The Chirality-Flow Formalism for SM Amplitudes

Andrew Lifson, Lund University/UC Louvain, LHCP 2021 poster

In collaboration with Joakim Alnefjord, Christian Reuschle, and Malin Sjödahl

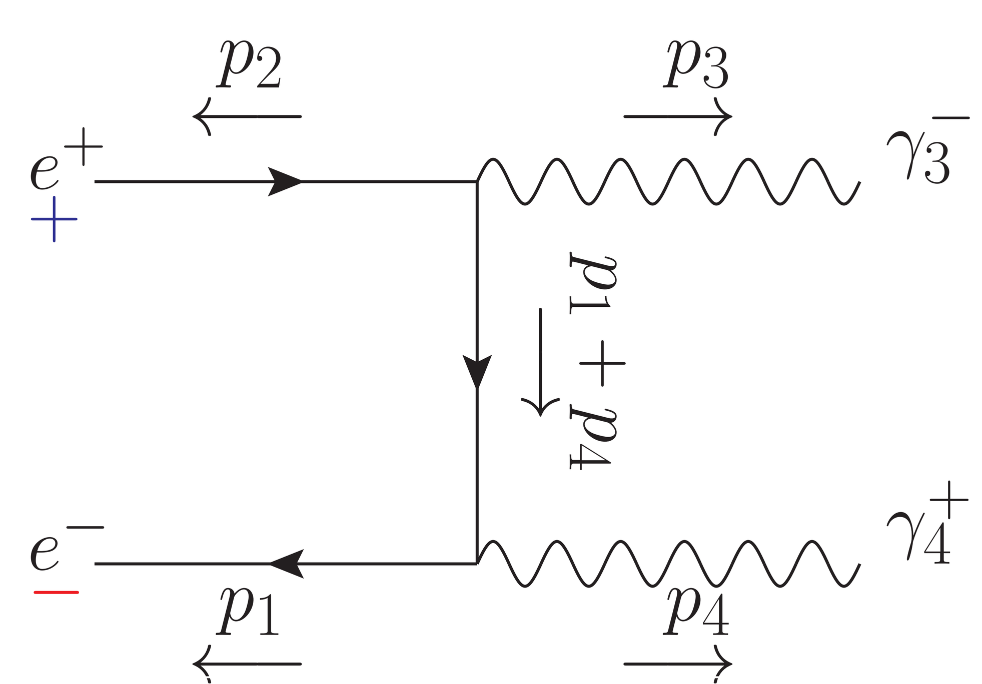
Aim of Chirality Flow

Explore if spinor-helicity $\simeq su(2) \oplus su(2)$ calculations can be done analogously to colour flow $\equiv su(3)$

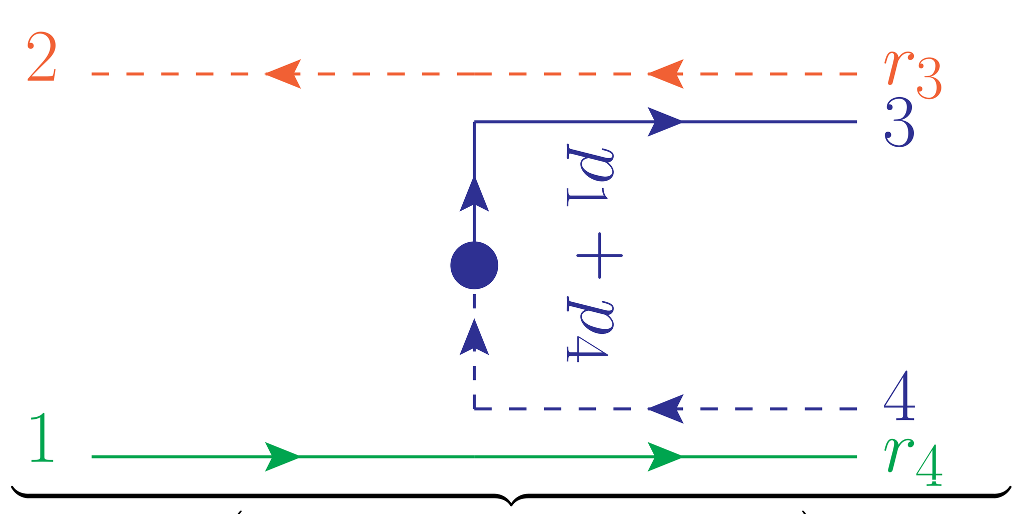
Key Conclusion of Chirality Flow

You can (often) **go from Feynman diagram to complex number in one line**

Ex: Calculate $ee \to \gamma\gamma$ in One Line



$$\sim \frac{1}{\langle r_3 3\rangle [4 r_4]}$$



$$\langle 1 r_4\rangle \big([41]\langle 13\rangle + [44]\langle 43\rangle\big)[r_3 2]$$

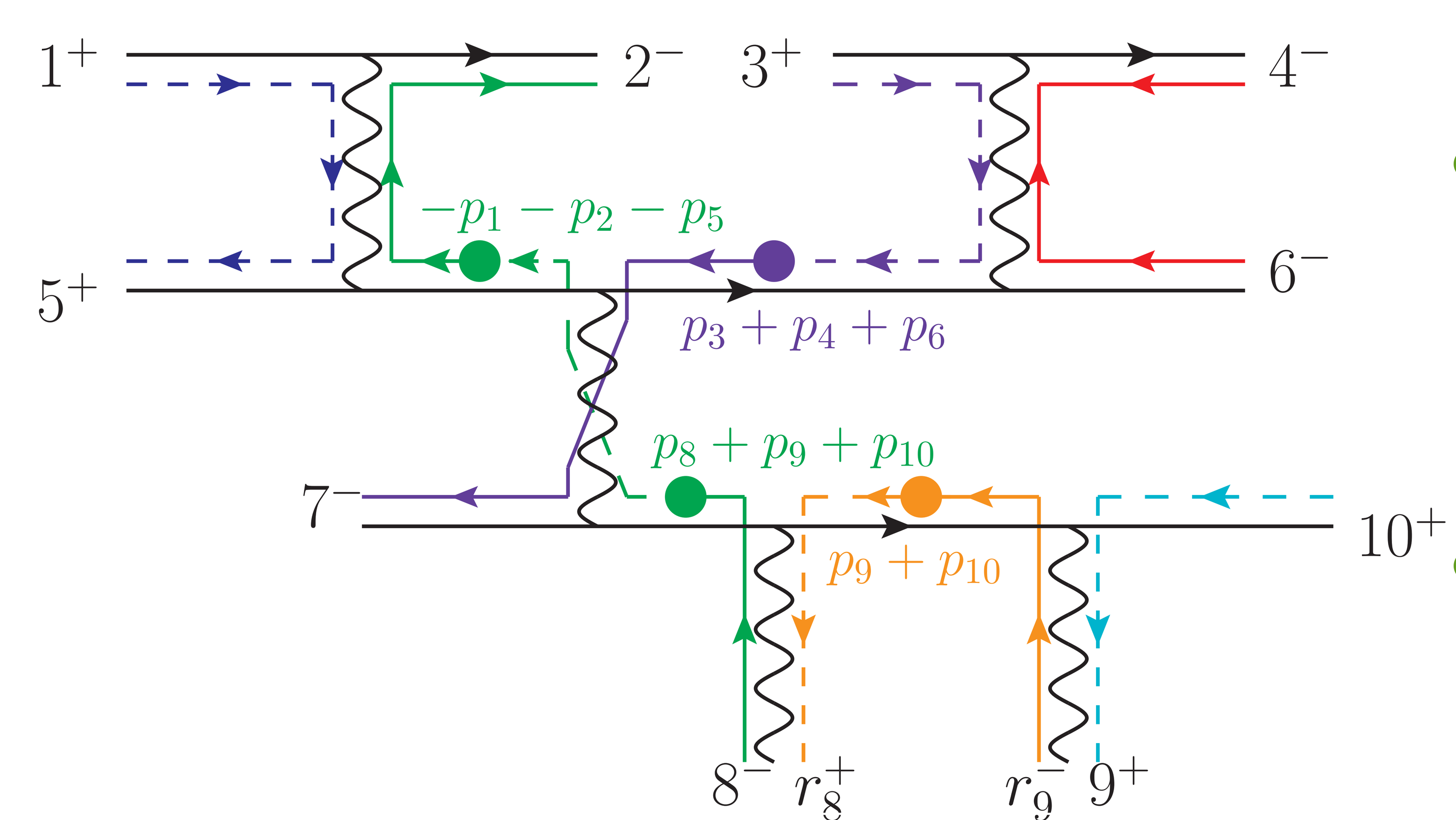
In both examples:

Feynman diagram in black

Coloured flow lines $\equiv$ coloured inner products

Inner products $\equiv$ well known complex numbers

Ex: 10-pt Feynman Diagram in One Line



Compare to:

- Textbook QFT: $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_{12}})$, $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_4})$, $2 \times$ photon spin sum
- Spinor-helicity: 5 charge conjugation/Fierz + rearranging

$$= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{s_{1\,2}\, s_{3\,4}\, s_{8\,9\,10}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{s_{1\,2\,5}\, s_{3\,4\,6}\, s_{8\,9\,10}\, s_{9\,10}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8 r_8]\langle r_9 9\rangle}}_{\text{polarization vectors}} [15]\langle 64\rangle[10\;9]$$

$$\times \big(\langle r_9 9\rangle[9 r_8] + \langle r_9 10\rangle[10 r_8]\big)\big(\underbrace{[33]}_{0}\langle 37\rangle + [34]\langle 47\rangle + [36]\langle 67\rangle\big)$$

$$\times \big(-\langle 89\rangle[91]\langle 12\rangle - \langle 89\rangle[95]\langle 52\rangle - \langle 8\,10\rangle[10\;1]\langle 12\rangle - \langle 8\,10\rangle[10\;5]\langle 52\rangle\big)$$

To find out details see next few slides

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Representations of Lorentz Group

- Lorentz group generators $\simeq$ 2 copies of su(2) generators, i.e. $so(3,1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

For rest of poster:

- Blue $su(2) \equiv$ left chiral
- Red $su(2) \equiv$ right chiral

Lorentz Reps expressed using $su(2) \oplus su(2)$:

- $(0,0)$ scalar
- $(\frac{1}{2},0)$ left-chiral and $(0,\frac{1}{2})$ right-chiral Weyl (2-component) rep
- $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$, Dirac (4-component) rep
- $(\frac{1}{2},\frac{1}{2})$ vector rep

Keys to Spinor-Helicity Formalism

Write all objects in terms of left- and right-chiral Weyl spinors, and Pauli matrices

Amplitude is a number, easy to square

The Spinor-Helicity Method

Massless **spinors** either left and right chiral (use chiral basis, $\gamma^5 = \mathrm{diag}(-1,1)$):

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \qquad u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = \begin{pmatrix} [p| & 0 \end{pmatrix} \qquad \bar{u}^-(p) = \bar{v}^+(p) = \begin{pmatrix} 0 & \langle p| \end{pmatrix}$$

Vectors ($r \equiv$ arbitrary ref spinor, $\tau^\mu = \sigma^\mu/\sqrt{2}$):

$$\epsilon_+^\mu(p,r) = \frac{\langle r|\bar{\tau}^\mu|p]}{\langle rp\rangle}, \qquad \epsilon_-^\mu(p,r) = \frac{[r|\tau^\mu|p\rangle}{[pr]}$$

$$\sqrt{2}p^\mu\tau_\mu \equiv \not{p} = |p]\langle p|, \qquad \sqrt{2}p^\mu\bar{\tau}_\mu \equiv \bar{\not{p}} = |p\rangle[p|$$

Algebraic maniputions to remove vector indices

$$\underbrace{\langle i|\bar{\tau}^\mu|j][k|\tau_\mu|l\rangle = \langle il\rangle[kj]}_{\text{Fierz identity}}, \qquad \underbrace{\langle i|\bar{\tau}^\mu|j] = [j|\tau^\mu|i\rangle}_{\text{Charge Conjugation}}$$

Amplitude $\equiv$ function of Lorentz-invariant spinor inner products (numbers)

$$\langle ij\rangle = -\langle ji\rangle \equiv \langle i||j\rangle \text{ and } [ij] = -[ji] \equiv [i||j],$$
$$\langle ij\rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$$

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Simplify using Chirality Flow

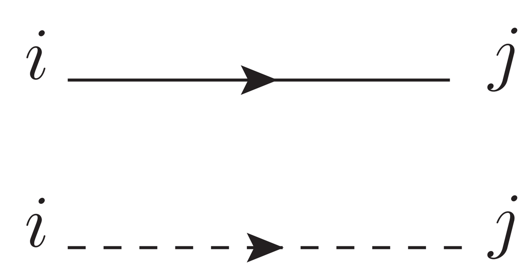
Left-chiral spinors ≡ dotted lines

Right-chiral spinors ≡ solid lines

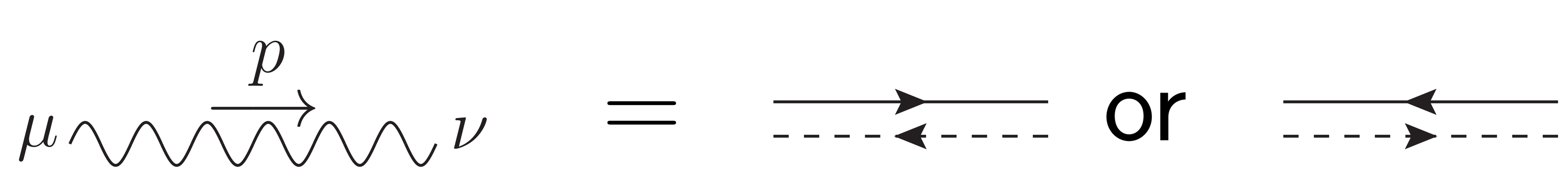
Inner products defined as:

$$\langle i|^{\alpha}|j\rangle_{\alpha} \equiv \langle ij\rangle = -\langle ji\rangle =$$

$$[i|_{\dot\beta}|j]^{\dot\beta} \equiv [ij] = -[ji] =$$



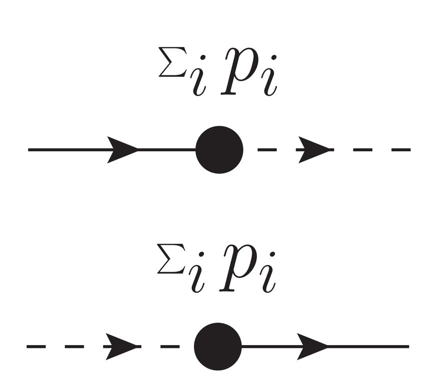
Vectors replaced by double lines



Momenta represented by momentum dot ($p = \Sigma_i\, p_i,\ p_i^2 = 0$)

$$\sqrt{2}p^{\mu}\bar\tau_{\mu} = \sum_i |i\rangle[i| = \quad ,$$

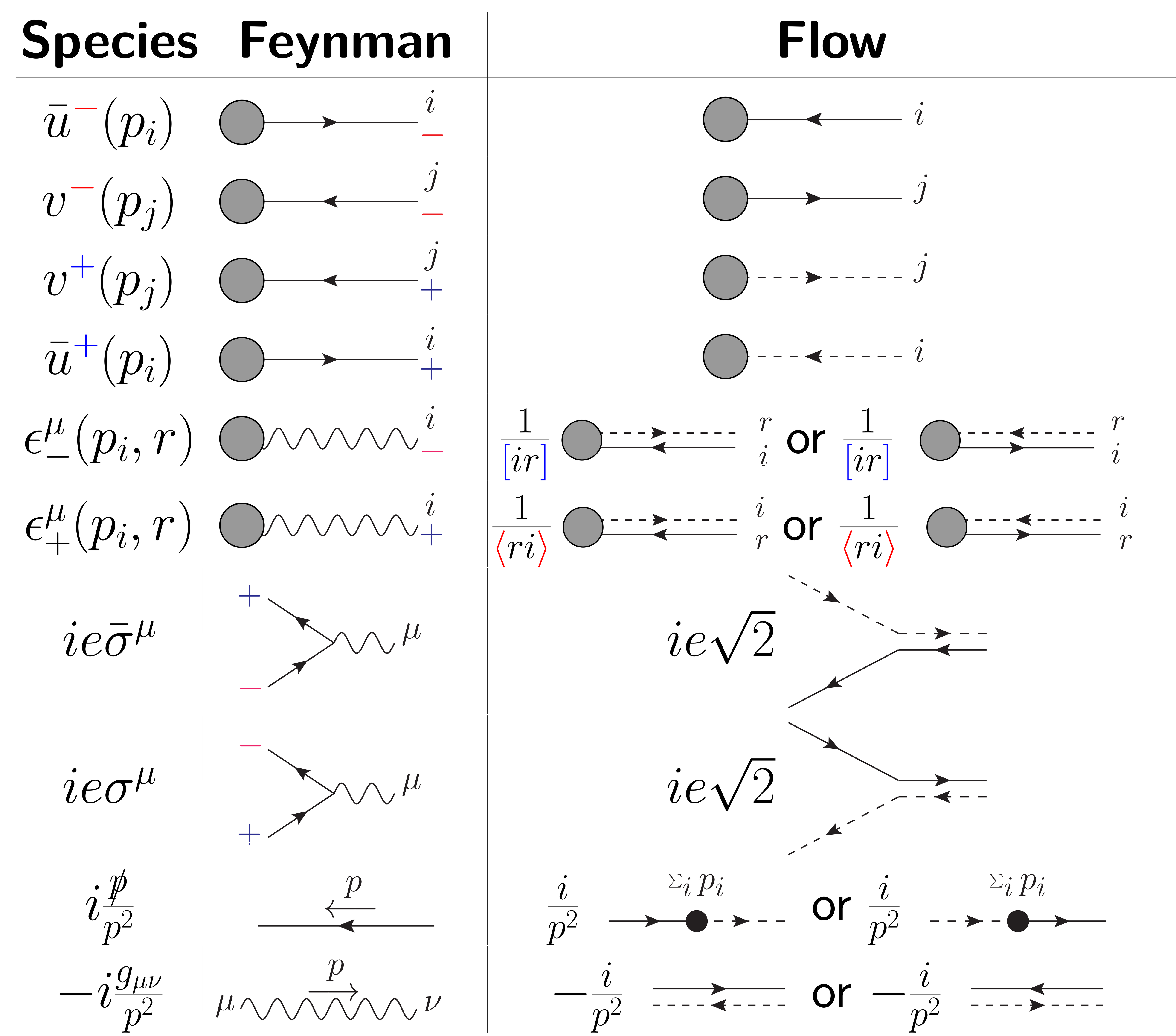
$$\sqrt{2}p^{\mu}\tau_{\mu} = \sum_i |i]\langle i| =$$



Important Takeaway

You can use these replacements to create new set of Feynman rules

(Massless) QED Chirality-Flow Rules



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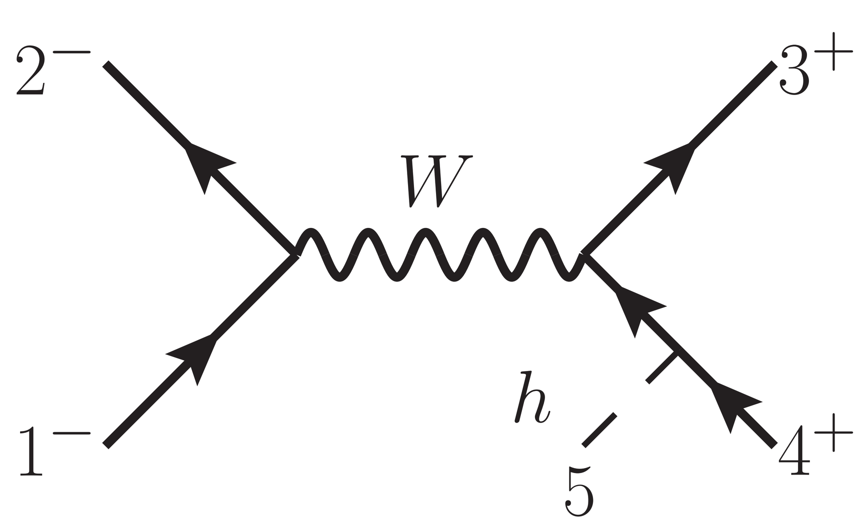
Application of Chirality Flow Rules

- Draw and connect flow lines *without* arrows
- Choose single arrow direction and follow it through diagram (vector double lines have arrows opposing)
- Read off inner products

Massive Chirality Flow + Example

- Massive particles ≡ combinations of massless ones ⇒ recycle massless chirality flow
- E.g. $p^\mu = p^{\flat,\mu} + \alpha q^\mu$, $(p^\flat)^2 = q^2 = 0, p^2 = m^2$

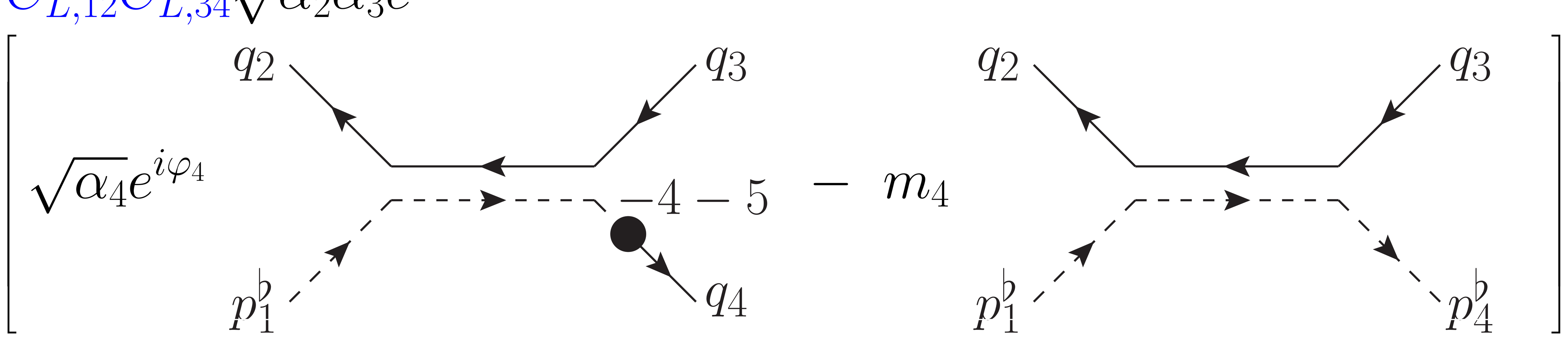


Note:

- W boson simplifies
- Simplify with choices of $q_1, \cdots, q_5$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i \rangle}$, $\quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$
- Scalar has no flow line

$\sim C_{L,12} C_{L,34} \sqrt{\alpha_2 \alpha_3} e^{i(\varphi_2 + \varphi_3)}$

$\times$



Conclusions

- Chirality flow offers the shortest possible journey from Feynman diagram to complex number
- Calculations often performed in a single step, *without* algebraic manipulations
- Full standard model at tree level understood

How to Find Out More Details?

- See hep-ph:2003.05877 (EPJC) and hep-ph:2011.10075 (EPJC)
- Listen to video recording of poster
- Join me in Zoom to discuss and ask questions
- Contact me at andrew.lifson@thep.lu.se

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