Poster : Pseudo-gauge dependence of quantum fluctuations of energy in a hot relativistic gas of fermions.

$\boxed{\text{Purpose :}}$ To study quantum fluctuations of energy and their dependence on the pseudo-gauge transformation. The latter refers to the freedom of choosing a specific form of the energy-momentum tensor to describe the behavior of the system.

Any energy-momentum tensor (EMT) can be defined (or redefined) by

$$T'^{\mu\nu} = T^{\mu\nu} + \partial_\lambda A^{\nu\mu\lambda} \quad , \quad \text{where,} \quad A^{\nu\mu\lambda} = -A^{\nu\lambda\mu} \quad \text{and} \quad \boxed{\partial_\mu T'^{\mu\nu} \equiv \partial_\mu T^{\mu\nu} = 0}$$

Our findings shed new light on the concept of energy density used in the classical description of fluids, in the context of relativistic hydrodynamics of hot matter (QGP) produced in relativistic heavy-ion collisions. Concept of quantum fluctuations of energy in very small thermodynamic systems has no absolute physical meaning and depends on specific pseudo-gauge, only for large subsystems such fluctuations become pseudo-gauge independent.

Spin-$\frac{1}{2}$ field operator

$$\hookrightarrow \psi(t,\vec{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3 \sqrt{2\omega_{\vec{k}}}} \left[U_r(\vec{k})\, a_r(\vec{k})\, e^{-ik.x} + V_r(\vec{k})\, b_r^\dagger(\vec{k})\, e^{ik.x} \right]$$

where, a_r and $b_r^\dagger$ are annilation and creation operator respectively.

Energy density operator is defined as:

$$T_a^{00} = \frac{1}{(a\sqrt{\pi})^3} \int d^3\vec{x}\, T^{00}(x) \exp\left(-\frac{\vec{x}^2}{a^2}\right)$$

'a' is the system size

To determine the fluctuation of the energy density of the subsystem S_a, we consider the variance:

$$\sigma^2(a,m,T) := \langle \underbrace{:T_a^{00}:}_{\text{normal ordered}} :T_a^{00}:\rangle - \underbrace{\langle :T_a^{00}: \rangle}_{\text{thermal averaging}}^2$$

Here, we study 4 different forms of EMT, currently being analysed in the context of relativistic heavy-ion collisions.

1:→ Canonical EMT

$$T_{can}^{\mu\nu} = \frac{i}{2}\bar{\Psi}\gamma^\mu(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu)\Psi$$

where, thermal expectation value of $T_{can,a}^{00}$ is $\langle :T_{can,a}^{00}: \rangle = \underbrace{4}_{\text{because of spin degeneracy.}} \int \frac{d^3k}{(2\pi)^3}\, \omega_{\vec{k}}\, f(\omega_{\vec{k}})$

And,

$$\sigma^2_{can}(a,m,T) = 2\int dK\, dK'\, f(\omega_{\vec{k}})\,(1-f(\omega_{\vec{k}'}))\times\Big[(\omega_{\vec{k}}+\omega_{\vec{k}'})^2\,(\omega_{\vec{k}}\omega_{\vec{k}'}+\vec{k}\cdot\vec{k}'+m^2)\,e^{-\frac{a^2}{2}(\vec{k}-\vec{k}')^2} - (\omega_{\vec{k}}-\omega_{\vec{k}'})^2\,(\omega_{\vec{k}}\omega_{\vec{k}'}+\vec{k}\cdot\vec{k}'-m^2)\,e^{-\frac{a^2}{2}(\vec{k}+\vec{k}')^2}\Big]$$

where, $dK \equiv \frac{d^3k}{(2\pi)^3\, 2\omega_{\vec{k}}}$

2: → <u>Belinfante- Rosenfeld</u> framework

$$T^{\mu\nu}_{BR} = \frac{i}{2}\bar{\psi}\gamma^\mu(\overrightarrow{\partial^\nu}-\overleftarrow{\partial^\nu})\psi - \frac{i}{16}\partial_\lambda\left(\bar{\psi}\{\gamma^\lambda,[\gamma^\mu,\gamma^\nu]\}\psi\right)$$

We find that, $T^{00}_{BR} \equiv T^{00}_{can}$, hence the results will be same.

3: → <u>de Groot-van Leeuwen-van Weert</u> (GLW) framework

$$T^{\mu\nu}_{GLW} = \frac{1}{4m}\left[-\bar{\psi}(\partial^\mu\partial^\nu\psi) + (\partial^\mu\bar{\psi})\partial^\nu\psi + (\partial^\nu\bar{\psi})\partial^\mu\psi - (\partial^\mu\partial^\nu\bar{\psi})\psi\right].$$

and,
$$\sigma^2_{GLW}(a,m,T) = \frac{1}{2m^2}\int dK\, dK'\, f(\omega_{\vec{k}})\,(1-f(\omega_{\vec{k}'}))\times\Big[(\omega_{\vec{k}}+\omega_{\vec{k}'})^4\,(\omega_{\vec{k}}\omega_{\vec{k}'}-\vec{k}\cdot\vec{k}'+m^2)\,e^{-\frac{a^2}{2}(\vec{k}-\vec{k}')^2} - (\omega_{\vec{k}}-\omega_{\vec{k}'})^4\,(\omega_{\vec{k}}\omega_{\vec{k}'}-\vec{k}\cdot\vec{k}'-m^2)\,e^{-\frac{a^2}{2}(\vec{k}+\vec{k}')^2}\Big]$$

We find that, $\langle :T^{00}_{can,a}:\rangle$, $\langle :T^{00}_{BR,a}:\rangle$ and $\langle :T^{00}_{GLW,a}:\rangle$ are same.

Finally,

4: → Hilgevoord - Wouthuysen (HW) framework

$$T^{\mu\nu}_{HW} = T^{\mu\nu}_{can} + \frac{i}{2m}\left(\partial^{\nu}\bar{\Psi}\sigma^{\mu\beta}\partial_{\beta}\Psi\right) + \frac{i}{2m}\partial_{\alpha}\bar{\Psi}\sigma^{\alpha\mu}\partial^{\nu}\Psi - \frac{i}{4m}g^{\mu\nu}\partial_{\lambda}\left(\bar{\Psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_{\alpha}\Psi\right)$$

Here, also, $\langle :T^{00}_{HW,a}: \rangle \equiv \langle :T^{00}_{can,a}: \rangle \equiv \langle :T^{00}_{BR,a}: \rangle \equiv \langle :T^{00}_{GLW,a}: \rangle$

where, $\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_{\mu},\gamma_{\nu}]$

and, $$\sigma^2_{HW}(a,m,T) = \frac{2}{m^2}\int dK\, dK'\, f(\omega_{\vec{K}})\left(1-f(\omega_{\vec{K}'})\right) \times$$
$$\Big[\left(\omega_{\vec{K}}\omega_{\vec{K}'} + \vec{k}\cdot\vec{k}' + m^2\right)^2\left(\omega_{\vec{K}}\omega_{\vec{K}'} - \vec{k}\cdot\vec{k} + m^2\right)e^{-\frac{a^2}{2}(\vec{k}-\vec{k}')^2} -$$
$$\left(\omega_{\vec{K}}\omega_{\vec{K}'} + \vec{k}\cdot\vec{k}' - m^2\right)^2\left(\omega_{\vec{K}}\omega_{\vec{K}'} - \vec{k}\cdot\vec{k}' - m^2\right)e^{-\frac{a^2}{2}(\vec{k}+\vec{k}')^2}\Big]$$

Conclusion :→ We find that quantum fluctuations of energy density depends on different forms of EMT (or pseudo-gauge choices), but their thermal average value is independent of pseudo-gauge transformation.
For very large subsystems (i.e., in classical limit), each quantum fluctuation expression converges and becomes pseudo-gauge independent. On the practical side, our results can be used to determine a scale of ~~coarg~~ coarse graining for which the choice of pseudo-gauge become irrelevant, which may be useful in the context of hydrodynamic modelling of high-energy collisions.

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