

CP violation in beauty with the LHCb experiment

A. Bertolin on behalf of the LHCb collaboration

LHCP2021

Online, 7-12 June 2021



Outlook:

✚ short introduction

✚ LHCb results:

- Δm_s mass difference: LHCb individual measurements and LHCb combination
- CKM γ : LHCb individual measurements and LHCb combination
- CPV in 2-body neutral B meson (Bd/Bs) decays
- CPV for baryons in LHCb

✚ take home message

CPV in the SM

CPV is one of the requirements for explaining the baryon asymmetry we observe today

in the SM the CKM matrix, V , 3×3 , fulfilling $V V^* = I$, is describing quark charged current weak interactions
 \Leftrightarrow 3 angles and 1 phase (or 3 reals and 1 imaginary parameters)

$$V = \begin{matrix} & \begin{matrix} d & s & b \end{matrix} \\ \begin{matrix} u \\ c \\ t \end{matrix} & \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} \end{matrix} + \mathcal{O}(\lambda^6) \quad \lambda \approx 0.22$$

the term $\rho + i \eta$ gives the CKM phase: **only** source of CPV in the SM quark sector
“intrinsic” connection between CPV in the beauty and charm sectors
however the imaginary part of:

$V_{cd} \propto \lambda^5$
 $V_{ub} \propto \lambda^3$

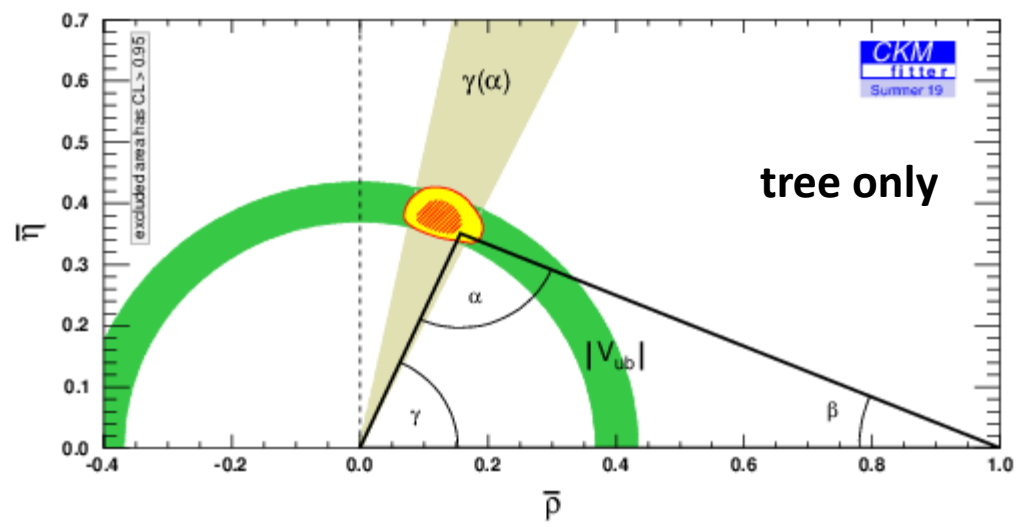
expect CPV suppression in charm w.r.t beauty ...

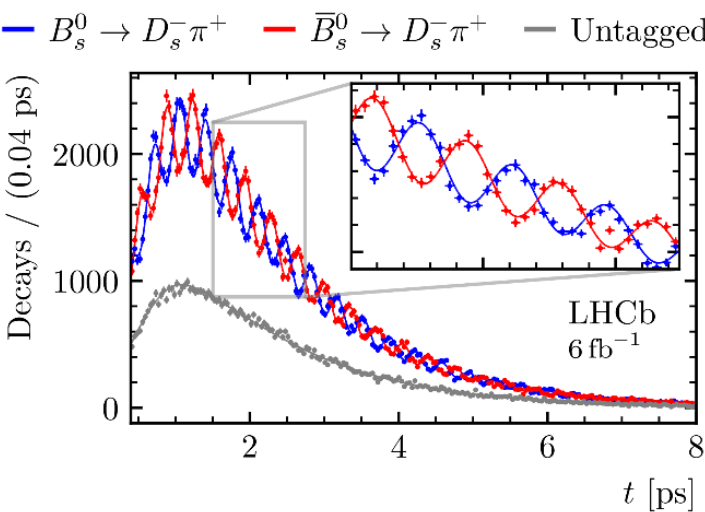
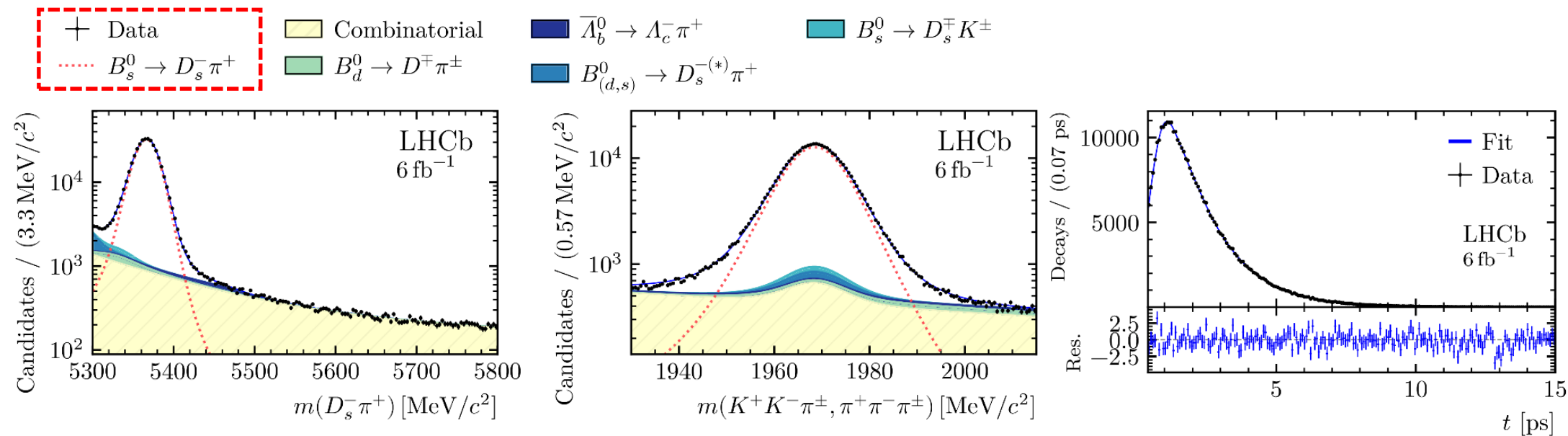
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

unitary condition relevant for beauty decays
can be represented as a triangle in a complex plane, with angles α , β and γ

- $\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$
a.k.a. ϕ_3
- only CKM angle easily accessible in tree-level decays
 - assuming no new physics in tree-level decays, has negligible theoretical uncertainty

http://ckmfitter.in2p3.fr/www/results/plots_summer19/ckm_res_summer19.html





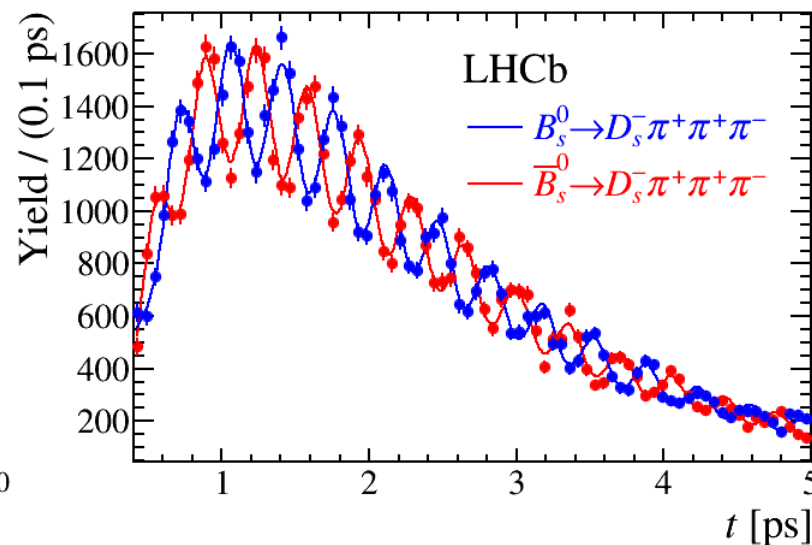
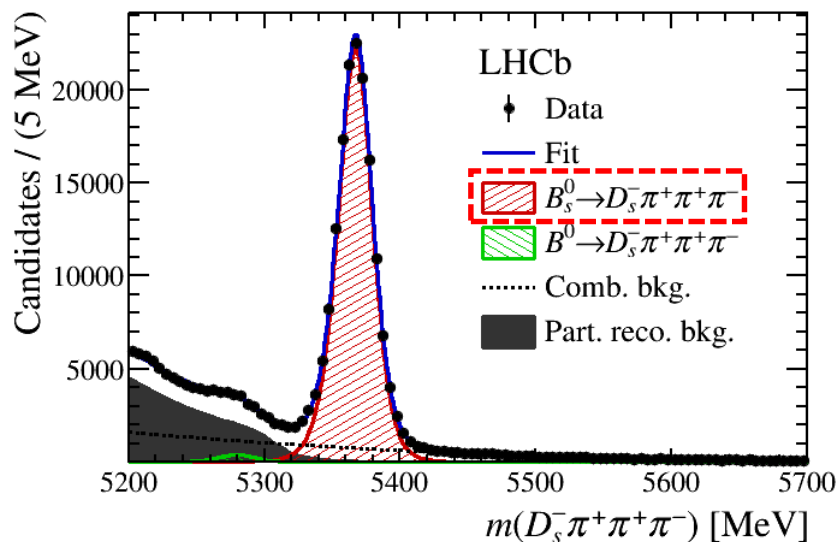
- exploit the flavor specific nature of this decay i.e. just oscillations
- full Run 2 statistic (6 /fb), partial Run 1 (1 /fb) result already published
- signal yield for a simultaneous fit to the beauty and charm mass distributions: **378.7 k events**
- Δm_s from a fit to the background subtracted decay time distribution

$$\Delta m_s = 17.7683 \pm 0.0051 \text{ (stat)} \pm 0.0032 \text{ (syst)} \text{ ps}^{-1}$$

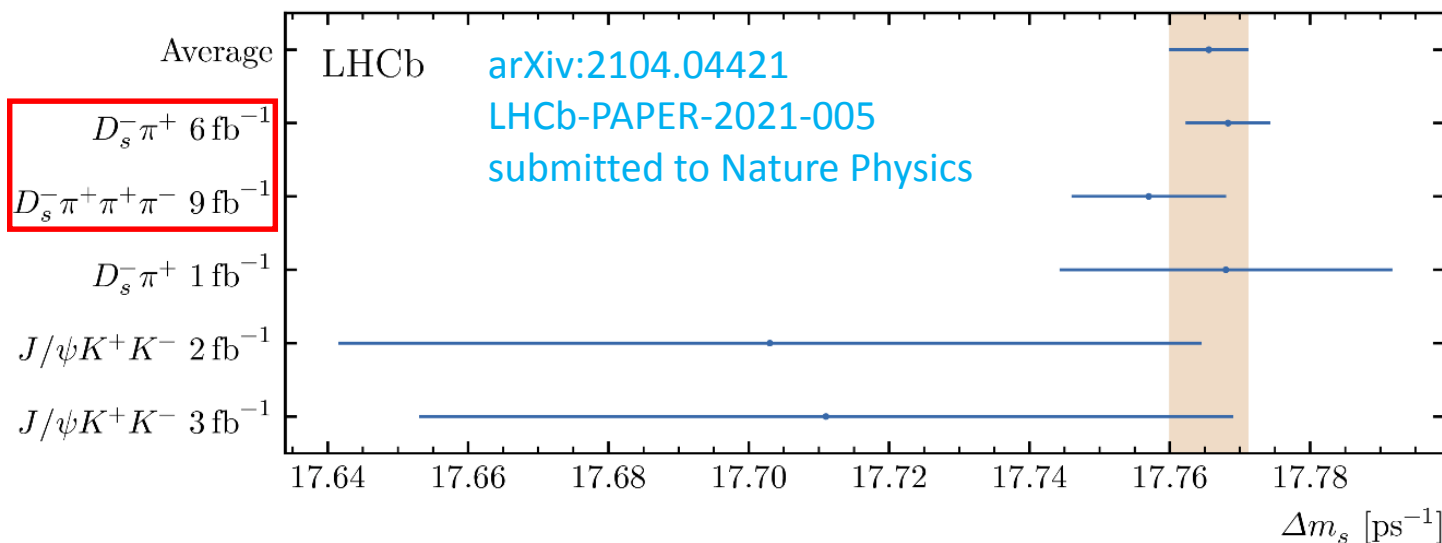
- spectacular decay time asymmetry distribution

$$A(t) = \frac{N(B_s^0 \rightarrow D_s^- \pi^+, t) - N(\bar{B}_s^0 \rightarrow D_s^- \pi^+, t)}{N(B_s^0 \rightarrow D_s^- \pi^+, t) + N(\bar{B}_s^0 \rightarrow D_s^- \pi^+, t)}$$

LHCb Δm_s mass difference update (cont.)



- full Run 1+2 statistic (9 /fb)
- procedure as in the previous analysis
- signal yield: **148 k events**
- spectacular decay time asymmetry distribution



LHCb average: 17.7656 ± 0.0057 ps⁻¹

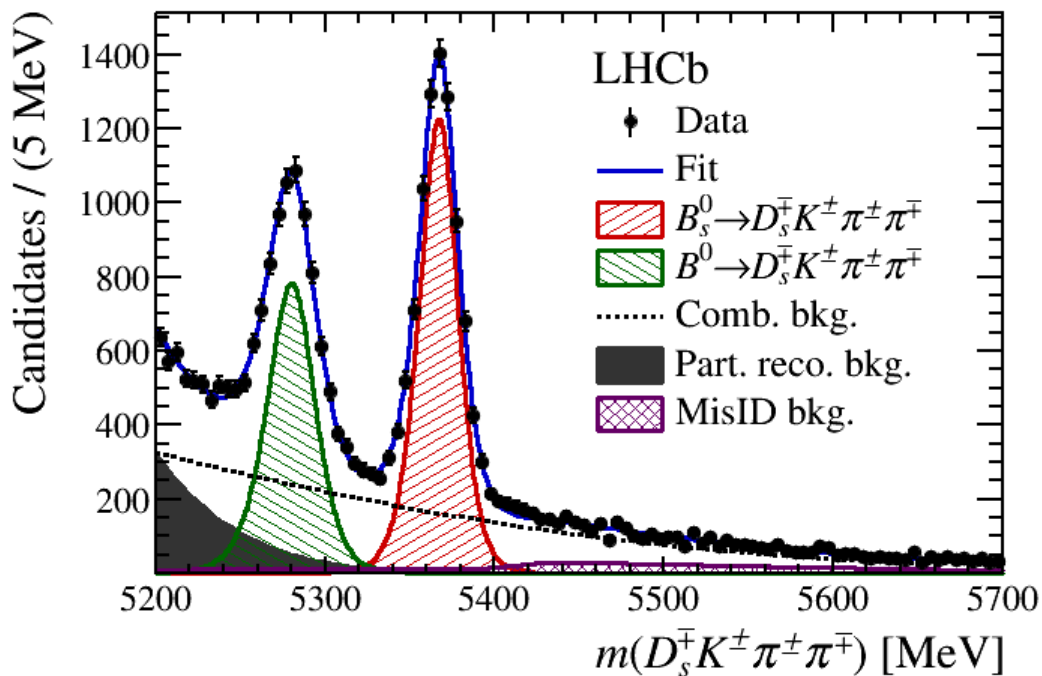
lattice QCD + sum rule: $18.4^{+0.7}_{-1.2}$ ps⁻¹

JHEP 12 (2019) 009

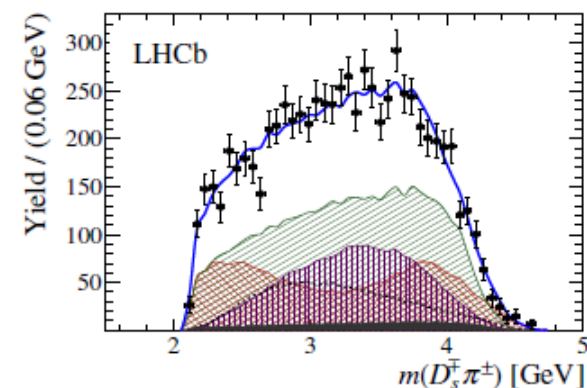
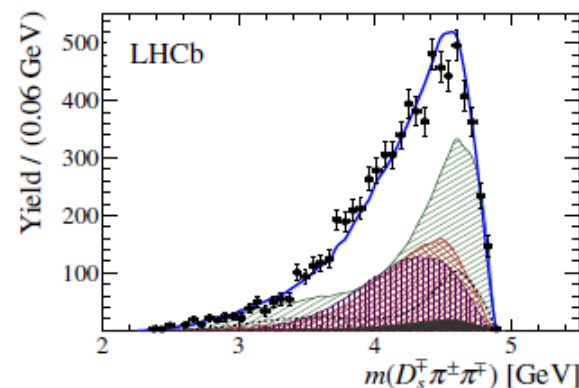
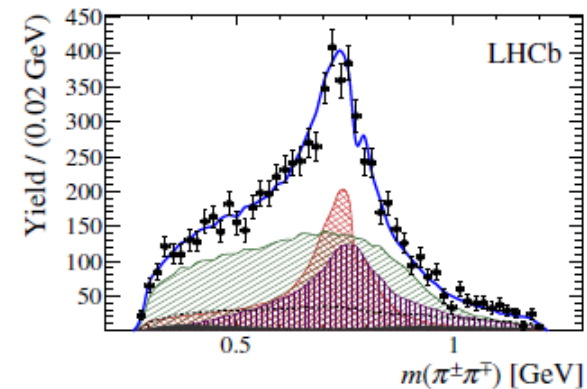
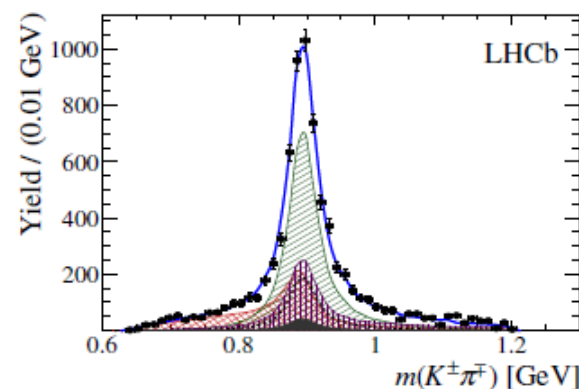
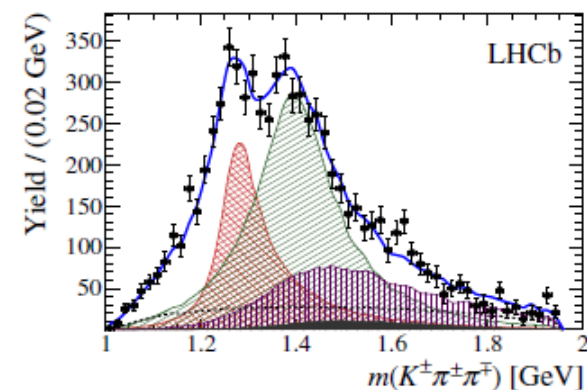
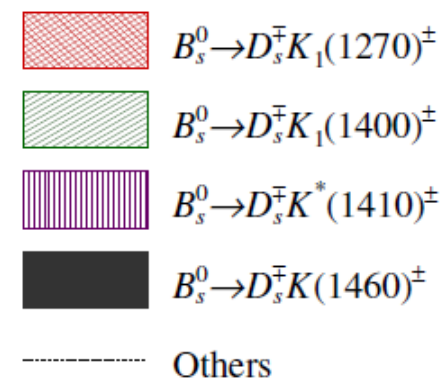
- interesting measurement on its own
- key input for many LHCb analyses: γ in primis

recent LHCb results on CMK γ : a TD measurement

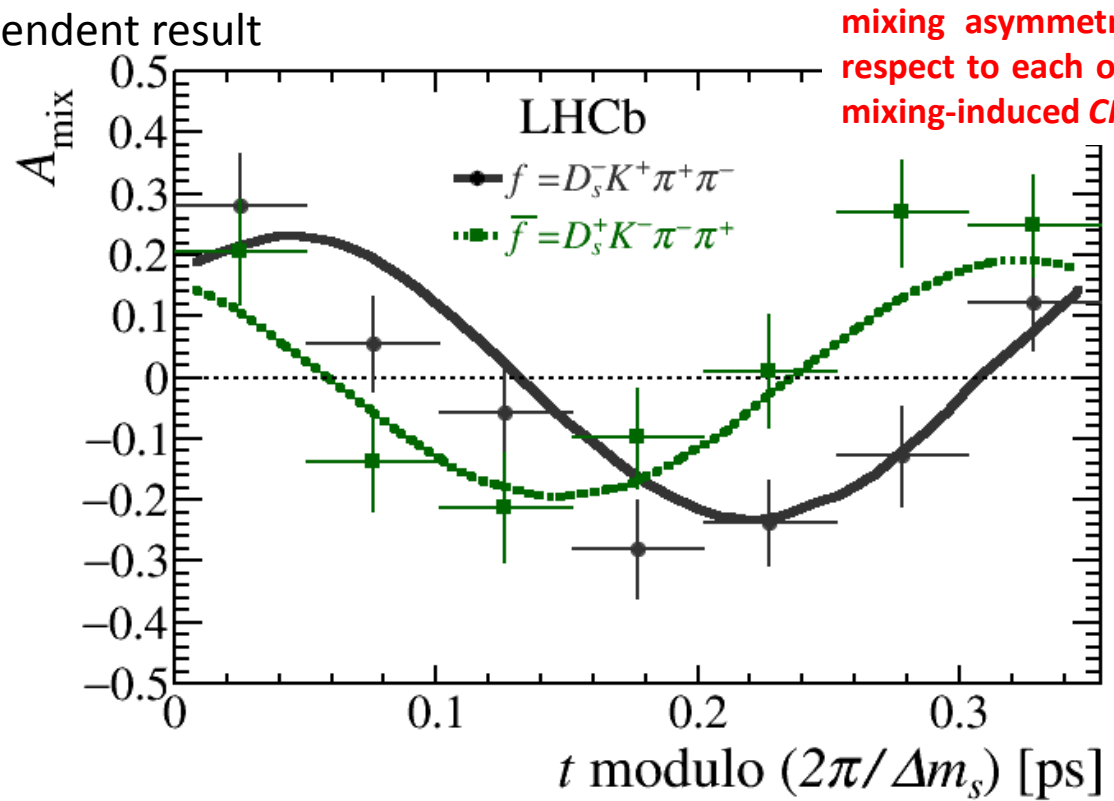
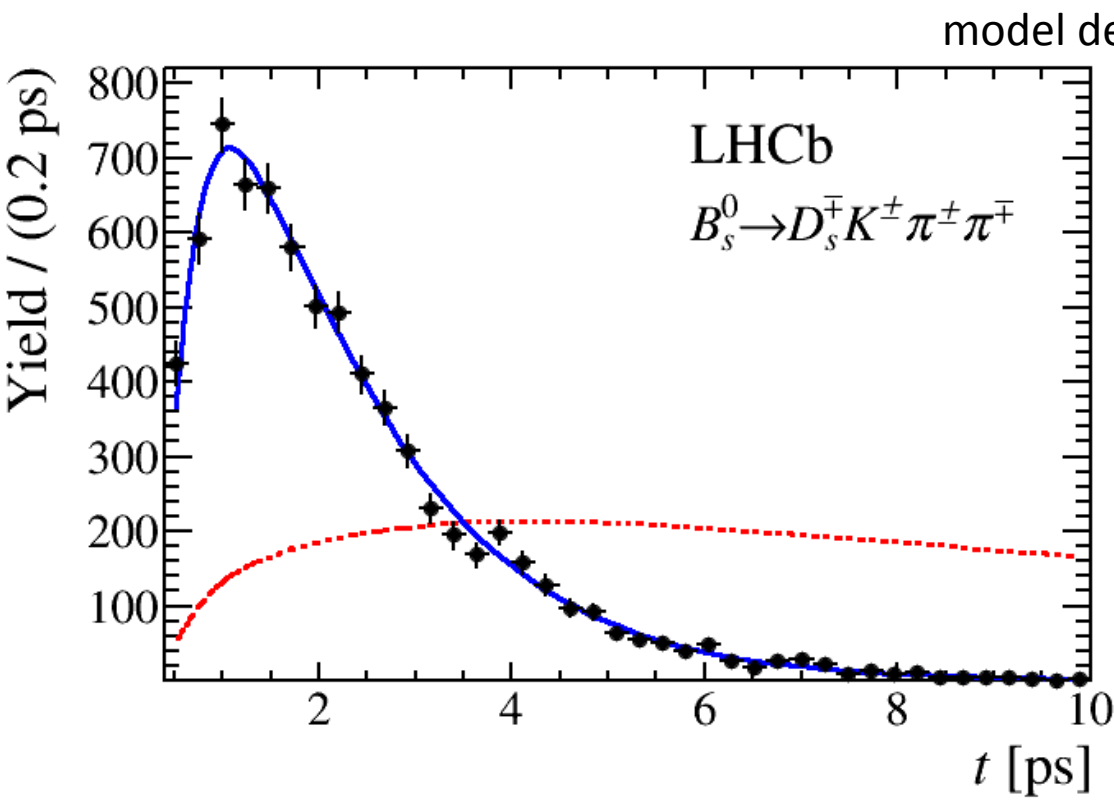
- replace a π with a K in the previous analysis
- CPV due to interference between mixing and decay to the same final state
- several contributing final states: amplitude analysis
- full Run 1+2 statistic (9 /fb)



- model-dependent approach:
describe resonance contributions, 4, with an amplitude model
- model-independent approach:
integrate over phase-space space



recent LHCb results on CMK γ : a TD measurement (cont.)



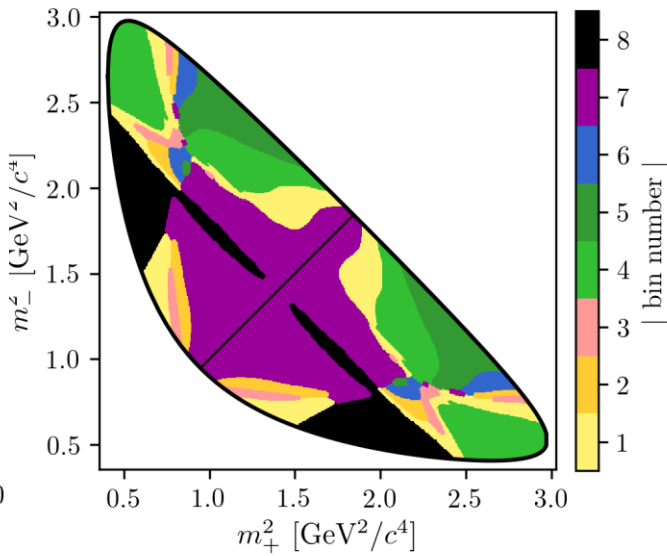
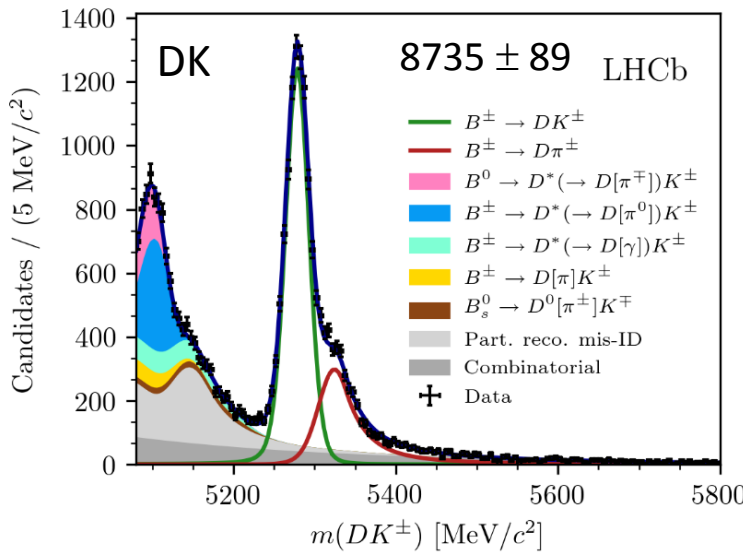
mixing asymmetries shifted with respect to each other is indicating mixing-induced CP violation

Parameter	Model-independent	Model-dependent
r	$0.47^{+0.08}_{-0.08}^{+0.02}_{-0.03}$	$0.56 \pm 0.05 \pm 0.04 \pm 0.07$
κ	$0.88^{+0.12}_{-0.19}^{+0.04}_{-0.07}$	$0.72 \pm 0.04 \pm 0.06 \pm 0.04$
δ [°]	$-6^{+10}_{-12}^{+2}_{-4}$	$-14 \pm 10 \pm 4 \pm 5$
$\gamma - 2\beta_s$ [°]	$42^{+19}_{-13}^{+6}_{-2}$	$42 \pm 10 \pm 4 \pm 5$

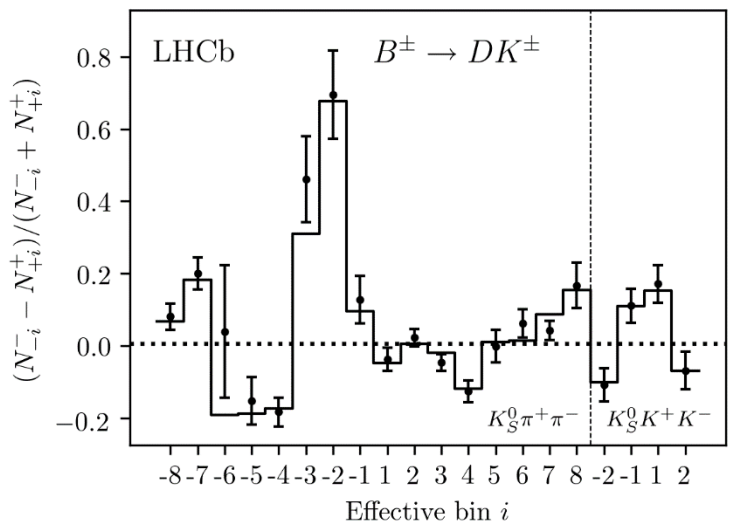
- alternative amplitude models considered
- ratio of the decay amplitudes to the same final state
 - coherence factor, fitted (computed) in M-i (M-d)
 - strong phase difference
 - weak phase difference

LHCb CMK γ : time integrated measurements

$$B^- \rightarrow D^0(\rightarrow K_S^0 h^+ h^-) K^- \propto V_{cb}$$
$$B^- \rightarrow \bar{D}^0(\rightarrow K_S^0 h^+ h^-) K^- \propto V_{ub}$$
$$m_{\pm}^2 = m(K_S^0, h^{\pm})$$



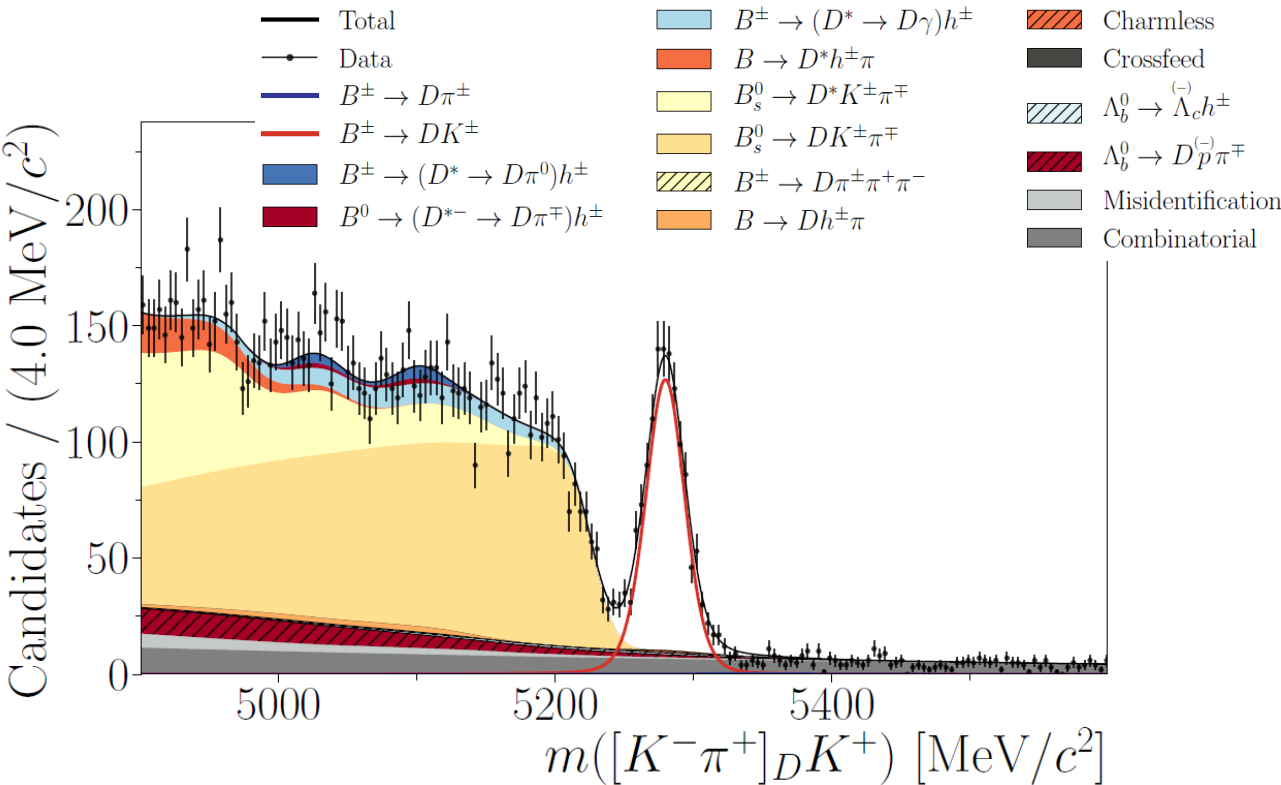
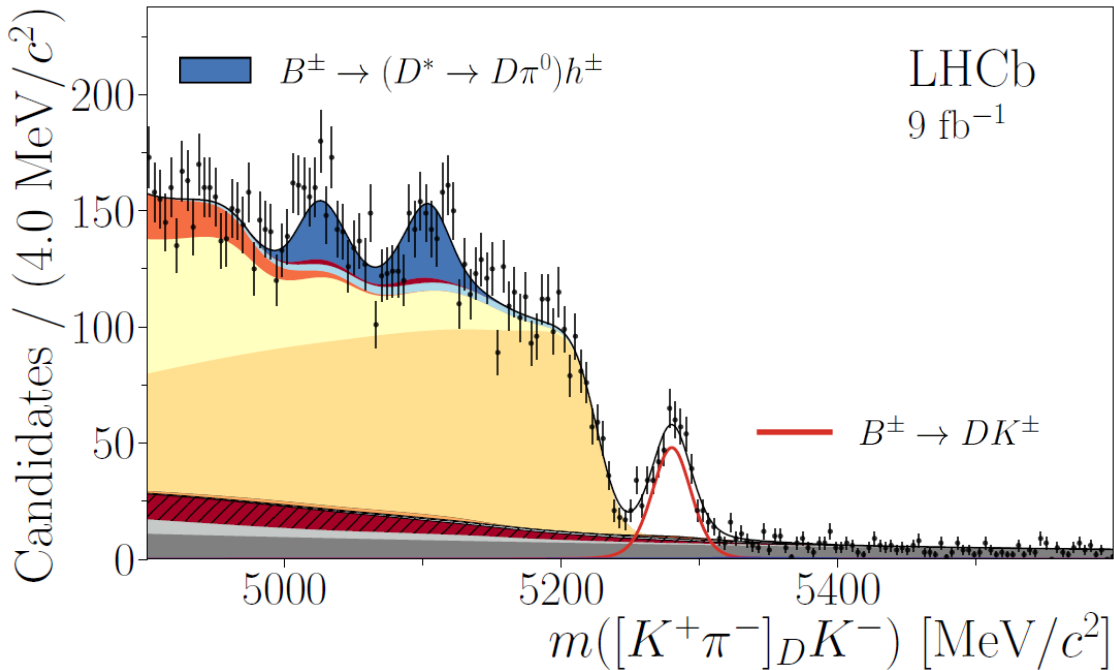
- full Run 1+2 statistic (9 /fb)
- external input: strong-phase difference between the D decay amplitudes at any given point of the Dalitz plot from CLEO and BESIII combined data
- CPV parameters from the distribution of events in the Dalitz plot: very large asymmetries in bins population
- most precise γ measurement from a single analysis



$$\gamma = (68.7_{-5.1}^{+5.2})^{\circ},$$
$$r_B^{DK^{\pm}} = 0.0904_{-0.0075}^{+0.0077},$$
$$\delta_B^{DK^{\pm}} = (118.3_{-5.6}^{+5.5})^{\circ},$$
$$r_B^{D\pi^{\pm}} = 0.0050 \pm 0.0017,$$
$$\delta_B^{D\pi^{\pm}} = (291_{-26}^{+24})^{\circ}.$$

LHCb CMK γ : time integrated measurements

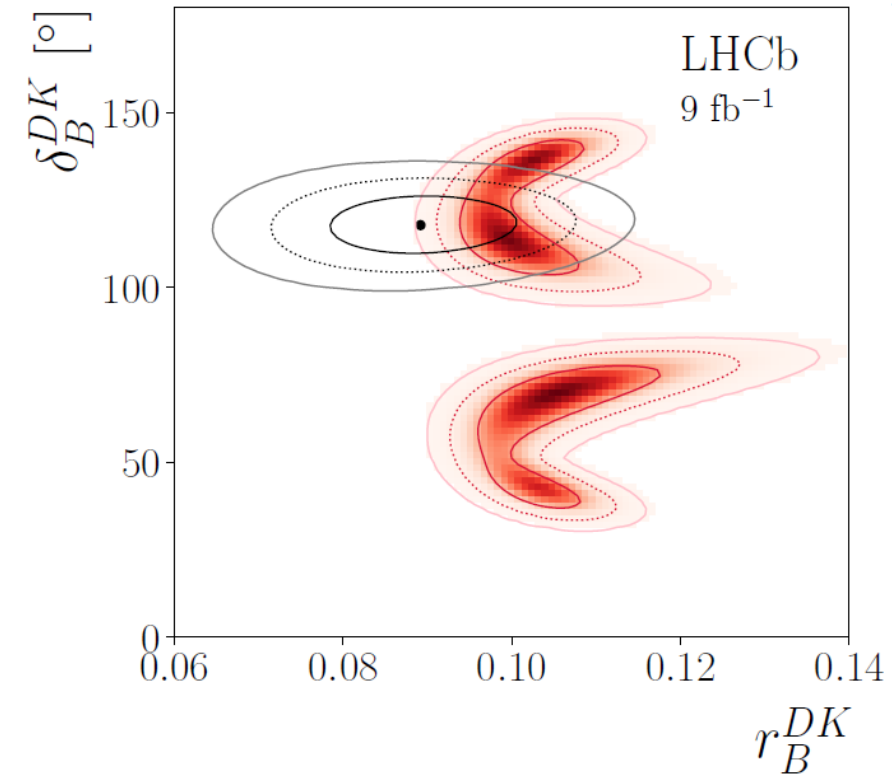
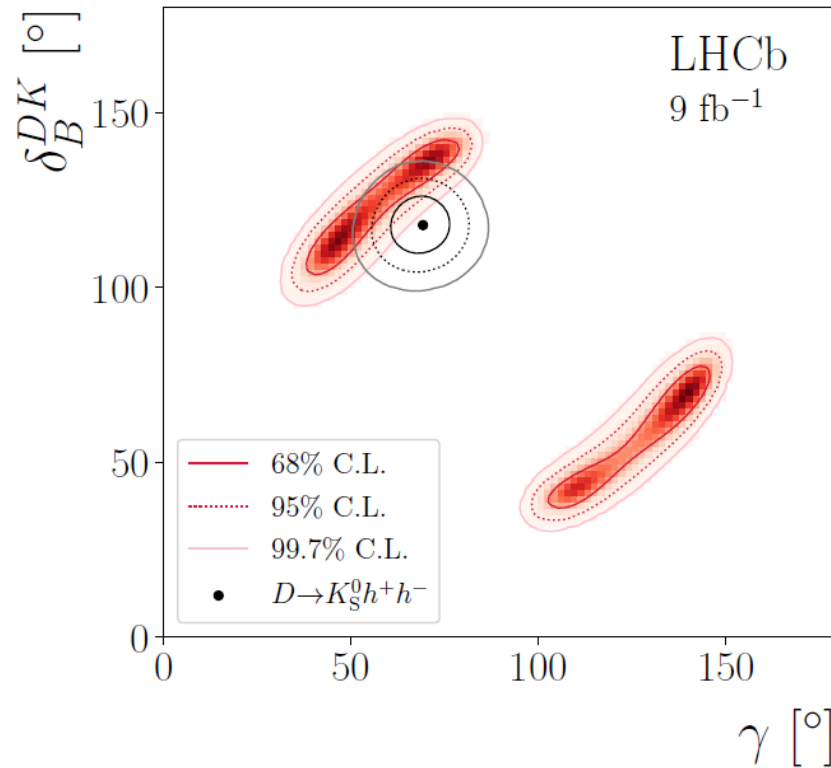
$$B^- \rightarrow D^0(\rightarrow K^+\pi^-)K^- \propto V_{cb}$$
$$B^- \rightarrow \bar{D}^0(\rightarrow K^+\pi^-)K^- \propto V_{ub}$$



- full Run 1+2 statistic (9 /fb)
- partial decay rates are related to the underlying physical parameters
- spectacular differences in peaks height
- 9 CP observables related to fully reconstructed decays
- 19 CP observables related to partially reconstructed decays (missing neutral particle)

Observable	Definition
A_K^{CP}	$\frac{\Gamma(B^- \rightarrow [h^+h^-]_D K^-) - \Gamma(B^+ \rightarrow [h^+h^-]_D K^+)}{\Gamma(B^- \rightarrow [h^+h^-]_D K^-) + \Gamma(B^+ \rightarrow [h^+h^-]_D K^+)}$
A_π^{CP}	$\frac{\Gamma(B^- \rightarrow [h^+h^-]_D \pi^-) - \Gamma(B^+ \rightarrow [h^+h^-]_D \pi^+)}{\Gamma(B^- \rightarrow [h^+h^-]_D \pi^-) + \Gamma(B^+ \rightarrow [h^+h^-]_D \pi^+)}$
$A_K^{K\pi}$	$\frac{\Gamma(B^- \rightarrow [K^-\pi^+]_D K^-) - \Gamma(B^+ \rightarrow [K^+\pi^-]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+]_D K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-]_D K^+)}$

LHCb CMK γ : time integrated measurements



- measurements on partially reconstructed decays are the first of their kind
- all CP observables are measured with world-best precision
- two-fold ambiguity solved combining results with the previous analysis

$$\gamma = (68.7_{-5.1}^{+5.2})^\circ \rightarrow (61.8 \pm 4.0)^\circ$$

LHCb input data:

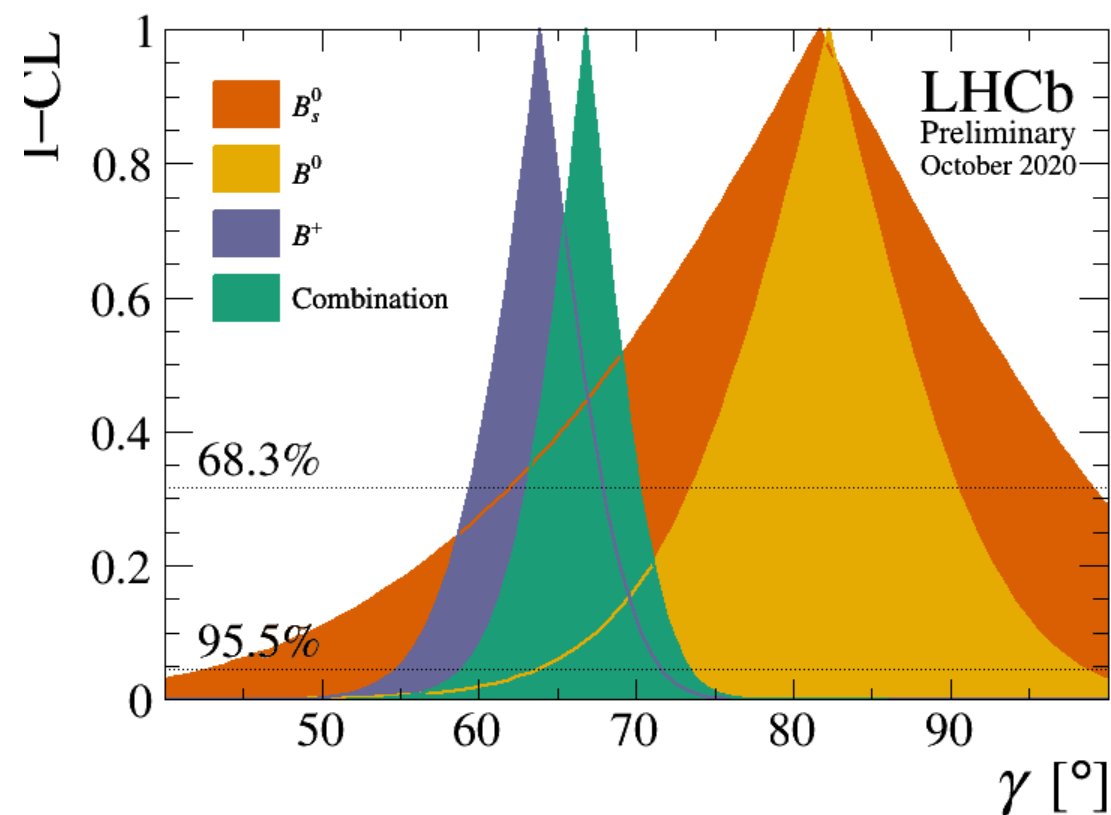
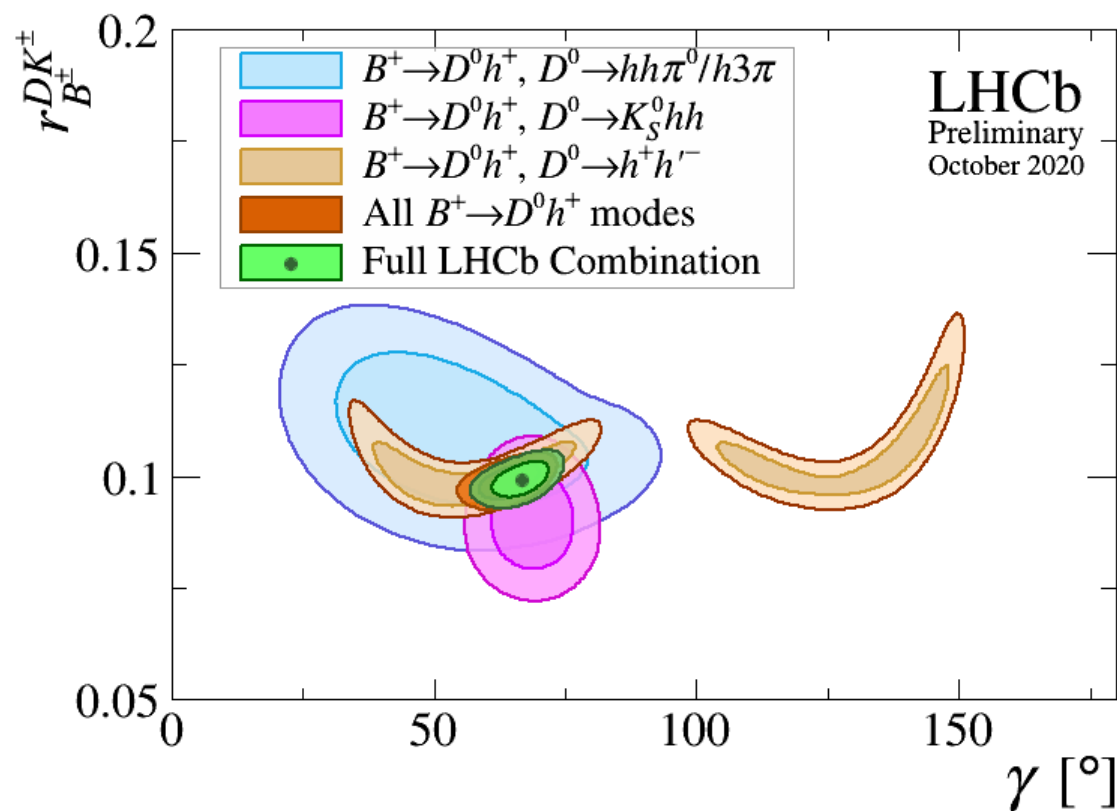
B decay	D decay	Method	Ref.	Dataset	Status since Ref. [3]
$B^+ \rightarrow Dh^+$	$D \rightarrow h^+h^-$	GLW/ADS	[16]	Run 1+2	Updated
$B^+ \rightarrow Dh^+$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	GLW/ADS	[24]	Run 1	As before
$B^+ \rightarrow Dh^+$	$D \rightarrow h^+h^-\pi^0$	GLW/ADS	[25]	Run 1	As before
$B^+ \rightarrow Dh^+$	$D \rightarrow K_S^0 h^+h^-$	BPGGSZ	[17]	Run 1+2	Updated
$B^+ \rightarrow Dh^+$	$D \rightarrow K_S^0 K^\pm \pi^\mp$	GLS	[20]	Run 1+2	Updated
$B^+ \rightarrow D^* h^+$	$D \rightarrow h^+h^-$	GLW/ADS	[16]	Run 1+2	Updated
$B^+ \rightarrow DK^{*+}$	$D \rightarrow h^+h^-$	GLW/ADS	[26]	Run 1+2(*)	As before
$B^+ \rightarrow DK^{*+}$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	GLW/ADS	[26]	Run 1+2(*)	As before
$B^+ \rightarrow DK^+\pi^+\pi^-$	$D \rightarrow h^+h^-$	GLW/ADS	[27]	Run 1	As before
$B^0 \rightarrow DK^{*0}$	$D \rightarrow h^+h^-$	GLW/ADS	[21]	Run 1+2(*)	Updated
$B^0 \rightarrow DK^{*0}$	$D \rightarrow h^+\pi^-\pi^+\pi^-$	GLW/ADS	[21]	Run 1+2(*)	New
$B^0 \rightarrow DK^+\pi^-$	$D \rightarrow h^+h^-$	GLW-Dalitz	[22]	Run 1	Superseded
$B^0 \rightarrow DK^{*0}$	$D \rightarrow K_S^0 \pi^+\pi^-$	BPGGSZ	[28]	Run 1	As before
$B^0 \rightarrow D^\mp \pi^\pm$	$D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$	TD	[29]	Run 1	As before
$B_s^0 \rightarrow D_s^\mp K^\pm$	$D_s^\pm \rightarrow h^\pm h^\mp \pi^\pm$	TD	[30]	Run 1	As before
$B_s^0 \rightarrow D_s^\mp K^\pm \pi^\pm \pi^\mp$	$D_s^\pm \rightarrow h^\pm h^\mp \pi^\pm$	TD	[23]	Run 1+2	New

auxiliary inputs:

Decay	Parameters	Source	Ref.
$D^0\text{--}\bar{D}^0$ -mixing	x_D, y_D	HLFAV	[29]
$D \rightarrow K^+\pi^-$	$r_D^{K\pi}, \delta_D^{K\pi}$	HLFAV	[29]
$D \rightarrow h^+h^-$	$A_{CP}^{\text{dir}}(KK), A_{CP}^{\text{dir}}(\pi\pi)$	HLFAV	[29]
$D \rightarrow K^\pm \pi^\mp \pi^+\pi^-$	$\delta_D^{K3\pi}, \kappa_D^{K3\pi}, r_D^{K3\pi}$	CLEO+LHCb	[30]
$D \rightarrow \pi^+\pi^-\pi^+\pi^-$	$F_{\pi\pi\pi\pi}$	CLEO	[31]
$D \rightarrow K\pi\pi^0$	$\delta_D^{K2\pi}, \kappa_D^{K2\pi}, r_D^{K2\pi}$	CLEO+LHCb	[30]
$D \rightarrow h^+h^-\pi^0$	$F_{\pi\pi\pi^0}, F_{KK\pi^0}$	CLEO	[31]
$D \rightarrow K_S^0 K^+\pi^-$	$\delta_D^{K_S K\pi}, \kappa_D^{K_S K\pi}, r_D^{K_S K\pi}$	CLEO	[32]
$D \rightarrow K_S^0 K^+\pi^-$	$r_D^{K_S K\pi}$	LHCb	[33]
$B^0 \rightarrow DK^{*0}$	$\kappa_B^{DK^{*0}}, \bar{R}_B^{DK^{*0}}, \bar{\Delta}_B^{DK^{*0}}$	LHCb	[23]
$B^+ \rightarrow DK^{*+}$	$\kappa_B^{DK^{*+}}$	LHCb	[20]
$B_s^0 \rightarrow D_s^\mp K^\pm$	ϕ_s	HFLAV	[29]
$B^0 \rightarrow D^\mp \pi^\pm$	β	HFLAV	[29]
$B^0 \rightarrow D^\mp \pi^\pm$	$r_B^{D^\mp \pi^\pm}$	See text	[26]

- whenever possible from experiment
- often from LHCb

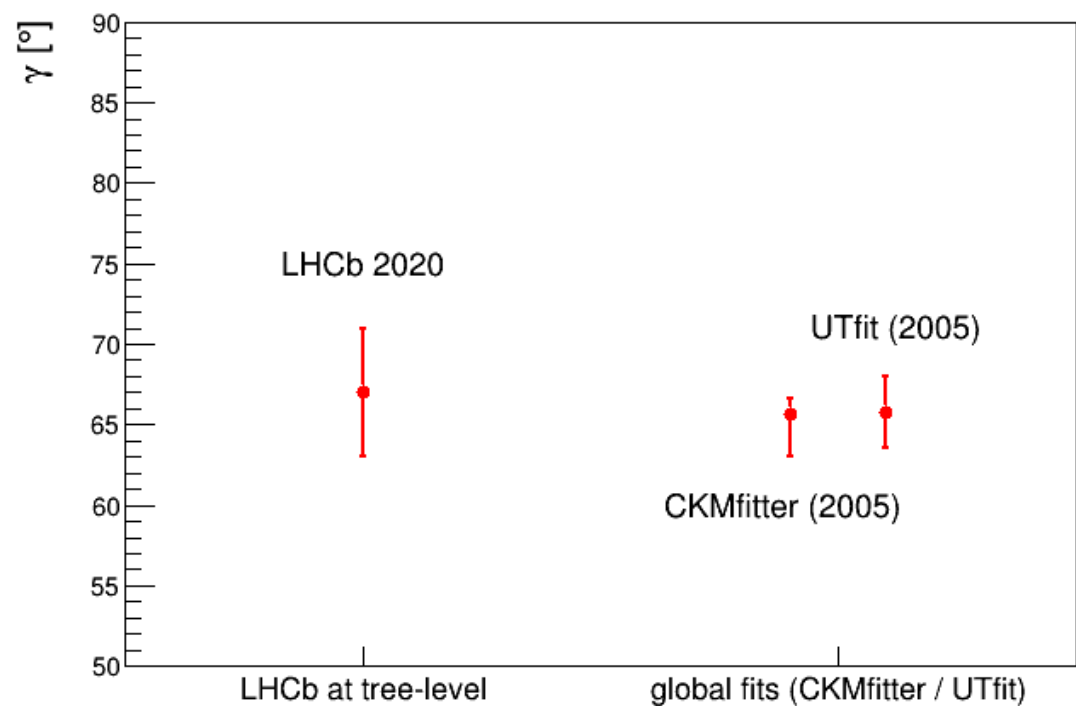
LHCb CMK γ combination



- complementarity between the different results
- most of the sensitivity from B^\pm
- LHCb unique sensitivity from B_s^0

$$\gamma = (67 \pm 4)^\circ$$

CMK γ : from trees vs global fits

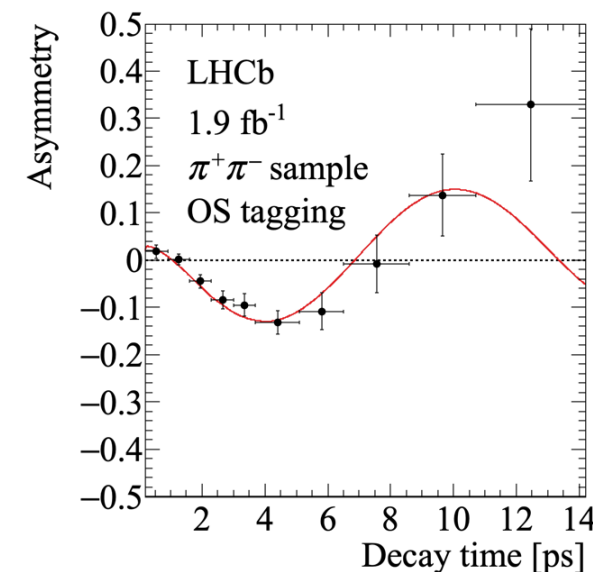
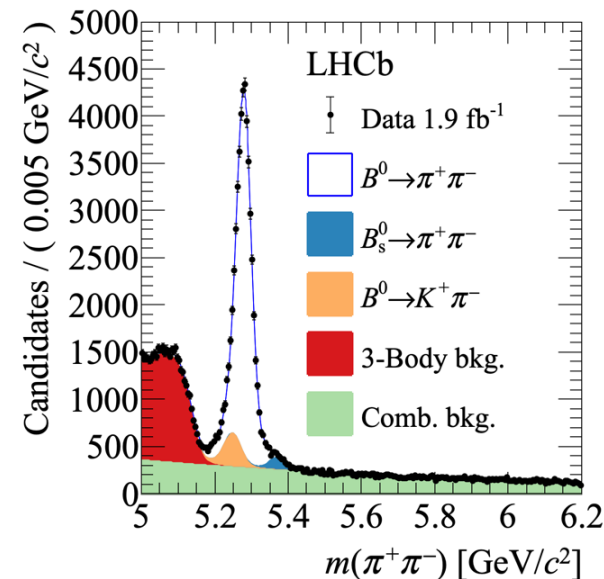
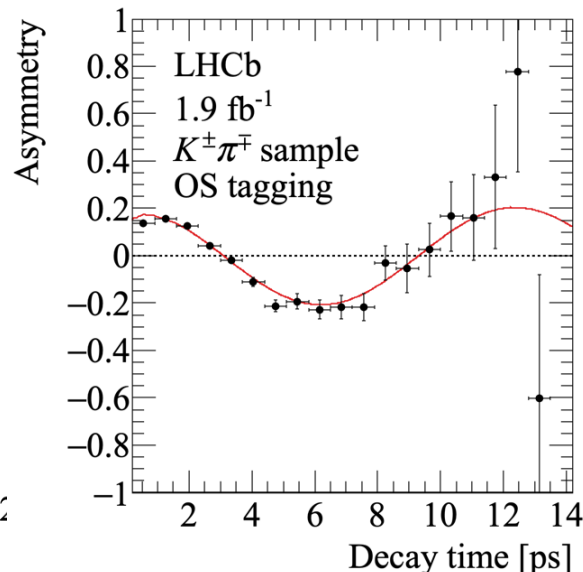
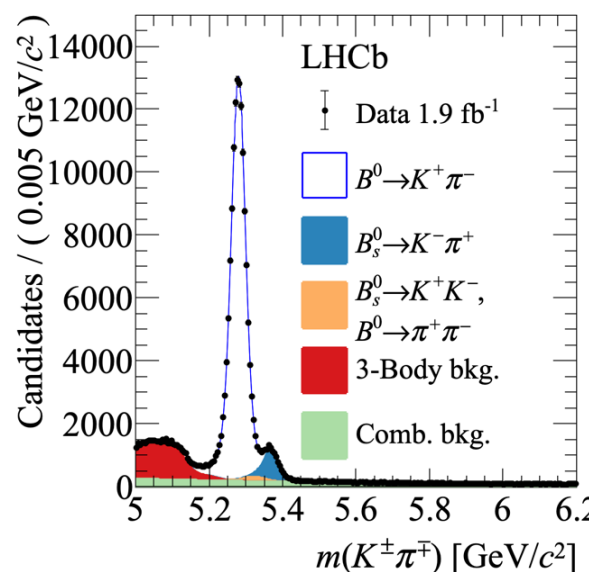


- **any disagreement between tree-level determinations and the value inferred from global CKM fits would indicate physics beyond the SM** due for example to new particles / mediators being exchanged in loops
- LHCb is nicely closing the sensitivity gap between direct measurements and global fits
- much more to come from LHCb:
 - extend already used channels to full Run 1 + 2 data
 - add new channels
- no show stopper on γ accuracy from the experimental side (importance of BESIII data)
- outstanding experimental task but LHCb will reach accuracies < 1 deg. already with 50 /fb

CPV from 2-body neutral B meson (Bd/Bs) decays

CPV from the interference between decay (to the same final state) and mixing

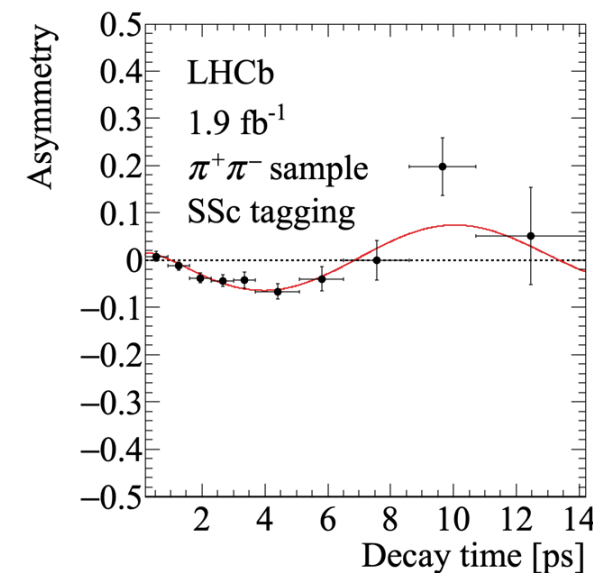
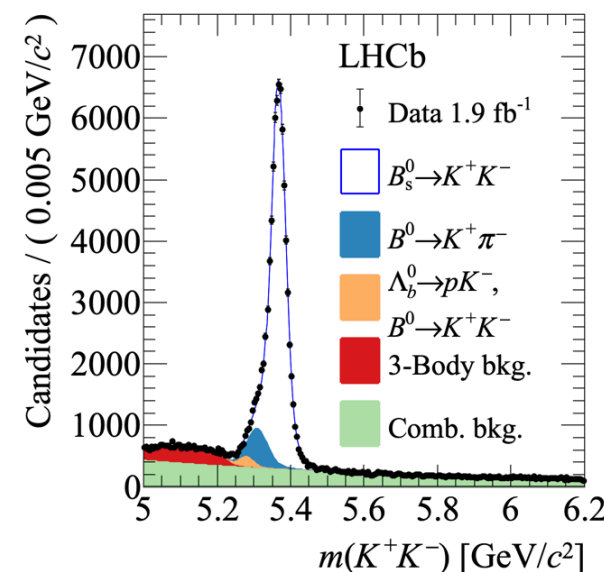
$$A_{CP}(t) = \frac{\Gamma_{\bar{B}_{(s)}^0 \rightarrow f}(t) - \Gamma_{B_{(s)}^0 \rightarrow f}(t)}{\Gamma_{\bar{B}_{(s)}^0 \rightarrow f}(t) + \Gamma_{B_{(s)}^0 \rightarrow f}(t)} = \frac{-C_f \cos(\Delta m_{d(s)} t) + S_f \sin(\Delta m_{d(s)} t)}{\cosh\left(\frac{\Delta\Gamma_{d(s)}}{2} t\right) + A_f^{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma_{d(s)}}{2} t\right)}$$



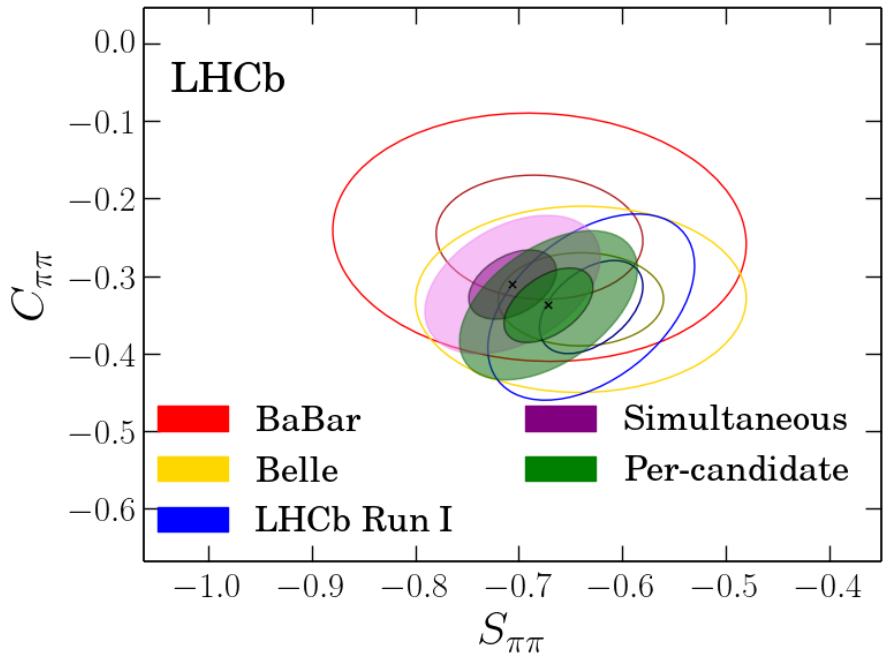
- 1.9 /fb (2015-2016)
- simultaneous fit to invariant mass, decay time, tagging decision, mistag probabilities distributions for Kπ ππ and KK samples

$$A_{CP} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2}$$

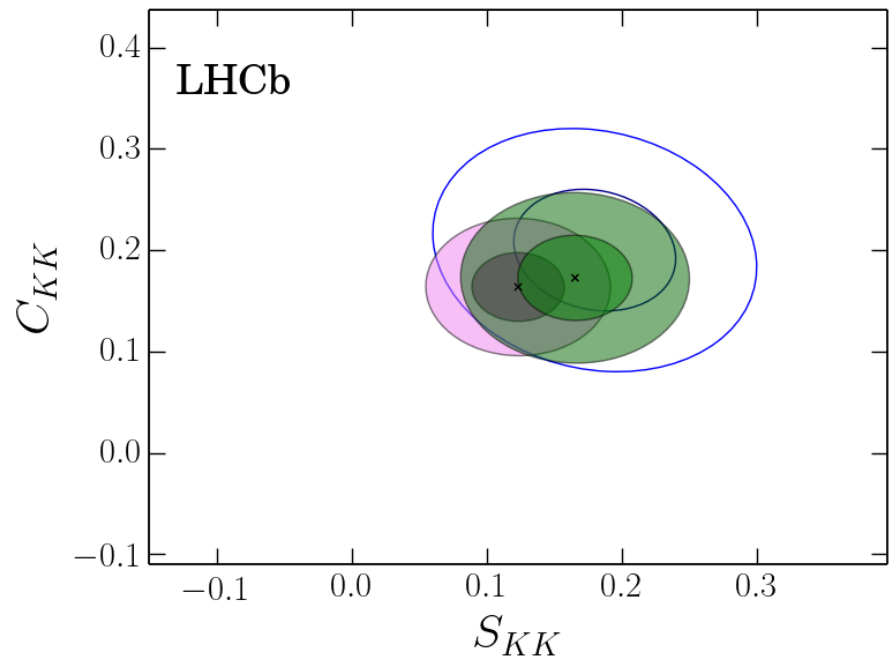
time-integrated
CP asymmetry



CPV from 2-body neutral B meson (Bd/Bs) decays (cont.)

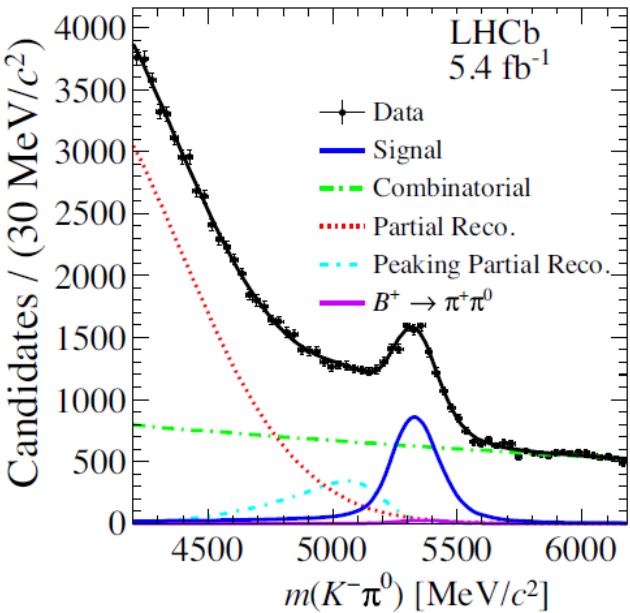
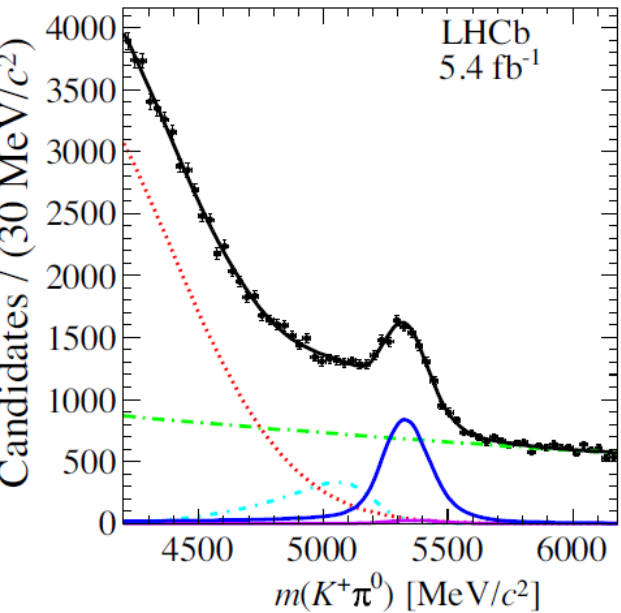


$$\begin{aligned} C_{\pi\pi} &= -0.311 \pm 0.045, \\ S_{\pi\pi} &= -0.706 \pm 0.042, \\ A_{CP}^{B^0} &= -0.0824 \pm 0.0033, \\ A_{CP}^{B_s^0} &= 0.236 \pm 0.013, \\ C_{KK} &= 0.164 \pm 0.034, \\ S_{KK} &= 0.123 \pm 0.034, \\ \mathcal{A}_{KK}^{\Delta\Gamma} &= -0.833 \pm 0.054, \end{aligned}$$



- per-candidate is an alternative analysis technique applied to the same data set, results are consistent
 - new results are in agreement with previous LHCb Run 1 measurements
 - good agreement with BaBar / Belle (where applicable)
 - most precise results from a single experiment to date
 - the KK mixing parameters are differing from 0 0 -1 by 6.5 standard deviations
- ⇒ first observation of time-dependent CP violation in decays of the B_s^0 meson

Parameter	Value
Δm_d	$0.5065 \pm 0.0019 \text{ ps}^{-1}$
Γ_d	$0.6579 \pm 0.0017 \text{ ps}^{-1}$
$\Delta\Gamma_d$	0 ps^{-1}
Δm_s	$17.757 \pm 0.021 \text{ ps}^{-1}$
Γ_s	$0.6562 \pm 0.0021 \text{ ps}^{-1}$
$\Delta\Gamma_s$	$0.082 \pm 0.005 \text{ ps}^{-1}$
$\rho(\Gamma_s, \Delta\Gamma_s)$	-0.170



$$B^+ \rightarrow K^+ \pi^0$$

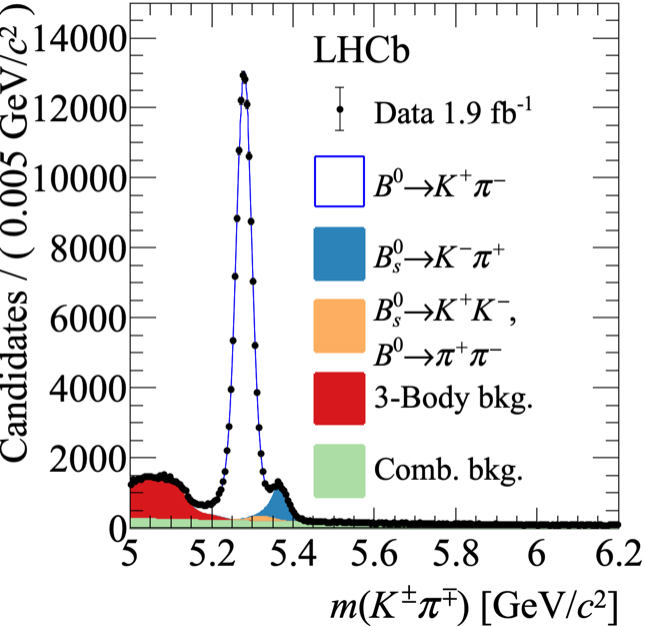
$$A_{CP} = \frac{\Gamma(B^- \rightarrow K^- \pi^0) - \Gamma(B^+ \rightarrow K^+ \pi^0)}{\Gamma(B^- \rightarrow K^- \pi^0) + \Gamma(B^+ \rightarrow K^+ \pi^0)}$$

- 5.4 /fb
- first analysis of a one-track decay at a hadron collider

$$A_{CP}(B^+ \rightarrow K^+ \pi^0) = 0.025 \pm 0.015 \pm 0.006 \pm 0.003$$

stat., syst., ext. inputs

- exceeding the precision of the current world average (HFLAV)



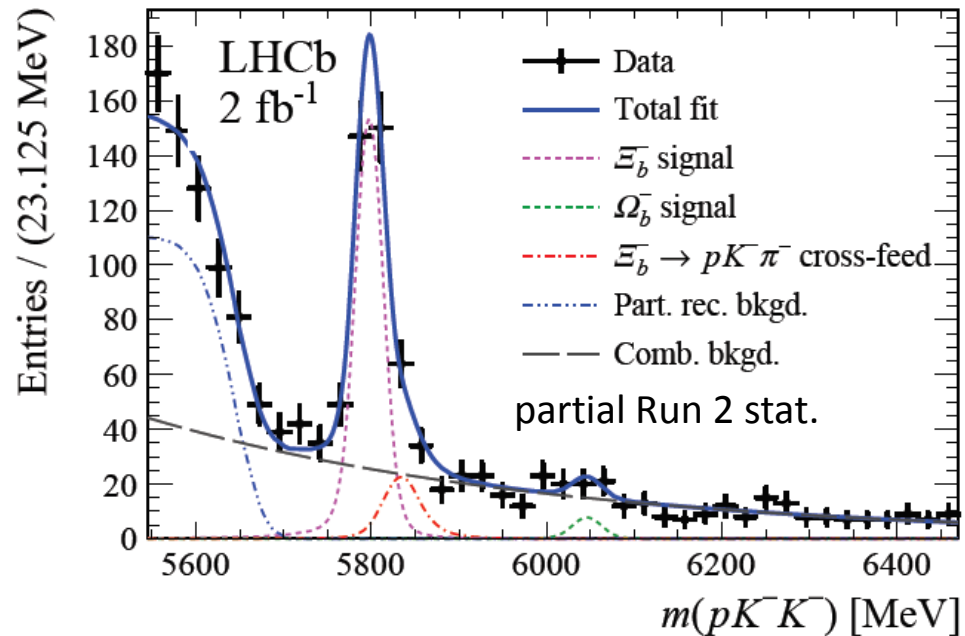
$$A_{CP}^{B^0} = -0.0831 \pm 0.0034$$

- average of Phys. Rev. D 98 (2018) 032004 (3 /fb, Run 1) and JHEP 03 (2021) 075 (1.9 /fb, 2016-2016)

$$\Delta A_{CP}(K\pi) \equiv A_{CP}(B^+ \rightarrow K^+ \pi^0) - A_{CP}(B^0 \rightarrow K^+ \pi^-)$$

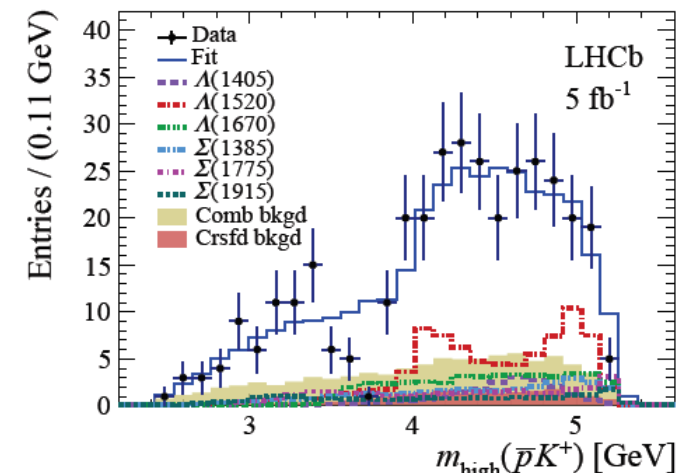
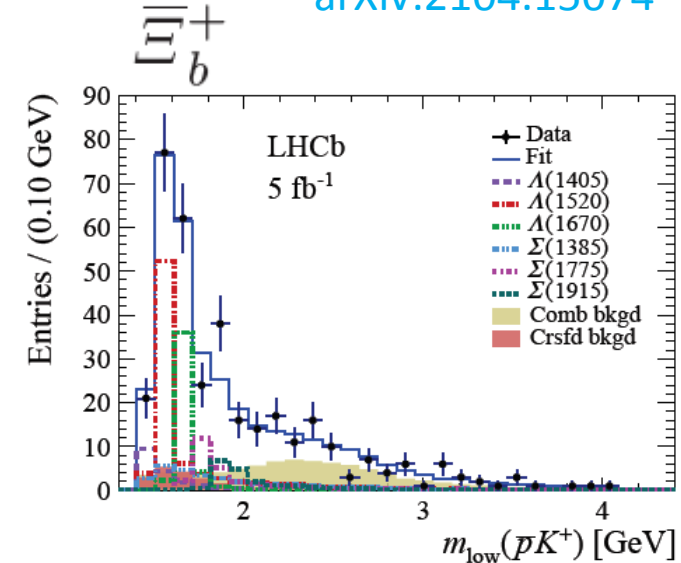
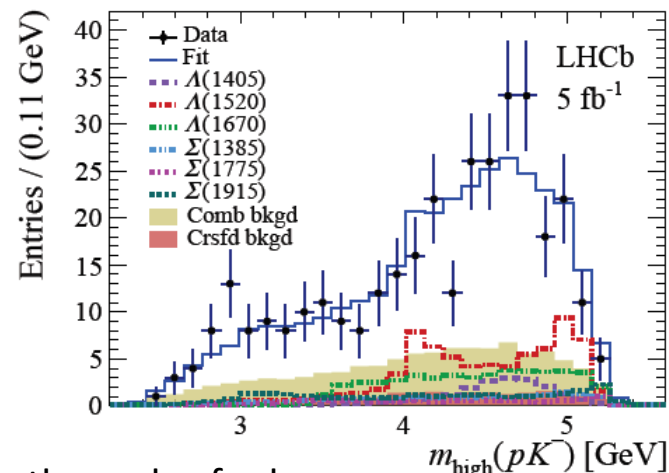
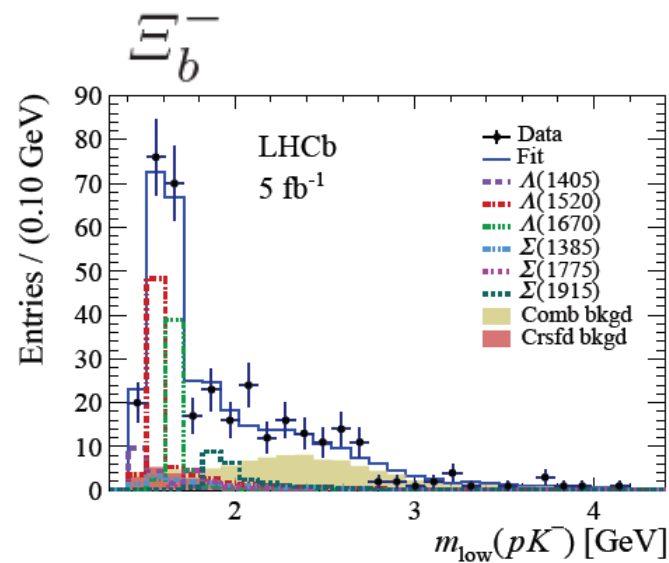
- is nonzero with a significance of more than 8 standard deviations (using the HFLAV averages updated with the above LHCb results) \Rightarrow isospin symmetry breaking
- accuracy substantially enhanced wrt previous measurement

B → K π puzzle



$$A_i^{CP} = \frac{\int_{\Omega} (d\Gamma_i^+ / d\Omega - d\Gamma_i^- / d\Omega) d\Omega}{\int_{\Omega} (d\Gamma_i^+ / d\Omega + d\Gamma_i^- / d\Omega) d\Omega}$$

Component	$A^{CP} (10^{-2})$
$\Sigma(1385)$	$-27 \pm 34 \text{ (stat)} \pm 73 \text{ (syst)}$
$\Lambda(1405)$	$-1 \pm 24 \text{ (stat)} \pm 32 \text{ (syst)}$
$\Lambda(1520)$	$-5 \pm 9 \text{ (stat)} \pm 8 \text{ (syst)}$
$\Lambda(1670)$	$3 \pm 14 \text{ (stat)} \pm 10 \text{ (syst)}$
$\Sigma(1775)$	$-47 \pm 26 \text{ (stat)} \pm 14 \text{ (syst)}$
$\Sigma(1915)$	$11 \pm 26 \text{ (stat)} \pm 22 \text{ (syst)}$



- CPV should be there also for baryons
- Run 1 + partial Run 2 statistic (5 /fb overall)
- amplitude analysis: several, 6, contributing resonances
- CP asymmetry for each
- results consistent with 0, expect a large boost in stat. from Run 3 data

take home message

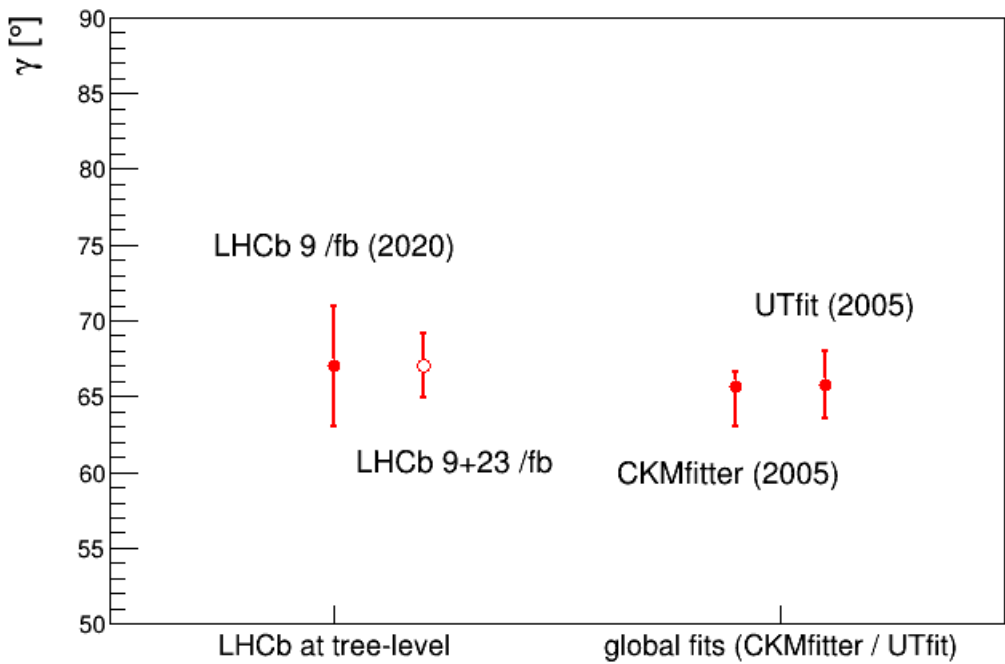
LHCb has performed outstanding measurements in beauty:

- Δm_s
- CKM γ
- CPV in 2-body neutral B meson (Bd/Bs) decays
- hunting CPV in the baryon sector

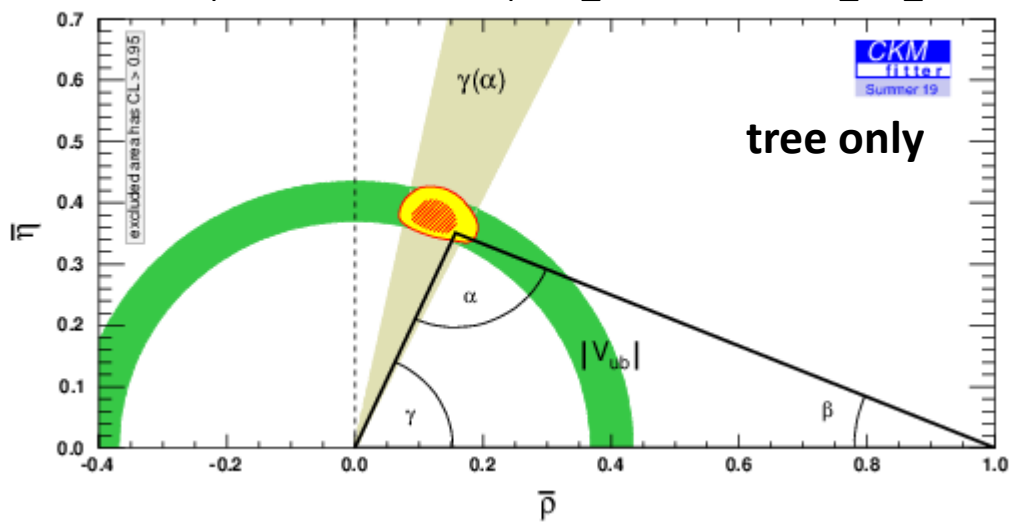
often:

- different and/or complementary decay channels
- different and/or complementary analysis techniques
- word best precision

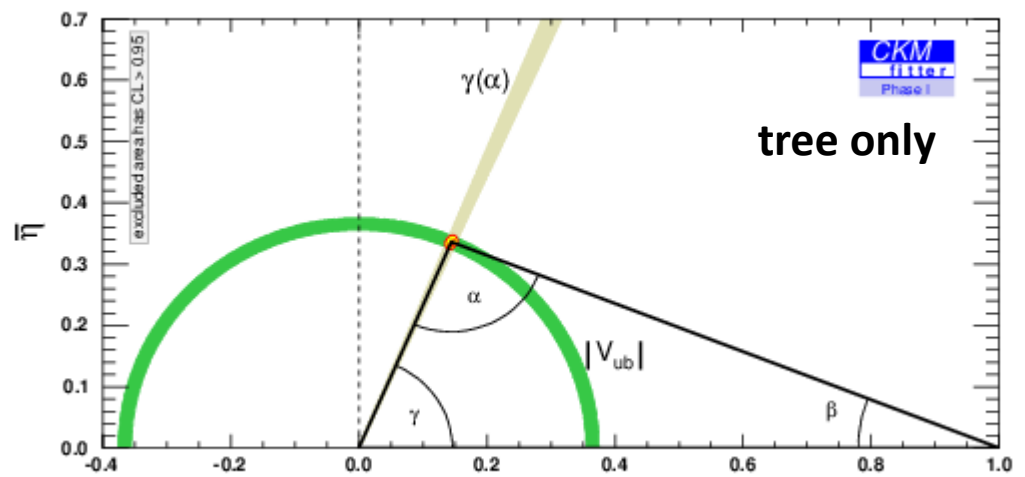
- given the LHC / LHCb upcoming upgrade era this was just an ... **appetizer**
- expected to boost statistics far beyond 9 /fb on a relatively short time scale
- looking forward to contributions from Belle II



http://ckmfitter.in2p3.fr/www/results/plots_summer19/ckm_res_summer19.html



same plot after phase I: LHCb at 23 /fb, CMS/ATLAS at 300 /fb and Belle II at 50 /ab



http://ckmfitter.in2p3.fr/www/studies/plots_hllhc18/phase1/ckm_plots_hllhc18_phase1.html

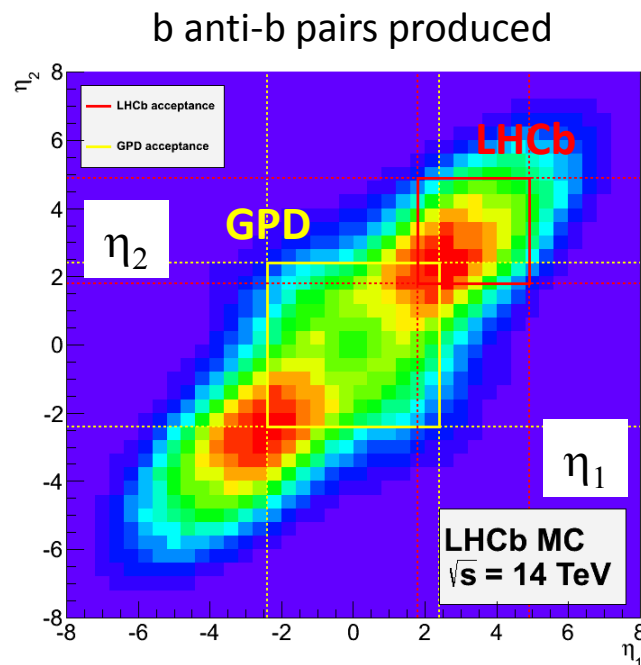
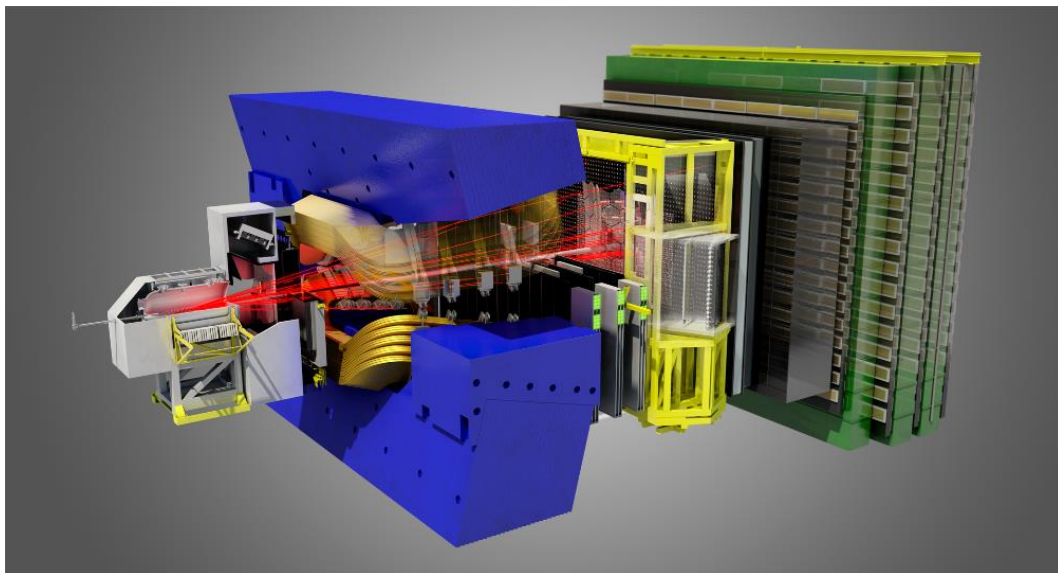
Thank you for your attention !

advertisement: **CPV and semileptonic in b-hadrons**,
plenary session tomorrow , by my colleague Khanji
Basem

backups

LHCb: the detector and its performance so far

single-arm forward spectrometer at the LHC



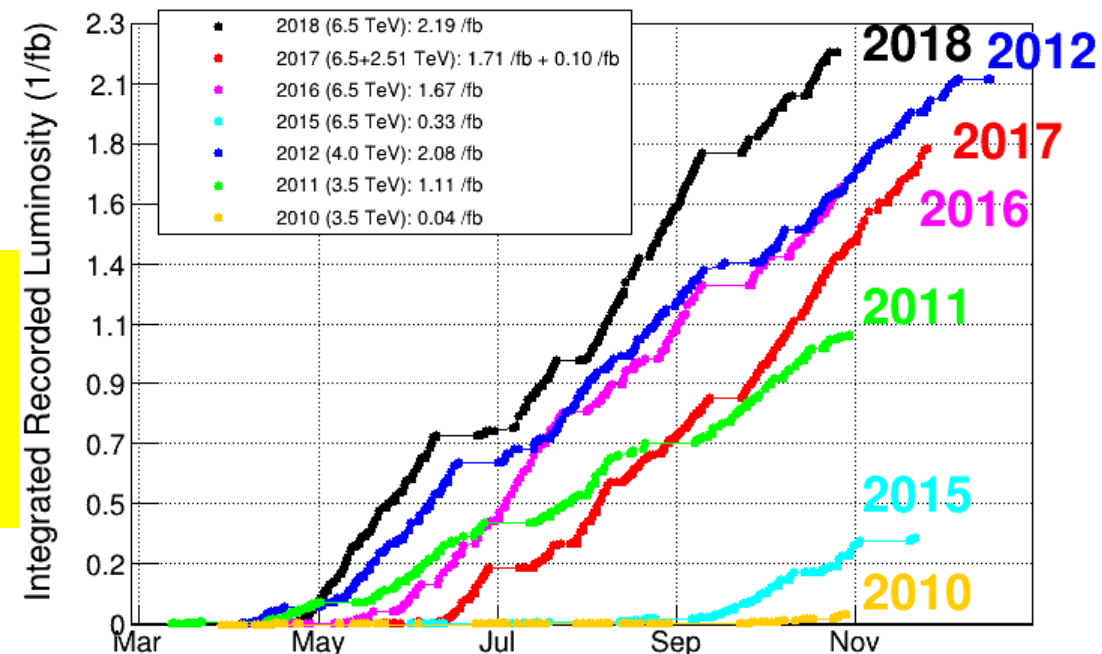
- detector paper: JINST 3 (2008) S08005
- Run 1 performance: Int. J. Mod. Phys. A30 (2015) 1530022
- Run 2 performance: JINST 14 (2019) P04013

optimized for beauty and charm physics at $2 < \eta < 5$

key points:

- momentum resolution
($\sigma(p)/p \approx 0.5\%$ (low momentum) to 1% @ $200\text{ GeV}/c$)
- impact parameter resolution
($\sigma(IP) \approx 15\text{ }\mu\text{m}$ at high p_T)
- primary and secondary vertices reco.
- decay time resolution ($\sigma(t) \approx 50\text{ fs}$)
- 'global' PID: $e / \mu / \pi / K$
($K\text{ id} \approx 95\%$ $\pi\text{ mis-id} \approx 5\%$, $p < 100\text{ GeV}/c$)
- γ and π^0 reconstruction

recorded lumi.:
 2011 \rightarrow 2012 (Run 1): 3.19 /fb
 $\sim 3 \times 10^{11}$ b anti-b pairs prod.
 2015 \rightarrow 2018 (Run 2): 5.9 /fb
 $\sim 2 \times 6 \times 10^{11}$ b anti-b pairs prod.



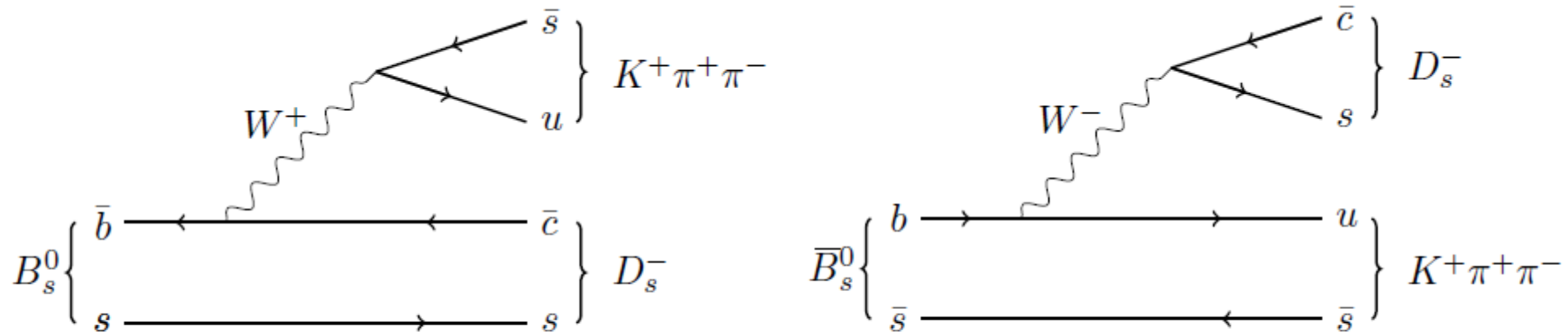


Figure 1. Leading-order Feynman diagrams for (left) B_s^0 and (right) \bar{B}_s^0 decays to the $D_s^- K^+ \pi^+ \pi^-$ final state, where the $\pi^+ \pi^-$ subsystem exemplarily hadronises in conjunction with the kaon.

$$\begin{aligned}
\frac{d\Gamma(B_s^0 \rightarrow f)}{dt} &\propto \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) \right. \\
&\quad \left. + A_f^{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - S_f \sin(\Delta m_s t) \right] e^{-\Gamma_s t}, \\
\frac{d\Gamma(\bar{B}_s^0 \rightarrow f)}{dt} &\propto \left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) \right. \\
&\quad \left. + A_{\bar{f}}^{\Delta\Gamma} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + S_{\bar{f}} \sin(\Delta m_s t) \right] e^{-\Gamma_s t}.
\end{aligned}$$

$$\begin{aligned}
C_f &= \frac{1 - r^2}{1 + r^2}, \\
A_f^{\Delta\Gamma} &= -\frac{2 r \kappa \cos(\delta - (\gamma - 2\beta_s))}{1 + r^2}, & A_{\bar{f}}^{\Delta\Gamma} &= -\frac{2 r \kappa \cos(\delta + (\gamma - 2\beta_s))}{1 + r^2}, \\
S_f &= +\frac{2 r \kappa \sin(\delta - (\gamma - 2\beta_s))}{1 + r^2}, & S_{\bar{f}} &= -\frac{2 r \kappa \sin(\delta + (\gamma - 2\beta_s))}{1 + r^2}.
\end{aligned}$$