Precision Computations for Higgs Production

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9th Edition of the Large Hadron Collider Physics Conference CERN, 10 June



Room for Coffee Break Discussion: https://mit.zoom.us/s/95962370657 (open from 15:45 CET, passcode same as for this session)

Motivation

- Measure fiducial & differential Higgs cross sections at the LHC
 - Most model-independent way we have to search for BSM in the Higgs sector
- Total fiducial cross section measures deviations from SM gluon-fusion rate

- p_T^H spectrum is the most important differential quantity
- High $p_T^H \sim \sqrt{\hat{s}} \gg m_H$ increases sensitivity to new operators [...for SM precision theory, see next talk by R. Röntsch]
- Focus of this talk: $p_T^H \lesssim m_H \sim \sqrt{\hat{s}} \ll 2m_t$ (or p_T^H integrated over)
 - Measure or put bounds on anomalous b, c, and light quark Yukawa couplings [Bishara, Haisch, Monni, Re '16; Soreq, Zhu, Zupan '16; see e.g. ATLAS-CONF-2019-029]

• Uncertainty $\Delta \sigma$ on SM prediction translates into discovery reach:

$$rac{\Delta\sigma}{\sigma} \sim rac{v^2}{\Lambda_{
m BSM}^2} ~~ \Leftrightarrow ~~ \Lambda_{
m BSM} \sim v ~ \sqrt{rac{\sigma}{\Delta\sigma}}$$

Challenges for theory

- QCD corrections to gg
 ightarrow H are large: $\sigma/\sigma_{
 m LO}pprox 3$
 - Calculation of inclusive cross section has been pushed to N³LO [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '15-'18]
- But LHC experiments apply kinematic selection cuts on Higgs decay products
 - Need complete interplay of QCD corrections and $\mathcal{O}(1)$ fiducial acceptance

Consider $gg
ightarrow H
ightarrow \gamma\gamma$ with ATLAS fiducial cuts:

 $p_T^{\gamma 1} \geq 0.35 \, m_H \,, \quad p_T^{\gamma 2} \geq 0.25 \, m_H \,, \quad |\eta^{\gamma}| \leq 2.37 \,, \quad |\eta^{\gamma}| \notin [1.37, 1.52]$

Focus of this talk

[Billis, Dehnadi, Ebert, JM, Tackmann, 2102.08039]

- Compute fiducial spectrum for $q_T\equiv p_T^H=p_T^{\gamma\gamma}$ at N 3 LL'+N 3 LO
- Compute total fiducial cross section at N³LO, and improved by resummation



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Focus of this talk

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- Compute fiducial spectrum for $q_T\equiv p_T^{H}=p_T^{\gamma\gamma}$ at N³LL'+N³LO
- Compute total fiducial cross section at N³LO, and improved by resummation
- Previous state of the art was N³LL(+NNLO₁) and NNLO, respectively [Chen et al. '18; Bizoń et al. '18; Gutierrez-Reyes et al. '19; Becher, Neumann '20]

Kicked off a recent push for fiducial color singlet at complete three-loop accuracy:

- Complementary N³LO results for fiducial $Y_{\gamma\gamma}$, $\eta_{\gamma1}$, $\Delta\eta_{\gamma\gamma}$ (with different method) [Chen, Gehrmann, Glover, Huss, Mistlberger, 2102.07607; see plenary talk by B. Mistlberger later today]
- Fiducial N³LL' results for Higgs (and Drell-Yan) q_T spectrum [Re, Rottoli, Torrielli, 2104.07509; see also Camarda, Cieri, Ferrera, 2103.04974]



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Method

q_T subtractions

Compute cross section from $\sigma=\int\!\mathrm{d}q_T\, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$ and power expand around IR, $q_T o 0$:

$$egin{array}{lll} rac{\mathrm{d}\sigma}{\mathrm{d}q_T} &=& rac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + & rac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T} + & rac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}q_T} + \cdots \ & & \sim rac{1}{q_T} iggl[\, \mathcal{O}(1) + \mathcal{O}\Big(rac{q_T}{m_H}\Big) + \mathcal{O}\Big(rac{q_T^2}{m_H^2}\Big) + \cdots iggr] \end{array}$$

$$rac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} = \sigma_{\mathrm{LO}}\,\delta(q_T) + \sum_n lpha_s^n igg\{\sigma_n^V\delta(q_T) + \sum_m \sigma_{n,m}^{(0)} \Big[rac{\mathrm{ln}^m(q_T/m_H)}{q_T}\Big]_+ igg\}$$

- Contains LO contribution, virtual corrections, and log-enhanced singular terms
- Predicted by factorization $\Rightarrow q_T$ subtractions [Catani, Grazzini '07]

$$rac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T} = \sum_n lpha_s^n \sum_m \sigma^{(1)}_{n,m} rac{1}{m_H} \ln^m(q_T/m_H)$$

- Still logarithmically divergent, only present if decay products are resolved
- Also predicted by factorization [Ebert, JM, Stewart, Tackmann '20]

q_T subtractions

Compute cross section from $\sigma=\int\!\mathrm{d}q_T\, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$ and power expand around IR, $q_T o 0$:

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ight) + \mathcal{O}\!\left(rac{q_T^2}{m_H^2}
ight) + \cdots
ight] \ & rac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}q_T} = \sum lpha_r^n \sum \sigma_r^{(2)} rac{q_T}{T} \ln^m (q_T/m_H) + \cdots \end{array}$$

$$\frac{1}{\mathrm{d}q_T} = \sum_n \alpha_s \sum_m \sigma_{n,m} \frac{1}{m_H^2} \prod_{m=1}^m (q_T/m_H) + \cdot$$

• Finite as $q_T
ightarrow 0$, extract from H+1j calculation

Set up some notation, use that production and decay (acceptance) factorize:

$$rac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \int \mathrm{d}Y \, \pmb{A}(\pmb{q_T},\pmb{Y};\pmb{\Theta}) \, W(q_T,Y) \,, \quad \pmb{A_{\mathrm{incl}}} = 1 \,, \quad W(q_T,Y) = rac{\mathrm{d}\sigma_{\mathrm{incl}}}{\mathrm{d}q_T \, \mathrm{d}Y}$$

Leading-power factorization & resummation to N³LL'

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:



- Predicts singular structure of $\frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$ as $q_T \to 0$
- ► Enables all-order resummation ⇒ Sudakov peak

Leading-power factorization & resummation to N³LL'

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$egin{aligned} W^{(0)}(q_T,Y) &= m{H}(m{m}_H^2,m{\mu}) \int\!\mathrm{d}^2ec{k}_a\,\mathrm{d}^2ec{k}_b\,\mathrm{d}^2ec{k}_s\,\deltaig(q_T - |ec{k}_a + ec{k}_b + ec{k}_s|ig) \ & imes B_g^{\mu
u}(x_a,ec{k}_a,m{\mu},m{
u})\,B_{g\,\mu
u}(x_b,ec{k}_b,m{\mu},m{
u})\,m{S}(ec{k}_s,m{\mu},m{
u}) \end{aligned}$$

To reach $N^{3}LL'$ for $W^{(0)}$, implemented in SCETlib:

- Three-loop soft and hard function ...includes in particular the three-loop virtual form factor [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
 [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- N³LL solutions to virtuality/rapidity RGEs in b_T space
- Hybrid profile scales for fixed-order matching [Lustermans, JM, Tackmann, Waalewijn '19]

 \dots are the power corrections coming from the q_T -dependent acceptance:

$$rac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T} \equiv \int \mathrm{d}Y \Big[A(q_T,Y;\Theta) - A^{(0)}(Y;\Theta) \Big] W^{(0)}(q_T,Y)$$

• Uniquely predict all linear power corrections $\mathrm{d}\sigma^{(1)}$ because

$$egin{aligned} W(q_T,Y) &= W^{(0)}(q_T,Y) \Big[1 + \mathcal{O}\Big(rac{q_T^2}{m_H^2}\Big) \Big] \ A(q_T,Y;m{\Theta}) &= A^{(0)}(Y;m{\Theta}) \ \left[1 + \mathcal{O}\Big(rac{q_T}{m_H}\Big)
ight] \end{aligned}$$

• Resummed to the same N³LL' accuracy as leading-power terms by resumming $W^{(0)}$ and keeping exact $A(q_T, Y; \Theta)$

[Presence of linear terms pointed out in Ebert, Tackmann '20] [Factorization demonstrated in Ebert, JM, Stewart, Tackmann '20]

Key point

Fiducial power corrections induce resummation effects in the total cross section

Compare fixed-order series, isolating the effect of
$$\int dq_T \frac{d\sigma^{fpc}}{dq_T}$$
:

$$\begin{split} \sigma_{\rm incl}^{\rm FO} &= 13.80 \left[1 + 1.291 \right. + 0.783 \left. + 0.299 \right] \rm pb \\ \sigma_{\rm fid}^{\rm FO} &= 6.928 \left[1 + 1.429 \right. + 0.723 \left. + 0.481 \right] \rm pb \\ &= 6.928 \left[1 + (1.300 + 0.129_{\rm fpc}) + (0.784 - 0.061_{\rm fpc}) + (0.331 + 0.150_{\rm fpc}) \right] \rm pb \end{split}$$

Fiducial power corrections show no convergence, remainder is similar to inclusive

Key point

Fiducial power corrections induce resummation effects in the total cross section

Two ways to understand this:

1. Acceptance acts as a weight under the q_T integral



 $\sigma_{
m incl} = \int \mathrm{d} q_T \, W(q_T) \qquad \sigma_{
m fid} = \int \mathrm{d} q_T \, \pmb{A(q_T)} \, W(q_T)$

Key point

Fiducial power corrections induce resummation effects in the total cross section

Two ways to understand this:

- 1. Acceptance acts as a weight under the q_T integral
- 2. We're cutting on the resummation-sensitive photon p_T



Differential q_T subtractions

$$\sigma = \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}q_T} + \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\text{nons}}_{\text{FO}}}{\mathrm{d}q_T} + \int_{q_T^{\text{off}}}^{1} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\text{FO}_1}}{\mathrm{d}q_T}$$

Include $\mathrm{d}\sigma^{\mathrm{fpc}}$ in differential subtraction:

$$rac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}q_T} = \int\!\mathrm{d}Y\, \pmb{A}(\pmb{q_T},\pmb{Y};\pmb{\Theta})\, \pmb{W}^{(0)}(\pmb{q_T},\pmb{Y}) = rac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\pmb{q_T}} + rac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}\pmb{q_T}}$$

Remaining (nonsingular) terms:

$$\frac{\mathrm{d}\sigma_{\mathrm{FO}}^{\mathrm{nons}}}{\mathrm{d}q_T} = \int \mathrm{d}Y \, \boldsymbol{A}(\boldsymbol{q}_T, \boldsymbol{Y}; \boldsymbol{\Theta}) \left[W_{\mathrm{FO}}^{(2)}(\boldsymbol{q}_T, \boldsymbol{Y}) + \cdots \right] = \left[\frac{\mathrm{d}\sigma_{\mathrm{FO}_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma_{\mathrm{FO}}^{\mathrm{sing}}}{\mathrm{d}q_T} \right]_{q_T > 0}$$

Challenges:

- Obtaining stable H+1j results for $q_T
 ightarrow 0$ is hard ... in particular at <code>NNLO_1</code>
- Dropping the nonsingular below $q_T \leq q_T^{ ext{cut}}$ is not viable, either ...as we'll see shortly
 - Crucial to use differential subtraction, not slicing

Differential q_T subtractions

$$\sigma = \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}q_T} + \int_0^{q_T^{\text{off}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\text{nons}}_{\text{FO}}}{\mathrm{d}q_T} + \int_{q_T^{\text{off}}}^{1} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\text{FO}_1}}{\mathrm{d}q_T}$$

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Key idea

Fit nonsingular data to known form at subleading power and integrate analytically:

$$\left. q_T rac{\mathrm{d}\sigma_{\mathrm{FO}}^{\mathrm{nons}}}{\mathrm{d}q_T}
ight|_{lpha_s^n} = rac{q_T^2}{m_H^2} \sum_{m{k}=0}^{m{2n-1}} \Bigl(a_k + b_k rac{q_T}{m_H} + c_k rac{q_T^2}{m_H^2} + \cdots \Bigr) \ln^{m{k}} rac{q_T^2}{m_H^2}$$

- Include higher-power b_k, c_k to get unbiased a_k
- Allows us to use more precise data at higher q_T as lever arm in the fit



Setup:

- Combined fit to existing binned inclusive and fiducial NNLO₁ data from NNLOjet [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
- Empirically find $0.4 \leq a_k^{
 m fid}/a_k^{
 m incl} \leq 0.55$ at (N)NLO \Rightarrow use as weak 1σ constraint
- Add $\sigma_{\rm incl}(q_T \le q_T^{\rm cut}) = \sigma_{\rm incl}^{\rm N^3LO} \sigma_{\rm incl}(q_T > q_T^{\rm cut})$ as additional incl. data point [Mistlberger '18]

Comparison to other methods: q_T slicing



Slicing approach to q_T subtractions:

[used e.g. in Cieri, Chen, Gehrmann, Glover, Huss, 1807.11501; Camarda, Cieri, Ferrera, 2103.04974]

$$\sigma = \sigma^{(0)}(q_T^{\text{cut}}) + \sigma^{\text{fpc}}(q_T^{\text{cut}}) + \sigma^{\text{nons}}(q_T^{\text{cut}}) + \int_{q_T^{\text{cut}}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma_{\text{FO}_1}}{\mathrm{d}q_T}$$

- Slicing uses finite $q_T^{\text{cut}} \sim 2 \,\text{GeV}$ and neglects both $\sigma^{\text{fpc}}(q_T^{\text{cut}}), \sigma^{\text{nons}}(q_T^{\text{cut}}) \approx 0$
- This is a catastrophic approximation even at $lpha_s^2$, and definitely at $lpha_s^3$
- Even without $\sigma^{
 m fpc}$ (incl. cross section), this is a bad approximation at $lpha_s^3$
 - q_T^{cut} variations only scan local maximum around $2 \text{GeV} \dots$

Comparison to other methods: Projection to Born



Projection-to-Born method:

[used e.g. in Chen, Gehrmann, Glover, Huss, Mistlberger, 2102.07607]

$$rac{\mathrm{d}\sigma}{\mathrm{d}Y} = A(0,Y) \, rac{\mathrm{d}\sigma_{\mathrm{incl}}}{\mathrm{d}Y} + \int_{pprox q_T^{\mathrm{cut}}} \mathrm{d}q_T \left[A(q_T,Y) - A(0,Y)
ight] W(q_T,Y) \, .$$

- First term from analytic (threshold expansion of) inclusive rapidity spectrum
- Second term numerically from H+1j MC, dominated by $\sigma^{
 m fpc}$ at small q_T
- Need to integrate down to $q_T^{
 m cut} \ll 0.1 {
 m GeV!}$

Results and future directions

The fiducial q_T spectrum at N³LL'+N³LO



- Total uncertainty is $\Delta_{tot} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{match} \oplus \Delta_{FO} \oplus \Delta_{nons}$ [See also Ebert, JM, Stewart, Tackmann, 2006.11382 for details]
- Observe excellent perturbative convergence & uncertainty coverage
 - Crucial to consider *every* variation to probe all parts of the prediction
- Divide $H o \gamma\gamma$ branching ratio ${\cal B}_{\gamma\gamma}$ out of data [LHC Higgs Cross Section WG, 1610.07922]
- Data are corrected for other production channels, photon isolation efficiency [ATLAS, 1802.04146]

The total fiducial cross section at N³LO and N³LL′+N³LO



- Large N³LO correction to fiducial cross section (worse than inclusive)
 - Caused by fiducial power corrections, *not* captured by rescaling inclusive N³LO result
- Resummation restores convergence
- Allows for the first direct comparison to experimentally measured total Higgs cross section at genuine three-loop order

Outlook: Resummation effects in other $H ightarrow \gamma \gamma$ observables

- "Infrared sensitivity" observed also in other Higgs observables at N³LO [Chen, Gehrmann, Glover, Huss, Mistlberger, 2102.07607]
- ⇔ Precisely the fiducial power corrections we can analytically deal with and resum



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- Presented N³LL'+N³LO and N³LO predictions for fiducial p_T^H spectrum and total fiducial cross section for $gg \rightarrow H \rightarrow \gamma\gamma$ at the LHC
 - First direct comparison to LHC data at this order and level of precision



- Resummed large fiducial power corrections induced by experimental acceptance
 - Even *total* fiducial cross sections are sensitive to q_T resummation effects
 - \blacktriangleright Enables best-possible combined predictions for other $H
 ightarrow \gamma\gamma$ observables
- Nonsingular extraction and matching to total cross section made possible by combining all information from N³LO σ_{incl} , fixed-order NNLOjet data, fiducial power corrections, and known functional form at subleading power
- N³LL' fiducial power corrections to be part of upcoming public SCETlib release

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Thank you for your attention!

Backup

Uncertainty breakdown



 $\begin{array}{ll} \Delta_{q_T} & \mbox{36 independent scale variations in } W^{(0)} \mbox{ factorization} \\ \Delta_{\varphi} & \mbox{Vary phase of hard scale over } \arg \mu_H \in \{\pi/4, 3\pi/4\} \\ \Delta_{\rm match} & \mbox{Vary transition points governing resummation turn-off} \\ \Delta_{\rm FO} & \mbox{Vary } \mu_R/m_H \in \{1/2, 2\} \mbox{ (dominates over } \mu_F \mbox{ due to overall } \alpha_s^2) \\ \Delta_{\rm nons} & \mbox{Uncertainty on nonsingular extraction} \end{array}$

Leading-power factorization & resummation to N³LL'

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

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u}(x_a,ec{k}_a,\mu,
u)\,B_{g\,\mu
u}(x_b,ec{k}_b,\mu,
u)\,m{S}(ec{k}_s,\mu,
u) \end{aligned}$$

Ingredients satisfy 2D renormalization group equations, e.g. soft function:

$$\mu rac{\mathrm{d}}{\mathrm{d}\mu} \ln ilde{m{S}}(ec{m{b}}_T, \mu, m{
u}) = ilde{\gamma}^g_S(\mu,
u) \qquad
u rac{\mathrm{d}}{\mathrm{d}
u} \ln ilde{m{S}}(ec{m{b}}_T, \mu, m{
u}) = ilde{\gamma}^g_
u(b_T, \mu)$$

- Solve recursively at fixed order
 - Complete log structure of $\mathrm{d}\sigma^{(0)}$
- Closed-form all-order solution
 - Resummed Sudakov peak
- Resummation order specified by perturbative order of anom. dims. and boundary conditions





Fixed-order inputs:

- NLO contribution to $W(q_T, Y)$ at $q_T > 0$ is easy
- At NNLO, renormalize & implement bare analytic results for $W(q_T, Y)$ [Dulat, Lionetti, Mistlberger, Pelloni, Specchia '17]



- Perform separate χ^2 fits of $\{a_k^{\text{incl,fid}}\}$ to inclusive and fiducial nonsingular data [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combination to ensure fit is stable [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu '15-'16]



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• Check the purely hadronic a_k^{fid} by directly fitting them to

$$q_T \int \mathrm{d}Y \, A^{(\mathbf{0})}(Y;\Theta) ig[W-W^{(0)}ig] = rac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \Bigl(a_k^{\mathrm{fid}} + c_k' rac{q_T}{m_H^2} + \cdots \Bigr) \ln^k rac{q_T^2}{m_H^2} \, \checkmark$$

- Recover analytic (N)NLO coefficient of $\sigma_{
 m incl}$ at 10^{-5} (10^{-4}) 🗸
- Analytic implementation gives us awesome precision on all NLP coefficients (all logs at NLO and NNLO, also differential in Y, broken down by color structure, ...)
 - Can serve as benchmark for q_T resummation at subleading power