Probing the SMEFT using the electroweak sector

Shankha Banerjee

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(with R. S. Gupta, J. Reiness, O. Ochoa-Valeriano, E. Venturini, S. Seth, C. Englert, and M. Spannowsky)
Motivations for Effective Field Theories (EFTs)

Several experimental and theoretical considerations do not support the SM as a complete theory. LHC has not yet found conclusive evidence of any BSM physics.

- Two phenomenological approaches:
  - *Model dependent*: study the signatures of each model individually
  - *Model independent*: low energy effective theory formalism – analogous to Fermi’s theory of beta decay

- The SM here is a low energy effective theory valid below a cut-off scale $\Lambda$

- A bigger theory (either weakly or strongly coupled) is assumed to supersede the SM above the scale $\Lambda$

- At the perturbative level, all heavy ($> \Lambda$) DOF are decoupled from the low energy theory (Appelquist-Carazzone theorem)

- Appearance of HD operators in the effective Lagrangian valid below $\Lambda$

$$\mathcal{L} = \mathcal{L}^{d=4}_{\text{SM}} + \sum_{d \geq 5} \sum_i \frac{f_i}{\Lambda^{d-4}} O^{d}_i$$

[EFT plot courtesy F. Riva]
Classification of anomalous Higgs interactions

The following terms are not constrained by LEP. First time probed at the LHC

\[ \mathcal{L}^\text{primary}_h = g^h_{VV} h \left[ W^+ \mu W^-_\mu + \frac{1}{2c^2_\theta_W} Z^\mu Z_\mu \right] + g_{3h} h^3 + g^h_{ff} (h \bar{f}_L f_R + h.c.) \]

\[ + \kappa_{GG} \frac{h}{v} G^{A\mu\nu} G^A_{\mu\nu} + \kappa_{\gamma\gamma} \frac{h}{v} A^{\mu\nu} A_{\mu\nu} + \kappa_{Z\gamma} t_\theta W \frac{h}{v} A^{\mu\nu} Z_{\mu\nu}, \]

In contrast, the following interactions were constrained by LEP

\[ \Delta \mathcal{L}_h = \delta g^h_{ZZ} \frac{v}{2c^2_\theta_W} h Z^\mu Z_\mu + g^h_{Zff} \frac{h}{2v} (Z_\mu J^\mu_N + h.c.) + g^h_{Wff} \frac{h}{v} (W^{\mu}_\mu J^\mu_C + h.c.) \]

\[ + \kappa_{WW} \frac{h}{v} W^+ \mu W^-_{\mu\nu} + \kappa_{ZZ} \frac{h}{v} Z^{\mu\nu} Z_{\mu\nu}, \]

[Pomarol, 2014]
Couplings constrained by LEP

The coefficients of the following

\[
\Delta \mathcal{L}_h = \delta g_{ZZ}^h \frac{v}{2c_{\theta_W}^2} h Z^\mu Z_\mu + g_{Zff}^h \frac{h}{2v} \left( Z_\mu J_\mu^N + h.c. \right) + g_{Wff'}^h \frac{h}{v} \left( W_\mu^+ J_C^\mu + h.c. \right) \\
+ \kappa_{WW} \frac{h}{v} W^{+\mu\nu} W_{\mu\nu} - \kappa_{ZZ} \frac{h}{v} Z^{\mu\nu} Z_{\mu\nu} ,
\]

can be written as

\[
\delta g_{ZZ}^h = \delta g_1^Z e^2 - \delta \kappa_{\gamma} \frac{e^2}{c_{\theta_W}^2} \\
g_{Zff}^h = 2 \delta g_{ff}^Z - 2 \delta g_1^Z (g_f^Z c_{2\theta_W} + e Q_f s_{2\theta_W}) + 2 \delta \kappa_{\gamma} Y_f \frac{e s_{\theta_W}}{c_{\theta_W}^3} , \\
\kappa_{ZZ} = \frac{1}{2c_{\theta_W}^2} (\delta \kappa_{\gamma} + \kappa_{Z\gamma} c_{2\theta_W} + 2 \kappa_{\gamma} c_{\theta_W}^2) ,
\]

\[\text{[Gupta, Pomarol, Riva, 2014]}\]
Proof of principle

- If one of these predictions is not confirmed then either
- Our Higgs is not a part of the doublet
- $\Lambda$ may not be very high and D8 operators need to be seriously considered
The multiple dimensions and the multiple variables

- For each process, we have **various EFT coefficients** contributing, which form our **various dimensions**

- Linear combinations of various coefficients can contribute to a vertex deformation

- The multiple variables are our observables, like the invariant masses, transverse momenta, angular variables, etc

- The full differential information can be carried in the relevant angles and the total energy pumped in the process


- The different regimes: **Total rates**, **High energy tails**, and **Angular observables**
Precision physics in the gauge-Higgs sector
(Standard Model Effective Field Theory)

Higgs-strahlung, Weak Boson Fusion, Gluon Fusion
### The four dibosonic channels

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>High-energy primaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{u}_Ld_L \rightarrow W_LZ_L, W_Lh$</td>
<td>$\sqrt{2}a_q^{(3)}$</td>
</tr>
<tr>
<td>$\bar{u}_Lu_L \rightarrow W_LW_L$</td>
<td>$a_q^{(1)} + a_q^{(3)}$</td>
</tr>
<tr>
<td>$\bar{d}_Ld_L \rightarrow Z_Lh$</td>
<td>$a_q^{(1)} - a_q^{(3)}$</td>
</tr>
<tr>
<td>$\bar{f}_Rf_R \rightarrow W_LW_L, Z_Lh$</td>
<td>$a_f$</td>
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</tr>
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</tr>
<tr>
<td>$\bar{f}_Rf_R \rightarrow W_LW_L, Z_Lh$</td>
<td>$g_{Z_{fR}f_R}^h$</td>
</tr>
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</table>

**VH** and **VV** channels are entwined by symmetry and they constrain the same set of observables at High energies but may have different directions [Franceschini, Panico, Pomarol, Riva, Wulzer, 2017 & SB, Gupta, Reiness, Seth, Spannowsky]
Lagrangian: Higgs-Strahlung, Weak-Boson Fusion and Gluon Fusion

\[ \Delta \mathcal{L}_6 \supset \delta \hat{g}^h_{WW} \frac{2m^2_W}{v} hW^+ \mu W^- + \delta \hat{g}^h_{ZZ} \frac{2m^2_Z}{v} h \frac{Z^\mu Z_\mu}{2} + \delta g^W_Q (W^+ \bar{u}_L \gamma^\mu d_L + h.c.) \\
+ \delta g^W_L (W^+ \bar{\nu}_L \gamma^\mu e_L + h.c.) + \frac{g^h_{WL}}{v} (W^+ \bar{\nu}_L \gamma^\mu e_L + h.c.) \\
+ g^h_{WQ} \frac{h}{v} (W^+ \bar{u}_L \gamma^\mu d_L + h.c.) + \sum_f \delta g^Z_f Z_\mu \bar{f} \gamma^\mu f + \sum_f g^h_{Zf} \frac{h}{v} Z_\mu \bar{f} \gamma^\mu f \\
+ \kappa_{WW} \frac{h}{v} W^{\mu\nu} W^-_{\mu\nu} + \tilde{\kappa}_{WW} \frac{h}{v} W^{+\mu\nu} \bar{W}^-_{\mu\nu} + \kappa_{ZZ} \frac{h}{2v} Z^{\mu\nu} Z_{\mu\nu} \\
+ \tilde{\kappa}_{ZZ} \frac{h}{2v} Z^{\mu\nu} \tilde{Z}_{\mu\nu} + \kappa_{Z\gamma} \frac{h}{v} A^{\mu\nu} Z_{\mu\nu} + \tilde{\kappa}_{Z\gamma} \frac{h}{v} A^{\mu\nu} \tilde{Z}_{\mu\nu} + \delta \hat{g}^h_{bb} \frac{\sqrt{2m_b}}{v} h b \bar{b} \\
+ \delta \hat{g}^h_{bb} \frac{\sqrt{2m_b}}{v} h b \bar{b} + \kappa_{\gamma\gamma} \frac{h}{v} A^{\mu\nu} A_{\mu\nu} \]

Specifically for gluon fusion: [Deutschmann, Durh, Maltoni, Vryonidou, 2017]

\[ (1 + \delta \hat{g}^h_{ZZ}) \to (1 + \delta \hat{g}^h_{ZZ})(1 + f(c_{GG}, c_{yb}, c_{yt}, c_{HtG}, c_{HbG})) \]

- The energy growth occurs because there is no propagator
- Terms in blue → deviations in SM amplitude suppressed by $O(m^2_Z/\hat{s})$ w.r.t. $g^h_{WW}$
The EFT space directions

- $\delta g^Z_f$ and $\delta \hat{g}^h_{ZZ} \rightarrow$ deviations in SM amplitude

- These do not grow with energy and are suppressed by $O(m_Z^2/\hat{s})$ w.r.t. $g^h_{Vf}$

- Five directions: $g^h_{Zf}$ with $f = u_L, u_R, d_L, d_R$ and $g^h_{WQ} \rightarrow$ only four operators in Warsaw basis $\rightarrow g^h_{WQ} = c_0 \frac{\hat{g}^h_{ZuL} - \hat{g}^h_{ZdL}}{\sqrt{2}}$

- Knowing proton polarisation is not possible and hence in reality there are two directions Also, upon only considering interference terms, we have

$$g^Z_u = g^h_{ZuL} + \frac{g^Z_{uR}}{g^h_{uL}} g^h_{ZuR}$$

$$g^Z_d = g^h_{ZdL} + \frac{g^Z_{dR}}{g^h_{dL}} g^h_{ZdR}$$

$$g^Z_P = g^Z_u + \frac{\mathcal{L}_d(\hat{s})}{\mathcal{L}_u(\hat{s})} g^Z_d$$

$$g^Z_f = g(T^f_3 - Q_f s^2_{\theta_W})/c_{\theta_W}$$

$$g^Z_P = 2\delta g^h_{ZuL} - 1.52 g^h_{ZdL} - 0.90 g^h_{ZuR} + 0.28 g^h_{ZdR} - 0.14 \delta \kappa_\gamma - 0.89 \delta g^Z_{1}$$

**HL-LHC:** $g^h_{ZP} = -0.14 (\delta \kappa_\gamma - \hat{S} + \hat{Y}) - 0.89 \delta g^Z_{1} - 1.3 W$

**FCC-hh:** $g^h_{ZP} = -0.15 (\delta \kappa_\gamma - \hat{S} + \hat{Y}) - 0.97 \delta g^Z_{1} - 1.4 W$
Till now, we have dropped the $gg \rightarrow Zh$ contribution which is $\sim 15\%$ of the $qq$ rate.

It doesn't grow with energy in presence of the anomalous couplings.

We estimate the scale of new physics for a given $\delta g^h_Z f$.

Example: Heavy $SU(2)_L$ triplet (singlet) vector $W' a (Z')$ couples to SM fermion current $\bar{f} \sigma^a \gamma_\mu f \ (\bar{f} \gamma_\mu f)$ with $g_f$ and to the Higgs current $i H^\dagger \sigma^a \gamma_\mu H \ (i H^\dagger \gamma_\mu H)$ with $g_H$.

$$g^h_{Z u L, d L} \sim \frac{g_H g^2 v^2}{2 \Lambda^2},$$

$$g^h_{Z u R, d R} \sim \frac{g_H g^' Y_{u R, d R} v^2}{\Lambda^2},$$

$\Lambda \rightarrow$ mass scale of vector and thus cut-off for low energy EFT.

Assumed $g_f$ to be a combination of $g_B = g' Y_f$ and $g_W = g/2$ for universal case.
Higgs-Strahlung: Operators at play

\[ O_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \]
\[ O_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D_\mu H) \]
\[ O_{Hu} = iH^\dagger D_\mu H\bar{u}_R\gamma^\mu u_R \]
\[ O_{Hd} = iH^\dagger D_\mu H\bar{d}_R\gamma^\mu d_R \]
\[ O_{He} = iH^\dagger D_\mu H\bar{e}_R\gamma^\mu e_R \]
\[ O^{(1)}_{HQ} = iH^\dagger D_\mu H\bar{Q}\gamma^\mu Q \]
\[ O^{(3)}_{HQ} = iH^\dagger \sigma^a D_\mu H\bar{Q}\sigma^a\gamma^\mu Q \]
\[ O^{(1)}_{HL} = iH^\dagger D_\mu H\bar{L}\gamma^\mu L \]
\[ O^{(3)}_{HL} = iH^\dagger \sigma^a D_\mu H\bar{L}\sigma^a\gamma^\mu L \]

\[ O_{HB} = |H|^2 B_{\mu\nu} B^{\mu\nu} \]
\[ O_{HWB} = H^\dagger \sigma^a HW_{\mu\nu} B^{\mu\nu} \]
\[ O_{HW} = |H|^2 W_{\mu\nu} W^{\mu\nu} \]
\[ O_{H\bar{B}} = |H|^2 B_{\mu\nu} \bar{B}^{\mu\nu} \]
\[ O_{H\bar{W}B} = H^\dagger \sigma^a HW_{\mu\nu} \bar{B}^{\mu\nu} \]
\[ O_{H\bar{W}} = |H|^2 W_{\mu\nu} \bar{W}^{a\mu\nu} \]
\[ O_{\gamma_b} = y_b |H|^2 (\bar{Q}Hb_R + h.c.) \]

Table: D6 operators in Warsaw basis contributing to anomalous \( hVV^*/hV\bar{f}f \) couplings.

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\[ \delta g_f^W = \frac{g}{\sqrt{2}} \frac{v^2}{\Lambda^2} c_{\text{HF}}^{(3)} + \frac{\delta m_Z^2}{m_Z^2} \frac{\sqrt{2} g c_{\theta W}^2}{4 s_{\theta W}^2}, \quad \text{where} \quad \frac{\delta m_Z^2}{m_Z^2} = \frac{v^2}{\Lambda^2} \left(2 t_{\theta W} c_{WB} + \frac{c_{HD}}{2} \right) \]

\[ g_{Wf}^h = \sqrt{2} g \frac{v^2}{\Lambda^2} c_{\text{HF}}^{(3)}, \quad \delta \hat{g}_{WW}^h = \frac{v^2}{\Lambda^2} \left( c_H \square - \frac{c_{HD}}{4} \right) \]

\[ \delta g_f^Z = -\frac{g' Y_f}{c_{\theta W}} c_{WB} \frac{v^2}{\Lambda^2} - \frac{g}{c_{\theta W}} \frac{v^2}{\Lambda^2} \left( |T_3^f| c_{\text{HF}}^{(1)} - T_3^f \ c_{\text{HF}}^{(3)} + (1/2 - |T_3^f|) c_{Hf} \right) c_{\theta W} \]

\[ + \frac{\delta m_Z^2}{m_Z^2} \frac{g}{2 c_{\theta W} s_{\theta W}^2} \left( T_3 c_{\theta W}^2 + Y_f s_{\theta W}^2 \right) \]

\[ \delta \hat{g}_{ZZ}^h = \frac{v^2}{\Lambda^2} \left( c_H \square + \frac{c_{HD}}{4} \right), \quad g_{ZF}^h = -\frac{2 g}{c_{\theta W}} \frac{v^2}{\Lambda^2} \left( |T_3^f| c_{\text{HF}}^{(1)} - T_3^f \ c_{\text{HF}}^{(3)} + (1/2 - |T_3^f|) c_{Hf} \right) \]

\[ \kappa_{WW} = \frac{2 v^2}{\Lambda^2} c_{HW}, \quad \tilde{\kappa}_{WW} = \frac{2 v^2}{\Lambda^2} c_{H\bar{W}}, \quad \kappa_{ZZ} = \frac{2 v^2}{\Lambda^2} \left( c_{\theta W}^2 c_{HW} + s_{\theta W}^2 c_{HB} + s_{\theta W} c_{\theta W} c_{HWB} \right) \]

\[ \tilde{\kappa}_{ZZ} = \frac{2 v^2}{\Lambda^2} \left( c_{\theta W}^2 c_{H\bar{W}} + s_{\theta W}^2 c_{HB} + s_{\theta W} c_{\theta W} c_{H\bar{W}B} \right) \]

\[ \delta \hat{g}_{bb}^h = -\frac{v^2}{\Lambda^2} \frac{v}{\sqrt{2} m_b} c_{yb} + \frac{v^2}{\Lambda^2} \left( c_H \square - \frac{c_{HD}}{4} \right) \]
Higgs-strahlung: Differential in angles and energy

- $\varphi$, $\Theta$ and \{x, y, z\} in $Vh$ CoM frame ($z$ identified as direction of $V$-boson; $y$ identified as normal to the plane of $V$ and beam axis; $x$ defined to complete the right-handed set), $\theta$ in $V$ CoM frame

- Q: How much differential information can one extract from this process?

- For three body phase space, $3 \times 3 - 4 = 5$ kinematic variables completely define final state

- Barring boost factor, the variables are $\sqrt{s}, \Theta, \theta, \varphi$

- Considering 10 bins per variable $\rightarrow$ 1000 numbers per energy bin to obtain full information $\rightarrow$ can be reduced to 9 per energy bin

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Differential in energy and angles: $Vh \rightarrow 2\ell b\bar{b}$ and $ggF$

$(h \rightarrow ZZ^* \rightarrow 4\ell)$

$$Vh \left( \frac{d\sigma}{dEd\Theta d\theta d\phi} \right)$$

$$ggF \left( \frac{d\sigma}{dEd\theta_1 d\theta_2 d\phi} \right)$$

\[
\begin{align*}
  f_{LL} &= S_\Theta^2 S_\Theta^2, \\
  f_{LT}^1 &= C_\Theta C_\theta, \\
  f_{TT}^1 &= (1 + C_\Theta^2)(1 + C_\theta^2), \\
  f_{LT}^2 &= C_\varphi S_\Theta S_\theta, \\
  f_{TT}^2 &= C_\varphi S_\Theta S_\theta C_\Theta C_\theta, \\
  \tilde{f}_{LT}^1 &= S_\varphi S_\Theta S_\theta, \\
  \tilde{f}_{TT}^2 &= S_\varphi S_\Theta S_\theta C_\Theta C_\theta, \\
  f_{TT'} &= C_{2\varphi} S_\Theta^2 S_\theta^2, \\
  \tilde{f}_{TT'} &= S_{2\varphi} S_\Theta^2 S_\theta^2,
\end{align*}
\]
An analog of **Fourier analysis** utilised to extract the aforementioned angular moments.

Our squared amplitude can be parametrised as,

\[ |A|^2 = \sum_i a_i(E) f_i(\Theta, \theta, \varphi) \]

We look for weight functions, \( w_i(\Theta, \theta, \varphi) = \lambda_{ij} f_j(\Theta, \theta, \varphi) \), such that

\[ \int_0^\pi d\theta \int_0^\pi d\Theta \int_0^{2\pi} d\varphi f_i w_j \sin \theta \sin \Theta = \delta_{ij} \]
For the set of basis functions, we get the following matrix

$$ M = \begin{pmatrix}
\frac{512\pi}{225} & 0 & \frac{128\pi}{25} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{8\pi}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{128\pi}{25} & 0 & \frac{6272\pi}{225} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{16\pi}{9} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{16\pi}{225} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{16\pi}{9} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{256\pi}{225} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{256\pi}{225}
\end{pmatrix} $$

$w_i \propto f_j$ except for $i = 1, 3$

We rotate the (1,3) system to an orthogonal basis

$$ \hat{M} = \text{diag}\left( \frac{64\pi}{225}, \frac{8\pi}{9}, \frac{64\pi}{225}, \frac{16\pi}{9}, \frac{16\pi}{225}, \frac{16\pi}{9}, \frac{256\pi}{225}, \frac{256\pi}{225} \right) $$

with $\xi_\pm = (53 \pm 9\sqrt{29})$

Using discrete method, we find: $w_i = \hat{M}_{ij}^{-1} f_j$ and $\{ \hat{a}_1, a_{TT}, \hat{a}_3, a_{LT}, a_{LT}^2, \hat{a}_{LT}, \hat{a}_{LT}^2, a_{TT}', \hat{a}_{TT}' \}$

Events divided in bins of final state invariant mass ($M \rightarrow$ central value of bin), $N(M)(N(\hat{M})) \rightarrow$ number of MC (actual) events in that bin for a fixed integrated luminosity
Differential in angles: Constraining the LT terms \((Vh)\)

| \(a_{LL}\) | \(\frac{G^2}{4} \left[ 1 + 2\delta \hat{g}^h_{VV} + 4\kappa VV + 2\delta g^Z_f + \frac{\hat{g}^h_{Vf}}{g_f} (-1 + 4\gamma^2) \right] \) |
| \(a_{1TT}\) | \(\frac{G^2 \sigma e_{RL}}{2\gamma^2} \left[ 1 + 4 \left( \frac{\hat{g}^h_{Vf}}{g_f} + \kappa VV \right) \gamma^2 \right] \) |
| \(a_{2TT}\) | \(\frac{G^2}{8\gamma^2} \left[ 1 + 4 \left( \frac{\hat{g}^h_{Vf}}{g_f} + \kappa VV \right) \gamma^2 \right] \) |
| \(a_{1LT}\) | \(- \frac{G^2 \sigma e_{RL}}{2\gamma} \left[ 1 + 2 \left( \frac{2g^h_f}{g_f} + \kappa VV \right) \gamma^2 \right] \) |
| \(a_{2LT}\) | \(- \frac{G^2}{2\gamma} \left[ 1 + 2 \left( \frac{2g^h_f}{g_f} + \kappa VV \right) \gamma^2 \right] \) |
| \(\tilde{a}_{1LT}\) | \(- G^2 \sigma e_{RL} \hat{\kappa} VV \gamma \) |
| \(\tilde{a}_{2LT}\) | \(- G^2 \hat{\kappa} VV \gamma \) |
| \(a_{TT}'\) | \(\frac{G^2}{8\gamma^2} \left[ 1 + 4 \left( \frac{\hat{g}^h_{Vf}}{g_f} + \kappa VV \right) \gamma^2 \right] \) |
| \(\tilde{a}_{TT}'\) | \(\frac{G^2}{2} \hat{\kappa} VV \) |

**Table:** Contribution of the different anomalous couplings to the angular coefficients up to linear order. Contributions subdominant in \(\gamma = \sqrt{s}/(2m_V)\) are neglected, with the exception of the next-to-leading EFT contribution to \(a_{LL}\), which we retain in order to keep the leading effect of the \(\delta \hat{g}^h_{VV}\) term. 

\(G = gg_f^Z \sqrt{(g^Z_{lL})^2 + (g^Z_{lR})^2}/(c_{\theta_W} \Gamma_Z)\).
As anticipated, the parametrically-largest contribution is to the LT interference terms

\[ \frac{a_{LT}^2}{4} \cos \varphi \sin 2\theta \sin 2\Theta + \frac{\tilde{a}_{LT}^2}{4} \sin \varphi \sin 2\theta \sin 2\Theta \]

These terms vanish on integration of any angle.

Q: How to probe \( \kappa_{ZZ} \) and \( \tilde{\kappa}_{ZZ} \)?

A: Simplified approach → Flip sign in regions to maintain positive \( \sin 2\theta \sin 2\Theta \)

A: Sophisticated approach → Use method of moments

Expect \( \cos \varphi \) distribution for CP-even and \( \sin \varphi \) distribution for CP-odd
Q: Are the LO theoretical shapes preserved upon the inclusion of NLO effects, radiations, showering, experimental cuts, etc.?
A: For the azimuthal angles, they are.

[SB, Gupta, Reiness, Spannowsky; 2019], [SB, Gupta, Reiness, Seth, Spannowsky; 2019]
Differential in Energy: Constraining the Contact terms

Two-parameter $\chi^2$-fit (at 300 fb$^{-1}$) in $\delta g_Z^2 - (\delta \kappa \gamma - \hat{S})$ plane

From Zh

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Our 100 TeV Projection</th>
<th>Our 14 TeV Projection</th>
<th>LEP Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0003 \pm 0.0001$</td>
<td>$\pm 0.002 \pm 0.0007$</td>
<td>$-0.0026 \pm 0.0032$</td>
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<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0003 \pm 0.0001$</td>
<td>$\pm 0.003 \pm 0.001$</td>
<td>$0.0023 \pm 0.002$</td>
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<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0005 \pm 0.0002$</td>
<td>$\pm 0.005 \pm 0.001$</td>
<td>$-0.0036 \pm 0.0070$</td>
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<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0015 \pm 0.0006$</td>
<td>$\pm 0.016 \pm 0.005$</td>
<td>$0.016 \pm 0.0104$</td>
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<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0005 \pm 0.0002$</td>
<td>$\pm 0.005 \pm 0.001$</td>
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<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0035 \pm 0.0015$</td>
<td>$\pm 0.032 \pm 0.009$</td>
<td>$-0.016 \pm 0.085$</td>
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<tr>
<td>$\delta \kappa \gamma$</td>
<td>$\pm 0.0035 \pm 0.0002$</td>
<td>$\pm 0.003 \pm 0.001$</td>
<td>$0.0004 \pm 0.0007$</td>
</tr>
<tr>
<td>$\hat{S}$</td>
<td>$\pm 0.0035 \pm 0.0015$</td>
<td>$\pm 0.032 \pm 0.009$</td>
<td>$0.0003 \pm 0.0006$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\pm 0.0004 \pm 0.0002$</td>
<td>$\pm 0.003 \pm 0.001$</td>
<td>$0.0000 \pm 0.0006$</td>
</tr>
</tbody>
</table>

Directions from VBF, Zh, Wh, and WZ

1. $\left| -0.04 \ c_Q^1 + 1.4 \ c_Q^{(3)} + 0.1 \ c_{uR} - 0.03 \ c_{dR} \xi \right| < 0.003$ [VBF]
2. $\left| -0.18 \ c_Q^1 + 1.3 \ c_Q^{(3)} + 0.3 \ c_{uR} - 0.1 \ c_{dR} \xi \right| < 0.0005$ [Zh]
3. $\left| c_Q^{(3)} \xi \right| < 0.0004$ [Wh]
4. $\left| c_Q^{(3)} \xi \right| < 0.0003$ [WZ]

WZ analysis: [Franceschini et al, 2017]

- $\sigma_{ZH}/\sigma_{Zb\bar{b}}$ without cuts $\sim 4.6/165$
- With the cut-based analysis $\rightarrow 0.26$
- With BDT optimisation $\rightarrow 0.50$ (multiple distributions trained)

- $S/B$ changes from 1/40 to $O(1)$ $\rightarrow$ Close to 35 SM $Zh(\ell^+\ell^-)$ events left at 300 fb$^{-1}$ [SB, Englert, Gupta, Spannowsky, 2018]

Blue dashed line $\rightarrow$ direction of accidental cancellation of interference term; Gray region: LEP exclusion; pink band: exclusion from WZ [Franceschini et al, 2017]; Blue region: exclusion from Zh; dark (light) shade represents bounds at 3 ab$^{-1}$ (300 fb$^{-1}$) luminosity; Green region: Combined bound from Zh and WZ [SB, Englert, Gupta, Spannowsky, 2018]. Black lines show direction from VBF analysis in di-photon channel.
Results: Contact terms, $Zh$, $Wh$, $Zh + Wh$ combination

- The four-point contact vertex is constrained upon using the $E^2$ dependent terms.
- The $a_{LL}$ term dominates at high energies $\rightarrow |g^h_{WQ}| < 6 \times 10^{-4}$ and $\rightarrow |g^h_{Zf}| < 4 \times 10^{-4}$ at $\mathcal{L} = 3 \text{ ab}^{-1}$.

- Method of moments used to constrain the other couplings.
- We obtain percent level bounds on $\kappa_{WW}$ and in the $(\kappa_{ZZ}, \delta_{hZZ})$ plane.
- Competitive and complementary bounds to previous analyses.
- Independent bound on the $CP$-odd coupling, $|\tilde{\kappa}^{P}_{ZZ}| < 0.03$.

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- Upon assuming a linearly realised electroweak symmetry and correlations, we can combine the above bounds.
Angular moments: Gluon Fusion in golden channel

- Angular differential distributions, modified in the EFT

\[
\begin{align*}
a_1 &= g^4 \left( (1 + \delta a) + \frac{bm_Z \gamma_b^2}{m_Z \gamma_a} \right)^2 \\
a_2 &= g^4 \left( \frac{(1 + \delta a)^2}{2 \gamma_a^2} + \frac{2c^2 m_Z^2 \gamma_b^2}{m_Z^2 \gamma_a^2} \right) \\
a_3 &= -g^4 \left( \frac{1 + \delta a}{2 \gamma_a} + \frac{bm_Z \gamma_b^2}{2m_Z \gamma_a} \right)^2 \\
a_4 &= g^4 \left( \frac{(1 + \delta a)^2}{2 \gamma_a^2} - \frac{2c^2 m_Z^2 \gamma_b^2}{m_Z^2 \gamma_a^2} \right) \\
a_5 &= -\epsilon^2 g^4 \left( \frac{2(1 + \delta a)^2}{\gamma_a} + \frac{2(1 + \delta a)bm_Z \gamma_b^2}{m_Z \gamma_a^2} \right) \\
a_6 &= \epsilon^2 g^4 \left( \frac{2(1 + \delta a)^2}{\gamma_a^2} + \frac{8c^2 m_Z^2 \gamma_b^2}{m_Z^2 \gamma_a^2} \right) \\
a_7 &= g^4 \frac{2(1 + \delta a)cm_Z \gamma_b}{m_Z \gamma_a^2} \\
a_8 &= -\epsilon^2 g^4 \left( \frac{4(1 + \delta a)cm_Z \gamma_b}{m_Z \gamma_a} + \frac{4bm_Z^2 \gamma_b^3}{m_Z^2 \gamma_a^2} \right) \\
a_9 &= g^4 \left( \frac{(1 + \delta a)cm_Z \gamma_b}{m_Z \gamma_a} + \frac{bcm_Z^2 \gamma_b^3}{m_Z^2 \gamma_a^2} \right) \\
\end{align*}
\]

$1 \rightarrow$ SM

\[
\begin{align*}
\delta a &= \delta g_{ZZ}^h - \kappa_{ZZ} \gamma_a \frac{m_Z^*}{m_Z} \frac{m_Z^2 - m_{Z^*}^2}{2m_Z^2} \\
b &= \kappa_{ZZ} \\
c &= -\frac{\tilde{\kappa}_{ZZ}}{2}
\end{align*}
\]

\[
\begin{align*}
g^4 &= ((g_{tL}^Z)^2 + (g_{tR}^Z)^2)((g_{tL}^{Z^*})^2 + (g_{tR}^{Z^*})^2) \\
\epsilon^2 g^4 &= ((g_{tL}^Z)^2 - (g_{tR}^Z)^2)((g_{tL}^{Z^*})^2 - (g_{tR}^{Z^*})^2),
\end{align*}
\]

Small $\rightarrow a_5, a_6$ and $a_8$ suppressed

$a_7$, $a_8$, $a_9$ CP-odd

[Slide courtesy Elena Venturini]
Results: Gluon Fusion in golden channel

- MC estimated moments: $a_i = \hat{N}\bar{w}_i$ with $\bar{w}_i = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} w_i(\theta_1,n, \theta_2,n, \phi_n)$ and $\hat{N} = 3 ab^{-1}$

- $\chi^2(\delta g_{ZZ}^h, \kappa_{ZZ}) = \sum_{ij} (a_{i}^{EFT} - a_{i}^{SM}) \Sigma_{ij}^{-1} (a_{j}^{EFT} - a_{j}^{SM})$ with

  $\Sigma_{ij} = \left( \frac{\hat{N}_{SM}}{N_{SM}} \right)^2 + \kappa_{syst}^2 \sigma_{ij}^{SM}$

- With systematic uncertainty of $\kappa_{syst.} = 0.02$, the 68% bounds on CP-even couplings are shown as [Combination with $Vh$ (green) and $WW$ (yellow) [ATL-PHYS-PUB-2018-054]

- For $\hat{g}_{h}^h$, we obtain a bound $|\kappa_{ZZ}| < 0.05$ comparing to MELA’s bound of 0.04 [Anderson, et al 2013]

![Graph showing total rate and $\kappa_{ZZ}$ vs $\hat{g}_{h}^h$]
NLO EW corrections to EFT have been studied by a few groups [Hartmann, Trott; 2017 for $h \rightarrow \gamma \gamma$], [Vryonidou, Zhang; 2018 for Higgs production and decay].

The top sector and the Higgs+EW sectors are usually treated separately in SMEFT studies. But how much is it justified?

$$\sigma = C_H(\mu_{EFT})\sigma_{\text{tree}} + C_t\frac{\alpha_{EW}}{\pi}\left(\log\frac{Q^2}{\mu_{EFT}^2}\sigma_{\text{log}} + \sigma_{\text{fin}}\right)$$

Operators entering at NLO

$$O_{t\varphi} = \bar{Q}t\bar{\varphi}\left(\varphi^\dagger\varphi\right) + h.c., \quad O_{\varphi Q}^{(1)} = (\varphi^\dagger i\not\!D_\mu\varphi)(\bar{Q}\gamma^\mu Q),$$

$$O_{\varphi Q}^{(3)} = (\varphi^\dagger i\not\!D_\mu\varphi)(\bar{Q}\gamma^\mu\tau^I Q), \quad O_{\varphi t} = (\varphi^\dagger i\not\!D_\mu\varphi)(\bar{t}\gamma^\mu t),$$

$$O_{\varphi tb} = (\varphi^\dagger iD_\mu\varphi)(\bar{t}\gamma^\mu b) + h.c., \quad O_{tW} = (\bar{Q}\sigma^{\mu\nu}\tau^I t)\varphi W_{\mu\nu}^I + h.c.,$$

$$O_{tB} = (\bar{Q}\sigma^{\mu\nu}t)\varphi B_{\mu\nu} + h.c.,$$

Operators entering at LO

$$O_{\varphi WB} = \varphi^\dagger\tau^IB_{\mu\nu}W_{\mu\nu}^I, \quad O_{\varphi W} = \varphi^\dagger\varphi W_{\mu\nu}^I W^{I\mu\nu},$$

$$O_{\varphi B} = \varphi^\dagger\varphi B_{\mu\nu} B^{\mu\nu}, \quad O_{\varphi \Box} = \left(\varphi^\dagger\varphi\right) \Box \left(\varphi^\dagger\varphi\right),$$

$$O_{\varphi D} = \left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right), \quad O_W = iD^\mu\varphi^\dagger\tau^I D^\nu\varphi W_{\mu\nu}^I,$$

$$O_B = iD^\mu\varphi^\dagger D^\nu\varphi B_{\mu\nu}, \quad O_{b\varphi} = (\varphi^\dagger\varphi)\bar{Q}b\varphi,$$

$$O_{\mu\varphi} = (\varphi^\dagger\varphi)\bar{l}_2 e_2 \varphi, \quad O_{\tau\varphi} = (\varphi^\dagger\varphi)\bar{l}_3 e_3 \varphi,$$
Higher order EW corrections: SMEFT

[Vryonidou, Zhang; 2018 for Higgs production and decay]

\[\text{WH, ZH} \]
\[\text{VBF} \]
\[\text{H} \rightarrow \mu\mu, \tau\tau \]
\[\text{W, Z masses, oblique parameters} \]
\[\text{H} \rightarrow \gamma\gamma, \gamma Z \]
\[\text{H} \rightarrow ZZ, Wl\nu \]
\[\text{H} \rightarrow bb \]
\[\mu \text{ decay} \]
Higher order EW corrections: SMEFT

[Vryonidou, Zhang; 2018 for Higgs production and decay]

\[pp \to ZH \text{ LHC13} \]

\[\mu_{\text{EFT}} = 1 \text{ TeV} \]

\[pp \to ZH \text{ LHC13} \]

\[\mu_{\text{EFT}} = 125 \text{ GeV} \]

\[m(ZH) \text{ [GeV]} \]

\[C(1 \text{ TeV}/\Lambda^2) \]

- Current individual
- Loop-induced individual $\mu_{\text{EFT}}=M_H$
- Loop-induced individual $\mu_{\text{EFT}}=\Lambda$
- Loop-induced marginalized $\mu_{\text{EFT}}=M_H$
- Loop-induced marginalized $\mu_{\text{EFT}}=\Lambda$
Higher order EW corrections: SMEFT

[Vryonidou, Zhang; 2018 for Higgs production and decay]

- Top-quark operators can shift signal strength of loop-processes ($gg \rightarrow h$, $h \rightarrow \gamma\gamma$, $h \rightarrow Z\gamma$) by $\mathcal{O}(1) - \mathcal{O}(10)$
- For the tree level processes (production and decay) the shift can be between 5%-10% at LHC and up to 15% at future lepton colliders
- In a global fit for Higgs+EW couplings, theoretical uncertainties from top-quark operators must be included

![Plot of H→ZZ and ZH](image.png)
Summary and conclusions

- EFT’s essence shows that many anomalous Higgs couplings were already constrained by LEP through Z-pole and di-boson measurements.

- The full $hZZ$ tensor structure can be disentangled by using fully differential information and sophisticated techniques like the Method of moments.

- $Zh$, $Wh$, $WW$ and $WZ$ are important channels to disentangle various directions in the EFT space. They are intrinsically correlated.

- Studying complementary directions like the $WBF$ is also important.

- Multiple dimensions come about from the various correlated EFT coefficients. Blind directions need to be broken.

- Multi-variate analyses like BDT, MoM, OOT, NNs are useful in construction most discriminatory variables.
The story so far

- A \textbf{CP-even} spin zero hypothesis is favoured.
- Still to be measured: \( h \to Z \gamma \), \( h \to \mu^+ \mu^- \), \( \lambda_{hhh}, \lambda_{hhhh} \). New tantalising evidence in \( h \to \ell^+ \ell^- \gamma \).

For a more thorough analysis, EW interplay between Higgs+EW and top sector must be considered [Vryonidou, Zhang; 2018], Higgs without Higgs processes [F. Riva’s talk], EFTs in PDFs [M. Ubiali’s talk], Higgs+EW [Ilaria’s talk].
HD operators

- Higher-dimensional Operators: invariant under SM gauge group
- $d = 5$: Unique operator $\rightarrow$ Majorana mass to the neutrinos:
  \[ \frac{1}{\Lambda} (\Phi^\dagger L)^T C (\Phi^\dagger L) \]
- $d = 6$: 59 = 15 (bosonic) + 19 (single fermionic) + 25 (four fermion) independent $B$-conserving operators. Lowest dimension (after $d = 4$) which induces $HXY$, $HXYZ$ interactions, charged TGCs [W. Buchmuller and D. Wyler; B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek; K.Hagiwara, D. Zeppenfeld et. al., Azatov, et. al., Falkowski, et. al.]
- $d = 7$: Such operators appear in Higgs portal dark matter models
- $d = 8$: Lowest dimension inducing neutral TGC interactions [See Rick’s talk for more details]
Motivations for Effective Field Theories (EFTs)

- Precisely measuring the Higgs couplings → one of the most important LHC goals and that of the FCC-hh as well
- Indirect constraints can constrain much higher scales $S$, $T$ parameters being prime examples
- Q: How well does the FCC-hh compete with LEP in constraining precision physics?
  A: From EFT correlated variables, LEP already constrained certain anomalous Higgs couplings → $Z$-pole measurements, TGCs
  Going to higher energies in FCC-hh is the only way to obtain newer information after the HL-LHC
- EFT techniques show that many Higgs deformations aren’t independent from cTGCs and EW precision which were already constrained at LEP → Same operators affect TGCs and Higgs deformations
**ggF with $h \rightarrow 4\ell$: Operators at play**

\[
\begin{align*}
\mathcal{O}_{H\Box} & = (H^\dagger H)\Box(H^\dagger H) \\
\mathcal{O}_{HD} & = (H^\dagger D_\mu H)^*(H^\dagger D_\mu H) \\
\mathcal{O}_{H\ell} & = iH^\dagger D_\mu H\bar{e}_R\gamma^\mu e_R \\
\mathcal{O}^{(1)}_{HL} & = iH^\dagger D_\mu H\bar{L}\gamma^\mu L \\
\mathcal{O}^{(3)}_{HL} & = iH^\dagger \sigma^a D_\mu H\bar{L}\sigma^a\gamma^\mu L \\
\mathcal{O}_{HtG} & = \tilde{Q}_3\tilde{H} T^A\sigma_{\mu\nu} t_R G^{A\mu\nu} \\
\mathcal{O}_{HbG} & = \tilde{Q}_3\tilde{H} T^A\sigma_{\mu\nu} b_R G^{A\mu\nu} \\
\mathcal{O}_{HG} & = (H^\dagger H)G^{A\mu\nu} G^{A\mu\nu}
\end{align*}
\]

\[
\begin{align*}
\mathcal{O}_{HB} & = |H|^2 B_{\mu\nu} B^{\mu\nu} \\
\mathcal{O}_{HWB} & = H^\dagger \sigma^a H W^a_{\mu\nu} B^{\mu\nu} \\
\mathcal{O}_{HW} & = |H|^2 W_{\mu\nu} W^{\mu\nu} \\
\mathcal{O}_{H\bar{B}} & = |H|^2 B_{\mu\nu} \bar{B}^{\mu\nu} \\
\mathcal{O}_{H\bar{W}B} & = H^\dagger \sigma^a H W^a_{\mu\nu} \bar{B}^{\mu\nu} \\
\mathcal{O}_{H\bar{W}} & = |H|^2 W_{\mu\nu} \bar{W}^{a\mu\nu} \\
\mathcal{O}_{y_b} & = |H|^2 (\tilde{Q}_3 H b_R + h.c.) \\
\mathcal{O}_{y_t} & = |H|^2 (\tilde{Q}_3 H t_R + h.c.)
\end{align*}
\]

**Table:** D6 operators in the Warsaw basis contributing to anomalous $hVV^*/hV\bar{f}f$, effective Higgs-gluon, Yukawa and chromomagnetic couplings.
\[\delta g_Z^\ell = - \frac{g Y_\ell s_{\theta W}}{c_{\theta W}^2} \frac{v^2}{\Lambda^2} c_{HWB} - \frac{g}{c_{\theta W}} \frac{v^2}{\Lambda^2} (|T_3^\ell| c_{HL}^{(1)} - T_3^\ell c_{HL}^{(3)} + (1/2 - |T_3^\ell|) c_{HL}) \]
\[+ \frac{\delta m_Z^2}{m_Z^2} \frac{g}{2c_{\theta W}s_{\theta W}^2} (T_3 c_{\theta W}^2 + Y_\ell s_{\theta W}^2)\]
\[\delta g_{ZZ}^h = \frac{v^2}{\Lambda^2} \left( c_{H\Box} + \frac{c_{HD}}{4} \right)\]
\[g_{Z\ell}^h = - \frac{2g}{c_{\theta W}} \frac{v^2}{\Lambda^2} (|T_3^\ell| c_{HL}^{(1)} - T_3^\ell c_{HL}^{(3)} + (1/2 - |T_3^\ell|) c_{HL})\]
\[\kappa_{ZZ} = \frac{2v^2}{\Lambda^2} \left( c_{\theta W}^2 c_{HW} + s_{\theta W}^2 c_{HB} + s_{\theta W} c_{\theta W} c_{HWB} \right)\]
\[\kappa_{GG} = \frac{2v^2}{\Lambda^2} c_{HG}\]
\[\tilde{\kappa}_{ZZ} = \frac{2v^2}{\Lambda^2} \left( c_{\theta W}^2 c_{H\tilde{W}} + s_{\theta W}^2 c_{H\tilde{B}} + s_{\theta W} c_{\theta W} c_{H\tilde{W}B} \right)\]

\[(m_W, m_Z, \alpha_{em})\] are the input parameters and \[\frac{\delta m_Z^2}{m_Z^2} = \frac{v^2}{\Lambda^2} \left( 2t_{\theta W} c_{HWB} + \frac{c_{HD}}{2} \right)\]
Some notes on BDT/MVA analyses

- BDT algorithms used for all our analyses to discriminate the SM $Vh/VV/ggF$ processes versus the other SM processes.
- Types of variables used: $p_T$, $\Delta R$, $\Delta \Phi$, $\Delta \eta$, $\mathbb{E}_T$, invariant masses, etc.
- Once the trainings (overtraining avoided) are done, the EFT samples are used as signals. The final variable used isn’t used for training, example $M_{Vh}$ etc.
- BDT has difficulty handling negative weight events. For NLO samples dealt with, $-ve/+ve \sim \mathcal{O}(1\%)$. For interference only pieces, the fraction can be larger.
- More involved transformations required rather than the standard ones.
- For NNs, we need to choose a proper crossentropy function:

$$\mathcal{H}(p^{\text{truth}}, p^{\text{pred}}) = -\frac{1}{N} \sum_{i=1}^{N} p_i^{\text{truth}} \log p_i^{\text{pred}}$$

**Loss:**

$$\text{Loss} := \mathcal{H} + \lambda \sum_{i=1}^{N} (||\omega_i||^2 + ||b_i||^2)$$

- $\omega_i, b_i, \lambda \rightarrow$ weight of node, bias, penalty strength
Differential in angles: Reconstruction of angles for $Wh$

- Ambiguity in neutrino $p_z$

Comparison of true and mean inferred $\Theta$

Comparison of true and inferred $\varphi$

Comparison of true and mean inferred $\theta$
VBF: NN setup
Differential in Energy: Constraining the Contact terms

Two-parameter $\chi^2$-fit (at 300 fb$^{-1}$) in $\delta g_Z^1 - (\delta \kappa - \hat{S})$ plane

Blue dashed line $\rightarrow$ direction of accidental cancellation of interference term; Gray region: LEP exclusion; pink band: exclusion from WZ [Franceschini et al., 2017]; Blue region: exclusion from Zh; dark (light) shade represents bounds at 3 ab$^{-1}$ (300 fb$^{-1}$) luminosity; Green region: Combined bound from Zh and WZ [SB, Englert, Gupta, Spannowsky, 2018]. Black lines show direction from VBF analysis in di-photon channel.

From Zh

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Our 100 TeV Projection</th>
<th>Our 14 TeV projection</th>
<th>LEP Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta g_Z^1$</td>
<td>$\pm 0.0003 \ (\pm 0.0001)$</td>
<td>$\pm 0.002 \ (\pm 0.0007)$</td>
<td>$-0.0026 \pm 0.0032$</td>
</tr>
<tr>
<td>$\delta g_Z^2$</td>
<td>$\pm 0.0003 \ (\pm 0.0001)$</td>
<td>$\pm 0.003 \ (\pm 0.001)$</td>
<td>$0.0023 \pm 0.002$</td>
</tr>
<tr>
<td>$\delta g_Z^3$</td>
<td>$\pm 0.0005 \ (\pm 0.0002)$</td>
<td>$\pm 0.005 \ (\pm 0.001)$</td>
<td>$-0.0036 \pm 0.0070$</td>
</tr>
<tr>
<td>$\delta g_Z^4$</td>
<td>$\pm 0.0015 \ (\pm 0.0006)$</td>
<td>$\pm 0.016 \ (\pm 0.005)$</td>
<td>$0.016 \pm 0.0104$</td>
</tr>
<tr>
<td>$\delta g_Z^5$</td>
<td>$\pm 0.0005 \ (\pm 0.0002)$</td>
<td>$\pm 0.005 \ (\pm 0.001)$</td>
<td>$-0.009 \pm 0.043$</td>
</tr>
<tr>
<td>$\delta g_Z^6$</td>
<td>$\pm 0.0005 \ (\pm 0.0002)$</td>
<td>$\pm 0.005 \ (\pm 0.001)$</td>
<td>$-0.016 \pm 0.052$</td>
</tr>
<tr>
<td>$\delta \kappa - \hat{S}$</td>
<td>$\pm 0.0035 \ (\pm 0.0015)$</td>
<td>$\pm 0.032 \ (\pm 0.009)$</td>
<td>$0.0004 \pm 0.0007$</td>
</tr>
<tr>
<td>$\hat{S}$</td>
<td>$\pm 0.0035 \ (\pm 0.0015)$</td>
<td>$\pm 0.032 \ (\pm 0.009)$</td>
<td>$0.0003 \pm 0.0006$</td>
</tr>
<tr>
<td>$W$</td>
<td>$\pm 0.0004 \ (\pm 0.0002)$</td>
<td>$\pm 0.003 \ (\pm 0.001)$</td>
<td>$0.0000 \pm 0.0006$</td>
</tr>
<tr>
<td>$Y$</td>
<td>$\pm 0.0035 \ (\pm 0.0015)$</td>
<td>$\pm 0.032 \ (\pm 0.009)$</td>
<td>$0.0000 \pm 0.0006$</td>
</tr>
</tbody>
</table>

Directions from VBF, Zh, Wh, and WZ

$|(-0.04 \ c_Q^1 + 1.4 \ c_Q^{(3)} + 0.1 \ c_{uR} - 0.03 \ c_{dR})\xi| < 0.003$  
$\ [VBF]$ 

$|(-0.18 \ c_Q^1 + 1.3 \ c_Q^{(3)} + 0.3 \ c_{uR} - 0.1 \ c_{dR})\xi| < 0.0005$  
$\ [Zh]$ 

$| c_Q^{(3)}\xi| < 0.0004$  
$\ [Wh]$ 

$-0.0004 < c_Q^{(3)}\xi < 0.0003$  
$\ [WZ]$ 

WZ analysis: [Franceschini et al., 2017]

$\sigma_{Zh}^{SM} / \sigma_{Zbb}$ without cuts $\sim 4.6/165$

With the cut-based analysis $\rightarrow 0.26$

With BDT optimisation $\rightarrow 0.50$ (multiple distributions trained)

$S/B$ changes from 1/40 to $\mathcal{O}(1) \rightarrow$ Close to 35 SM Zh$(b\bar{b}\ell^+\ell^-)$ events left at 300 fb$^{-1}$  
[SB, Englert, Gupta, Spannowsky, 2018]
Statistically optimal observables

minimize the one-sigma ellipsoid in EFT parameter space
(joint efficient set of estimators, saturating the Cramér-Rao bound: $V^{-1} = I$, like MEM)

For small $C_i$, with a phase-space distribution $\sigma(\Phi) = \sigma_0(\Phi) + \sum_i C_i \sigma_i(\Phi)$, the stat. opt. obs. are the average values of $O_i(\Phi) = n \sigma_i(\Phi)/\sigma_0(\Phi)$.

\begin{itemize}
  \item e.g. $\sigma(\phi) = 1 + \cos(\phi) + C_1 \sin(\phi) + C_2 \sin(2\phi)$
  \item 1. asymmetries: $O_i \sim \text{sign}\{\sin(i\phi)\}$
  \item 2. moments: $O_i \sim \sin(i\phi)$
  \item 3. statistically optimal: $O_i \sim \frac{\sin(i\phi)}{1 + \cos \phi}$
\end{itemize}

$\Rightarrow$ area ratios $1.9 : 1.7 : 1$

Previous applications in $e^+e^- \rightarrow t \bar{t}$, on different distributions:

[Grzadkowski, Hioki '00] [Janot '15] [Khiem et al '15]
The story so far

- The nature of the discovered boson is more or less consistent with the *SM* Higgs
- Still to be measured: $h \rightarrow Z\gamma$, $h \rightarrow \mu^+\mu^-$, $\lambda_{hhh} \rightarrow$ Some of these will be achieved at the HL-LHC
- What is the level of precision that we seek? The HL-LHC has the potential of constraining several couplings at the percent or the per-mille level → How better can the FCC-hh do?
- Projected bounds from HL-LHC: $0.52 < \kappa_\lambda < 1.5$. Projected FCC-hh bound: $\Delta \kappa_\lambda < 5\%$ Study $hh, hhj, hhjj, t\bar{t}hh, Vhh$ in conjunction to obtain complete bounds on all these couplings → At HL-LHC as well as the FCC-hh.
- For a more thorough analysis, EW interplay between Higgs+EW and top sector must be considered [Vryonidou, Zhang; 2018]
SMEFT motivation

- Many reasons to go beyond the SM, viz. gauge hierarchy, neutrino mass, dark matter, baryon asymmetry etc.
- Plethora of BSM theories to address these issues
- Two phenomenological approaches:
  - *Model dependent*: study the signatures of each model individually
  - *Model independent*: low energy effective theory formalism – analogous to Fermi’s theory of beta decay
- The SM here is a low energy effective theory valid below a cut-off scale $\Lambda$
- A bigger theory (either weakly or strongly coupled) is assumed to supersede the SM above the scale $\Lambda$
- At the perturbative level, all heavy ($> \Lambda$) DOF are decoupled from the low energy theory (Appelquist-Carazzone theorem)
- Appearance of HD operators in the effective Lagrangian valid below $\Lambda$

$$\mathcal{L} = \mathcal{L}_{SM}^{d=4} + \sum_{d \geq 5} \sum_{i} \frac{f_i}{\Lambda^{d-4}} O_{i}^{d}$$
SMEFT motivation

- Precisely measuring the Higgs couplings → one of the most important LHC goals and that of the FCC-hh as well
- Indirect constraints can constrain much higher scales $S, T$ parameters being prime examples
- Q: How well does the FCC-hh compete with LEP in constraining precision physics?
  A: From EFT correlated variables, LEP already constrained certain anomalous Higgs couplings → $Z$-pole measurements, TGCs
  Going to higher energies in FCC-hh is the only way to obtain newer information after the HL-LHC
- EFT techniques show that many Higgs deformations aren’t independent from cTGCs and EW precision which were already constrained at LEP → Same operators affect TGCs and Higgs deformations
Motivation

- **Plethora of data** available/ will be available at the HL-LHC and significantly more at the FCC-hh
- **Q:** How do we reconstruct a TeV-Scale Lagrangian from this data?
- **Q:** How to extract the best observables to study the effects of a particular operator?
- **New vertices** ensuing from EFT can produce novel/ enhanced effects in parts of the phase space
- **Q:** What is the best way to extract every differential information for a particular process?
- These questions and ideas can be addressed in the regime of high energies/ luminosities
- **FCC-hh** will run at C.o.M. of 10s of TeV → If the scale of new-physics is in such a range, there will be an interesting transition from EFTs to actual models.
- Higher-dimensional Operators: invariant under SM gauge group
- $d = 5$: Unique operator $\rightarrow$ Majorana mass to the neutrinos:
  $\frac{1}{\Lambda} (\Phi^\dagger L)^T C (\Phi^\dagger L)$
- $d = 6$: $59 = 15$ (bosonic) + $19$ (single fermionic) + $25$ (four fermion) independent $B$-conserving operators. Lowest dimension (after $d = 4$) which induces $HXY, HXYZ$ interactions, charged TGCs [W. Buchmuller and D. Wyler; B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek; K.Hagiwara, D. Zeppenfeld et. al., Azatov, et. al., Falkowski, et. al.]
- $d = 7$: Such operators appear in Higgs portal dark matter models
- $d = 8$: Lowest dimension inducing neutral TGC interactions
Many deformations from a single operator: Correlated interactions

Let’s consider the operator \((H^\dagger \sigma^a H) W_{\mu\nu}^a B^{\mu\nu}\)

Upon expanding, we get terms like:
\[
\hat{h}^2[\hat{W}_{\mu\nu}^3 B^{\mu\nu} + 2igc_\theta W^-_\mu W^+_\nu (A^{\mu\nu} - t_{\theta_w} Z^{\mu\nu})]
\]

Considering \(\hat{h} = \nu + h\) and expanding further, we get the following deformations:

\[hA_{\mu\nu} A^{\mu\nu}, hA_{\mu\nu} Z^{\mu\nu}, hZ_{\mu\nu} Z^{\mu\nu}, hW^+_{\mu\nu} W^-_{\mu\nu} \rightarrow \text{Higgs deformations}\]

\[2igc_\theta W^-_\mu W^+_\nu (A^{\mu\nu} - t_{\theta_w} Z^{\mu\nu}) \rightarrow \delta\kappa_\gamma, \delta\kappa_Z \ (\text{TGCs})\]

\[\hat{W}_{\mu\nu} B^{\mu\nu} \rightarrow S\text{-parameter}\]

Hence, we obtain 7 deformations from a single operator.
The following terms are **not constrained by LEP**. First time probed at the LHC

\[
\mathcal{L}_{h,\text{primary}} = g^h_{VV} h \left[ W^+ \mu W^- \frac{1}{2c_{\theta_W}^2} Z^\mu Z_\mu \right] + g^h_{3h} h^3 + g^h_{ff} (h \bar{f}_L f_R + \text{h.c.}) + \kappa_{GG} \frac{h}{v} G^A \mu \nu G^A_{\mu \nu} + \kappa_{\gamma \gamma} \frac{h}{v} A^\mu \nu A_{\mu \nu} + \kappa_{Z \gamma} t_{\theta_W} \frac{h}{v} A^\mu \nu Z_{\mu \nu},
\]

In contrast, the following interactions were **constrained by LEP**

\[
\Delta \mathcal{L}_h = \delta g^h_{ZZ} \frac{v}{2c_{\theta_W}^2} h Z^\mu Z_\mu + g^h_{Zff} \frac{h}{2v} (Z_\mu J^\mu_N + \text{h.c.}) + g^h_{Wff} \frac{h}{v} (W^\mu J^\mu_C + \text{h.c.}) + \kappa_{WW} \frac{h}{v} W^+ \mu \nu W^-_{\mu \nu} + \kappa_{ZZ} \frac{h}{v} Z^\mu \nu Z_{\mu \nu},
\]

[Pomarol, 2014]
The coefficients of the following

\[ \Delta \mathcal{L}_h = \delta g^h_{ZZ} \, \frac{v}{2c_{\theta_W}^2} h Z^\mu Z_\mu + g^h_{Zff} \, \frac{h}{2v} \left( Z_\mu J^\mu_N + h.c. \right) + g^{h}_{Wff'} \, \frac{h}{v} \left( W^+ J^\mu_C + h.c. \right) \]

\[ + \, \kappa_{WW} \, \frac{h}{v} W^{+\mu\nu} W^-_{\mu\nu} + \kappa_{ZZ} \, \frac{h}{v} Z^{\mu\nu} Z_{\mu\nu} , \]

can be written as

\[ \delta g^h_{ZZ} = \delta g_1^Z e^2 - \delta \kappa_\gamma \frac{e^2}{c_{\theta_W}^2} \]

\[ g^h_{Zff} = 2\delta g_{ff}^Z - 2\delta g_1^Z (g_f^Z c_{2\theta_W} + e Q_f s_{2\theta_W}) + 2\delta \kappa_\gamma Y_f \frac{e s_{\theta_W}}{c_{\theta_W}^3} , \]

\[ g^h_{Wff'} = 2\delta g_{ff'}^W - 2\delta g_1^W g_f^W c_{\theta_W}^2 , \]

\[ \kappa_{ZZ} = \frac{1}{2c_{\theta_W}^2} (\delta \kappa_\gamma + \kappa_{Z\gamma} c_{2\theta_W} + 2\kappa_\gamma c_{\theta_W}^2) , \]

\[ \kappa_{WW} = \delta \kappa_\gamma + \kappa_{Z\gamma} + 2\kappa_\gamma , \]

[Gupta, Pomarol, Riva, 2014]
If one of these predictions is not confirmed then either

- Our Higgs is not a part of the doublet
- \( \Lambda \) may not be very high and D8 operators need to be seriously considered
Following are some of the Higgs observables (assuming flavour universality):

- $hW_{\mu\nu}^{+}W^{\mu\mu}$
- $hZ_{\mu\nu}Z^{\mu\nu}$, $hA_{\mu\nu}A^{\mu\nu}$, $hA_{\mu\nu}Z^{\mu\nu}$, $hG_{\mu\nu}G^{\mu\nu}$
- $hf\bar{f}$, $h^{2}f\bar{f}$
- $hW_{\mu}^{+}W^{-\mu}$
- $h^{3}$
- $hZ_{\mu}\bar{f}_{L,R}\gamma^{\mu}f_{L,R}$

These anomalous Higgs couplings are first probed at the LHC.
Electroweak Pseudo-Observables

- Following are the 9 EW precision observables (assuming flavour universality)
  \[ Z_\mu \bar{f}_{L,R} \gamma^\mu f_{L,R} \quad W^+_{\mu} \bar{u}_L \gamma^\mu d_R \]

- These couplings were measured very precisely by the \( Z/W \)-pole measurements through the \( Z/W \) decays

- Following are the 3 TGCs which were measured by the \( e^+e^- \rightarrow W^+W^- \) channel at LEP
  \[ g_1^Z c_{\theta_w} Z^\mu (W^{+\nu} \hat{W}^-_{\mu \nu} - W^{-\nu} \hat{W}^+_{\mu \nu}) \]
  \[ \kappa_\gamma s_{\theta_w} \hat{A}^{\mu \nu} W^+_{\mu} W^-_{\nu} \]
  \[ \lambda_\gamma s_{\theta_w} \hat{A}^{\mu \nu} W^{-\rho} W^+_{\rho \nu} \]

- Finally, following are the QGCs
  \[ Z^\mu Z^\nu W^-_{\mu} W^+_{\nu} \]
  \[ W^+_{-\mu} W^{+\nu} W^-_{\mu} W^+_{\nu} \]
There are only **18 independent operators** from which the aforementioned vertices ensue.

\[
\begin{align*}
\mathcal{O}_H &= \frac{1}{2} (\partial^\mu |H|^2)^2 \\
\mathcal{O}_T &= \frac{1}{2} \left( H^\dagger \bar{D}_\mu H \right)^2 \\
\mathcal{O}_6 &= \lambda |H|^6 \\
\mathcal{O}_W &= \frac{i g}{2} \left( H^\dagger \sigma^a \bar{D}^\mu H \right) D^\nu W^a_{\mu \nu} \\
\mathcal{O}_B &= \frac{i g'}{2} \left( H^\dagger \bar{D}^\mu H \right) \partial^\nu B_{\mu \nu} \\
\mathcal{O}_{BB} &= g^2 |H|^2 B_{\mu \nu} B^{\mu \nu} \\
\mathcal{O}_{GG} &= g_s^2 |H|^2 G^A_{\mu \nu} G^{A \mu \nu} \\
\mathcal{O}_{HW} &= ig (D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu \nu} \\
\mathcal{O}_{HB} &= ig' (D^\mu H)^\dagger (D^\nu H) B_{\mu \nu} \\
\mathcal{O}_{3W} &= \frac{1}{3!} g_\epsilon \epsilon_{abc} W^a_\mu W^b_\nu W^c_\rho \\
\mathcal{O}_{y_u} &= y_u |H|^2 \bar{Q}_L H u_R \\
\mathcal{O}_{y_d} &= y_d |H|^2 \bar{Q}_L H d_R \\
\mathcal{O}_{y_e} &= y_e |H|^2 \bar{L}_L H e_R \\
\mathcal{O}_R^u &= (i H^\dagger \bar{D}_\mu H)(\bar{u}_R \gamma^\mu u_R) \\
\mathcal{O}_R^d &= (i H^\dagger \bar{D}_\mu H)(\bar{d}_R \gamma^\mu d_R) \\
\mathcal{O}_R^q &= (i H^\dagger \sigma^a \bar{D}_\mu H)(\bar{Q}_L \sigma^a \gamma^\mu Q_L) \\
\mathcal{O}_R^{(3)} &= (i H^\dagger \sigma^a \bar{D}_\mu H)(\bar{Q}_L \sigma^a \gamma^\mu Q_L) \\
\mathcal{O}_R^e &= (i H^\dagger \bar{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)
\end{align*}
\]
There are 18 independent operators and many more pseudo-observables.

This implies correlations between the various pseudo-observables.

Besides, the following operators can not be constrained by LEP:

\[ |H|^2 G_{\mu\nu} G^{\mu\nu}, |H|^2 B_{\mu\nu} B^{\mu\nu}, |H|^2 W^a_{\mu\nu} W^{a,\mu\nu} \]

\[ |H|^2 |D_\mu H|^2, |H|^6 \]

\[ |H|^2 f_L H f_R + h.c. \]

It is thus necessary to redefine many parameters, viz.,
\[ e(\hat{h}), s_{\theta_w}(\hat{h}), g_s(\hat{h}), \lambda_h(\hat{h}), Z_h(\hat{h}), Y_f(\hat{h}), \]
where \( \hat{h} = \nu + h \)
Sensitivity at high-energy colliders

- We have seen that there are a fewer number of $SU(2)_L \times U(1)_Y$ invariant HD operators than the number of pseudo-observables.

- Hence, correlations between LEP and LHC measurements can be exploited.

- LEP measurements of $Z$-pole measurements and anomalous TGCs inform the Higgs observables at the LHC.

- Apart from the 8 “Higgs primaries“, all other Higgs observables can be already constrained by $Z$-pole and diboson measurements.

- For processes that grow with energy,
  \[
  \frac{\delta \sigma(\hat{s})}{\sigma_{SM}(\hat{s})} \sim \frac{\delta g_i \hat{s}}{m_Z^2},
  \]
  one can measure the coupling deviation to per-mille level if the fractional cross-section is $\mathcal{O}(30\%)$ for $\sqrt{\hat{s}} \sim 1$ TeV.
Higgs anomalous couplings: Dimension 6 effects

\[ \mathcal{L}_{h}^{\text{primary}} = g_{VV}^{h} h \left[ W^{+} \mu W_{\mu}^{-} + \frac{1}{2c_{\theta_{W}}^{2}} Z^{\mu} Z_{\mu} \right] + g_{h}^{3} h^{3} + g_{f f}^{h} (h f L f R + h.c.) + \kappa_{GG} \frac{h}{v} G^{A \mu \nu} G_{\mu \nu}^{A} + \kappa_{\gamma \gamma} \frac{h}{v} A^{\mu \nu} A_{\mu \nu} + \kappa_{Z \gamma} t_{\theta_{W}} \frac{h}{v} A^{\mu \nu} Z_{\mu \nu}, \]

\[ \Delta \mathcal{L}_{h} = \delta g_{ZZ}^{h} \frac{v}{2c_{\theta_{W}}^{2}} h Z^{\mu} Z_{\mu} + g_{Z f f}^{h} \frac{h}{2v} (Z_{\mu} J_{N}^{\mu} + h.c.) + g_{W f f}^{h} \frac{h}{v} (W_{\mu}^{+} J_{C}^{\mu} + h.c.) + \kappa_{WW} \frac{h}{v} W^{+ \mu \nu} W_{\mu \nu}^{-} + \kappa_{ZZ} \frac{h}{v} Z^{\mu \nu} Z_{\mu \nu}, \]

[Pomarol, 2014]

- Higgs interactions were directly measured for the first time at the LHC
**STU oblique parameters**

\[
\begin{align*}
\Pi_{\gamma\gamma}(q^2) &= q^2 \Pi'_{\gamma\gamma}(0) + \ldots \\
\Pi_{Z\gamma}(q^2) &= q^2 \Pi'_{Z\gamma}(0) + \ldots \\
\Pi_{ZZ}(q^2) &= \Pi_{ZZ}(0) + q^2 \Pi'_{ZZ}(0) + \ldots \\
\Pi_{WW}(q^2) &= \Pi_{WW}(0) + q^2 \Pi'_{WW}(0) + \ldots \\
\alpha S &= 4 s_w^2 c_w^2 \left[ \Pi'_{ZZ}(0) - \frac{c_w^2 - s_w^2}{s_w c_w} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right] \\
\alpha T &= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \\
\alpha U &= 4 s_w^2 \left[ \Pi'_{WW}(0) - c_w^2 \Pi'_{ZZ}(0) - 2 s_w c_w \Pi'_{Z\gamma}(0) - s_w^2 \Pi'_{\gamma\gamma}(0) \right]
\end{align*}
\]

1. Any BSM correction which is indistinguishable from a redefinition of $e, G_F$ and $M_2$ (or equivalently, $g_1, g_2$ and $v$) in the Standard Model proper at the tree level does not contribute to $S$, $T$ or $U$.

2. Assuming that the Higgs sector consists of electroweak doublet(s) $H$, the effective action term $\left| H^\dagger D_\mu H \right|^2 / \Lambda^2$ only contributes to $T$ and not to $S$ or $U$. This term violates custodial symmetry.

3. Assuming that the Higgs sector consists of electroweak doublet(s) $H$, the effective action term $H^\dagger W^{\mu\nu} B_{\mu\nu} H / \Lambda^2$ only contributes to $S$ and not to $T$ or $U$. (The contribution of $H^\dagger B^{\mu\nu} B_{\mu\nu} H / \Lambda^2$ can be absorbed into $g_1$ and the contribution of $H^\dagger W^{\mu\nu} W_{\mu\nu} H / \Lambda^2$ can be absorbed into $g_2$).

4. Assuming that the Higgs sector consists of electroweak doublet(s) $H$, the effective action term $\left( H^\dagger W^{\mu\nu} H \right) \left( H^\dagger W_{\mu\nu} H \right) / \Lambda^4$ contributes to $U$. 
ZH: Four directions in the EFT space (SILH Basis)

\[ g_{Z_u L u_L}^h = \frac{g}{c_{\theta_W}} \frac{m_W^2}{\Lambda^2} (c_W + c_{HW} - c_{2W} - \frac{t_{\theta_W}^2}{3} (c_B + c_{HB} - c_{2B})) \]

\[ g_{Z_d L d_L}^h = -\frac{g}{c_{\theta_W}} \frac{m_W^2}{\Lambda^2} (c_W + c_{HW} - c_{2W} + \frac{t_{\theta_W}^2}{3} (c_B + c_{HB} - c_{2B})) \]

\[ g_{Z_u R u_R}^h = -\frac{4 g s_{\theta_W}^2}{3 c_{\theta_W}^3} \frac{m_W^2}{\Lambda^2} (c_B + c_{HB} - c_{2B}) \]

\[ g_{Z_d R d_R}^h = \frac{2 g s_{\theta_W}^2}{3 c_{\theta_W}^3} \frac{m_W^2}{\Lambda^2} (c_B + c_{HB} - c_{2B}) \]
ZH: Four directions in the EFT space (Higgs Primaries Basis)

\[
\begin{align*}
    g_{Z_{uL}u_L}^h &= 2\delta g_{Z_{uL}u_L}^Z - 2\delta g_1^Z (g_f^Z c_{2\theta_W} + eQ s_{2\theta_W}) + 2\delta\kappa_\gamma g' Y_h \frac{s_{\theta_W}}{c_{\theta_W}^2} \\
    g_{Z_{dL}d_L}^h &= 2\delta g_{Z_{dL}d_L}^Z - 2\delta g_1^Z (g_f^Z c_{2\theta_W} + eQ s_{2\theta_W}) + 2\delta\kappa_\gamma g' Y_h \frac{s_{\theta_W}}{c_{\theta_W}^2} \\
    g_{Z_{uR}u_R}^h &= 2\delta g_{Z_{uR}u_R}^Z - 2\delta g_1^Z (g_f^Z c_{2\theta_W} + eQ s_{2\theta_W}) + 2\delta\kappa_\gamma g' Y_h \frac{s_{\theta_W}}{c_{\theta_W}^2} \\
    g_{Z_{dR}d_R}^h &= 2\delta g_{Z_{dR}d_R}^Z - 2\delta g_1^Z (g_f^Z c_{2\theta_W} + eQ s_{2\theta_W}) + 2\delta\kappa_\gamma g' Y_h \frac{s_{\theta_W}}{c_{\theta_W}^2}
\end{align*}
\]

[Gupta, Pomarol, Riva, 2014]
ZH: Four directions in the EFT space (Universal model Basis)

\begin{align*}
    g_{Z_uL_uL}^h &= -\frac{g}{c_{\theta_W}} \left( (c_{\theta_W}^2 + \frac{s_{\theta_W}^2}{3}) \delta g_1^Z + W + \frac{t_{\theta_W}^2}{3} (\hat{S} - \delta \kappa_\gamma - Y) \right) \\
    g_{Z_dL_dL}^h &= \frac{g}{c_{\theta_W}} \left( (c_{\theta_W}^2 - \frac{s_{\theta_W}^2}{3}) \delta g_1^Z + W - \frac{t_{\theta_W}^2}{3} (\hat{S} - \delta \kappa_\gamma - Y) \right) \\
    g_{Z_uR_uR}^h &= -\frac{4gs_{\theta_W}^2}{3c_{\theta_W}^3} (\hat{S} - \delta \kappa_\gamma + c_{\theta_W}^2 \delta g_1^Z - Y) \\
    g_{Z_dR_dR}^h &= \frac{2gs_{\theta_W}^2}{3c_{\theta_W}^3} (\hat{S} - \delta \kappa_\gamma + c_{\theta_W}^2 \delta g_1^Z - Y)
\end{align*}

[Franceschini, Panico, Pomarol, Riva, Wulzer, 2017]
Given a hard jet $j$, obtained with some radius $R$, we then use the following new iterative decomposition procedure to search for a generic boosted heavy-particle decay. It involves two dimensionless parameters, $\mu$ and $\eta_{\text{cut}}$:

1. Break the jet $j$ into two subjets by undoing its last stage of clustering. Label the two subjets $j_1, j_2$ such that $m_{j_1} > m_{j_2}$.

2. If there was a significant mass drop (MD), $m_{j_1} < \mu m_{j_2}$, and the splitting is not too asymmetric, $y = \frac{\Delta R^2_{R_{j_1 j_2}}}{m_{j_1}^2} > \eta_{\text{cut}}$, then deem $j$ to be the heavy-particle neighbourhood and exit the loop. Note that $y \simeq \min(\Delta R_{j_1 j_2}, \Delta R_{j_2 j_1}) \approx \max(\Delta R_{j_1 j_2}, \Delta R_{j_2 j_1})$.

3. Otherwise redefine $j$ to be equal to $j_1$ and go back to step 1.

The final jet $j$ is to be considered as the candidate Higgs boson if both $j_1$ and $j_2$ have $b$ tags. One can then identify $R_{bb}$ with $\Delta R_{j_1 j_2}$. The effective size of jet $j$ will thus be

In practice the above procedure is not yet optimal for LHC at the transverse momenta of interest, $p_T \sim 200 - 300$ GeV because, from eq. (1), $R_{bb} \gtrsim 2m_B/p_T$ is still quite large and the resulting Higgs mass peak is subject to significant degradation from the underlying event (UE), which scales as $R_{bb}^2$. A second novel element of our analysis is to filter the Higgs neighbourhood. This involves resolving it on a finer angular scale, $R_{bb} < R_{bb}$, and taking the three hardest objects (subjets) that appear — thus one captures the dominant $O(\alpha_s)$ radiation from the Higgs decay, while eliminating much of the UE contamination. We find $R_{bb} = \min(0.3, R_{bb}/2)$ to be rather effective. We also require the two hardest of the
Case study I: Higgs-Strahlung at the LHC and the FCC-hh

\[ \Delta L_6 \supset \delta \hat{g}_{WW} \frac{2m_W^2}{v} h W^+ \mu W^- \mu + \delta \hat{g}_{ZZ} \frac{2m_Z^2}{v} h \frac{Z^\mu Z^\mu}{2} + \delta g_Q^W (W^+ \mu \bar{u}_L \gamma^\mu d_L + h.c.) + \\
+ \delta g_L^W (W^+ \mu \nu_L \gamma^\mu e_L + h.c.) + g_{WL}^h \frac{h}{v} (W^+ \mu \nu_L \gamma^\mu e_L + h.c.) + \\
+ g_{WQ}^h \frac{h}{v} (W^+ \mu \bar{u}_L \gamma^\mu d_L + h.c.) + \sum_f \delta g_f^Z Z^\mu_f \bar{f}^\gamma \gamma f + \sum_f g_{Zf}^h \frac{h}{v} Z^\mu_f \bar{f}^\gamma \gamma f + \\
+ \kappa_{WW} \frac{h}{v} W^{+\mu\nu} W^-_{\mu\nu} + \tilde{\kappa}_{WW} \frac{h}{v} W^{+\mu\nu} \tilde{W}^-_{\mu\nu} + \kappa_{ZZ} \frac{h}{2v} Z^{\mu\nu} Z^\mu Z^\nu + \\
+ \tilde{\kappa}_{ZZ} \frac{h}{2v} Z^{\mu\nu} \tilde{Z}^\mu \tilde{Z}^\nu + \kappa_{Z\gamma} \frac{h}{v} A^{\mu\nu} Z^\mu Z^\nu + \tilde{\kappa}_{Z\gamma} \frac{h}{v} A^{\mu\nu} \tilde{Z}^\mu \tilde{Z}^\nu + \delta \hat{g}_{bb} \frac{\sqrt{2}m_b}{v} h b \bar{b} \]

The leading effect comes from contact interaction at high energies. The energy growth occurs because there is no propagator.
Important to consider NLO EW effects in the tails of the distributions for SMEFT studies \cite{SB, Schönherr, Spannowsky; Upcoming}

Important assumption: The $K$-factors do not change significantly for the FCC-hh scenario $\rightarrow$ Needs to be explicitly checked

\cite{Greljo, Isidori, Lindert, Marzocca, Zhang, 2017}
Differential in Energy: $pp \rightarrow Zh$ at high energies (Contact term)

- We study the impact of constraining TGC couplings at higher energies
- We study the channel $pp \rightarrow Zh \rightarrow \ell^+ \ell^- b\bar{b}$
- The backgrounds are SM $pp \rightarrow Zh, Zb\bar{b}, t\bar{t}$ and the fake $pp \rightarrow Zjj$ ($j \rightarrow b$ fake rate taken as 2%)
- Major background $Zb\bar{b}$ ($b$-tagging efficiency taken to be 70%)
- Boosted substructure analysis with fat-jets of $R = 1.2$ used

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$Zbb$</th>
<th>$Zh$ (SM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least 1 fat jet with 2 $B$-mesons with $p_T &gt; 15$ GeV</td>
<td>0.52</td>
<td>0.79</td>
</tr>
<tr>
<td>2 OSSF isolated leptons</td>
<td>0.44</td>
<td>0.48</td>
</tr>
<tr>
<td>$80$ GeV $&lt; M_{\ell\ell} &lt; 100$ GeV, $p_T,\ell\ell &gt; 260$ GeV, $\Delta R_{\ell\ell} &gt; 0.2$</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>At least 1 fat jet with 2 $B$-meson tracks with $p_T &gt; 220$ GeV</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>2 Mass drop subjets and $\geq 2$ filtered subjets</td>
<td>0.88</td>
<td>0.92</td>
</tr>
<tr>
<td>2 $b$-tagged subjets</td>
<td>0.35</td>
<td>0.48</td>
</tr>
<tr>
<td>$115$ GeV $&lt; m_h &lt; 135$ GeV</td>
<td>0.180</td>
<td>0.50</td>
</tr>
<tr>
<td>$\Delta R(b_i, \ell_j) &gt; 0.4, \not{E}_T &lt; 30$ GeV, $</td>
<td>y_h</td>
<td>&lt; 2.5, p_{T,h/Z} &gt; 300$ GeV</td>
</tr>
</tbody>
</table>

Shankha Banerjee (CERN) LHCP 2021
Differential in Energy: $pp \rightarrow Zh$ at high energies (Contact term) - HL-LHC result

- Next we perform a two-parameter $\chi^2$-fit (at 300 fb$^{-1}$) to find the allowed region in the $\delta g_1^Z - (\delta K_{1\gamma} - \hat{S})$

Blue dashed line → direction of accidental cancellation of interference term; Gray region: LEP exclusion; pink band: exclusion from $WZ$ [Franceschini, Panico, Pomarol, Riva and Wulzer, 2017]; Blue region: exclusion from $ZH$ Dark (light) shade represents bounds at 3 ab$^{-1}$ (300 fb$^{-1}$) luminosity; Green region: Combined bound from $Zh$ and $WZ$ [SB, Englert, Gupta, Spannowsky, 2018] Black dashed lines show the VBF direction at 3 ab$^{-1}$ [Araz, SB, Gupta, Spannowsky, 2020]
For a $2 \rightarrow 2$ process $f(\sigma)\bar{f}(-\sigma) \rightarrow Zh$, the helicity amplitudes are given by

$$M_{\sigma}^{\lambda \pm} = \sigma \frac{1 + \sigma \lambda \cos \Theta}{\sqrt{2}} G_V \frac{m_V}{\sqrt{s}} \left[ 1 + \left( \frac{g_{Vf}^h}{g_f} + \hat{\kappa}_{VV} - i \lambda \hat{\kappa}_{VV} \right) \frac{\hat{s}}{2m_V^2} \right]$$

$$M_{\sigma}^{\lambda = 0} = -\frac{\sin \Theta}{2} G_V \left[ 1 + \delta \hat{g}_{VV}^h + 2\hat{\kappa}_{VV} + \delta g_{f}^Z + \frac{g_{Vf}^h}{g_f} \left( -\frac{1}{2} + \frac{\hat{s}}{2m_V^2} \right) \right]$$

$$\hat{\kappa}_{WW} = \kappa_{WW}$$

$$\hat{\kappa}_{ZZ} = \kappa_{ZZ} + \frac{Q_f e}{g_f^Z} \kappa_{Z\gamma}$$

$$\hat{\kappa}_{\bar{Z}Z} = \tilde{\kappa}_{\bar{Z}Z} + \frac{Q_f e}{g_f^Z} \tilde{\kappa}_{Z\gamma}$$

- $\lambda = \pm 1$ and $\sigma = \pm 1$ are, respectively, the helicities of the $Z$-boson and initial-state fermions, $g_f^Z = g(T_f^3 - Q_f s_{\theta_W}^2)/c_{\theta_W}$

- Leading SM is longitudinal ($\lambda = 0$), Leading effect of $\kappa_{WW}$, $\kappa_{ZZ}$, $\tilde{\kappa}_{\bar{Z}Z}$ is in the transverse-longitudinal (LT) interference, LT term vanishes if we aren’t careful
The differential cross-section for the process $pp \rightarrow Z(\ell^+\ell^-)/W(\ell\nu)h(b\bar{b})$ is a differential in four variables, viz., $\frac{d\sigma}{dE d\Theta d\theta d\phi}$.

The amplitude at the decay level can be written as

$$A(\hat{s}, \Theta, \theta, \phi) = \frac{-ig_\ell^V + \delta g_\ell^V}{\Gamma_V} \sum_\lambda \mathcal{M}_\lambda^\sigma(\hat{s}, \Theta) d_{\lambda,1}^{J=1}(\theta) e^{i\lambda\phi}$$

$d_{\pm1,1}^{J=1} = \tau \frac{1 \mp \tau \cos \theta}{\sqrt{2}}$, $d_{0,1}^{J=1} = \sin \theta$ are the Wigner functions, $\tau$ is lepton helicity, $\Gamma_V$ is the $V$-width and $g_f^Z = g(\mathcal{T}_3 f - Q_f s_\theta^2)/c_W$ and $g_f^W = g/\sqrt{2}$

$\hat{\phi} \rightarrow$ azimuthal angle of positive helicity lepton, $\hat{\Theta} \rightarrow$ its polar angle in $Z$-rest frame

Polarisation of lepton is experimentally not accessible

$A_0 \sim \sin \Theta \sin \theta$
$A_+ \sim (1 + \cos \Theta)(1 + \cos \theta)e^{i\phi}$
$A_- \sim (1 - \cos \Theta)(1 - \cos \theta)e^{-i\phi}$
Helicity Amplitudes: Angular Moments

- We sum over lepton polarisations and express the analogous angles ($\theta$, $\varphi$) for the positively-charged lepton

$$\sum_{L,R} |\mathcal{A}(\hat{s}, \Theta, \theta, \varphi)|^2 = \alpha_L |\mathcal{A}_h(\hat{s}, \Theta, \theta, \varphi)|^2 + \alpha_R |\mathcal{A}_h(\hat{s}, \Theta, \pi - \theta, \pi + \varphi)|^2$$

- $\alpha_{L,R} = (g_{Z, L,R}^Z)^2 / [(g_{Z, L}^Z)^2 + (g_{Z, R}^Z)^2]$ → fraction of $Z \to \ell^+ \ell^-$ decays to leptons with left-handed (right-handed) chiralities $\epsilon_{LR} = \alpha_L - \alpha_R \approx 0.16$

- For left-handed chiralities, positive-helicity lepton → positive-charged lepton

- For right-handed chiralities, positive-helicity lepton → negative-charged lepton → ($\hat{\theta}, \hat{\varphi}$) → ($\pi - \theta, \pi + \varphi$) → Following 9 coefficients are 9 angular moments for $pp \to Z(\ell\ell)h$

$$\sum_{L,R} |\mathcal{A}(\hat{s}, \Theta, \theta, \varphi)|^2 = a_{LL} \sin^2 \Theta \sin^2 \theta + a_{TT}^1 \cos \Theta \cos \theta$$

$$+ a_{TT}^2 (1 + \cos^2 \Theta)(1 + \cos^2 \theta) + \cos \varphi \sin \Theta \sin \theta$$

$$\times (a_{LT}^1 + a_{LT}^2 \cos \Theta \cos \Theta) + \sin \varphi \sin \Theta \sin \theta$$

$$\times (\tilde{a}_{LT}^1 + \tilde{a}_{LT}^2 \cos \Theta \cos \Theta) + a_{TT'} \cos 2\varphi \sin^2 \Theta \sin^2 \theta$$

$$+ \tilde{a}_{TT'} \sin 2\varphi \sin^2 \Theta \sin^2 \theta$$

See also [Azatov, Elias-Miro, Reyimuaji, Venturini; 2017]
Bounds on Pseudo-observables at HL-LHC and FCC-hh

**HL-LHC: @ 95% CL**

\[ g_{Zp}^h \in [-0.004, 0.004] \quad (300 fb^{-1}) \]

\[ g_{Zp}^b \in [-0.001, 0.001] \quad (3000 fb^{-1}) \]

Directions: (\( \xi = v^2 / \Lambda^2 \)) [Araz, SB, Gupta, Spannowsky, 2020]

\[
|(-0.04 c_Q^1 + 1.4 c_Q^{(3)} + 0.1 c_{uR} - 0.03 c_{dR})\xi| < 0.003 \quad [VBF]
\]

\[
|(-0.18 c_Q^1 + 1.3 c_Q^{(3)} + 0.3 c_{uR} - 0.1 c_{dR})\xi| < 0.0005 \quad [Zh]
\]

\[
|c_Q^{(3)}\xi| < 0.0004 \quad [Wh]
\]

\[ -0.0004 < c_Q^{(3)}\xi < 0.0003 \quad [WZ] \]

**FCC-hh: @ 95% CL**

\[ g_{Zp}^{h,\ell\ell \bar{b} \bar{b}} \in [-0.00051, 0.00054] \quad \text{([-0.00021, 0.00023]) with 5\% systematic uncertainty.} \]

\[ g_{Zp}^{h,\ell\ell \gamma \gamma} \in [-0.00047, 0.00049] \quad \text{([-0.00016, 0.00017]) with 1\% systematic uncertainty.} \]

\[ g_{Zp}^{h,\ell\ell \gamma \gamma} \in [-0.001, 0.001] \quad \text{([-0.0004, 0.0004]) with 1\% systematic uncertainty at 3 (30) ab^{-1}.} \]

| \( \delta g_{uR}^Z \) | \( \pm 0.0003 \ (\pm 0.0001) \) | \( \pm 0.002 \ (\pm 0.0007) \) | \(-0.0026 \pm 0.0032 \)
| \( \delta g_{dL}^Z \) | \( \pm 0.0003 \ (\pm 0.0001) \) | \( \pm 0.003 \ (\pm 0.001) \) | \(0.0023 \pm 0.002 \)
| \( \delta g_{dR}^Z \) | \( \pm 0.0005 \ (\pm 0.0002) \) | \( \pm 0.005 \ (\pm 0.001) \) | \(-0.0036 \pm 0.0070 \)
| \( \delta g_{dL}^B \) | \( \pm 0.0015 \ (\pm 0.0006) \) | \( \pm 0.016 \ (\pm 0.005) \) | \(0.016 \pm 0.0104 \)
| \( \delta g_{dR}^B \) | \( \pm 0.0005 \ (\pm 0.0002) \) | \( \pm 0.005 \ (\pm 0.001) \) | \(-0.009 \pm 0.043 \)
| \( \delta \kappa \) | \( \pm 0.0035 \ (\pm 0.0015) \) | \( \pm 0.032 \ (\pm 0.009) \) | \(-0.016 \pm 0.085 \)
| \( \psi \) | \( \pm 0.0035 \ (\pm 0.0015) \) | \( \pm 0.032 \ (\pm 0.009) \) | \(0.0004 \pm 0.0007 \)
| \( W \) | \( \pm 0.0004 \ (\pm 0.0002) \) | \( \pm 0.003 \ (\pm 0.001) \) | \(-0.0003 \pm 0.0006 \)
| \( Y \) | \( \pm 0.0035 \ (\pm 0.0015) \) | \( \pm 0.032 \ (\pm 0.009) \) | \(0.0000 \pm 0.0006 \)

[SB, Englert, Gupta, Spannowsky, 2018] LEP bounds: [Falkowski, Riva, 2014], [Baak et al., 2012], [Barbieri, Pomarol, Rattazzi, Strumia, 2004]
The four di-bosonic channels

- The four directions, viz., $Zh$, $Wh$, $W^+ W^-$ and $W^\pm Z$ can be expressed (at high energies) respectively as $G^0 h$, $G^+ h$, $G^+ G^-$ and $G^\pm G^0$ and the Higgs field can be written as

$$\left( \begin{array}{c} G^+ \\ h + iG^0 \\ 2 \end{array} \right)$$

- These four final states are intrinsically connected
- At high energies $W/Z$ production dominates
- With the Goldstone boson equivalence it is possible to compute amplitudes for various components of the Higgs in the unbroken phase
- Full SU(2) theory is manifest [Franceschini, Panico, Pomarol, Riva, Wulzer, 2017]
Evolving from signal strength measurements performed during Run I, by reducing uncertainties and by providing finely-grained measurements.

Allows combination of measurements in several decay channels. Several stages proposed. **Measuring cross-sections instead of signal strengths**

Stage 0 corresponds to production mode categorisation; Stage 1 defines complete setup with potential bin merging etc.

From the various bins, one can translate to signal strength measurements, measurements on EFT coefficients, BSM coefficients etc.
STXS (slide from S. Jiggins)

→ **VHbb resolved**: Stage 1.2 STXS scheme merged down to 5 bins

![Graph showing VHbb resolved](image)

→ **VHbb Resolved**: Increased precision of 5 POI $\sigma^{W/ZH}$ results:
  - 80 fb$^{-1}$: 50%-125% uncertainty on $\sigma^{W/ZH}$
  - 139 fb$^{-1}$: 30%-85% uncertainty on $\sigma^{W/ZH}$

→ **VHbb boosted**: Stage 1.2 STXS scheme merged down to 4 bins
  - Measurement of $p_T^V$(truth) > 400 GeV

→ **VHbb resolved + boosted not orthogonal!**

→ See talk by Nikita Belyaev for more STXS/EFT results
  → Or backup for questions!

Stephen Jiggins

See slides from Stephen Jiggins, Jonathan M. Langford and Nikita Belyaev
The STXS method evolves with increasing statistics and requires intuition and systematic understanding of the data. The various stages of binning help us with an excellent understanding of the present data. STXS gradually moves forward to a fully differential analysis with shape information. STXS can be connected to EFTs, $\kappa$ framework, various BSM scenarios, etc. It is a powerful tool. The Matrix Element Method (MEM) is one of the most powerful tools to discern the full structure of any process. The Method of Moments (MoM), as described in this talk, has comparable sensitivity to the Matrix Element method. MoM exploits the full angular structure for the squared amplitude in a transparent and experiment-friendly manner. MoM combines the advantages of both STXS and the MEM, to a certain extent.