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Renormalization of scalar EFTs at higher orders (arXiv:2105.12742)

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Background & questions

Scalar effective field theories

— easier to get higher-order results

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{4!}\phi^4 + \sum_{n>4}\frac{1}{\Lambda^{n-4}}\sum_i c_i^{(n)}\mathcal{O}_i^{(n)}$$

Anomalous dimension matrix (ADM)

— capturing the scale dependence of the coefficients and the mixing between EFT operators

$$\mu\frac{dc_i^{(n)}}{d\mu} = \gamma_{ij}c_j^{(n)}$$

The EFT operators are redundant under integration by part (IBP) and equation of motion (EOM). How to **count** and **construct** the **physical basis** (without any redundancies)?

How to calculate the ADM of EFT operators at **higher mass dimension** to **higher loops**?

Operator basis

1. We count the physics basis of EFT operators by **Hilbert series**
— by state-operator correspondence, counting operators in physical basis is the same as counting conformal primary states

2. It is natural to use the **conformal primary operators** to form the physical basis.

3. Primary operators can be annihilated by special conformal generator K_μ , e.g.

$$K_\mu\left(-\frac{2}{4} + 9 - \frac{72}{6} - \frac{72}{6} - \frac{126}{4} + \frac{108}{2}\right) = 0$$

(Graph notation: dot represents ϕ; line represents the contracted derivatives)

4. Inner product defined by $\mathrm{Im}\,\langle\mathcal{O}_i,\,\mathcal{O}_j\rangle_{P\neq 0} =: \left[\prod_{i=1}^{N}\int d^4p_i\;\delta^+(p_i^2)\right]\delta(P-\Sigma_i p_i)\;\mathcal{O}_1(p_i)\cdot\mathcal{O}_2(\text{-}\,p_i)$

R^* method

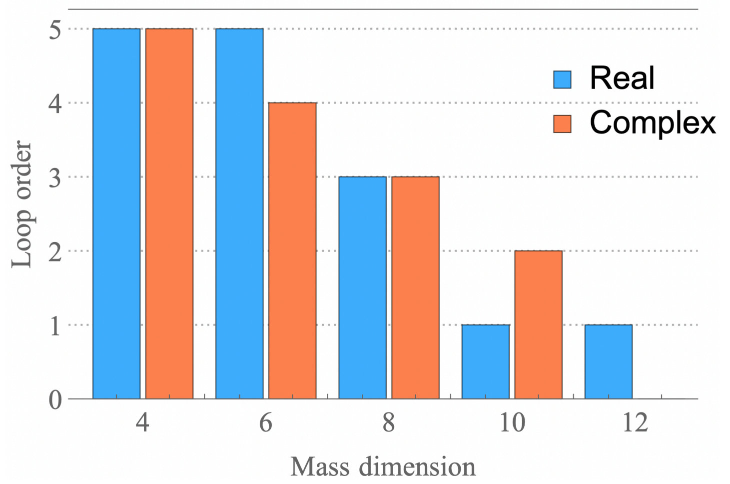
— R^* recursively subtracts UV subdivergences and IR divergences

— Allows for **Taylor expanding** the Feynman diagrams and '**Infrared rearrangement**'

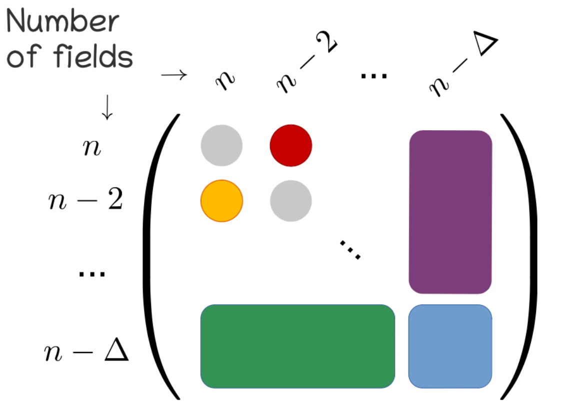
— Transforms the calculation of complicated diagrams to simple diagrams by the above tricks

— Only needs physical basis as input

Scope of calculation



General results of ADM



- (red) 1-loop zero only when $\mathcal{O}^{(n)}_{n-2}$ is primary
- (purple) Generally non-zero in a basis of primaries
- (green) Zeros up to $L=\Delta$ loops
- (yellow) Unexpected 3-loop zero in many bases
- (blue) Depends only on primary parts, symmetric at 1-loop when orthonormal

Appendix: calculation process

Original: Chetyrkin & Tkachov 1982
Chetyrkin & Smirnov 1984
Smirnov & Chetyrkin 1985
Recent: Herzog & Ruijl [1703.03776]
Beekveldt, Borinsky & Herzog [2003.04301]

Primary operators → Physical basis

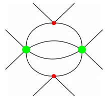
R^* method

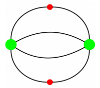
Additional IR divergences will be subtracted by R^*

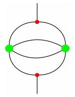
Taylor expansion of the diagram

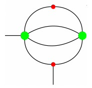
Infrared rearrangment

Diagram with degree of divergences ω → Logarithmic scaleless vacuum diagrams → Self-energy diagram or vacuum diagram with massive edges









Same counterterms but simpler diagrams

(Off-shell basis)

Counterterms in basis that are IBP independent

Field redefinition and EoM operators

Field redefinition that contributes to mixing in dimension n

$$\phi \to \phi + \frac{1}{\Lambda^{n-4}} F^{(n)}(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, ..)$$

$$I[\phi] \xrightarrow{\text{FR}} I[\phi] + \int d^D x \, \frac{1}{\Lambda^{n-4}} F^{(n)}(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, ...) \frac{\delta I^{(4)}[\phi]}{\delta \phi} + O(1/\Lambda^{n-2}, F)$$

Classical EOM at dimension 4: $E^{(4)} := \dfrac{\delta I^{(4)}[\phi]}{\delta \phi}$

EoM operators: $\mathcal{E}^{b(n)} = \dfrac{1}{\Lambda^{n-4}} E^{(4)} F^{(n)}(\phi, \partial_\mu \phi, \partial_\mu \partial_\nu \phi, ..)$

Off-shell basis = physical operators + EoM operators

EoM operators do not mix into physical operators!

H. David Politzer Nucl. Phys. B 172 (1980) 349

Physical operators + EoM operators

Field redefinition

ADM for physical basis

Real

$$\gamma_{\mathrm{C}}^{(6)} = 9g - \frac{359g^2}{6} + \left(216\zeta_3 + \frac{5773}{12}\right)g^3 - \left(\frac{5283\zeta_3}{2} - 459\zeta_4 + 3960\zeta_5 + \frac{1312907}{288}\right)g^4 + \left(1755\zeta_3^2 + \frac{510193}{16}\zeta_3 - \frac{25483}{4}\zeta_4 + 54621\zeta_5 - \frac{29025}{2}\zeta_6 + 73584\zeta_7 + \frac{333811365}{6912}\right)g^5$$

$$\gamma_{\mathrm{C}}^{(8)} = \begin{array}{c} \begin{array}{cc} g^3\,\mathcal{O}_8^{(8)c} & g\,\mathcal{O}_4^{(8)c} \end{array} \\ \begin{pmatrix} 19g - 169g^2 + \left(636\zeta_3 + \frac{46255}{24}\right)g^3 & 2352g - \frac{76720g^2}{3} + \left(65856\zeta_3 + \frac{8676248}{27}\right)g^3 \\ 0g + 0g^2 + 0g^3 & \frac{g}{3} + \frac{49g^2}{27} - \left(\frac{12017}{1944} + \frac{20\zeta_3}{3}\right)g^3 \end{pmatrix} \end{array}$$

$$\gamma_{\mathrm{C}}^{(10)} = \begin{array}{c} \begin{array}{ccc} g^4\,\mathcal{O}_{10}^{(10)c} & g^2\,\mathcal{O}_6^{(10)c} & g\mathcal{O}_4^{(10)c} \end{array} \\ \begin{pmatrix} 33g & 23100g & 233520g \\ 0g & 5g & \frac{112g}{5} \\ 0g & 0g & -2g \end{pmatrix} \end{array} \quad \gamma_{\mathrm{C}}^{(12)} = \begin{array}{c} \begin{array}{ccccc} g^5\,\mathcal{O}_{12}^{(12)c} & g^3\,\mathcal{O}_8^{(12)c} & g^2\,\mathcal{O}_6^{(12)c}{}_{(1,0)} & g^2\,\mathcal{O}_6^{(12)c}{}_{(0,1)} & g\,\mathcal{O}_4^{(12)c} \end{array} \\ \begin{pmatrix} 51g & \frac{1121120g}{9} & \frac{477400g}{3} & \frac{284900g}{3} & \frac{7037295100g}{147} \\ 0g & \frac{41g}{3} & -\frac{4675g}{2} & \frac{395g}{4} & \frac{45981307g}{1372} \\ 0g & 0g & 2g & 0g & -\frac{26779g}{490} \\ 0g & 0g & 0g & 2g & -\frac{445574g}{245} \\ 0g & 0g & 0g & 0g & -\frac{g}{5} \end{pmatrix} \end{array}$$

Complex

$$\gamma_{\mathrm{C}}^{(6)} = \begin{array}{c} \begin{array}{cc} g^2\,\mathcal{O}_6^{(6)c} & g\,\mathcal{O}_4^{(6)c} \end{array} \\ \begin{pmatrix} 14g - \frac{297g^2}{2} + \left(816\zeta_3 + \frac{14981}{8}\right)g^3 - \left(\frac{32087\zeta_3}{2} - 2892\zeta_4 + 23320\zeta_5 + \frac{888983}{32}\right)g^4 & 0g - \frac{45g^2}{2} + \left(216\zeta_3 + \frac{6153}{16}\right)g^3 - \left(783\zeta_3 + 8100\zeta_5 + \frac{74079}{16}\right)g^4 \\ 0g + 0g^2 + 0g^3 + \frac{5g^4}{6} & -g + \frac{13g^2}{2} - \left(36\zeta_3 + \frac{383}{12}\right)g^3 + \left(\frac{769\zeta_3}{2} - 123\zeta_4 + 560\zeta_5 + \frac{7893}{32}\right)g^4 \end{pmatrix} \end{array}$$

$$\gamma_{\mathrm{C}}^{(8)} = \begin{array}{c} \begin{array}{cccc} g^3\,\mathcal{O}_8^{(8)c} & g^2\,\mathcal{O}_6^{(8)c} & g\,\mathcal{O}_4^{(8)c}{}_{(1,0)} & g\,\mathcal{O}_4^{(8)c}{}_{(0,1)} \end{array} \\ \begin{pmatrix} 29g - 409g^2 + \left(2352\zeta_3 + \frac{57765}{8}\right)g^3 & 0g - 240g^2 + \left(2304\zeta_3 + 5882\right)g^3 & \frac{54216g}{5} - \frac{958314g^2}{5} + \left(\frac{4083264\zeta_3}{5} + \frac{55731313}{15}\right)g^3 & -\frac{8856g}{5} + \frac{159894g^2}{5} - \left(\frac{713664\zeta_3}{5} + \frac{9281093}{15}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 4g - \frac{122g^2}{3} + \left(216\zeta_3 + \frac{4559}{12}\right)g^3 & -\frac{679g}{5} + \frac{14209g^2}{12} - \left(\frac{29216\zeta_3}{5} + \frac{3248605}{324}\right)g^3 & \frac{164g}{5} - \frac{9233g^2}{36} + \left(\frac{6056\zeta_3}{5} + \frac{2740291}{1296}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g + 0g^2 + \frac{5g^3}{108} & \frac{11g}{3} - \frac{29g^2}{180} - \left(\frac{32\zeta_3}{3} + \frac{97091}{9720}\right)g^3 & -\frac{4g}{3} + \frac{1057g^2}{540} - \left(\frac{16\zeta_3}{3} + \frac{10868}{1215}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g + 0g^2 + \frac{115g^3}{324} & \frac{46g}{3} - \frac{14557g^2}{540} + \left(96\zeta_3 + \frac{198001}{2430}\right)g^3 & -\frac{19g}{3} + \frac{2809g^2}{180} - \left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{pmatrix} \end{array}$$

$$\gamma_{\mathrm{C}}^{(10)} = \begin{array}{c} \begin{array}{cccccccc} g^4\,\mathcal{O}_{10}^{(10)c} & g^3\,\mathcal{O}_8^{(10)c} & g^2\,\mathcal{O}_6^{(10)c}{}_{(1,0,0,0)} & g^2\,\mathcal{O}_6^{(10)c}{}_{(0,1,0,0)} & g^2\,\mathcal{O}_6^{(10)c}{}_{(0,0,1,0)} & g^2\,\mathcal{O}_6^{(10)c}{}_{(0,0,0,i)} & g\,\mathcal{O}_4^{(10)c}{}_{(1,0)} & g\,\mathcal{O}_4^{(10)c}{}_{(0,1)} \end{array} \\ \begin{pmatrix} 50g - \frac{1725g^2}{2} & 0g - 1125g^2 & \frac{412500g}{7} - \frac{61613665g^2}{42} & \frac{3300g}{7} - \frac{19655g^2}{6} & \frac{141900g}{7} - \frac{6734025g^2}{14} & 0 & -\frac{500460g}{7} + \frac{31376515g^2}{21} & -64740g + \frac{16819700g^2}{21} \\ 0g + 0g^2 & 15g - \frac{385g^2}{2} & -\frac{1474g}{7} + \frac{83689g^2}{45} & -\frac{234g}{7} + \frac{386467g^2}{315} & \frac{326g}{7} - \frac{4577g^2}{7} & 0 & -\frac{43732g}{35} + \frac{5863532g^2}{315} & \frac{14272g}{5} - \frac{15354406g^2}{315} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{35g}{6} - \frac{678389g^2}{15120} & -\frac{g}{6} + \frac{24379g^2}{15120} & \frac{7g}{6} - \frac{28265g^2}{3024} & 0 & \frac{1006g}{105} + \frac{1249571g^2}{15120} & -\frac{967g}{30} + \frac{1903001g^2}{3780} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{4g}{3} - \frac{331g^2}{630} & -\frac{2g}{3} - \frac{5401g^2}{945} & 2g - \frac{14965g^2}{756} & 0 & -\frac{4159g}{105} + \frac{3146033g^2}{7560} & \frac{914g}{15} - \frac{103037g^2}{315} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{15g}{2} - \frac{247109g^2}{5040} & -\frac{g}{2} - \frac{4127g^2}{1680} & \frac{7g}{2} - \frac{31333g^2}{1008} & 0 & -\frac{1769g}{35} + \frac{243011g^2}{336} & \frac{573g}{10} + \frac{37049g^2}{1260} \\ 0 & 0 & 0 & 0 & 0 & \frac{10g}{3} - \frac{770g^2}{27} & 0 & 0 \\ 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0 & -2g + \frac{223g^2}{24} & -g + \frac{7g^2}{24} \\ 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0 & -g + \frac{7g^2}{12} & -3g + \frac{251g^2}{24} \end{pmatrix} \end{array}$$

Results: ADM and Hilbert series for physical basis in both real and complex scalar EFTs

Hilbert series

$$H_{S\text{-matrix}}^{\text{real}}(\Delta, t, \phi) = \Delta^6\phi^6 + \Delta^8\left(\phi^4 t^4 + \phi^8\right) + \Delta^{10}\left(\phi^4 t^6 + \phi^6 t^4 + \phi^{10}\right) + \Delta^{12}\left(\phi^4 t^8 + 2\phi^6 t^6 + \phi^8 t^4 + \phi^{12}\right)$$

$$H_{S\text{-matrix}}^{\text{complex}}(\Delta, t, \phi, \phi^\dagger) = \Delta^6\left(\phi^2\phi^{\dagger 2}t^2 + \phi^3\phi^{\dagger 3}\right) + \Delta^8\left(2\phi^2\phi^{\dagger 2}t^4 + \phi^3\phi^{\dagger 3}t^2 + \phi^4\phi^{\dagger 4}\right) + \Delta^{10}\left(2\phi^2\phi^{\dagger 2}t^6 + 4\phi^3\phi^{\dagger 3}t^4 + \phi^4\phi^{\dagger 4}t^2 + \phi^5\phi^{\dagger 5}\right)$$