# Pseudo-gauge dependence of quantum fluctuations of energy in a hot relativistic fermionic gas

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# Motivation:

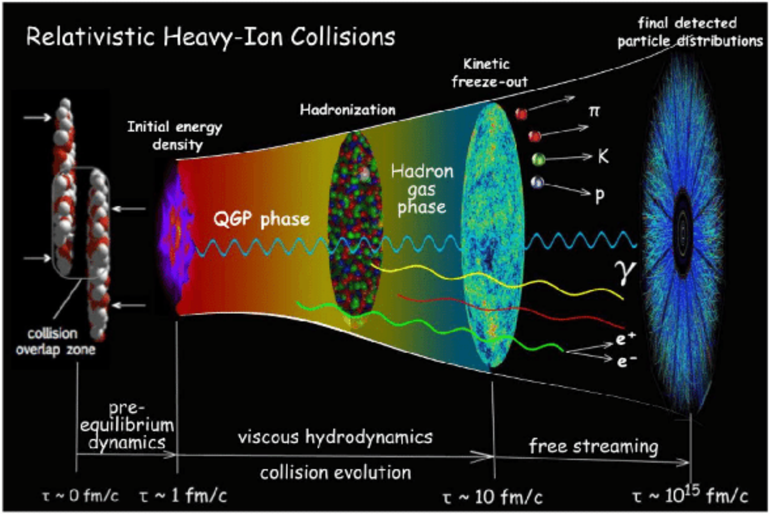



Figure: By Prof. Chun Shen

# Motivation:

Fluctuations of various physical quantities play a very important role in all fields of physics, as they reveal the information about

- possible phase transitions
- formation of structures in the Early Universe
- dissipative phenomena

Most common fluctuations we deal with are those arising from quantum uncertainty relation or those present in thermodynamic systems.

# Motivation:

- Space-time evolution of matter produced in relativistic heavy-ion collision is very well described by relativistic (dissipative) hydrodynamics.

- One of the concepts used in hydrodynamics are those of energy density and pressure, both are defined locally – formally, the fluid element has zero size.

- Successful hydro models are then used to conclude about the energy density attained in the collision processes, usually such values are very large.

# Basic concepts and definitions:

A quantum field operator for spin-$\frac{1}{2}$ particle has the standard form:

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}} \Big( U_r(\boldsymbol{k}) a_r(\boldsymbol{k}) e^{-ik\cdot x} + V_r(\boldsymbol{k}) b_r^\dagger(\boldsymbol{k}) e^{ik\cdot x} \Big),$$

where $a_r(\boldsymbol{k})$ and $b_r^\dagger(\boldsymbol{k})$ are annihilation and creation operators for particles and antiparticles, respectively, satisfying the canonical commutation relations $\{a_r(\boldsymbol{k}), a_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$ and $\{b_r(\boldsymbol{k}), b_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3\delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$, whereas $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2+m^2}$ is the energy of a particle.

# Basic concepts and definitions:

To perform thermal averaging, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators

$$\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}),$$

$$\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle = (2\pi)^6 \Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p})$$

$$-\,\delta_{rr'}\delta_{ss'}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\ \delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}')\Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}).$$

Here $f(\omega_{\boldsymbol{k}})$ is the Fermi–Dirac distribution function for particles.

# Basic concepts and definitions:

We define an operator $\hat{T}_a^{00}$ that represents the energy density of a subsystem $S_a$ placed at the origin of the coordinate system

$$\hat{T}_a^{00} = \frac{1}{(a\sqrt{\pi})^3} \int d^3\boldsymbol{x} \;\hat{T}^{00}(x)\; \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right).$$

In above eq., a smooth Gaussian profile has been used to define the subsystem $S_a$.

To determine the fluctuation of the energy density of the subsystem $S_a$, we consider the variance

$$\sigma^2(a, m, T) = \langle :\hat{T}_a^{00} :: \hat{T}_a^{00}: \rangle - \langle :\hat{T}_a^{00}: \rangle^2$$

and the normalized standard deviation

$$\sigma_n(a, m, T) = \frac{\left(\langle :\hat{T}_a^{00} :: \hat{T}_a^{00}: \rangle - \langle :\hat{T}_a^{00}: \rangle^2\right)^{1/2}}{\langle :\hat{T}_a^{00}: \rangle}.$$

# Different forms of energy-momentum tensors:

For any original EM tensor $\hat{T}^{\mu\nu}$ satisfying the continuity equation $\partial_\mu \hat{T}^{\mu\nu} = 0$ we can construct a different one by adding the divergence of an antisymmetric object, namely

$$\hat{T}'^{\,\mu\nu} = \hat{T}^{\mu\nu} + \partial_\lambda \hat{A}^{\nu\mu\lambda}$$

with $\hat{A}^{\nu\mu\lambda} = -\hat{A}^{\nu\lambda\mu}$. By construction, the new tensor is also conserved, i.e., $\partial_\mu \hat{T}'^{\,\mu\nu} \; = \; 0$.

- We aim here to study the dependence of quantum fluctuations of energy on the so-called pseudo-gauge transformation.

# Different forms of energy-momentum tensors:

Canonical energy-momentum tensor:

$$\hat{T}^{\mu\nu}_{\text{Can}} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi.$$

Belinfante-Rosenfeld form:

$$\hat{T}^{\mu\nu}_{\text{BR}} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}\psi - \frac{i}{16}\partial_{\lambda}\Big(\bar{\psi}\Big\{\gamma^{\lambda},\Big[\gamma^{\mu},\gamma^{\nu}\Big]\Big\}\psi\Big).$$

The de Groot-van Leeuwen-van Weert (GLW) form:

$$\hat{T}^{\mu\nu}_{\text{GLW}} = \frac{1}{4m}\Big[-\bar{\psi}(\partial^{\mu}\partial^{\nu}\psi) + (\partial^{\mu}\bar{\psi})(\partial^{\nu}\psi) + (\partial^{\nu}\bar{\psi})(\partial^{\mu}\psi) - (\partial^{\mu}\partial^{\nu}\bar{\psi})\psi\Big].$$

Hilgevoord-Wouthuysen form:

$$\hat{T}^{\mu\nu}_{\text{HW}} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{i}{2m}\left[\partial^{\nu}\bar{\psi}\sigma^{\mu\beta}\partial_{\beta}\psi + \partial_{\alpha}\bar{\psi}\sigma^{\alpha\mu}\partial^{\nu}\psi\right] - \frac{ig^{\mu\nu}}{4m}\partial_{\lambda}\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_{\alpha}\psi\right)$$

# Energy densities and quantum fluctuation expressions:

Energy densities obtained for different pseudo-gauge choices are the same, i.e., $\varepsilon_{\mathrm{Can}}(T) = \varepsilon_{\mathrm{BR}}(T) = \varepsilon_{\mathrm{GLW}}(T) = \varepsilon_{\mathrm{HW}}(T)$.

$$\langle :\ \hat{T}^{00}_{\mathrm{Can},a}\ : \rangle = 4 \int \frac{d^3k}{(2\pi)^3}\ \omega_{\boldsymbol{k}}\ f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\mathrm{Can}}(T).$$

On the other hand, the fluctuations of $:\ \hat{T}^{00}_a\ :$ are in general different for different pseudo-gauge choices.

# Comparison of normalized standard deviation for various pseudo-gauges:

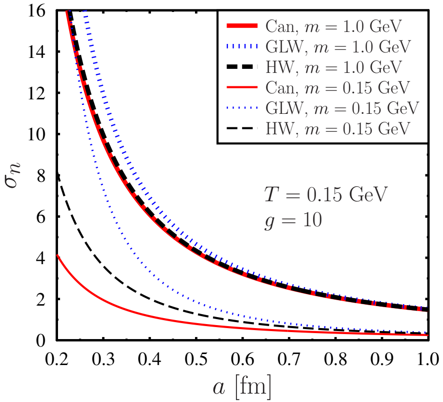



Figure: Comparison of normalized standard deviation for various pseudo-gauges for $T = 0.15$ GeV, $m = 1.0$ GeV (thick lines) and $m = 0.15$ GeV (thin lines).

# Comparison of normalized standard deviation for various pseudo-gauges:

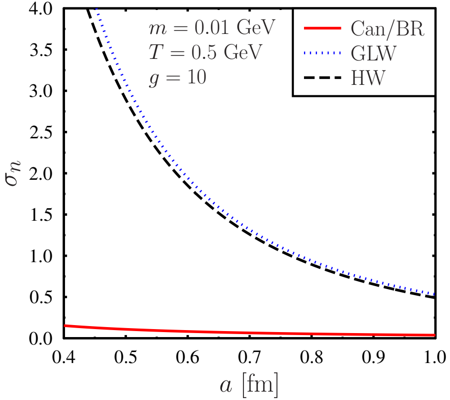



Figure: Comparison of normalized standard deviation for various pseudo-gauges for $T = 0.5$ GeV and $m = 0.01$ GeV.

# Variation of the normalized energy fluctuation:

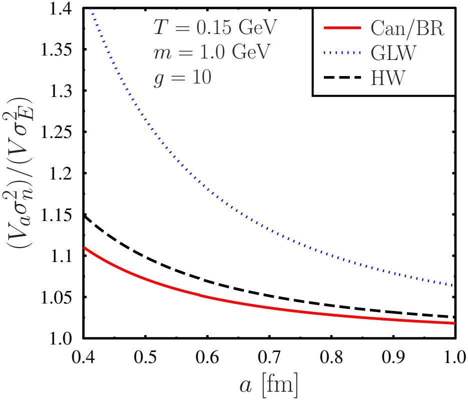



Figure: Variation of the normalized energy fluctuation in the subsystem $S_a$ with the length scale $a$ for $T = 0.15$ GeV and $m = 1.0$ GeV.

# Variation of the normalized energy fluctuation:

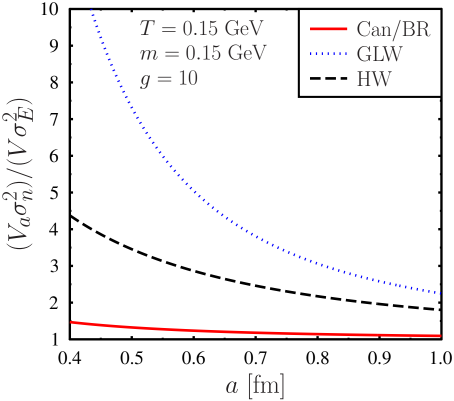
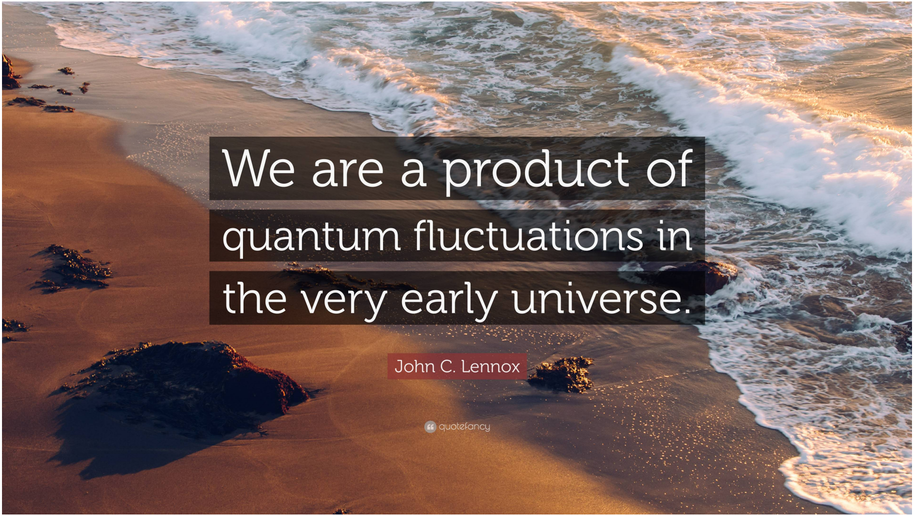



Figure: Same as above but for $T = m = 0.15$ GeV.

# Summary:

- Expressions for quantum fluctuations of energy density in subsystems of a hot relativistic gas of particles with spin $\frac{1}{2}$ derived.
- They depend on the form of the energy-momentum tensor.
- For sufficiently large subsystems the results obtained in different pseudo-gauges converge and agree with the canonical-ensemble formula known from statistical physics.
- On the practical side, the results of our calculations can be used to determine a scale of coarse graining for which the choice of the pseudo-gauge becomes irrelevant, which may be useful, in particular, in the context of hydrodynamic modeling of high-energy collisions.




Thank you for your attention!