CKM Metrology: Theory

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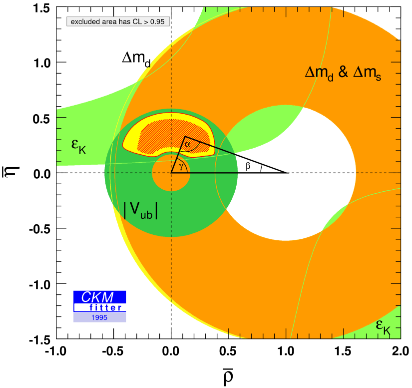
CKM Metrology: A success story

[CKMfitter, http://ckmfitter.in2p3.fr]



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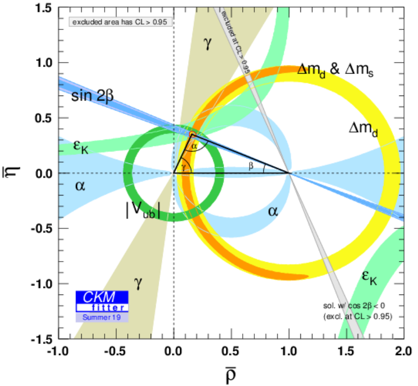
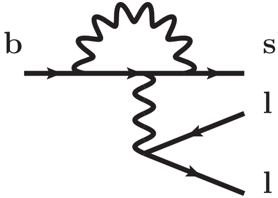


EPS-HEP 2019 in Ghent, Belgium

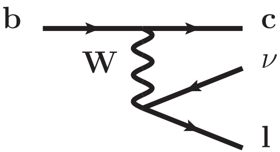
This talk: concentrate on new developments in V_{cb}.

B Anomalies

- There are anomalies in
  - flavor changing neutral current (FCNC) decays $b \to s l^+ l^-$
  - charged current decays $b \to c\tau\nu$.



Loop-level



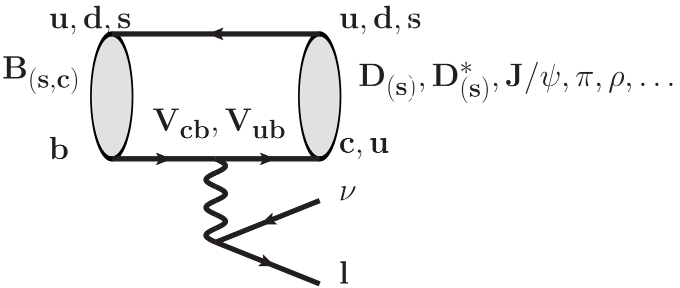
Tree-level

Actually, there are two charged current anomalies

- Exclusive/inclusive V_{cb}, V_{ub} and
- Lepton flavor universality violation.

What makes life harder

- Bound states of QCD introduce hadronic uncertainties.



- The good thing:
  Decay modes w/ same underlying quark transition
  $\Rightarrow$ Cross-checks.

Importance of $|V_{cb}|$

- V_{cb} is a fundamental parameter of the Standard Model.
- V_{cb} plays an important role in the Unitarity Triangle.
  ➥We want to overconstrain the triangle as a new physics test.
- V_{cb} goes into the prediction of ε_K via

$$\varepsilon_K \propto x\,|V_{cb}|^4 + \ldots .$$

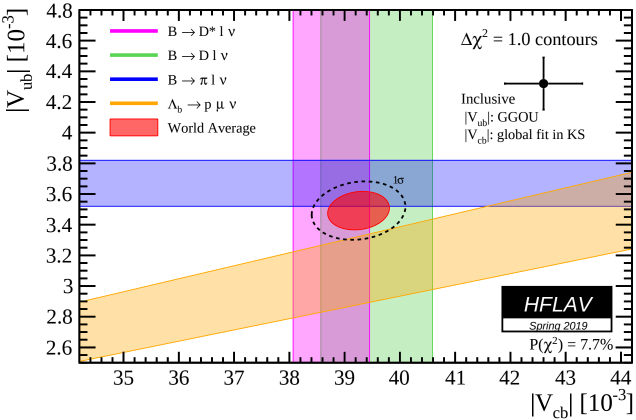
- V_{cb} goes into the predictions of flavor changing neutral currents.
- The ratio

$$\left|\frac{V_{ub}}{V_{cb}}\right|$$

  directly constrains one side of the Unitarity Triangle.

Exclusive vs. Inclusive V_{cb}, V_{ub}

[HFLAV 1909.12524]



- Recent years: Enter era of precision measurements.
  New results from B-factories, new lattice form factor results.
- Λ_b decays constrain $\dfrac{\mathcal{B}(\Lambda_b^0 \to p\mu\nu)}{\mathcal{B}(\Lambda_b \to \Lambda_c\mu\nu)} \propto \dfrac{V_{ub}^2}{V_{cb}^2}$.

Inclusive V_{cb}

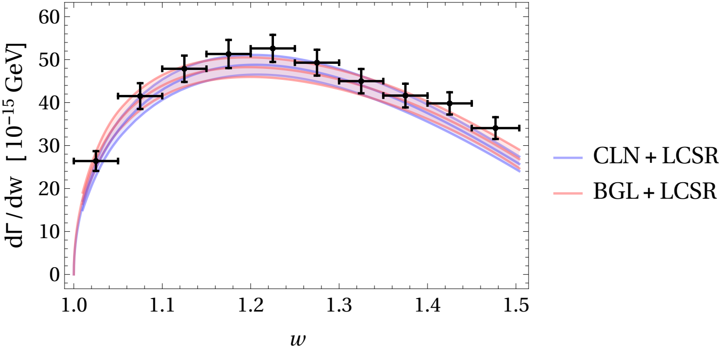
- Operator product expansion: Double expansion in α_s and $\Lambda_{\mathrm{QCD}}/m_b$.
- Parametrization of non-pert. physics by B meson matrix elements.
- Most recent global fit: [Gambino Healey Turzcyk 1606.06174]

$$|V_{cb}| = 42.00(64) \times 10^{-3}$$

New developments

- Methodology with smaller number of HQE parameters. [Fael Mannel Vos 1812.07472]
- Reducing uncertainties due to background processes in the inclusive determination of V_{cb}. [Mannel Rahimi Vos 2105.02163]
- Inclusive semi-leptonic decays on the lattice. [Gambino Hashimoto 2005.13730]

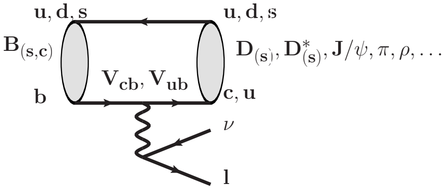
Three-body decay: Nontrivial kinematical dependences



Fit: [Bigi Gambino StS 1703.06124] Data: [Belle, 1702.01521]

- Invariant lepton mass squared $q^2 = (p_B - p_{D^*})^2 = (p_l + p_\nu)^2$.
- Dimensionless quantity $w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$.
- High $q^2 \Leftrightarrow$ low w. Low $q^2 \Leftrightarrow$ high w.

Form Factors for Exclusive Decays



# of FFs	$B \to P l \nu_l$	$B \to V l \nu_l$
$l = e, \mu$	1	3
$l = \tau$	2	4

- Example:

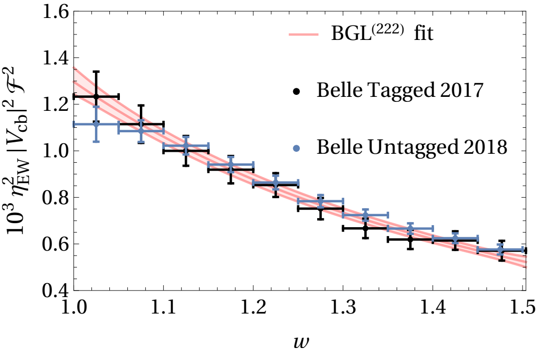
$$\langle D(k)|\, \bar{c}\gamma^{\mu} b \,|\bar{B}(p)\rangle = \left((p+k)^{\mu} - \frac{m_B^2 - m_D^2}{q^2}\right) f_+^{B\to D}(q^2) + \left(\frac{m_B^2 - m_D^2}{q^2} q^{\mu}\right) f_0^{B\to D}(q^2)$$

- Information on form factors:
    - Lattice QCD.
    - Light Cone Sum Rules (LCSR, at low q^2 ⇔ high w).
    - Heavy Quark Expansion.
    - Experiment.

Fit to 2017/18 Belle Data + Lattice $A_1(1)$

[Bigi Gambino StS 1703.06124, Grinstein Kobach 1703.08170, Bigi Gambino StS 1707.09509, Gambino Jung StS 1905.08209]

BGL$^{(222)}$	Data + lattice (weak)
χ^2/dof	$80.1/72$
$\lvert V_{cb}\rvert 10^3$	$39.6\left(^{+1.1}_{-1.0}\right)$
a_0^f	$0.01221(16)$
a_1^f	$0.006\left(^{+32}_{-45}\right)$
a_2^f	$-0.2\left(^{+12}_{-8}\right)$
$a_1^{\mathcal{F}_1}$	$0.0042\left(^{+22}_{-22}\right)$
$a_2^{\mathcal{F}_1}$	$-0.069\left(^{+41}_{-37}\right)$
a_0^g	$0.024\left(^{+21}_{-9}\right)$
a_1^g	$0.05\left(^{+39}_{-72}\right)$
a_2^g	$1.0\left(^{+0}_{-20}\right)$



- Our global $B \to D^* l\nu$ fit gives $|V_{cb}| = (39.6^{+1.1}_{-1.0}) \times 10^{-3}$.
- Differs from inclusive determination by 1.9σ.
- Truncation of BGL series when additional terms do not change fit.
  ↳No overfitting, stable fit.

Additional Theory Input on Form Factors: HQET

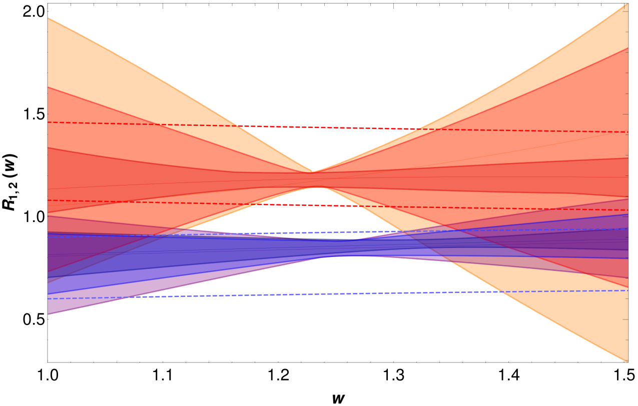
Heavy Quark Effective Theory and QCD sum rules (HQET)

[Bernlochner Ligeti Papucci Robinson 1703.05330, Caprini Lellouch Neubert hep-ph/9712417, Luke Phys.Lett B252,447 (1990), Neubert Rieckert Nucl. Phys. B382, 97 (1992) Neubert hep-ph/9306320, Ligeti Neubert Nir hep-ph/9209271, 9212266, 9305304]

- Important constraints for all $B^{(*)} \to D^{(*)}$ form factors.
- In the heavy quark limit $m_{c,b} \gg \Lambda_{\text{QCD}}$ all $B^{(*)} \to D^{(*)}$ form factors either vanish or are proportional to 1 Isgur-Wise (IW) function.
- NLO corrections at $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$ known, expressible with 3 subleading IW functions, which are extracted using QCDSRs.

Form factor ratios $R_{1,2}$ compared to HQET estimates

[Gambino Jung StS 1905.08209]



Red: R_1. Blue: R_2. Dashed: HQET

- Weak unitarity; strong unitarity; weak unitarity+LCSR, respectively.
- Good agreement with HQET.

Heavy Quark Expansion at Higher orders

Meson decays: “Update” of CLN

[Bernlochner Ligeti Papucci Robinson 1703.05330, Gubernari Kokulu van Dyk 1811.00983, Bordone Jung van Dyk 1908.09398, Bordone Gubernari Jung van Dyk 1912.09335]

- Heavy-Quark Expansion to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$.
- Combining new LCSR results and lattice input.
- Consistency between HQE and BGL fits.

Baryon decays

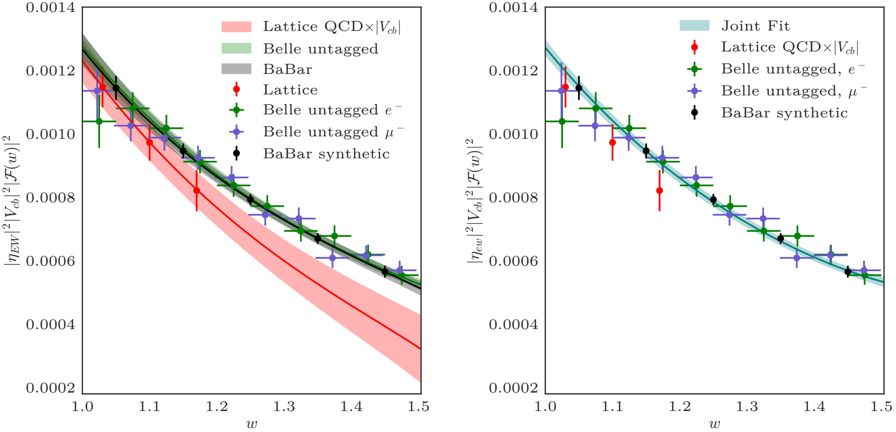
- Heavy quark expansion of $\Lambda_b \to \Lambda_c l\nu$ to $\mathcal{O}(\alpha_s/m_{b,c}, 1/m_c^2)$. Predictions in terms of only 2 sub-subleading Isgur-Wise functions.

  [Bernlochner Ligeti Robinson Sutcliffe 1812.07593]
- Heavy quark expansion for $\Lambda_b \to \Lambda_c^* l\nu$ to $\mathcal{O}(\alpha_s, 1/m_{b,c})$.

  [Papucci Robinson 2105.09330]
- Baryon FFs from lattice. [Meinel et al 2103.08775, 1702.02243, 1503.01421]

NEW: Non-zero recoil $B \to D^*$ FFs from Lattice QCD

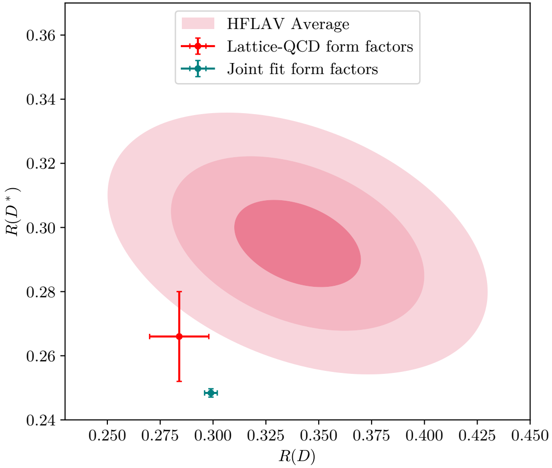
[FNAL/MILC 2105.14019]



- Tension between slope from lattice and experimental data.
- Agreement over whole kinematic range at $\approx 2\sigma$.
- Result: $|V_{cb}| = \left(38.57 \pm 0.70_{\text{th}} \pm 0.34_{\text{exp}}\right) \times 10^{-3}$.

Lattice only prediction $R(D^*)_{\mathrm{Lat}} = 0.266 \pm 0.014$

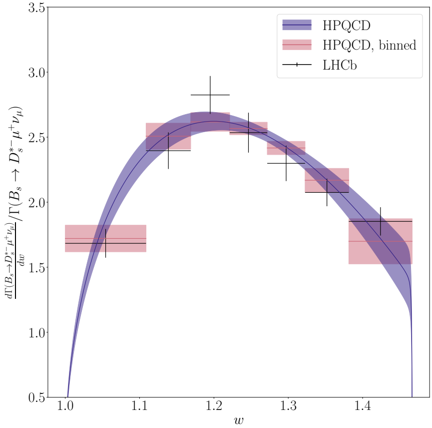
[FNAL/MILC 2105.14019]



- More from lattice: [JLQCD 1912.11770], [LANL/SWME 2003.09206], [Martinelli et al 2105.08674, 2105.07851]

NEW: Full q^2 range $B_s \to D_s^*$ FFs from Lattice QCD

[HPQCD 2105.11433]



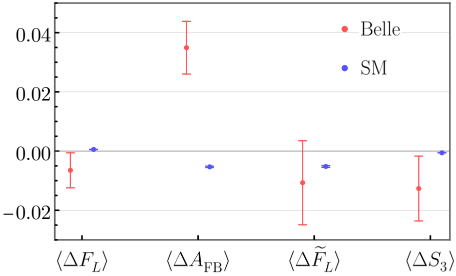
- Results: $|V_{cb}| = 43.0(2.1)_{\text{latt}}(1.7)_{\text{exp}}(0.4)_{\text{EM}} \cdot 10^{-3}$. Lattice-only $R(D_s^{*-})$.
- Also available: $B_c \to J/\psi l\nu$ in full q^2 range [HPQCD 2007.06956]

$e - \mu$ non-universality in $B \to D^* l\nu$?

- [Belle 1809.03290] finds $\frac{\mathcal{B}(B^0 \to D^{*-} e^+ \nu_e)}{\mathcal{B}(B^0 \to D^{*-} \mu^+ \nu_\mu)} = 1.01 \pm 0.01 \pm 0.03$.
- [Bobeth van Dyk Bordone Jung Gubernari 2104.02094] extract from angular distributions of same Belle data set the observable

$$\Delta A_{FB} \equiv A_{FB}^{(\mu)} - A_{FB}^{(e)} \neq 0 \quad \text{at} \sim 4\sigma.$$

- Physical or artifact? More study needed.



[Bobeth et al 2104.02094]

Conclusion

For now, the two anomalies in charged currents persist:

- Exclusive/inclusive V_{cb}, V_{ub} and
- Lepton flavor universality violation.

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https://doi.org/10.1140/epjc/s10052-020-08490-x

THE EUROPEAN PHYSICAL JOURNAL C

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Review

Challenges in semileptonic B decays

P. Gambino[1,a], A. S. Kronfeld[2], M. Rotondo[3], C. Schwanda[4], F. Bernlochner[5], A. Bharucha[6], C. Bozzi[7], M. Calvi[8], L. Cao[5], G. Ciezarek[9], C. T. H. Davies[10], A. X. El-Khadra[11], S. Hashimoto[12], M. Jung[1], A. Khodjamirian[13], Z. Ligeti[14], E. Lunghi[15], V. Lüth[16], T. Mannel[13], S. Meinel[17], G. Paz[18], S. Schacht[1,19], S. Simula[20], W. Sutcliffe[5], A. Vaquero Avilés-Casco[21]

Everything you always wanted to ask about semileptonic decays

2006.07287

BACK-UP

Unitarity Constraints

[Boyd Grinstein Lebed 1994, 1997]

- Use dispersion relations to relate physical semileptonic region

$$m_l^2 \leq q^2 \leq (m_B - m_D)^2 \,, \qquad q^2 \equiv (p_B - p_{D^*})^2 \,,$$

  to pair-production region beyond threshold

$$q^2 \geq (m_B + m_D)^2 \,, \quad \text{with poles at } q^2 = m_{B_c}^2 \,.$$

- Constrain form factors in pair-production region with pert. QCD.

- Translate constraint to semileptonic region using analyticity.

Model independent form factor parametrization

[Boyd Grinstein Lebed (BGL), hep-ph/9412324, hep-ph/9504235, hep-ph/9705252]

Boyd Grinstein Lebed parametrization

$$f_i(z) = \frac{1}{B_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_n^i z^n ,$$

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} , \qquad w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} .$$

- $0 < z < 0.056$ for $B \to D^* l\nu \Rightarrow$ truncation at $N = 2$ enough, $z^3 \sim 10^{-4}$.
- $B_i(z)$: “Blaschke factor”: removes poles.
- $\phi_i(z)$: phase space factors.

- Where should one truncate the series?
- Where adding more terms becomes irrelevant:
  Stability of central values and errors, especially of V_{cb}.

Unitarity Constraints

Use basic properties of QCD:
Unitarity, crossing symmetry, analyticity, dispersion relations.

(Weak) Unitarity Conditions

- Vector current:

$$\sum_{i=0}^{\infty} \left(a_n^{V_4}\right)^2 \leq 1\,.$$

- Axial vector current:

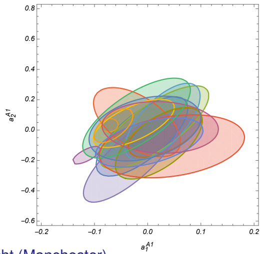
$$\sum_{i=0}^{\infty} \left(\left(a_n^{A_1}\right)^2 + \left(a_n^{A_5}\right)^2\right) \leq 1\,.$$

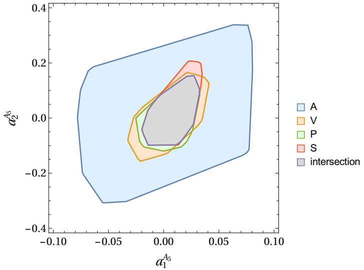
Strong Unitarity Constraints

- Use HQET information on further $b \to c$ channels: $B \to D$, $B^* \to D$, $B^* \to D^*$, to relate them to $B \to D^*$.
- Make the unitarity bounds stronger: [BGL, hep-ph/9705252]

$$\sum_{i=1}^{H} \sum_{n=0}^{\infty} b_{in}^2 \leq 1 \,. \qquad \text{for } S, P, V, A \text{ currents}$$

- Vary QCDSR parameters + higher order corrections: obtain many different unitarity bounds.
- Take their envelope as side condition in the fit.





Parametrization employing unitarity+heavy-quark exp.

Practical Caprini Lellouch Neubert parametrization as employed in experimental analyses

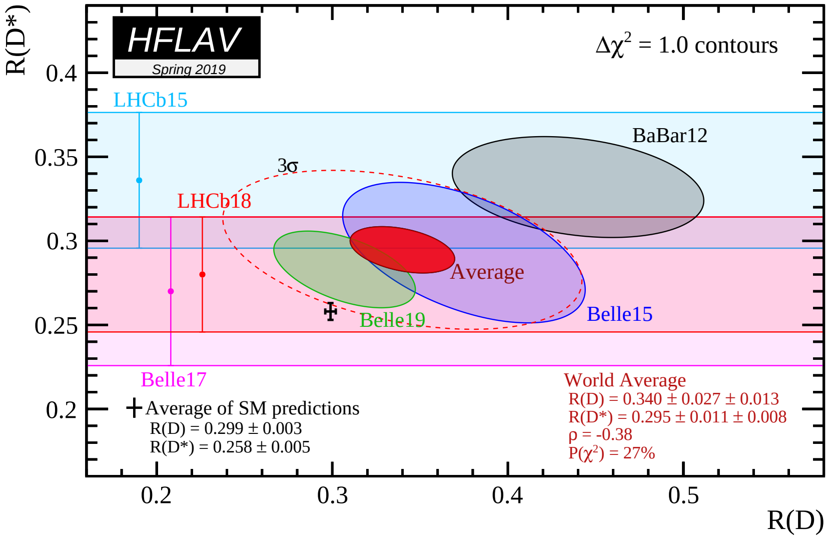
$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right),$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2.$$

- Theoretical uncertainties pointed out in [CLN, hep-ph/9712417], e.g. for slope and curvature of form factor ratios R_1 and R_2 were set to zero.
- Appropriate at the time.
- At current experimental precision, theoretical uncertainties of fixed parameters need to be taken into account.

Status $R(D^{(*)})$ combined: 3.08σ [HFLAV 1909.12524]



Anatomy of $R(D^*) \equiv \frac{\int_1^{w_{\tau,\max}} dw\, d\Gamma_\tau/dw}{\int_1^{w_{\max}} dw\, d\Gamma/dw}$

Differential decay rate for $B \to D^* \tau \nu_\tau$

[BGL, hep-ph/9705252]

$$\frac{d\Gamma_\tau}{dw} = \frac{d\Gamma_{\tau,1}}{dw} + \frac{d\Gamma_{\tau,2}}{dw}$$

$$\frac{d\Gamma_{\tau,1}}{dw} = \left(1 - m_\tau^2/q^2\right)^2 \left(1 + m_\tau^2/(2q^2)\right) \frac{d\Gamma}{dw}$$

$$\frac{d\Gamma_{\tau,2}}{dw} = |V_{cb}|^2 m_\tau^2 \times \text{kinematics} \times P_1(z)^2$$

- $d\Gamma/dw$: Measured differential decay rate of $B \to D^* l \nu$ with $m_l = 0$, depends on axial vector form factors A_1, A_5 and vector form factor V_4.
- P_1 : Additional unconstrained pseudoscalar form factor.
- $d\Gamma_{\tau,2}/dw$ contributes $\sim 10\%$ to $R(D^*)$.
- Common normalization/notation:

$$R_0 = \frac{P_1}{A_1} = 1 \quad \text{in heavy quark limit}$$

[BGL, hep-ph/9705252]