Polarized multiboson production:
At the LHC and beyond
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What exactly do I mean by “polarized boson production”?

\[ p \rightarrow p + W^+_{\lambda_{W^+}} + W^-_{\lambda_{W^-}} \]
Intuitively, the production of on-shell EW bosons ($V$) in specific helicity eigenstates ($\lambda_V = \pm, 0$), defined in a particular frame ($\mathcal{R}$), in spin-averaged proton collisions.
In practice, technical details, e.g., def. of $R$, causes ambiguities when comparing predictions and measurements.

In response, updates on theoretical formalism and event generators ($mg5aMC$) to disambiguate things!
WHY?!?!?!?!?!!
Measuring polarized (multi)boson processes is important

**Practical Considerations:**
- Polarization is excellent test of $V \pm A$ (chiral) structure in (B)SM
- Polarization is excellent test of gauge+unitarity structure in (B)SM

**Future Proofing:**
- $W_0/Z_0$ and $W_T/Z_T$ PDFs (needed at $\sqrt{s} \gtrsim 50$ TeV) couple differently to bosons and massless fermions

  Note that rationale studies for $\sqrt{s} = 27 - 100$ TeV are being done today!

- (N)NLO QCD + NLO EW PDFs will eventually be needed to match precision of (N)NLO QCD + NLO EW predictions

  DGLAP evolution for LH/RH quarks is asymmetric $\implies$ polarized PDFs

**Important:** While formally clear, technical implementation is difficult due to relaxing of Lorentz invariance / reference frame independence
So what is new?\textsuperscript{1}

\textsuperscript{1}w/ D. Buarque Franzosi, O. Mattelaer, S. Shil [1912.01725]
To get $pp$ scattering rates, one uses the **Collinear Factorization Thm**

Collins, Soper, Sterman ('85,'88,'89); Collins, Foundations of pQCD (2011)

$$d\sigma(pp \rightarrow W\gamma + X) = \sum_{i,j} f_i \otimes f_j \otimes \Delta_{ij} \otimes d\hat{\sigma}(ij \rightarrow W\gamma) + O\left(\Lambda_{NP}^p / Q^{p+2}\right)$$

hadron-level scattering probabilities are the product (convolution) of parton-dist. (PDFs), -emission (Sudakov), and -scattering probs. ($|M|^2$)
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The partonic scattering rate is given by the usual (textbook) expression:

$$d\hat{\sigma}(ij \rightarrow W\gamma) = \frac{1}{2Q^2} \underbrace{|M(ij \rightarrow W\gamma)|^2}_{\text{hard scale}} \underbrace{\text{dof avg./summed.}}_{\text{hard scale}}$$
The *unpolarized* external parton scattering rate is given by the *dof-averaged* (initial states) and *dof-summed* (final state) matrix element:

\[
|M_{ij \rightarrow W\gamma}|^2 = \frac{1}{S_i S_j} \frac{1}{N_c N_c'} \sum_{\text{dof}} |M_{i\lambda j\lambda' \rightarrow W\tilde{\chi}\gamma\tilde{\chi}'}|^2
\]

---

2 Degrees of freedom = all discrete quantum numbers, e.g., color, spin, electric charge.
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For *polarized* scattering, truncate the *spin* averaging/summing

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|\mathcal{M}(i\lambda j\lambda' \to W\tilde{\chi}\gamma\tilde{\chi}')[|\mathcal{M}(i\lambda j\lambda' \to W\tilde{\chi}\gamma\tilde{\chi}')[|^2
\]

The two are related by reintroducing *spin averaging/summing*

\[
|\mathcal{M}(ij \to W\gamma)|^2 = \frac{1}{S_i S_j} \sum_{\lambda, \lambda', \tilde{\lambda} \tilde{\lambda}'} |\mathcal{M}(i\lambda j\lambda' \to W\tilde{\chi}\gamma\tilde{\chi}')[|\mathcal{M}(i\lambda j\lambda' \to W\tilde{\chi}\gamma\tilde{\chi}')[|^2
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\(^2\)Degrees of freedom = all discrete quantum numbers, e.g., color, spin, electric charge
Polarized External Parton Scattering (3/3)

Polarized parton scattering in LHC collisions is given by

\[ d\sigma(pp \rightarrow W_\lambda' \gamma_\lambda' + X)|_{i_\lambda,j_\lambda'} = f_{i_\lambda} \otimes f_{i_\lambda'} \otimes \Delta_{i_\lambda,j_\lambda'} \otimes d\hat{\sigma}(i_\lambda j_\lambda' \rightarrow W_\lambda \gamma_\lambda') \]

- \( f_{i_\lambda} \) is the PDF for parton \( i \) with helicity \( \lambda \) in \textit{unpolarized proton} \( p \)
- \( \Delta_{i_\lambda,j_\lambda'} \) is the parton shower / evolution for \( i, j \) with helicities \( \lambda, \lambda' \)
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Again, \textit{unpolarized scattering} is recovered by \textit{spin averaging/summing}

\[ d\sigma(pp \rightarrow W_\gamma + X) = \sum_{i_\lambda,j_\lambda'} \frac{1}{S_i S_j} \sum_{\lambda,\lambda',\bar{\lambda},\bar{\lambda}'} d\sigma(pp \rightarrow W_{\bar{\lambda}}\gamma_{\bar{\lambda}'}, + X)|_{i_\lambda,j_\lambda'} \]
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Again, unpolarized scattering is recovered by spin averaging/summing

\[ d\sigma(pp \rightarrow W^{\gamma} + X) = \sum_{i\lambda,j\lambda'} \frac{1}{S_i S_j} \sum_{\lambda,\lambda',\bar{\lambda}\bar{\lambda}'} d\sigma(pp \rightarrow W_{\bar{\lambda}}^{\gamma} \bar{\gamma}_{\bar{\lambda}'} + X)|_{i\lambda,j\lambda'} \]

Hence, for \textit{unpolarized initial states} and \textit{polarized final states}:

\[ d\sigma(pp \rightarrow W_{\bar{\lambda}}^{\gamma} \bar{\gamma}_{\bar{\lambda}'} + X) = \sum_{i\lambda,j\lambda'} \frac{1}{S_i S_j} \sum_{\lambda,\lambda'} d\sigma(pp \rightarrow W_{\bar{\lambda}}^{\gamma} \bar{\gamma}_{\bar{\lambda}'} + X)|_{i\lambda,j\lambda'} \]
Polarized multo-boson production

with MadGrap5_aMC@NLO (mg5amc)
Generating polarized events at LO in QCD with mg5amc is as difficult as unpolarized computations now (NLO still in development!)

- \( z\{T\} \) denotes LH \((-\)) and RH \((+\)) transverse \( Z \) bosons
- \( \text{ww}\{0\} \) denotes longitudinal \( W^\pm \) bosons
- **Important**: Just be careful to know in which frame the helicity polarizations are defined

Details and instructions in [1912.01725]
Vector boson scattering at the LHC
Quick advert: new review on VBS/F for (B)SM at the LHC and beyond!

- Covers machine learning, BSM models, EW PDFs/parton showers, SMEFT, muon colliders, ILC, FCC, hardware, and more!

The work of many great people! [2106.01393]
With \( \text{mg5amc} \), fully differential events are possible, e.g.,

\[
pp \rightarrow W^{+}_{\lambda^{+}} W^{-}_{\lambda^{-}} jj \rightarrow \ell^{+} \ell^{-} \nu \bar{\nu} jj \text{ at } \mathcal{O}(\alpha_W^4)
\]

For \((\lambda^{+}, \lambda^{-})\) defined in the \((W^{+}W^{-})\)-frame and after VBF cuts role of individual polarizations clear

\( (\lambda^{+}, \lambda^{-}) \) can be defined in most any frame [1912.01725]
With mg5amc, on-the-fly uncertainty weights and $\gamma$-PDF compatibility

\[ \gamma\gamma \rightarrow W_{\pm}^+ W_{\pm}^- \]
Beyond the LHC
Preliminary Effective Vector Boson Approximation (EVA):
EW bosons as partons of the $\mu^{\pm3}$

$\mu^+$

$\mu^-$

$V'$$V$

$\ell$

$X$

$\bar{\ell}$

w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer, et al [soon]
At very high scales \( Q \gg M_W, M_Z \), EW bosons can be treated as partons

a.k.a. the Effective \( W \) Approximation [Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)]

- Treatment of \( V_T \) identical to gluons in QCD; \( V_0 \) is novel complication
- \( W/Z \) PDFs will be released very soon in MadGraph5

\[
\frac{f_{V+}/f_L(z, \mu_f^2)}{f_{V-}/f_L(z, \mu_f^2)} = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)^2}{2z} \log \left[ \frac{\mu_f^2}{M_V^2} \right],
\]

\[
\frac{f_{V+}/f_R(z, \mu_f^2)}{f_{V-}/f_R(z, \mu_f^2)} = \left( \frac{g_R}{g_L} \right)^2 \times \frac{f_{V-}/f_L(z, \mu_f^2)}{f_{V+}/f_L(z, \mu_f^2)}
\]

\[
\frac{f_{V0}/f_L(z, \mu_f^2)}{f_{V0}/f_R(z, \mu_f^2)} = \left( \frac{g_R}{g_L} \right)^2 \times \frac{f_{V0}/f_L(z, \mu_f^2)}{f_{V0}/f_R(z, \mu_f^2)}
\]

\[
f_{V0}/f_L(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)}{2z},
\]

\[
f_{V0}/f_R(z, \mu_f^2) = \frac{g_V^2}{4\pi^2} \frac{g_L^2(1-z)}{2z}
\]
some results on scattering of polarized EW bosons $V_\lambda V'_\lambda \to X$ \[^4\]

\[^4\] w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2105.????]
Diboson production
$V_\lambda V'_\lambda \rightarrow VV'$: (L) all polarizations (R) $V_0 V_0$

- Only minor role played by $V_0 V'_0$ scattering
- At $\mathcal{L} = 1 \text{ ab}^{-1}/\text{yr} \implies 10^6 W\bar{W}/\text{yr}$ driven by non-Abelian couplings
Triboson production in EVA
$V_\lambda V'_\lambda \rightarrow VV'V'': \ (L) \ all \ polarizations \ \ (R) \ V_T V_T$

- Major role played by $V_T V'_T$ scattering
- At $\mathcal{L} = 1 \text{ ab}^{-1}/\text{yr} \implies 10^3 \ VV'V''/\text{yr}$
Summary

Searches and measurements of polarized EW bosons remains a powerful probe of SM and BSM physics

- Formalism ready for multiboson, VBF/VBS, etc, tests of (B)SM, in place for future studies and future colliders!
  
  w/ D. Buarque Franzosi, O. Mattelaer, S. Shil [1912.01725]

- Polarized scattering amplitudes and cross sections now possible with MadGraph5 simulation framework

  w/ D. Buarque Franzosi, O. Mattelaer, S. Shil [1912.01725]

- Using EVA, $V_\lambda V'_\lambda$, scattering reveal a new picture of the EW sector at high energies (EVA in MadGraph5 will be released soon!)

  w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer, et al [soon!]

- Lots not shown: polarization propagation via MadSpin, polarization in different reference frames Stay tuned! New results out soon!
Thank you for listening!