Associated top-pair and (heavy) V Calculations

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CUNY

LHCP
June 9, 2021
Outline

• Review the status of the calculations for the associated production of a top pair and a heavy electroweak boson

\[
pp \rightarrow t\bar{t}W^\pm \\
pp \rightarrow t\bar{t}Z \\
pp \rightarrow t\bar{t}H
\]

• NLO QCD + NLO EW + NNLL results for on-shell tops and weak vector bosons, for total cross section and differential distributions

• Pointers to recent NLO corrections including decays, off-shell and non resonant effects
Motivation

- These three processes are extremely important in the searches for BSM effects, both as components of the background and of the signal itself.
- $t\bar{t}W^\pm$ and $t\bar{t}Z$ production are backgrounds in the measurement of the leptonic signatures in $t\bar{t}H$ production.
- $t\bar{t}H$ production can be used to measure the top Yukawa coupling.
- $t\bar{t}Z$ production can be employed to detect anomalies in the top quark Z boson coupling.
Tree level partonic processes

Quark annihilation channel

Gluon fusion channel

ttZ and ttH production

ttW production

Only quark antiquark annihilation channel at LO
Goal

- In the last five years there was an effort to improve predictions on $ttV$ production (for on shell tops and V) within the SM
- NLO QCD corrections for these processes have been known for more than a decade (cutting edge calculations that played a crucial role in developing and testing automated tools)
Goal

- In the last five years there was an effort to improve predictions on \( \text{ttX} \) production (for on shell tops and X) within the SM
- NLO QCD corrections for these processes have been known for more than a decade (cutting edge calculations that played a crucial role in developing an testing automated tools)

**A (incomplete) history of top pair + Higgs calculations**

- Cross section and some distributions evaluated to NLO QCD
  - Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01-'02)
  - Dawson, Reina, Wackeroth, Orr, Jackson ('01,'03)
- In 2 \( \rightarrow \) 3 processes ("multileg processes"), analytic NLO calculations become cumbersome: top pair + Higgs production was one of the first processes to be used to test automated tools
  - Frixione et al ('11), Hirshi et al.'('11)
  - Garzelli et al.'('11), Bevilacqua et al.'('11)

Many important contributions, unfortunately no time to review them here
Goal

- In the last five years there was an effort to improve predictions on $t\bar{t}X$ production (for on shell tops and X) within the SM

- NLO QCD corrections for these processes have been known for more than a decade (cutting edge calculations that played a crucial role in developing and testing automated tools)

- NLO EW corrections are also known

  Frixione, Hirshi, Pagani, Shao, Zaro ('14)
  Zhang, Ma, Chen, Guo ('14)
  Frixione, Hirshi, Pagani, Shao, Zaro ('15)

- NNLL soft gluon emission corrections in the partonic threshold limit were evaluated in “direct QCD” and SCET approach

  Kulesza, Motyka, Stebel, Theeuwes ('15, '16, '17)
  Broggio, AF, Ossola, Pecjak, Sameshima, Yang ('15, '16, '17)
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Combination of NLO QCD + NLO EW + NNLL for several differential distributions available

- Broggio, AF, Frederix, Pagani, Pecjak, Tsinikos ('19)
- Kulesza, Motyka, Schwartländer Stebel, Theeuwes ('20)
- Kulesza, Motyka, Schwartländer Stebel, Theeuwes ('15, '16, '17)
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NLO electroweak corrections to $ttV$ can be phenomenologically important

$$\Sigma_{\text{NLO}}(\alpha_s, \alpha) = \alpha_s^3 \alpha \Sigma_{4,0} + \alpha_s^2 \alpha^2 \Sigma_{4,1} + \alpha_s \alpha^3 \Sigma_{4,2} + \alpha^4 \Sigma_{4,3}$$

$$\equiv \Sigma_{\text{NLO}_1} + \Sigma_{\text{NLO}_2} + \Sigma_{\text{NLO}_3} + \Sigma_{\text{NLO}_4}$$

- NLO QCD
- Dominant NLO EW corrections for $ttZ$ and $ttH$
- Dominant EW correction in $ttW$

$m_t = 173.34$ GeV, $m_W = 80.385$ GeV, $m_Z = 91.1876$ GeV, $m_H = 125$ GeV, $G_\mu = 1.16639 \cdot 10^{-5}$ GeV$^{-2}$
Soft emission corrections

- The partonic cross section for top pair (+Higgs, W or Z) production receives potentially large corrections from soft gluon emission diagrams.

- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:
  \[ L \equiv \ln \left( \frac{\text{"hard" scale}}{\text{"soft" scale}} \right) \]
  \[ z = \frac{M^2}{\hat{s}} \]
  \[ \hat{s} - M^2 \]

- It can be that \( \alpha_s L \sim 1 \)

- One needs to reorganize the perturbative series: Resummation

- The resummation of soft emission corrections can be carried out by means of effective field theory methods

\[ \alpha_s^n \left[ \frac{\ln^k (1-z)}{1-z} \right] + \]

Plus distributions in the partonic cross section can be accounted for systematically.
Soft emission corrections

- The partonic cross section for top pair (+Higgs) production receives potentially large corrections from soft gluon emission diagrams.

- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:
  - It can be that
  - One needs to reorganize the perturbative series:
  - The resummation of soft emission corrections can be carried out by means of effective field theory methods.

Renormalization group improved perturbation theory schematically:

- Separation of scales ↔ factorization
- Evaluate each (single-scale) factor in fixed order perturbation theory at a scale for which it is free of large logs
- Use Renormalization Group Equations to evolve the factors to a common scale
NLO (QCD+EW)+NNLL results for ttH

In the following, when not otherwise specified, results are taken from

- A. Broggio, AF, R. Frederix, D. Pagani, B. Pecjak, and I. Tsinikos
  JHEP 08 (2019) 039

Fixed order calculations: MadGraph5_aMC@NLO
NNLL threshold resummation: In-house parton level MC + OpenLoops + Collier
Scales and settings

- The PDFs employed are LUXQED17 (82200)
- Theoretical predictions depend on a set of scale choices, which are the scales at which the individual components of the factorization formula are free from large logarithmic corrections:
  - Hard scale $\mu_h$
  - Soft scale $\mu_s$

- The renormalization group equation is used to run everything to the factorization scale $\mu_f$, that is the scale at which the PDFs are evaluated.

Need to choose a default value for each of the scales, and then vary the default value to estimate the residual scale uncertainty $\mu_i \in [\mu_i,0/2, 2\mu_i,0]$. 

If all of the elements in the cross sections were known to all orders, there would be no scale dependence. The residual scale dependence of the predictions can be used to assess the theoretical uncertainty.
Scale choices

The default values of the scales can be chosen as function of the invariant mass of the final state

\[ \mu^0_f = \frac{m(tt\bar{V})}{2}, \quad \mu^0_h = m(tt\bar{V}), \quad \mu^0_s = \frac{m(tt\bar{V})}{N} \]

Alternatively, the scale choices can be parameterized in terms of H_T rather than on the invariant mass of the three massive objects in the final state

\[ H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_{t'}^2 + p_{T,t'}^2} + \sqrt{m_V^2 + p_{T,V}^2} \]

\[ \mu^0_f = \frac{H_T}{2}, \quad \mu^0_h = \frac{H_T}{2}, \quad \mu^0_s = \frac{H_T}{N} \]

A conservative (and safe) approach consists in taking the scale uncertainty as the envelope of the scale uncertainties obtained by varying the M and H_T based scales
# Total cross section - $ttH$

## $13$ TeV

<table>
<thead>
<tr>
<th>Combined scales</th>
<th>Monte Carlo integration uncertainty</th>
<th>Scale uncertainty</th>
<th>PDF uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Order</strong></td>
<td>$\sigma_{[fb]}$</td>
<td>$A_{C} [%]$</td>
<td></td>
</tr>
<tr>
<td>LO$_{QCD}$</td>
<td>$336.25(3)$</td>
<td>$0.88(1)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$+109.98(+32.7%)$</td>
<td>$+0.25(+28.9%)$</td>
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<tr>
<td></td>
<td>$-77.07(-22.9%)$</td>
<td>$-0.17(-19.2%)$</td>
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<tr>
<td></td>
<td>$-7.42(-2.2%)$</td>
<td>$-0.04(-4.2%)$</td>
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</tr>
<tr>
<td>NLO$_{QCD}$</td>
<td>$467.96(5)$</td>
<td>$1.05(1)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$+45.57(+9.7%)$</td>
<td>$+0.27(+25.5%)$</td>
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<tr>
<td></td>
<td>$-53.98(-11.5%)$</td>
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<tr>
<td></td>
<td>$-55.42(-11.5%)$</td>
<td>$-0.01(-1.5%)$</td>
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<td>$484.33(7)$</td>
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For the total cross section, PDF uncertainties are smaller than scale uncertainties.

$$A_{C} = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)}$$

$$\Delta \equiv |y(t)| - |y(t)|$$

$mt = 173.34$ GeV  
$m_H = 125$ GeV

NLO complete $\rightarrow$ QCD+EW

NLO complete → QCD+EW
Total cross section - ttH

13 TeV

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<td>490.27(6) +18.56(+3.8%) +11.93(+2.4%) −9.50(−1.9%) −11.93(−2.4%)</td>
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ATLAS (arXiv:1806.00425)

\[ \sigma = 670^{+90+110}_{-90-100} \text{ fb} \]

CMS (arXiv:1804.02610)

\[ \frac{\sigma_{\text{exp}}}{\sigma_{\text{SM}}} = 1.26^{+0.31}_{-0.26} \]

\[ A_C = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)} \]

\[ \Delta \equiv |y(t)| - |y(t)| \]
# Total cross section - $ttH$

## 13 TeV

### Agreement between different implementations of the resummation

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### ATLAS (arXiv:1806.00425)

\[
\sigma_{\text{NLO}+\text{NNLL}} = 504^{+7.6\%+2.4\%}_{-7.1\%-2.4\%} \text{ fb}
\]

### CMS (arXiv:1804.02610)

\[
\frac{\sigma_{\text{exp}}}{\sigma_{\text{SM}}} = 1.26^{+0.31}_{-0.26}
\]

\[
A_C = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)}
\]

\[
\Delta \equiv |y(t)| - |y(\tilde{t})|
\]

Kulesza et al. (‘20)
Invariant mass distributions - ttH

Scale uncertainty → dark bands, PDF uncertainty → light bands

Shows the impact of the resummation on the distribution shape

Shows the impact of the EW corrections on the distribution shape
Invariant mass distributions - ttH

Scale uncertainty → dark bands, PDF uncertainty → light bands

NNLL resummation increases the cross section in all bins.

EW corrections are more relevant near the production threshold.
NLO+NNLL calculations have a smaller scale uncertainty than NLO calculations. EW corrections reduce the cross section at high pT.
ttH with decays

The full NLO QCD and EW corrections to the process

\[ pp \to e^+ \nu_e \mu^- \bar{\nu}_\mu \bar{b}bH \]

are known

A. Denner and R. Feger (‘15)

A. Denner, J. N. Lang, M. Pellen and S. Uccirati (‘17)

Non-resonant and off-shell contributions included, ex.

EW corrections and off-shell effects are found to be most relevant in the tail of the differential distributions

NLO QCD calculation of \[ pp \to e^+ \nu_e \mu^- \bar{\nu}_\mu \bar{b}bH \] important for measurements of \[ t\bar{t}H(H \to \bar{b}b) \]

A Denner, J.N. Lang and M. Pellen (‘20)


See talk by M. Worek on Tuesday
NLO (QCD+EW)+NNLL results for ttW
**Total cross section - ttW+**

<table>
<thead>
<tr>
<th>13 TeV</th>
<th>[ m_t = 173.34 \text{ GeV} \quad m_W = 80.385 \text{ GeV} ]</th>
</tr>
</thead>
</table>
| Combined scales | \[
\begin{array}{ccc}
\text{Order} & \sigma [\text{fb}] & A_C [%] \\
\hline
\text{LO}_{QCD} & 233.297(8) & +64.88(+27.8\%) & +6.16(+2.6\%) & 0 \\
\text{NLO}_{QCD} & 365.66(3) & +57.95(+15.85\%) & +8.35(+2.3\%) & \text{2.68}(1) \\
\text{NLO} & 387.24(4) & +62.05(+16.0\%) & +8.25(+2.1\%) & \text{2.85}(1) \\
\text{nNLO}_{QCD} & 371.72(3) & +51.11(+13.8\%) & +8.50(+2.3\%) & \text{3.30}(2) \\
\text{nNLO} & 393.29(4) & +55.21(+14.0\%) & +8.40(+2.1\%) & \text{3.43}(2) \\
\text{NLO}_{QCD} + \text{NNLL} & 362.59(8) & +47.94(+13.2\%) & +8.26(+2.3\%) & \text{—} \\
\text{NLO} + \text{NNLL} & 384.17(9) & +51.52(+13.4\%) & +8.16(+2.1\%) & \text{—} \\
\end{array}
\]|
| Sizable NLO corrections due to the opening of the qg channel and to the large gluon luminosity at the LHC. |
| Only the quark annihilation channel contributes at tree level, resummation has a modest impact |
| Relatively large EW corrections due to tW→ tW scattering diagrams |

\[
A_C = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)} \\
\Delta \equiv |y(t)| - |y(t^-)|
\]
## Total cross section - $tt\bar{W}$-

<table>
<thead>
<tr>
<th>Order</th>
<th>(\sigma_{tt\bar{W}}) [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO_QCD</td>
<td>(870^{+190}_{-190}) fb</td>
</tr>
<tr>
<td>NLO_QCD</td>
<td>(770^{+120+130}_{-110-120}) fb</td>
</tr>
<tr>
<td>NLO</td>
<td></td>
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<tr>
<td>nNLO_QCD</td>
<td></td>
</tr>
<tr>
<td>nNLO</td>
<td></td>
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<tr>
<td>NLO_QCD+NNLL</td>
<td></td>
</tr>
<tr>
<td>NLO+NNLL</td>
<td></td>
</tr>
</tbody>
</table>

Combined prediction NLO+NNLL

\[
\sigma_{\text{NNLO+NNLL}}^{tt\bar{W}} = 581.9^{+91.5(16\%)}_{-62(11\%)} \text{ fb}
\]

\[
\sigma_{\text{NLO+NNLL}}^{tt\bar{W}} = 592^{+26.1+2.1\%}_{-16.2-2.1\%} \text{ fb}
\]

\[
A_C = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)}
\]

\[
\Delta \equiv |y(t)| - |y(\bar{t})|
\]

Kulesza et al. (2020)
Invariant mass distributions - $ttW^+$

Small impact of the resummation on the shape of the distribution. Minor reduction of the scale uncertainty. In the tail the distribution is dominated by the $qg$ channel, which is subleading in the threshold limit.
A number of recent papers aimed to improve predictions while accounting for the $W$ decay.

Estimates of the NNLO QCD corrections indicate that they could be large

S. von Buddenbrock, R. Ruiz, B. Mellado ('20)

The impact of $\mathcal{O}(\alpha_s^3)$ corrections was examined in multilepton final states. These corrections do not have a flat impact on jet multiplicities and dijet invariant mass distributions.

R. Frederix and I. Tsinikos ('20)
F. F. Cordero, M. Kraus and L. Reina ('20)

Calculations considering off-shell top pairs were also carried out. In this approach, given a fixed observed final state (eg. $b\bar{b}e^+\nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu + X$) all resonant and non resonant diagrams are considered. Off shell effects can be $\sim 10\%$ in the tail of some distributions (ex. $p_T$ distribution of the leading $b$ jet).

Two independent off-shell non resonant calculations of the NLO QCD corrections in the multilepton channel are available.

G. Bevilacqua, H. Y. Bi, H. B. Hartanto, M. Kraus, J. Nasufi and M. Worek ('20)
A. Denner and G. Pelliccioli ('20, '21)

See also talks by A. Kulesza and M. Worek on Monday and Tuesday.
NLO (QCD+EW)+NNLL results for $t\bar{t}Z$
### Total cross section - $ttZ$

<table>
<thead>
<tr>
<th>Combined scales</th>
<th>$\sigma$ [fb]</th>
<th>$A_C$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Order</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LO$_{QCD}$</td>
<td>$484.26(4)$</td>
<td>$-0.09(1)$ $+0.01(-11.1%)$ $+0.02(-16.3%)$</td>
</tr>
<tr>
<td></td>
<td>$+171.26(+35.4%)$ $+11.05(+2.3%)$ $-117.32(-24.2%)$ $-11.05(-2.3%)$</td>
<td>$-0.009(9.9%)$ $-0.02(+16.3%)$</td>
</tr>
<tr>
<td>NLO$_{QCD}$</td>
<td>$751.2(1)$</td>
<td>$0.79(2)$  $+0.23(+29.0%)$ $+0.05(+6.3%)$</td>
</tr>
<tr>
<td></td>
<td>$+110.1(+14.8%)$ $+17.7(+2.4%)$ $-108.5(-14.4%)$ $-17.7(-2.4%)$</td>
<td>$-0.15(-19.1%)$ $-0.05(-6.3%)$</td>
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<tr>
<td>NLO</td>
<td>$759.5(1)$</td>
<td>$0.87(2)$  $+0.22(+25.0%)$ $+0.05(+5.3%)$</td>
</tr>
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<td>$+110.1(+14.8%)$ $+17.9(+2.4%)$ $-107.8(-14.2%)$ $-17.9(-2.4%)$</td>
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<tr>
<td>nNLO$_{QCD}$</td>
<td>$817.1(1)$</td>
<td>$0.96(4)$  $+0.02(+1.7%)$ $+0.06(+5.8%)$</td>
</tr>
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<td>$+42.3(+5.2%)$ $+19.3(+2.4%)$ $-29.9(-3.7%)$ $-19.3(-2.4%)$</td>
<td>$-0.07(-7.5%)$ $-0.06(-5.8%)$</td>
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<td>nNLO</td>
<td>$825.4(1)$</td>
<td>$1.03(4)$  $+0.01(+1.4%)$ $+0.05(+5.2%)$</td>
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<td>$+41.3(+5.0%)$ $+19.5(+2.4%)$ $-29.3(-3.5%)$ $-19.5(-2.4%)$</td>
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<td>NLO$_{QCD}+NNLL$</td>
<td>$802.6(2)$</td>
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<td>$+89.4(+11.1%)$ $+19.0(+2.4%)$ $-78.1(-9.7%)$ $-19.0(-2.4%)$</td>
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$\sigma_{NLO+NNLL}^{ttZ} = 859_{-9.5\%}^{+8.6\%}+2.3\%$ fb  

$m_t = 173.34$ GeV  
$m_Z = 91.1876$ GeV  

Kulesza et al. (‘20)
Total cross section - $tt\bar{Z}$

13 TeV

<table>
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<tr>
<th>Combined scales</th>
<th>Order</th>
<th>$\sigma$ [fb]</th>
<th>ATLAS hep-ex:2103.12603</th>
<th>CMS JHEP 03 (2020) 056</th>
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<tr>
<td></td>
<td>$\sigma_{tt\bar{Z}} = 990^{+50+80}_{-50-80}$ fb</td>
<td>$\sigma_{tt\bar{Z}} = 950^{+50+60}_{-50-60}$ fb</td>
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<td>LO$_{QCD}$</td>
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<td>$+171.26(+63)$</td>
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Statistics uncertainties already smaller than systematic uncertainties for both experiments

\[
A_C = \frac{\sigma(\Delta > 0) - \sigma(\Delta < 0)}{\sigma(\Delta > 0) + \sigma(\Delta < 0)}
\]

\[
\Delta \equiv |y(t)| - |y(\bar{t})|
\]
ATLAS predictions vs theory $ttZ$

$$\sigma(pp \rightarrow t\bar{t}Z) = 0.99 \pm 0.05 \text{ (stat.)} \pm 0.08 \text{ (syst.) pb.}$$

Measurements performed by considering final states with 3 or 4 isolated leptons, 2015-2018 data, full Run 2 data set.

See talk by Barbara Alvarez Gonzales on Monday

**hep-ex:2103.12603**
ttZ with decays

The full NLO QCD corrections to the process

\[ pp \rightarrow b\bar{b}e^+\nu_e\mu^-\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau + X \]

"ttZ(\(Z \rightarrow \nu_\ell\bar{\nu}_\ell\))"

are known. The channel with invisible decay of the Z boson is an important background for dark matter searches

G. Bevilacqua, H. B. Hartanto, M. Kraus, T. Weber and M. Worek (‘20)

The shape of the distribution is sensitive to the choice of the scale
Conclusions and Outlook

• NLO QCD + EW + NNLL results are available for top pair + W, top pair + H, top pair + Z production (total cross section + differential distributions)

• Predictions by two different groups using different resummation techniques are in good agreement, good agreement with data

• Ongoing efforts to obtain NLO QCD + EW accuracy for off-shell tops and non-resonant contributions
Back up material
Resummation

(“Direct QCD” approach)

Resummation = (re-)arrangement of large logarithms in perturbative expansion

\[ \hat{O} = 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3) \]

\[ = \exp\left( Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots \right) C(\alpha_s) \]

+ suppressed terms

\[ Lg_1 \rightarrow \alpha_s^n L^{n+1}, \quad g_2 \rightarrow \alpha_s^n L^n, \quad \alpha_s g_3 \rightarrow \alpha_s^{n+1} L^n \]

Resummation reduces the theoretical uncertainty on a given observable.
Scale uncertainty

• In fixed order results, the scale uncertainty is evaluated by varying $\mu_i \in \{\mu_i^0/2, 2\mu_i^0\}$ ($i = s, h, f$)

• For resummed results, we vary all scales (hard, soft and factorization) independently in the range $\mu_i \in [\mu_{i,0}/2, 2\mu_{i,0}]$

• For an observable $O$ (the total cross section, or the value of a differential cross section in a given bin) one evaluates (for $i = s, f, h$ and $\kappa_i = \mu_i/\mu_{i,0}$)

\[
\Delta O_{i}^{+} = \max\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)
\]
\[
\Delta O_{i}^{-} = \min\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)
\]

• The quantities $\Delta O_{i}^{+}$ ($\Delta O_{i}^{-}$) are then combined in quadrature in order to obtain the scale uncertainty above (below) the central value
The acronym nNLO indicates approximate NNLO calculations obtained by expanding the resummation formulas. Corrections beyond NLO enhance the cross section for large forward and backward rapidities.
Transverse momentum distributions - ttW+

EW corrections increase the cross section at low pT reduce the cross section at high pT
Antitop quarks are produced more centrally than top quarks. This property is responsible for the large charge asymmetry.
Invariant mass distributions - ttW-
Transverse momentum distributions

- ttW-
Rapidity distributions - ttW-
Beyond NLO+NNLL in $ttW$

S. von Buddenbrock, R. Ruiz, B. Mellado (‘20)

An estimate of the NNLO QCD corrections suggests that these corrections could increase the $ttW$ NLO rate by $\sim 10\%$.

Estimated NNLO QCD corrections to both $ttZ$ and $ttW$ seem to reduce the current tension between theory and experiment.

NNLO calculations highly desirable

$$\sigma_{ttW^\pm}^{FxFx1j+EW} = \sigma_{ttW^\pm}^{FxFx1j} + \delta \sigma_{EW}^{NLO} = 690 \text{ fb} +12\% +1.6\% -12\% -1.6\%.$$
Invariant mass distributions - ttZ
Transverse momentum distributions
- ttZ
Predictions vs theory $t\bar{t}Z$

**ATLAS**

$\sqrt{s} = 13$ TeV, 139 fb$^{-1}$

4l SR combination

- **Data**
- MG5_aMC@NLO + Pythia8
- MG5_aMC@NLO + Herwig7
- Sherpa NLO inclusive
- Sherpa NLO multi-leg
- NLO JHEP 08 (2019) 039

**Theoretical vs Data**

- **Theory**
- **Data**

- Parton-level $|\Delta \phi(t\bar{t}, Z)|/\pi$
- Parton-level $p_\text{T}^{\ell\ell}$ [GeV]

**hep-ex: 2103.12603**
Predictions vs theory ttZ

Azimuthal separation between the Z boson and the top quark (antiquark) featuring the $W \rightarrow \ell \nu$ decay

![Graphs showing predictions vs theory for ttZ](image)

**ATLAS**

$\sqrt{s} = 13$ TeV, 139 fb$^{-1}$

3l-Z-2b3j

**Theory** vs **Data**

Absolute rapidity difference between the Z boson and the top quark (antiquark) featuring the $W \rightarrow \ell \nu$ decay

**hep-ex:** 2103.12603
Scale choices in top-pair production

The hard function include terms proportional to $\ln \left( \frac{\mu^2}{-t_1} \right)$

$$|t_1| = \frac{M_{tt}}{2} \left(1 - \sqrt{1 - \frac{4m_t^2}{M_{tt}^2}} \cos \theta \right)$$

$$|t_1| \xrightarrow{\theta \to 0, m_t \to 0} \frac{p_T^2}{p_T^2} + m_t^2 \sim m_t^2$$

if $\mu \sim M \to \ln \left( \frac{\mu^2}{-t_1} \right) \xrightarrow{\theta \to 0, m_t \to 0} \ln \left( \frac{M^2}{m_t^2} \right)$ large log!

One can choose instead

$$\mu \sim H_T \equiv \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_t^2}$$

So that $\ln \left( \frac{\mu^2}{-t_1} \right)$ remains small always
Scale choices in top-pair production

The hard function include terms proportional to

\[ \ln \left( \frac{\mu^2}{-t_1} \right) \]

\[ |t_1| = \frac{M_{tt}}{\mu^2} \left( 1 - \sqrt{1 - \frac{4m_t^2}{\cos \theta}} \right) \]

The calculation of the top pair productions at fixed order in perturbation theory converges better with a factorization scale equal to \( H_T/4 \) than with a factorization scale set equal to \( M/2 \).

Also for resummation in “boosted” top pair production there are arguments to prefer a scale choice related to \( H_T \) rather than \( M \).

One can choose instead

\[ \mu \sim H_T \equiv \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_{\bar{t}}^2} \]

M. Czakon, D. Heymes, A. Mitov
arXiv:1606.03350

arXiv:1803.07623
### ttH total cross section: H_T vs M combined

<table>
<thead>
<tr>
<th>Scale</th>
<th>m(t\bar{t}H)/2-based scales</th>
<th>(\sigma) [fb]</th>
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<tr>
<td>NLO+NNLL</td>
<td>(491.1(1)) (+27.8(+5.7%)) (-24.0(-4.9%)) (+11.6(+2.4%)) (-11.6(-2.4%))</td>
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<td>NLO+NNLL</td>
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ttH total cross section:  
H_T vs M combined

<table>
<thead>
<tr>
<th>Scale Type</th>
<th>Value</th>
<th>Error</th>
<th>Central Value</th>
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<th>Cross Section [fb]</th>
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<td></td>
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<tr>
<td>NLO QCD</td>
<td>467.96(5)</td>
<td>+45.57 (+9.7%)</td>
<td></td>
<td>-53.98 (-11.5%)</td>
<td>+11.31 (+2.4%)</td>
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<tr>
<td></td>
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Total cross section - $tt\bar{Z}$

13 TeV

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<th>Order</th>
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<tr>
<td>LO QCD</td>
<td>$461.5(1)$</td>
</tr>
<tr>
<td>NLO QCD (complete) NLO</td>
<td>$790.7(2)$</td>
</tr>
<tr>
<td>NLO QCD+NNLL</td>
<td>$799.3(2)$</td>
</tr>
<tr>
<td></td>
<td>$950^{+130}_{-130}$</td>
</tr>
<tr>
<td></td>
<td>$990^{+90+120}_{-80-100}$</td>
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</table>

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)