu$

Aebischer, Fael, Greub, Virto 2017

Relevant part of the W$_{\text{eak}}$E$_{\text{ffective}}$T$_{\text{theory}}$ for $b \to s\ell\ell$ transitions:

$$\mathcal{L}_W = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^\star \sum_i \mathcal{C}_i(\mu)\, \mathcal{O}_i(\mu)$$

$$\mathcal{O}_1 = (\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L c) \qquad \mathcal{O}_2 = (\bar{c}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a c)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu} \qquad \mathcal{O}_{7'} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}$$

$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{9'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \qquad \mathcal{O}_{10'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

Currently, global determinations of $\mathcal{C}_9$ (and -maybe- $\mathcal{C}_{10}$) seem discrepant with SM predictions, with an important statistical significance.

Spectrum of available $b \to s\ell\ell$ Observables – (Total $= 180$)

$B_s \to \mu^+\mu^-$	$B \to X_s\mu^+\mu^-$	$B \to K^*\gamma$	$B \to X_s\gamma$
$B \to K\mu\mu$	$B \to K^*\mu\mu$	$B_s \to \Phi\mu\mu$	$\Lambda_b \to \Lambda\mu\mu$
BRs	AOs	Low q^2	Large q^2
R_K	R_{K^*}	LFU (μ)	LFUV (μ vs e)
LHCb	Belle/BaBar	ATLAS	CMS

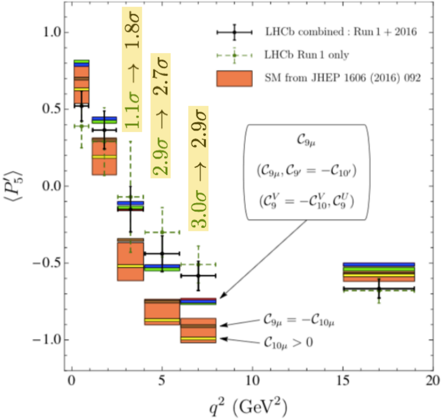
Latest updates: R_K^{LHCb}, R_K^{Belle} (2019), $B \to K^*\mu\mu$ (LHCb 2020 [Run 1 + 2016]).

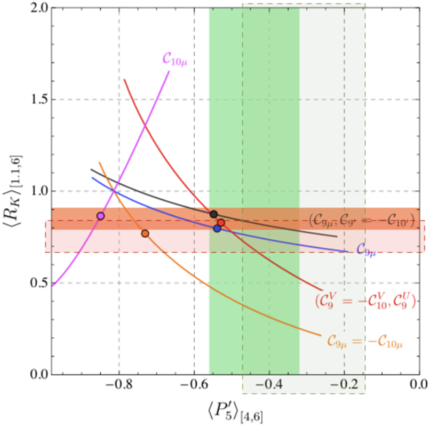
“Anomalies” (as of 2020)

Observable	Experiment	SM prediction	pull
$R_K^{[1.1,6]}$	0.85 ± 0.06	1.00 ± 0.01	$+2.5\sigma$
$R_{K*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07}$	0.92 ± 0.02	$+2.3\sigma$
$R_{K*}^{[1.1,6]}$	$0.69^{+0.12}_{-0.08}$	1.00 ± 0.01	$+2.6\sigma$
$\langle P_5' \rangle_{[4,6]}$	-0.44 ± 0.12	-0.82 ± 0.08	-2.7σ
$\langle P_5' \rangle_{[6,8]}$	-0.58 ± 0.09	-0.94 ± 0.08	-2.9σ
$\mathcal{B}_{\phi\mu\mu}^{[2,5]}$	0.77 ± 0.14	1.55 ± 0.33	$+2.2\sigma$
$\mathcal{B}_{\phi\mu\mu}^{[5.8]}$	0.96 ± 0.15	1.88 ± 0.89	$+2.2\sigma$

Global fit should accommodate these deviations within all other measurements

A closer look at new measurements of R_K and P_5' (LHCb 2019, 2020)





More details: Algeró et al. Addendum to Eur.Phys.J.C 79 (2019)