Precision QCD Theory

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QCD factorisation

- The 'master formula' for LHC observables:

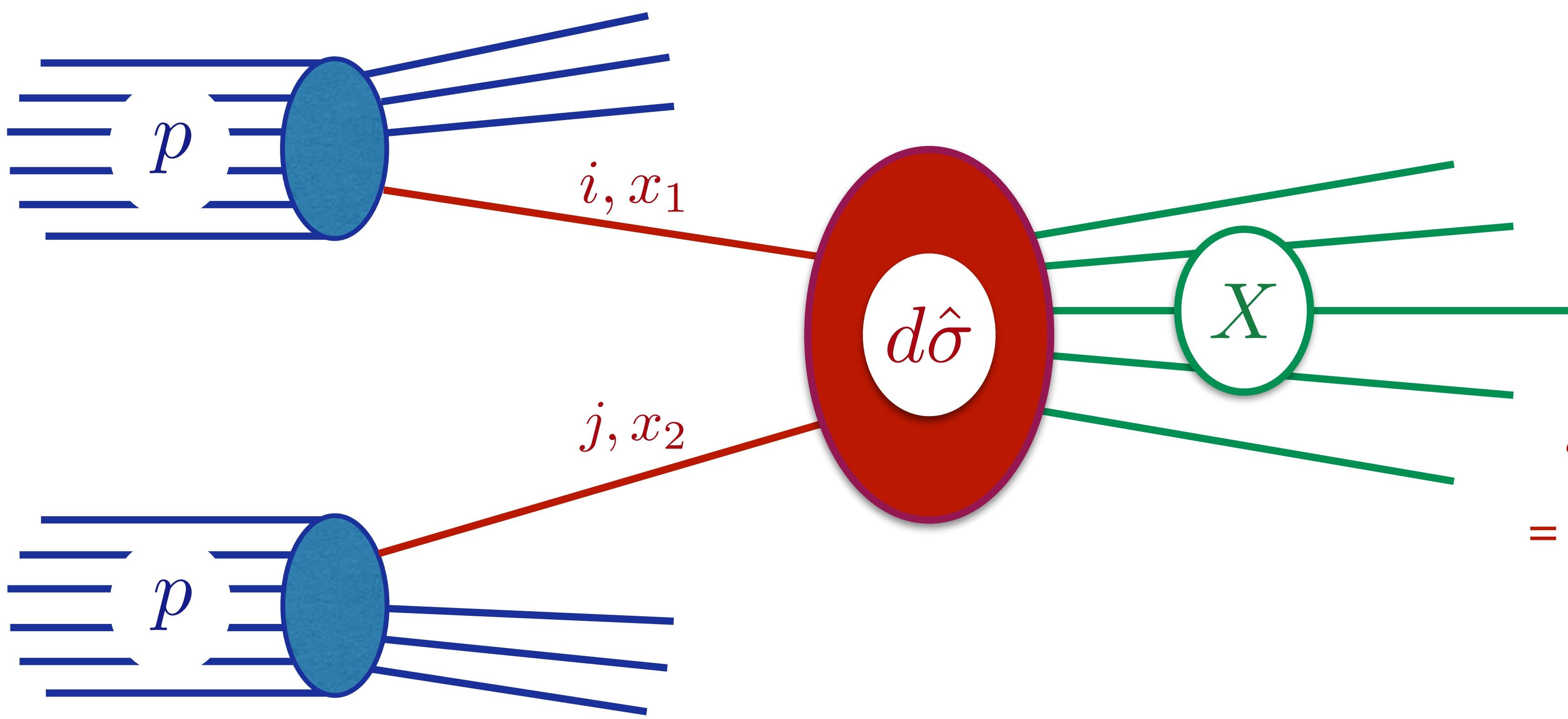
$$d\sigma(p\,p \to X) = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}(i\,j \to X)$$

Parton Distribution Functions

non-perturbative;
describe the structure of the proton

Partonic cross section

computable in perturbation theory
as collisions between quarks and gluons

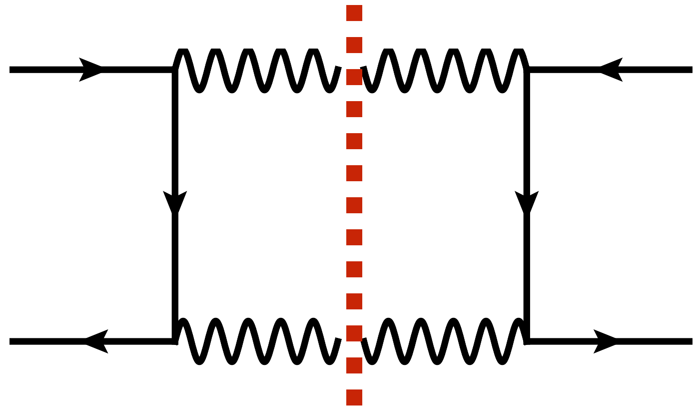


$$d\hat{\sigma} \sim \int dPS\, |\mathcal{A}|^2$$

$\mathcal{A}$ = scattering amplitude
= sum of Feynman diagrams

Anatomy of higher orders

- Leading order (LO):



Anatomy of higher orders

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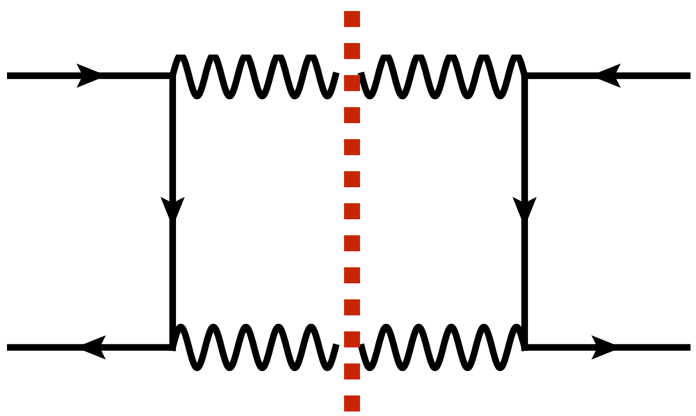
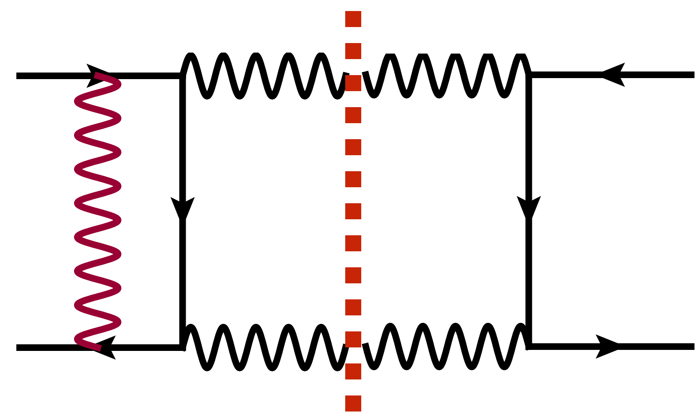
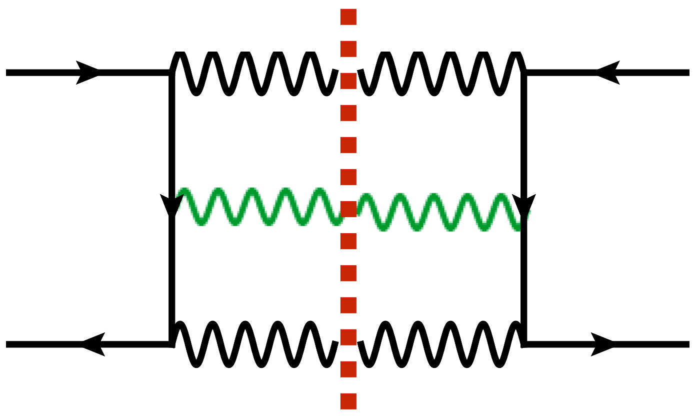


- Next-to-LO (NLO):



Virtual

+



Real

Individually divergent, but sum is finite.

Anatomy of higher orders

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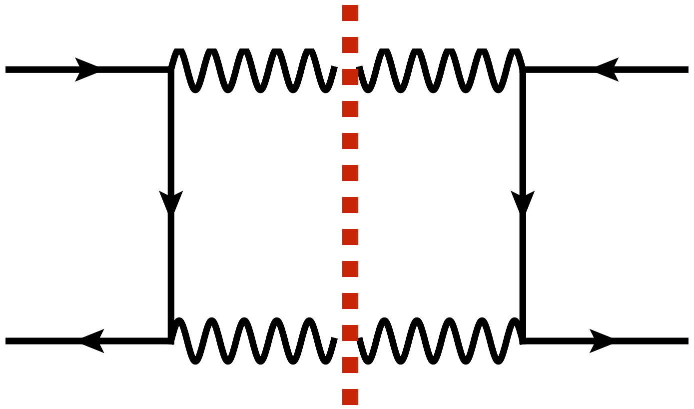
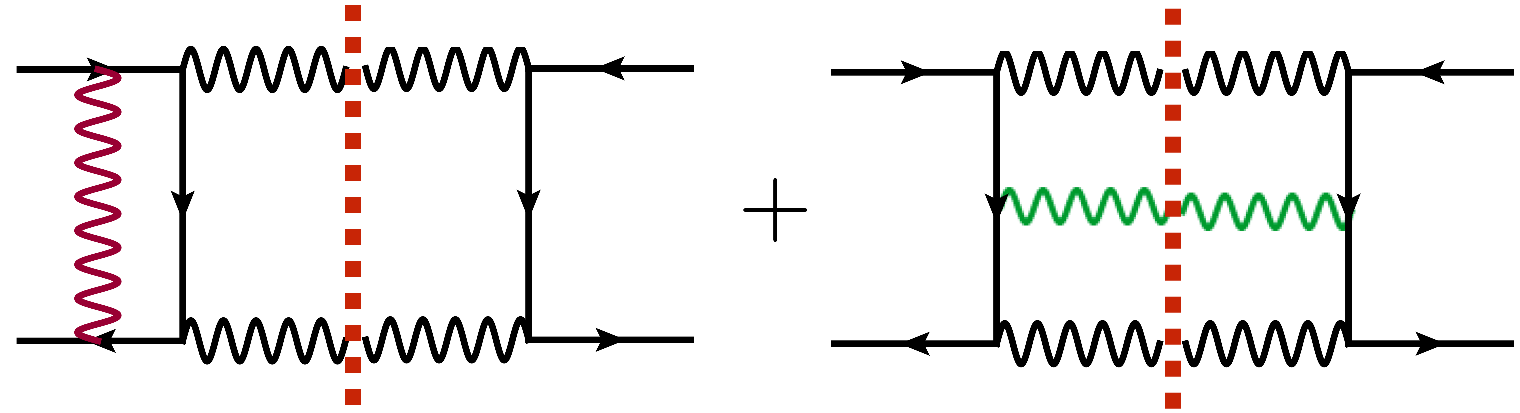
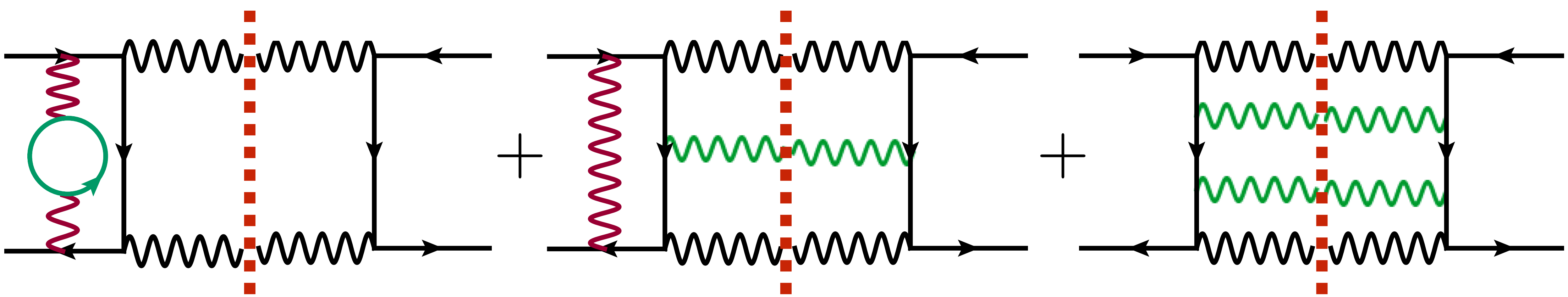


- Next-to-LO (NLO):



- Next-to-next-to-LO (NNLO):



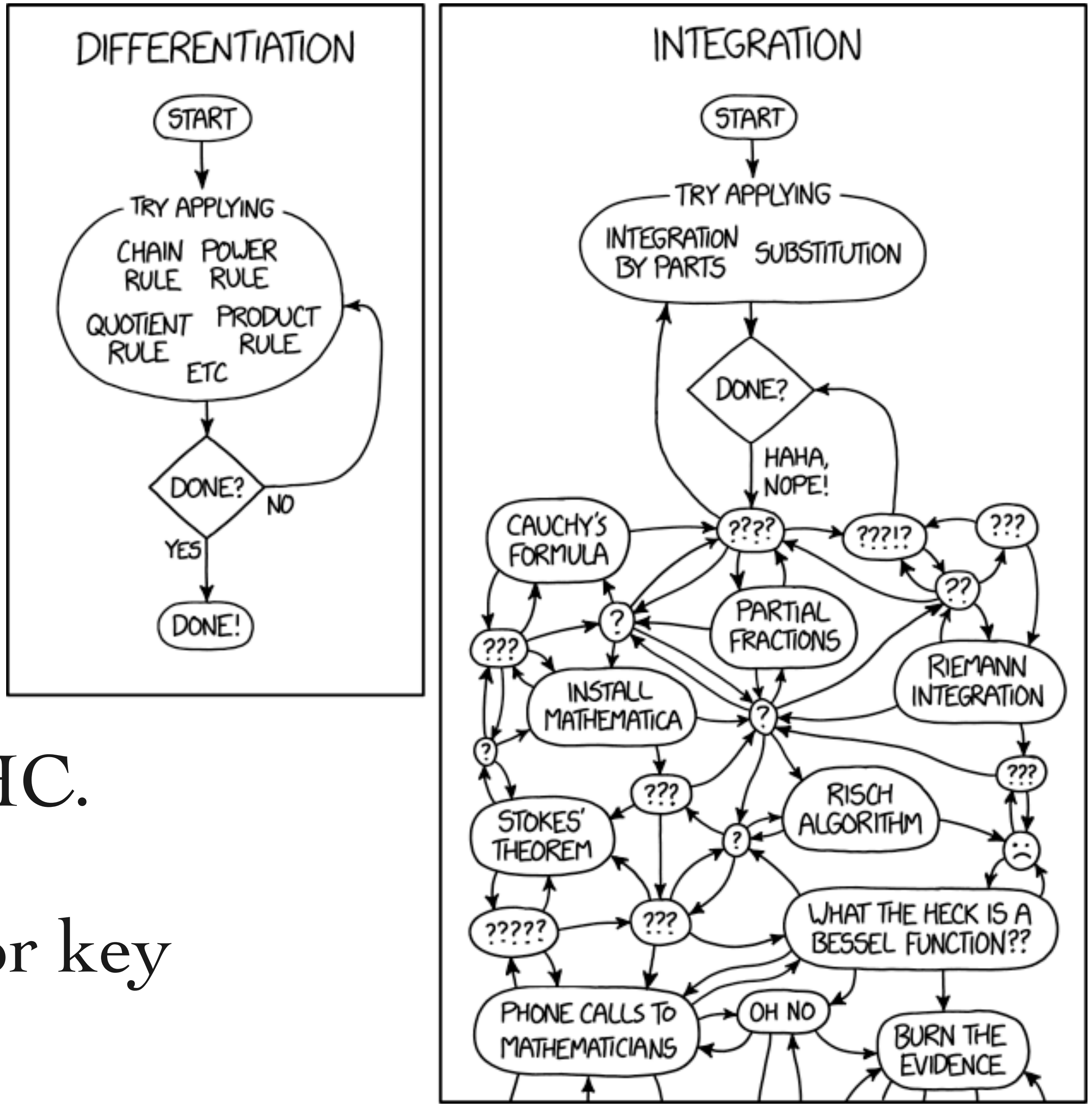
We know how to combine reals and virtuals at NLO and NNLO.

[NNLO: Anastasiou, Melnikov, Petriello; Catani, de Florian, Grazzini; Gehrmann, Gehrmann-de Ridder, Glover; Czakon; Czakon, Fiedler, Mitov; Caola, Melnikov, Schulze; Caola, Melnikov, Röntsch; Gaunt, Stahlhofen, Tackmann, Walsh; Boughezal, Focke, Giele, Liu, Petriello; Cacciari, Dreyer, Karlberg, Salam, Zanderighi; G.Bevilacqua, A.Kardos, G.Somogyi, Z.Trocsanyi, Z.Tulipant; L.Magnea, L.Maina, G.Pelliccioli, C.Signorile-Signorile, P.Torrielli, S.Uccirati, …]

Outline

- These computations are **hard**!
  - ➡ conceptionally.
  - ➡ practically.
- In the rest of the talk, I will focus on recent results in fixed order perturbation theory.
- Outline:
  - ➡ NNLO precision for the LHC.
  - ➡ Towards N3LO precision for key observables.

[© xkcd.com]



NNLO precision for the LHC

State-of-the-art NNLO

- Fully differential predictions for $2 \to 2$ processes at NNLO are becoming the standard, e.g.:

$pp \to t\bar{t}$

[Czakon, Fiedler, Mitov; Catani, Devoto, Grazzini, Kallweit, Mazzitelli]

$pp \to \gamma + j$

[Chen, Gehrmann, Glover, Höfer, Huss]

$pp \to H + j$

[Boughezal, Caola, Melnikov, Petriello; Schulze; Boughezal, Focke, Giele, Liu, Petriello; Chen, Gehrmann, Glover, Jaquier]

$pp \to j\,j$

[Currie, Gehrmann-de Ridder, Gehrmann, Glover, Huss, Pires; Czakon, van Hameren, Mitov, Poncelet]

$pp \to V + j$

[Boughezal, Focke, Liu, Petriello; Boughezal, Campbell, Ellis, Focke, Liu, Petriello; Gehrmann-de Ridder, Gehrmann, Glover, Huss, Morgan]

$pp \to \gamma\,\gamma$

[Catani, Cieri, de Florian, Ferrera, Grazzini]

$pp \to V\,V'$

[Cascioli, Gehrmann Grazzini, et al.; Gehrmann, Grazzini, Kallweit, et al.; Grazzini, Kallweit, Wiesemann, Yook]

$pp \to V\,H$

[Ferrera, Grazzini, Tramontano; Gauld, Gehrmann-de Ridder, Glover, Huss, Majer]

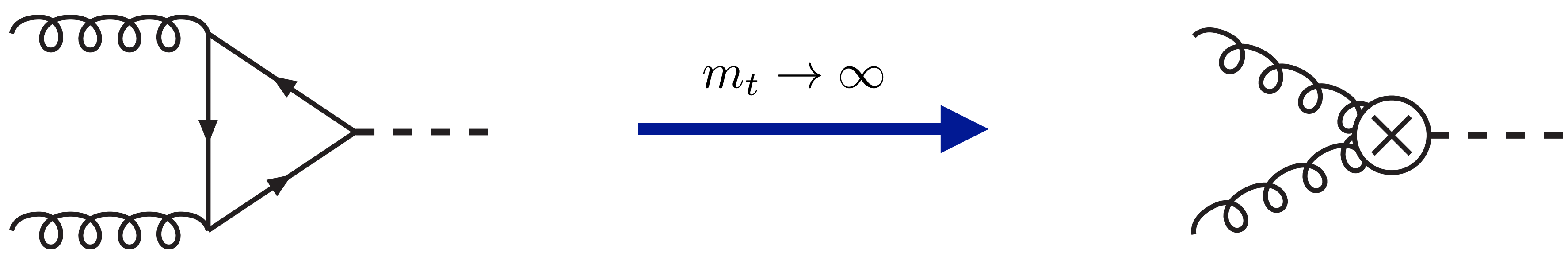
- The relevant two-loop virtual integrals are mostly known (analytically or numerically).

➡ Frontier: two-loop computations with massive propagators.

Gluon-fusion at NNLO

- Recent milestone: Higgs production in gluon-fusion at NNLO with full top-mass dependence. [Czakon, Harlander, Klappert, Niggetiedt]

➡ So far only available for $m_t \to \infty$, e.g., at LO:



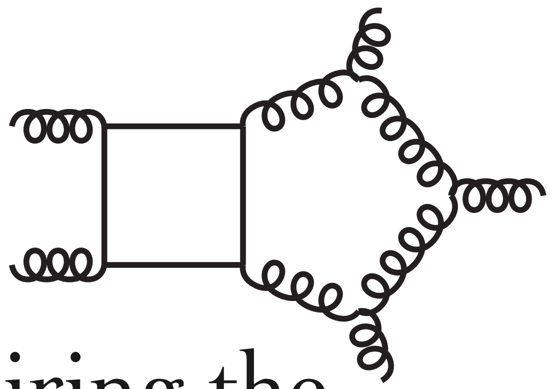
➡ Mass-effects previously estimated to be ~1%.

channel	$\sigma^{\rm NNLO}_{\rm HEFT}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}-\sigma^{\rm NNLO}_{\rm HEFT})$ [pb] $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^4)$	$(\sigma^{\rm NNLO}_{\rm exact}/\sigma^{\rm NNLO}_{\rm HEFT}-1)$ [%]
		$\sqrt{s}=8$ TeV		
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10
		$\sqrt{s}=13$ TeV		
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30+21.15+9.79$	-0.3029	$+0.1815\pm0.0023$	-0.26

The 2-to-3 frontier

- Two-loop integrals for 5-point functions (with massless propagators) are slowly becoming available.

  [Gehrmann, Henn, Lo Presti; Papadopoulos, Tommasini, Wever; Gehrmann, Henn, Wasser, Zhang, Zoia; Abreu, Ita, Moriello, Page, Tschernow]



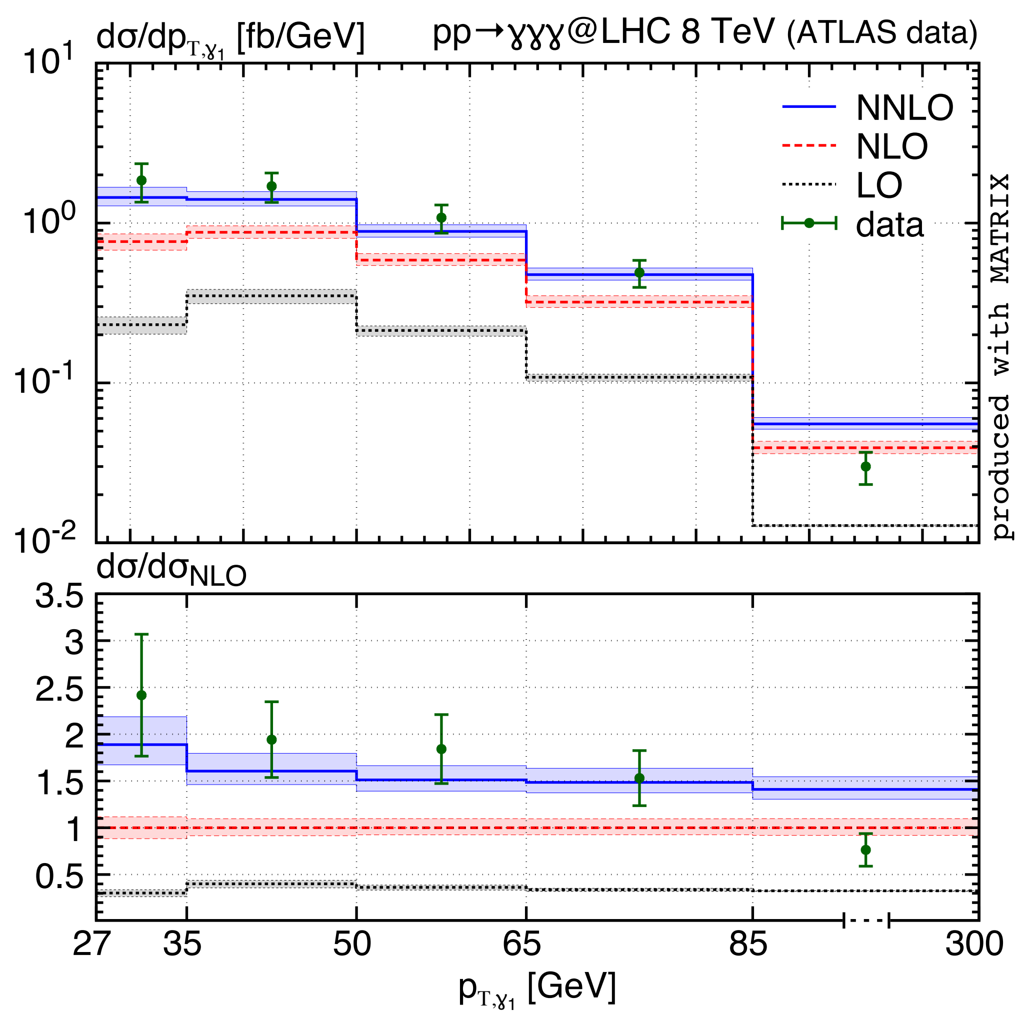
  ➡ Extremely challenging computation, often requiring the development of novel computational techniques and/or new insight from mathematics.

- This opens the way for two-loop amplitudes for $2 \to 3$ processes at the LHC:

$p\,p \to 3j$	$p\,p \to 3\gamma$	$p\,p \to 2\gamma + j$
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia; Abreu, Dormans, Frebres Cordero, Ita, Page, Sotnikov]	[Abreu, Page, Pascual, Sotnikov; Chawdhry, Czakon, Mitov, Poncelet]	[Agarwal, Buccioni, von Manteuffel, Tancredi; Chawdhry, Czakon, Mitov, Poncelet]

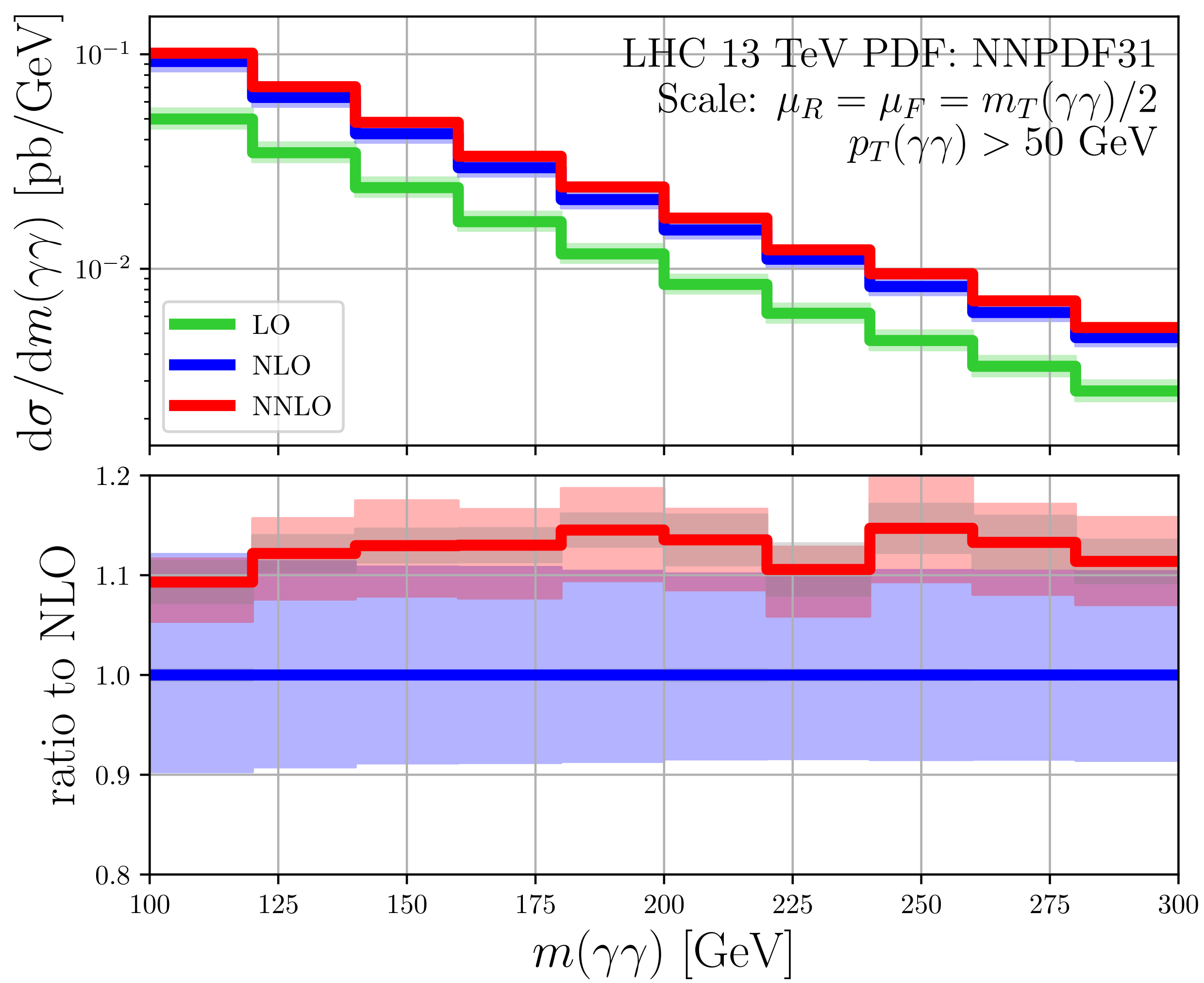
The 2-to-3 frontier

- Over the last year, the first NNLO predictions for $pp \to 3\gamma$ and $pp \to 2\gamma + j$ have been published…



$pp \to 3\gamma$ (using qT subtraction)

[Kallweit, Sotnikov, Wiesemann]

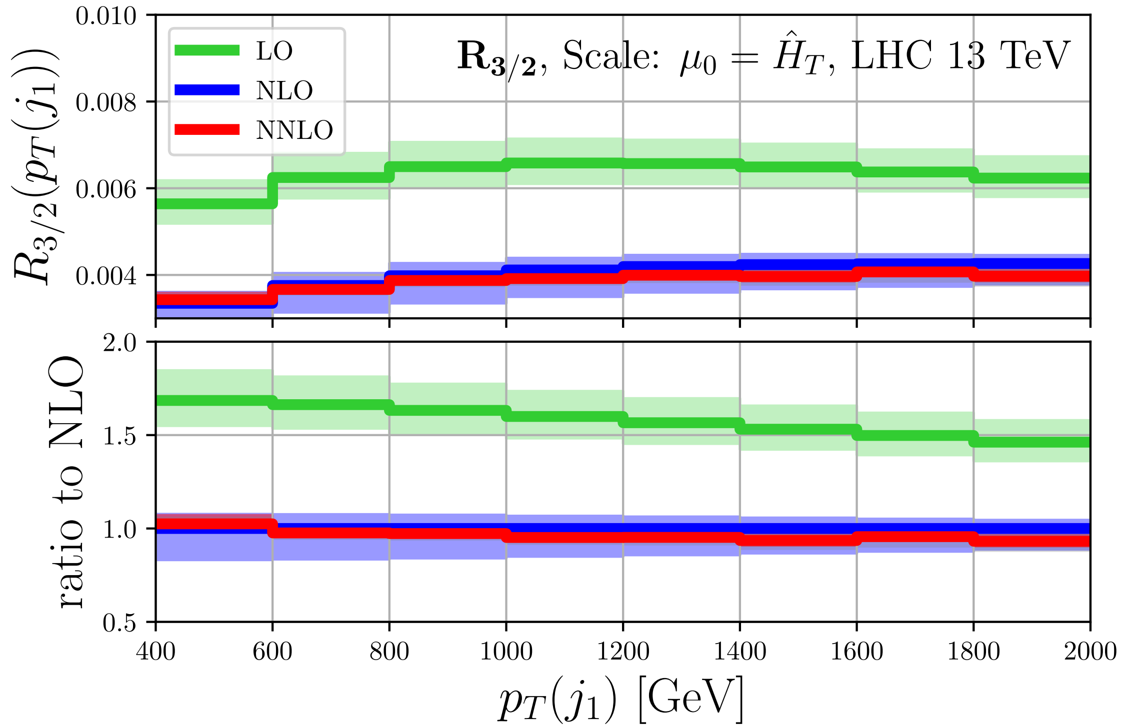


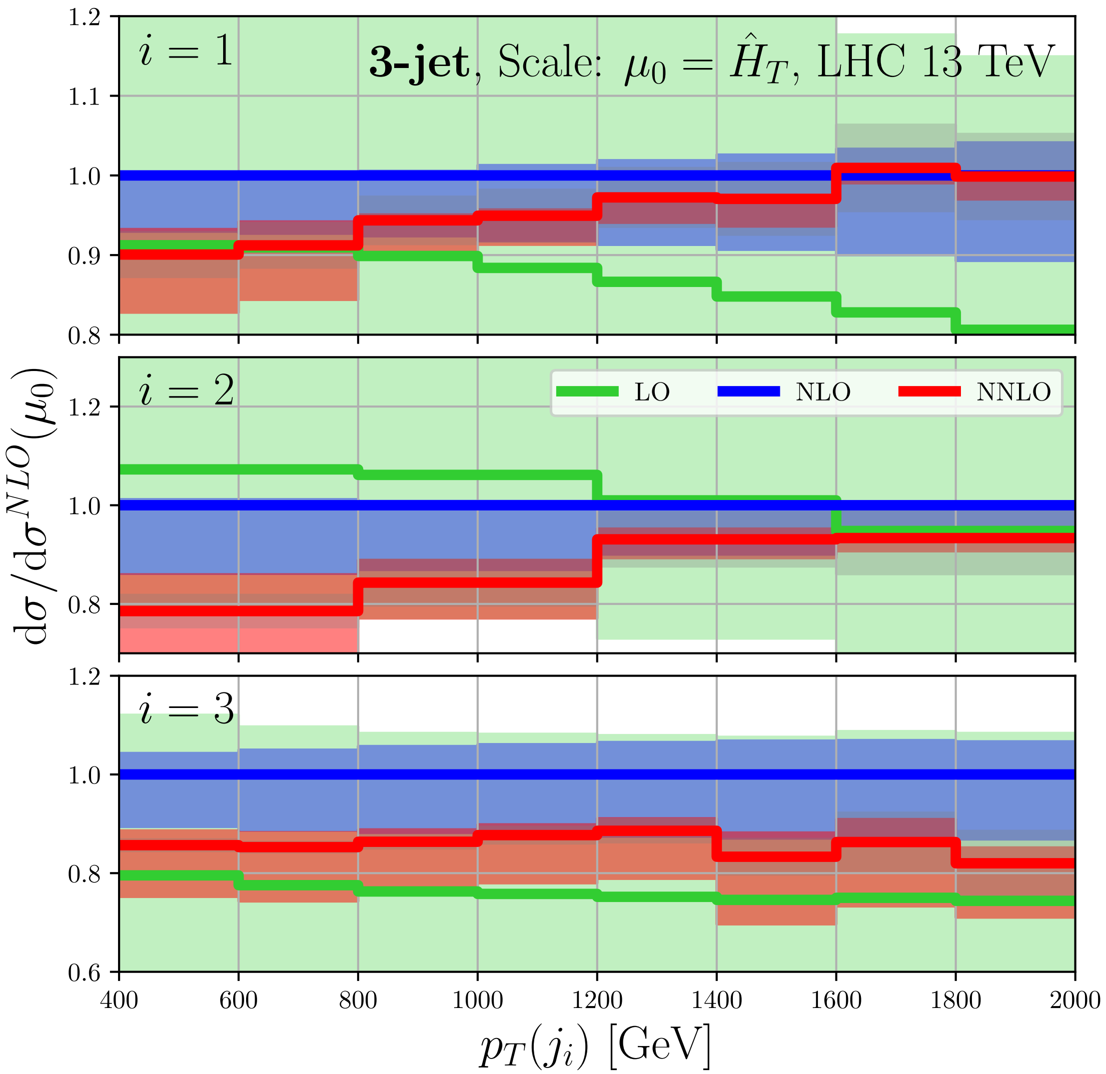
$pp \to 2\gamma + j$ (using STRIPPER)

[Chawdhry, Czakon, Mitov, Poncelet]

The 2-to-3 frontier

- … and just this morning $pp \to 3j$ has appeared on the arXiv!

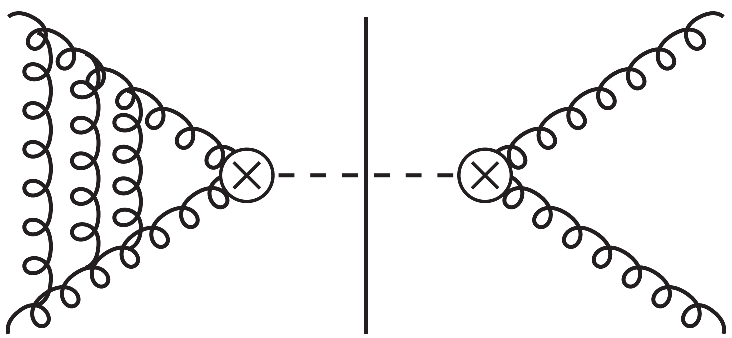




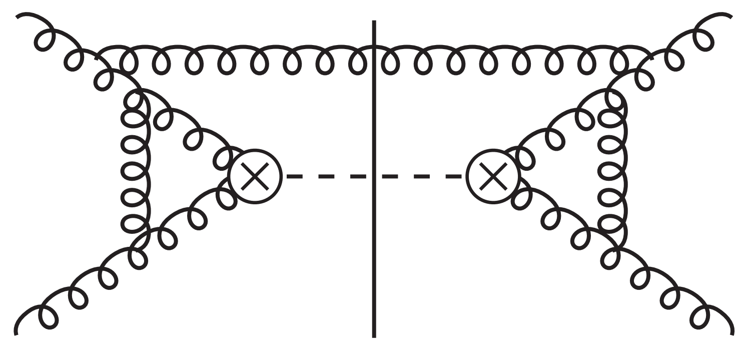
[Czakon, Mitov, Poncelet]

Towards N3LO precision for key observables

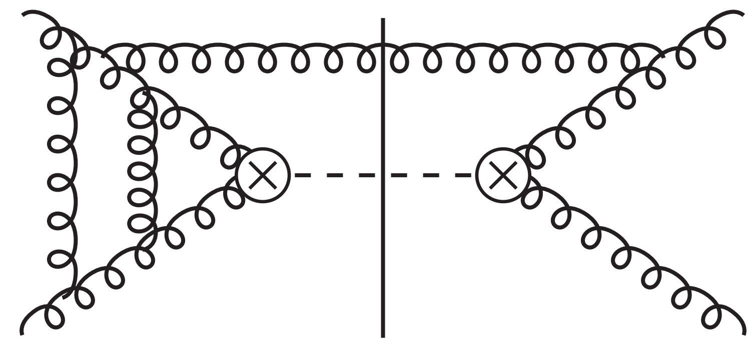
Anatomy of N3LO



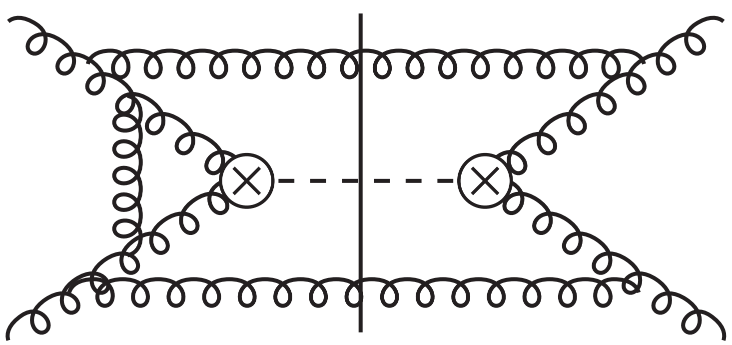
Triple virtual



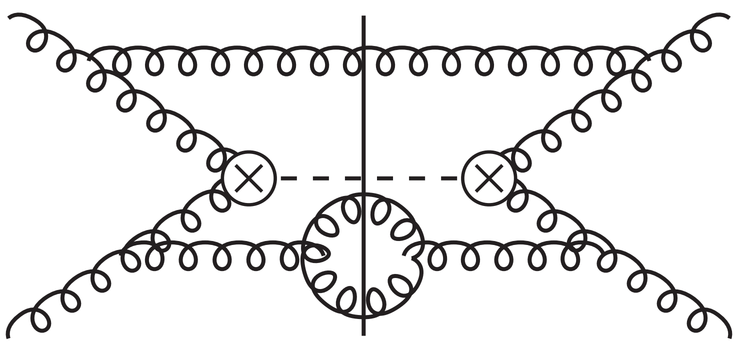
Real-virtual squared



Double virtual real



Double real virtual



Triple real

State-of-the-art

- State-of-the-art at N3LO: Inclusive color-singlet production.

$$g\,g \to H$$
[Anastasiou, CD, Dulat, Herzog, Mistlberger]

$$b\,\bar{b} \to H$$
[CD, Dulat, Mistlberger; CD, Dulat, Hirschi, Mistlberger]

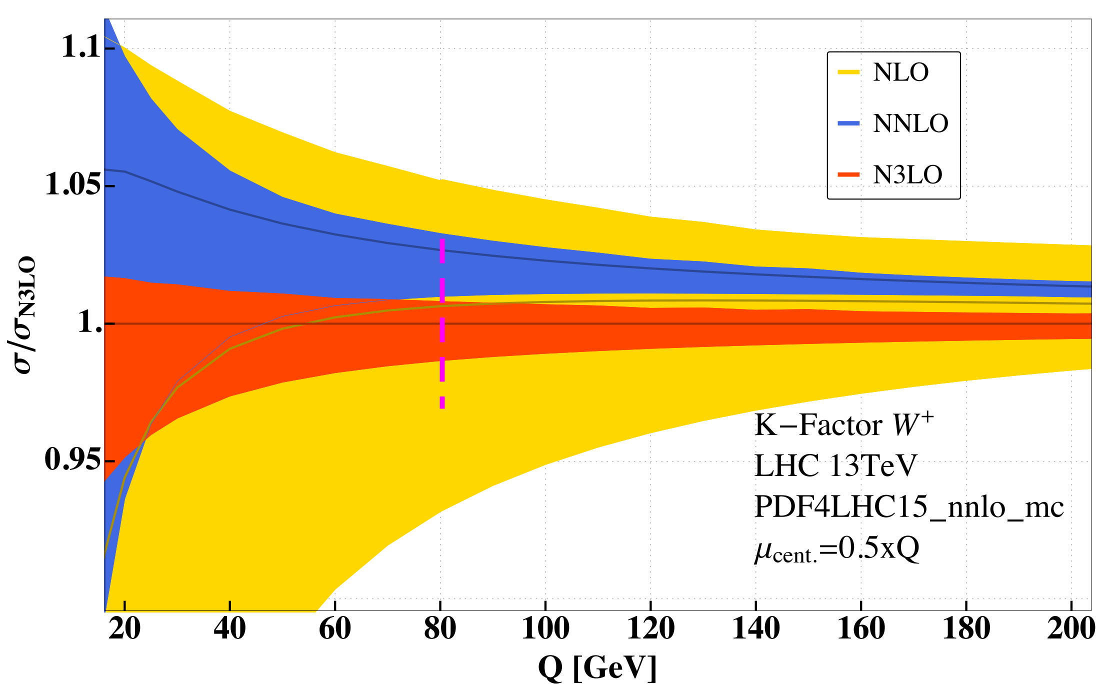
$$p\,p \to W\,H$$
[Baglio, CD, Mistlberger, Szafron] **NEW!**

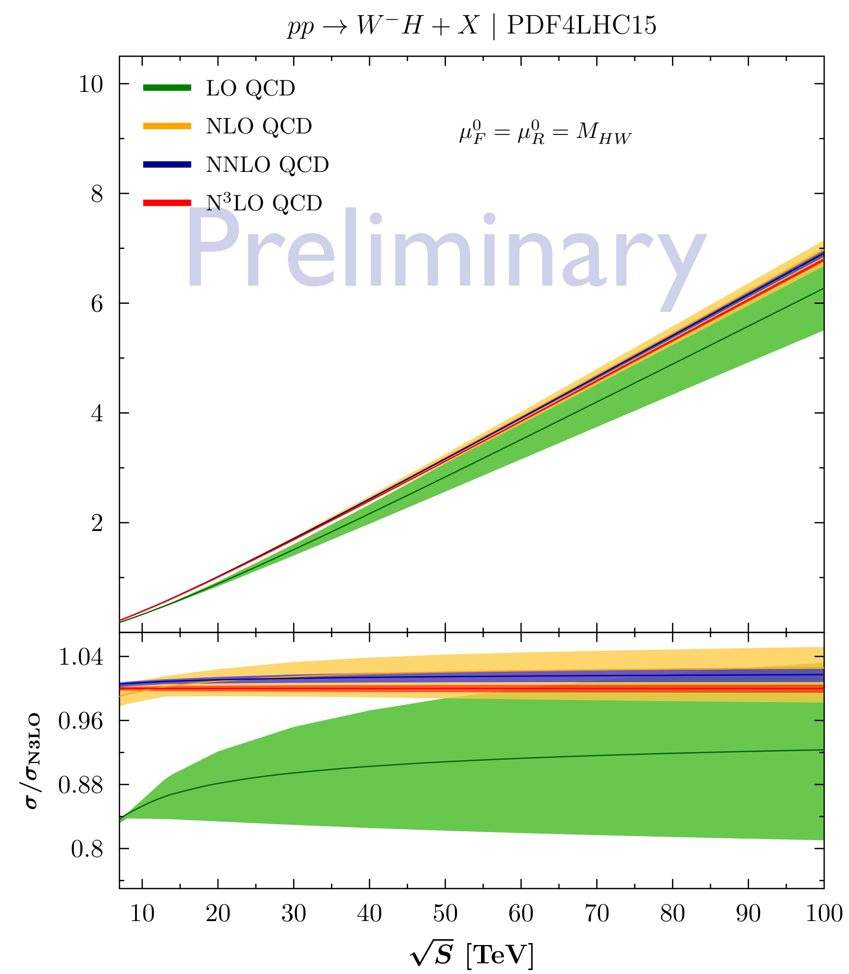
$$p\,p \to H + 2j$$
[Dreyer, Karlberg]

$$g\,g \to H\,H$$
[Chen, Li, Shao, Wang]

$$p\,p \to \gamma^* \to \ell^+\ell^-$$
[CD, Dulat, Mistlberger]

$$p\,p \to W \to \ell\nu$$
[CD, Dulat, Mistlberger]





Lessons learned from N3LO

	Q[GeV]	$K_{QCD}^{N^3LO}$	δ_{scale}
ggH		1.04	$^{+0.21\%}_{-2.37\%}$
bbH		0.978	$^{+3.0\%}_{-4.8\%}$
DY/W	30	0.952	$^{+1.5\%}_{-2.5\%}$
	90	0.978	$^{+0.75\%}_{-0.89\%}$
	150	0.985	$^{+0.50\%}_{-0.54\%}$
HW+		0.984	$^{+0.58\%}_{-0.30\%}$
HW-		0.994	$^{+0.33\%}_{-0.43\%}$
ggHH		1.03	$^{+0.66\%}_{-2.8\%}$
VBF (DIS, 14 TeV)		0.999	$^{+0.05\%}_{-0.05\%}$

Preliminary

Typical K-factors and scale dependence:

$$K_{QCD}^{N^3LO} = \frac{N^3LO}{NNLO} \sim 0.95 - 1.05$$

$$\delta_{scale} \sim \text{few } \%$$

Typical PDF uncertainty:

$$\delta_{PDF} \sim 2 - 9\%$$

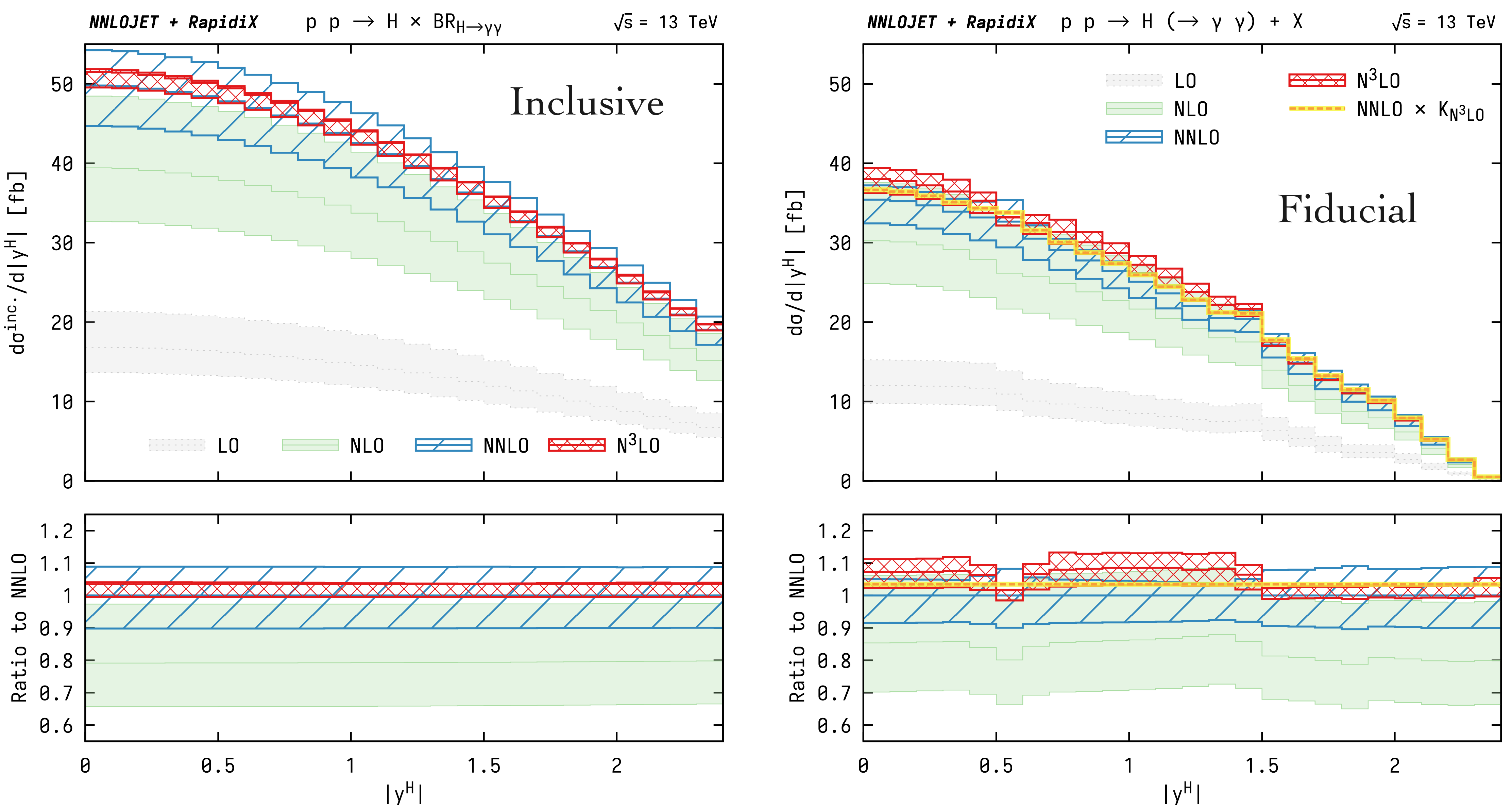
Missing N3LO PDFs:

$$\delta_{PDF}^{N^3LO} \sim 1 - 3\%$$

Towards differential N3LO

- Ultimate goal: Differential predictions at N3LO!
  - ➡ First results for gluon-fusion.

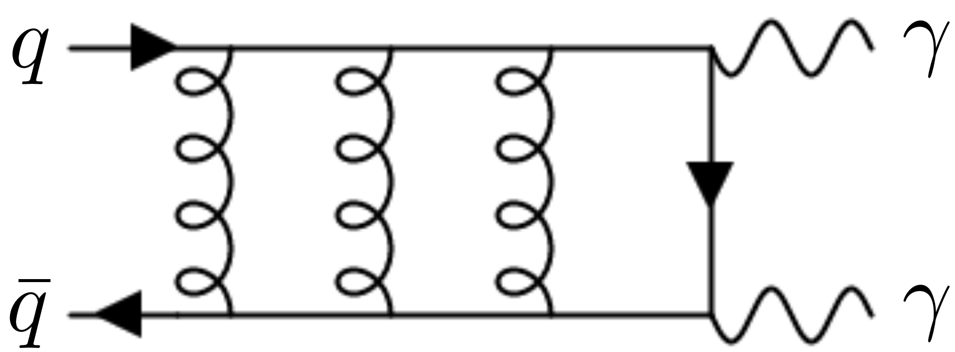
[Dulat, Mistlberger, Pelloni; Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni]

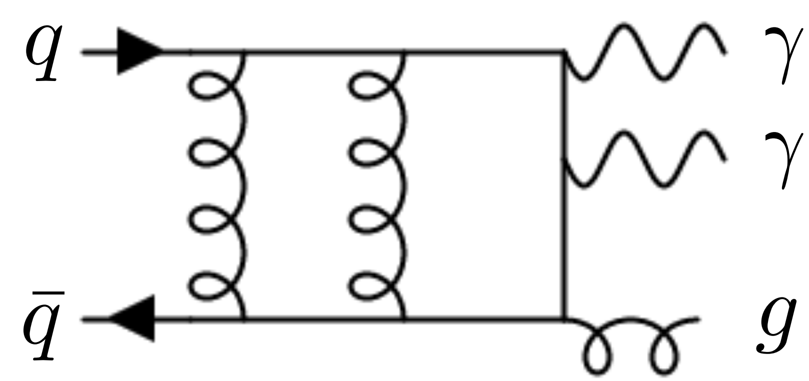


[Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni]

Towards differential N3LO

- We have all the ingredients to compute di-photon production at N3LO.
  - ➡ 3-loop integrals & 3-loop amplitude. [Henn, Mistlberger, Smirnov, Wasser; Caola, Tancredi, von Manteuffel]
  - ➡ NNLO computation for $pp \to 2\gamma + j$.





  - ➡ They can be combined using, e.g., qT subtraction (at least in principle).

- Still a lot to done here, but this shows that our technologies are becoming mature!

Tools for Higgs Physics

Cross Section

ggF

HIGLU (NNLO QCD+NLO EW)
iHixs (NNLO QCD+NLO EW)
FeHiPro (NNLO QCD+NLO EW)
HNNLO, HRes (NNLO+NNLL QCD)
SusHi (NNLO QCD)
RGHiggs (NNLO+NNNLL QCD)
ggHiggs (approx. NNNLO QCD)

VBF

VV2H (NLO QCD)
VBFNLO (NLO QCD)
HAWK (NLO QCD+EW)
VBF@NNLO (NNLO QCD)

WH/ZH

V2HV (NLO QCD)
HAWK (NLO QCD+EW)
VH@NNLO (NNLO)

ttH

HQQ (LO QCD)

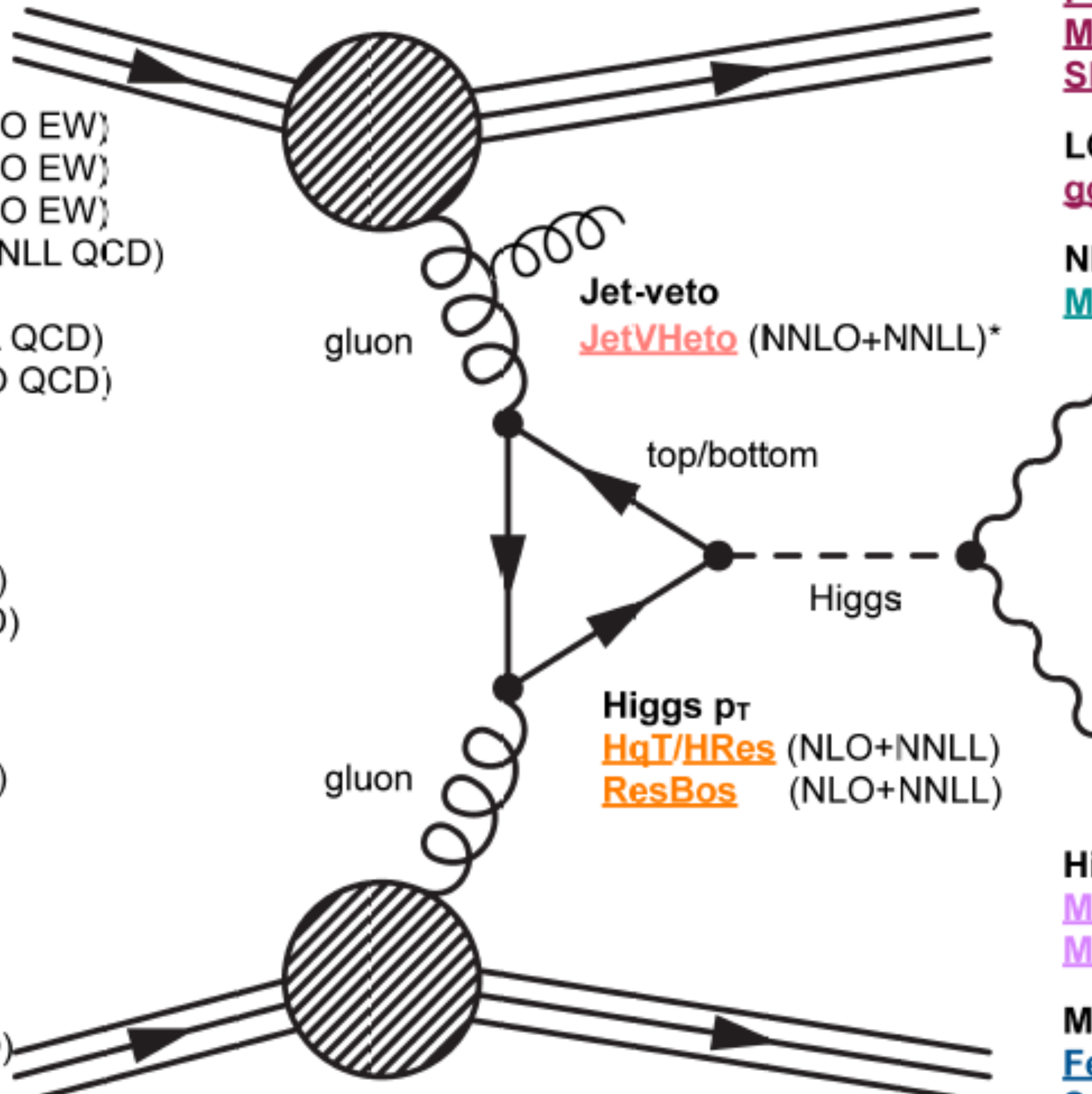
bbH

bbh@NNLO (NNLO QCD)

HH

HPAIR (NLO QCD)

+ private codes.



Jet-veto

JetVHeto (NNLO+NNLL)*

Higgs p_T

HqT/HRes (NLO+NNLL)
ResBos (NLO+NNLL)

PDF: MSTW, CTEQ, NNPDF, etc.
LHAPDF, HOPPET, APFEL

NLO MC

POWHEG MiNLO
MadGrapn5_aMC@NLO
SHERPA MEPS@NLO

LO MC

gg2VV

NLO ME

MCFM, MG5_aMC@NLO

Higgs Decay

HDECAY (NLO++)
Prophecy4f (NLO)

Higgs Properties

MELA/JHU, MEKD
MG5_aMC@NLO (HC)

MSSM/2HDM

FeynHiggs, CPSuperH
SusHi+2HDMC
HIGLU+HDECAY

Dittmaier

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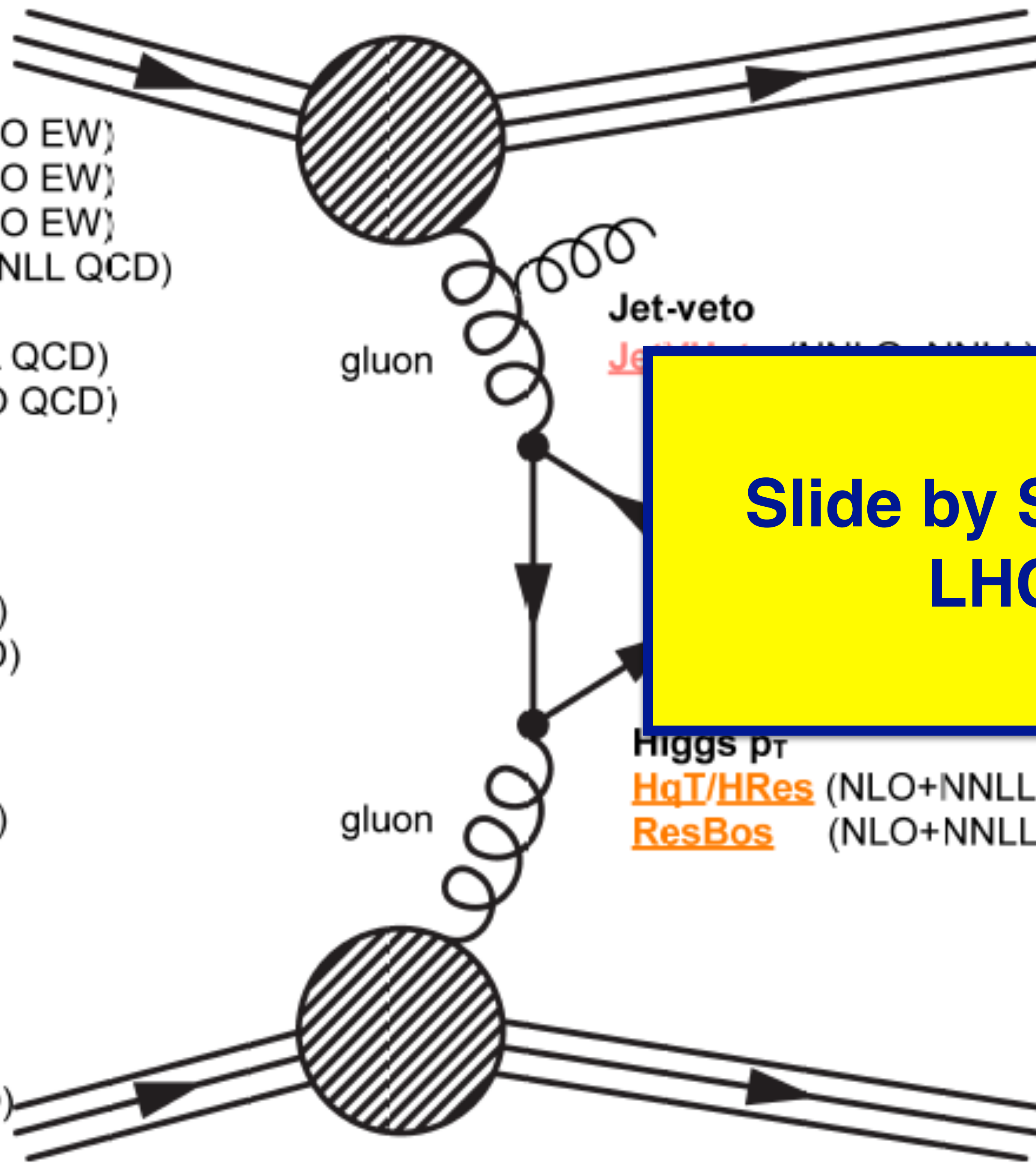
VH@NNLO (NNLO)

ttH
HQQ (LO QCD)

bbH
bbh@NNLO (NNLO QCD)

HH
HPAIR (NLO QCD)

+ private codes.



Jet-veto

Slide by S. Dittmaier, LHCP '14

Higgs p_T
HqT/HRes (NLO+NNLL)
ResBos (NLO+NNLL)

PDF: MSTW, CTEQ, NNPDF, etc.
LHAPDF, HOPPET, APFEL

NLO MC
POWHEG MiNLO
MadGrapn5_aMC@NLO
SHERPA MEPS@NLO

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Higgs Properties
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MG5_aMC@NLO (HC)

MSSM/2HDM
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Dittmaier

Tools for Higgs Physics

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iHixs (NNL
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ggHiggs (approx. NNNLO QCD)

VB
VBF@NNLO (NNLO QCD)

W
VH@NNLO (NNLO)

ttH
HQQ (LO

bbH
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HH
HPAIR (NLO

+ private cod

NLO MC
POWHEG MiNLO
MadGrapn5_aMC@NLO
SHERPA MEPS@NLO

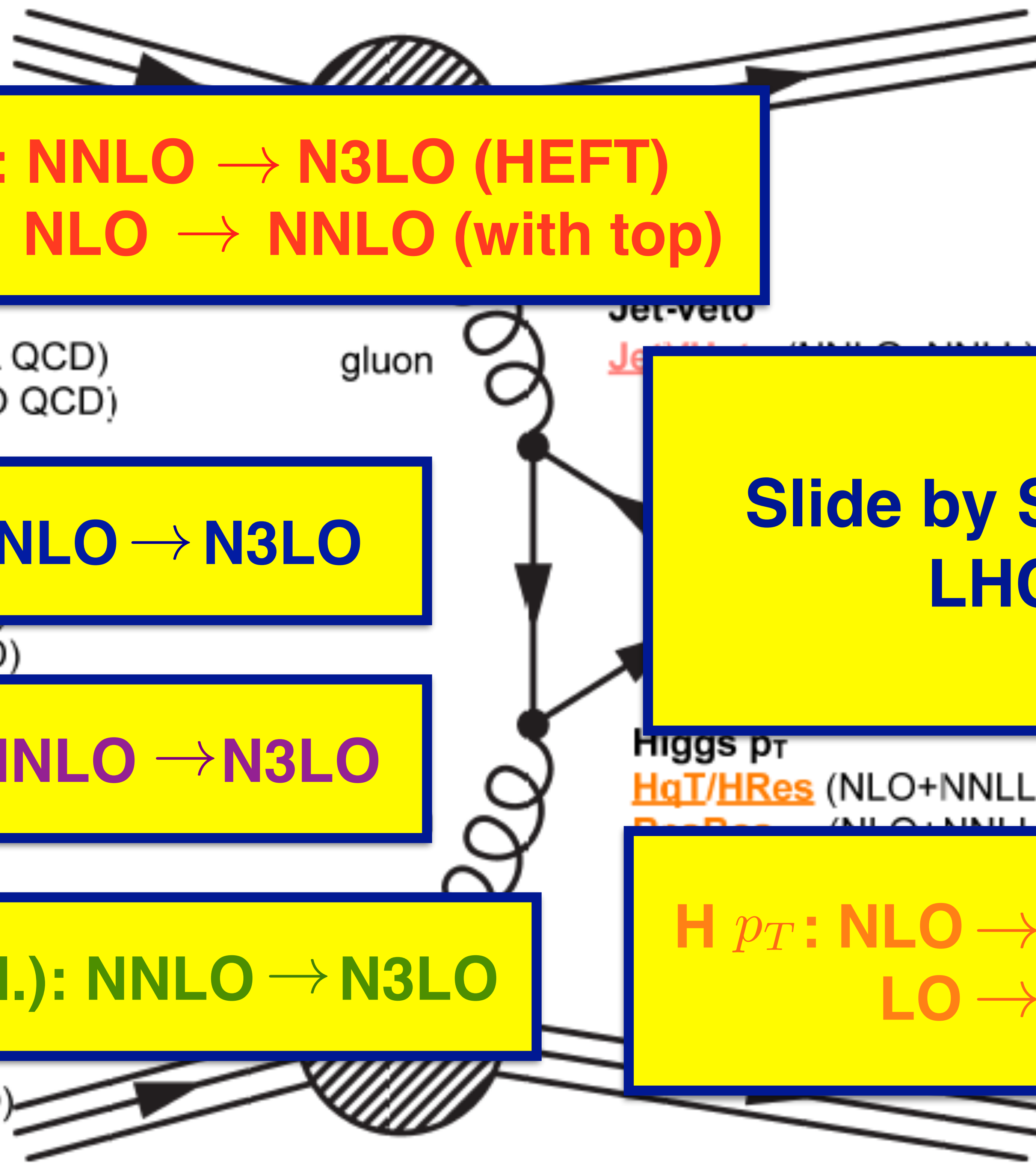
LO MC
gg2VV

NLO ME
MCFM, MG5_aMC@NLO

Jet-veto

Higgs p_T
HqT/HRes (NLO+NNLL)

FeynHiggs, CPSuperH
SusHi+2HDMC



ggH: NNLO → N3LO (HEFT)
NLO → NNLO (with top)

VBF-DIS: NNLO → N3LO

WH (incl.): NNLO → N3LO

bbH (incl.): NNLO → N3LO

ggHH: NLO → N3LO (HEFT); LO → NLO (with top)

Slide by S. Dittmaier, LHCP ‘14

H p_T: NLO → NNLO (HEFT)
LO → NLO (with top)

tmaier

Conclusion

- We have entered a new era for fixed-order computations for the LHC!
  - ➡ First NNLO result for 2-to-3 processes start to appear.
  - ➡ New results for observables at N3LO.
  - ➡ Impact of these higher-order calculations not negligible!
- Important questions (not covered here):
  - ➡ PDFs?
  - ➡ Matching NNLO computations to parton showers?
  - ➡ EW/mixed QCD-EW corrections?