Impact of Soft Photons on $B \to K\ell^+\ell^-$

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Abstract

In this precision era, it has become evident to have precise expressions and construct theoretically clean observables to match with the experiments. Here, we discuss $\mathcal{O}(\alpha)$ QED corrections to $B \to K\ell^+\ell^-$ modes. The structure of the contact term is fixed by demanding gauge invariance of the real emission amplitude. The calculation is done by providing fictitious mass (λ) to the photon, which acts as IR regulator, and results are shown to be independent of it. QED effects are found to be negative. Electron channel is shown to receive large correction $\mathcal{O}(20\%)$. We also discuss the impact on lepton flavour universality (LFU) ratio ($R_K^{\mu e}$).

Introduction

- Quark transitions due to Flavour Changing Neutral Currents (FCNCs) are both loop and CKM suppressed which make them important candidates to test the Standard Model (SM) and possibly the search for New Physics (NP).
- The quest of the precision test of SM leads us to construct theoretically clean observables. The decay modes $B \to K\ell^+\ell^-$ allow to test the lepton flavour universality (LFU), defined via

$$R_K^{\mu e} \equiv \frac{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0\to K^0\mu^+\mu^-)}{dq^2}}{\int_{1GeV^2}^{6GeV^2} dq^2 \frac{d\Gamma(B^0\to K^0 e^+e^-)}{dq^2}}$$

- The SM and experimental values are[1]

$$R_K^{\mu e}|_{SM} = 1.00 \pm 0.01, \quad R_K^{\mu e}|_{exp} = 0.846^{+0.060+0.016}_{-0.054-0.014}$$

- Within the SM, if the kinematical range is chosen such that the dilepton invariant mass is way larger than the mass of considered leptons then it is expected that the ratio of two branching fractions is unity to a high accuracy.

[1] LHCb collaboration, Test of lepton universality in beauty-quark decays, 2103.11769.

- The strong interaction effects are included via RGEs and form factors. The considered ratio in this kinematical range is less sensitive to uncertainties due to form factors.

Matrix element analysis

The effective Hamiltonian for $b \to s\ell^+\ell^-$ transition is

$$H_{eff} = 4\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu) \quad (1)$$

- **Non-radiative**:

The matrix element for non-radiative decay:

$$M_0(B \to K l^+ l^-) = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{ts}^* V_{tb} \Bigg[\Bigg(\left\{ C_9^{eff} f_+ + C_7^{eff} \frac{2 f_T m_b}{m_B + m_k} \right\} p^\mu + \left\{ C_9^{eff} f_- - C_7^{eff} \frac{2 f_T m_b}{q^2}(m_B - m_k) \right\} q^\mu \Bigg) (\bar{l}\gamma_\mu l) - (C_{10} f_+ p^\mu + C_{10} f_- q^\mu)(\bar{l}\gamma_\mu \gamma_5 l) \Bigg]$$

- **Radiative**:

The matrix element for the emission of photon from the external legs is:

$$\begin{aligned}\tilde{M} = & -e\epsilon_\alpha(k)\bar{u}(p_2)\Gamma_A^\mu \frac{(\not{p}_3 + \not{k}) - m_l}{2p_3.k}\gamma^\alpha v(p_3) \otimes H_{A\mu}(p_0, p_1) \\ & + e\epsilon_\alpha(k)\bar{u}(p_2)\gamma^\alpha \frac{(\not{p}_2 + \not{k}) + m_l}{2p_2.k}\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0, p_1) \\ & + eQ_B\epsilon_\alpha(k)\frac{2p_0^\alpha}{2p_0.k}\bar{u}(p_2)\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0 - k, p_1) \\ & - eQ_K\epsilon_\alpha(k)\frac{2p_1^\alpha}{2p_1.k}\bar{u}(p_2)\Gamma_A^\mu v(p_3) \otimes H_{A\mu}(p_0, p_1 + k)\end{aligned}$$

where, $H_\mu(p_i, p_j) = f_+(p_i + p_j)_\mu + f_-(p_i - p_j)_\mu$

Leptonic part	Gauge invariant
Hadronic part	Not gauge invariant
Total amplitude	Not gauge invariant

Main Objectives

1. To fix the gauge invariance of matrix element.
2. To get the $\mathcal{O}(\alpha)$ QED correction for the decay width and $R_K^{\mu e}$.
3. To discuss the collinear divergences and their cancellation.

Real Photon emission

- **Contact Term (CT)**: The addition of a CT (i.e. $e(Q_B+Q_K)\xi_A k_\mu\left[\bar{u}(p_2)\Gamma_A^\mu v(p_3)\right]$) is required to preserve gauge invariance.

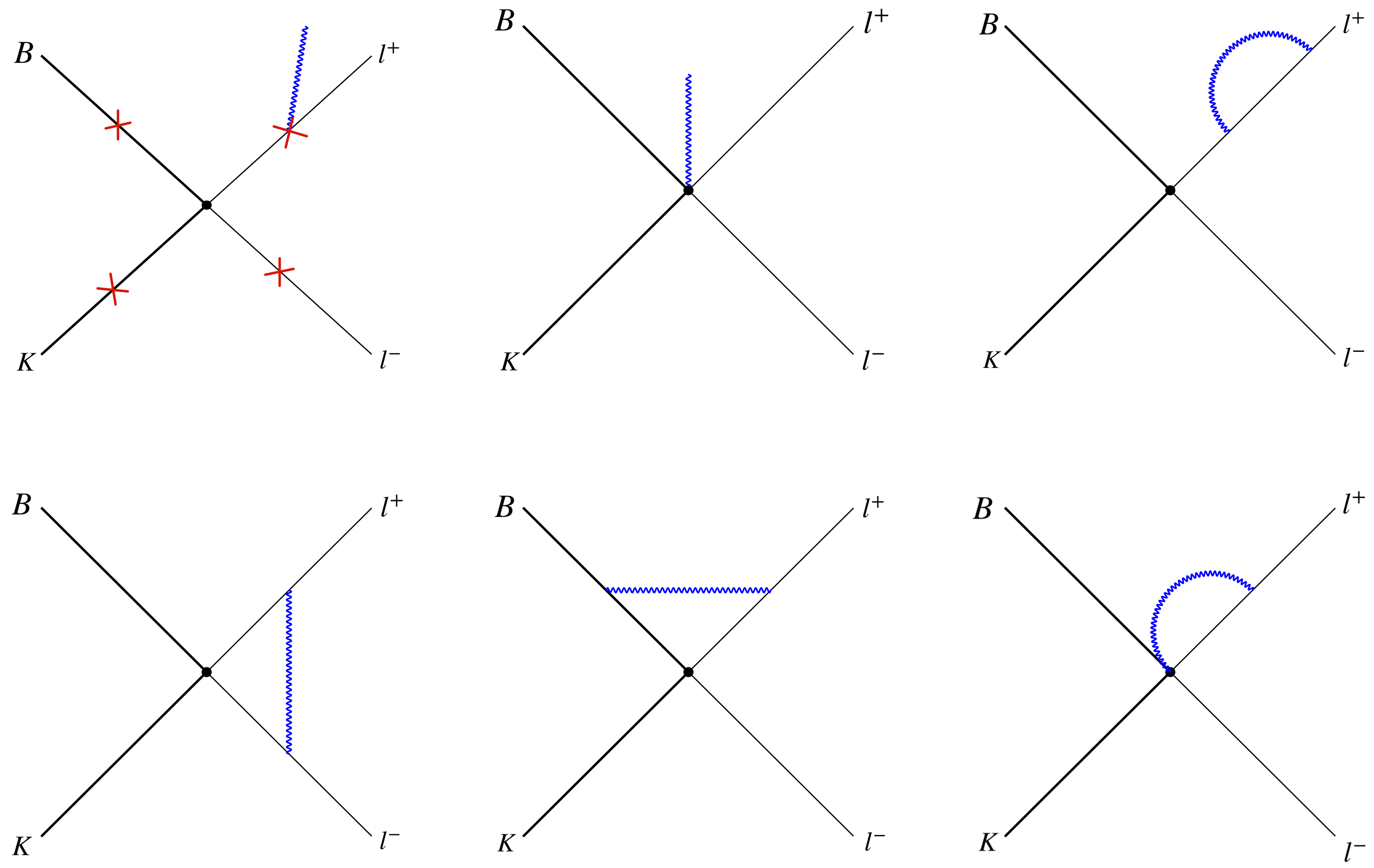

Figure: Representative diagrams contributing to real emission and virtual corrections(X :Photon emission)

- The total contribution to the real photon emission amplitude $B(p_0) \rightarrow K(p_1)\ell^+(p_2)\ell^-(p_3)\gamma(k)$ is sum of Low's IR term and $M'(k)$(contribution from the contact term added properly)

$$d\Gamma_{real} = \underbrace{d\Gamma_0}_{\text{Non-radiative decay width}}\left(1+2\alpha\underbrace{\tilde{B}}_{\text{Low's term}}\right)+\underbrace{d\Gamma'}_{\text{Non-IR contribution}} \qquad (2)$$

- The non-IR term is important to see the cancellation of collinear divergences.
- Charge conservation and integrating over photons momentum k gives:

$$\tilde{B_{ij}} = \frac{Q_iQ_j\eta_i\eta_j}{2\pi}\left\{\ln\left(\frac{k_{max}^2 m_i m_j}{\lambda^2 E_i E_j}\right) - \frac{p_i.p_j}{2}\left[\int_{-1}^{1}\frac{dx}{p_x^2}\ln\left(\frac{k_{max}^2}{E_x^2}\right)+\int_{-1}^{1}\frac{dx}{p_x^2}\ln\left(\frac{p_x^2}{\lambda^2}\right)\right]\right\}$$

where,

$$2p_x = (1+x)p_i + (1-x)p_j$$
$$2E_x = (1+x)E_i + (1-x)E_j$$
$$2p'_x = (1+x)p_i\eta_i - (1-x)p_j\eta_j$$

- There will be one such term for neutral mesons and six for charged mesons.

Virtual photon corrections

- **Virtual photon corrections due to contact term:** Contain ultraviolet divergences which get cancelled for leptons but remained for charged mesons.
- Our method to construct the contact term provides $\mathcal{O}(e)$ term whereas the obtained UV divergence is at $\mathcal{O}(e^2)$.
- **Proposed solution:** There may be higher dimensional operators to absorb this UV divergence or a new formalism is required to derive CT.
- Discarded the leftover UV divergences. The finite part is proportional to momenta of the particles and numerically it contributes to $\sim 1.4\%$.
- Evaluating the rest of virtual diagrams;

$$M_{\text{virtual}} = M_0\left[1+\alpha B+\frac{\alpha}{2\pi}\right]+M_{CT} \qquad (3)$$

with

$$B_{ij} = \frac{-1}{2\pi}Q_iQ_j\eta_i\eta_j\left[ln(\frac{m_i m_j}{\lambda^2})+\frac{1}{4}\int_{-1}^{1}dx ln(\frac{p_x'^2}{m_i m_j})+\frac{p_i.p_j\eta_i\eta_j}{2}\int_{-1}^{1}\frac{dx}{p_x^2}ln(\frac{p_x'^2}{\lambda^2})\right]$$

- The Coulomb factor (Sommerfeld enhancement factor):

$$\Omega_c = \prod_{i<j}\frac{-2\pi\alpha}{\beta_{ij}}\frac{1}{e^{\frac{-2\pi\alpha}{\beta_{ij}}}-1}\,; \qquad \beta_{ij} = \sqrt{1-\frac{m_i^2 m_j^2}{(p_i.p_j)^2}},$$

β_{ij}: Relative velocity between i^{th} and j^{th} particle.

Total $\mathcal{O}(\alpha)$ QED corrections to Γ_0 and observable $R_{k^{\mu e}}$

- The total decay rate: $d\Gamma_{real} = d\Gamma_0 \left(1 + 2\alpha \underbrace{(\tilde{\mathcal{B}} + \mathcal{B})}_{\mathcal{H}_{ij}} + \frac{\alpha}{\pi} \right) \Omega_c + d\Gamma'$

$$\mathcal{H}_{ij} = \frac{-Q_i Q_j \eta_i \eta_j}{2\pi} \left[-\ln\left(\frac{k_{max}^2}{E_i E_j}\right) + \frac{1}{4}\int_{-1}^{1} dx \ln\left(\frac{p_x'^2}{m_i m_j}\right) + \frac{p_i.p_j \eta_i \eta_j}{2} \int_{-1}^{1} \frac{dx}{p_x'^2} \ln\left(\frac{p_x'^2}{\lambda^2}\right) + \frac{p_i.p_j}{2} \int_{-1}^{1} \frac{dx}{p_x^2} \ln\left(\frac{k_{max}^2 p_x^2}{E_x^2 \lambda^2}\right) \right]$$

- The correction factor Δ^i to $\mathcal{O}(\alpha)$: $\Delta^i = \left(\frac{d^2\Gamma_0}{dsdq^2}\right)^{-1} \left(\frac{d^2\Gamma^i}{dsdq^2}\right) - 1$
- The shift, $\Delta_{R_K^{\mu e}}$: $\Delta^i_{R_K^{\mu e}} = R_K^{0\mu e} \left(\frac{\Delta\Gamma^i_\mu}{\Gamma^i_\mu} - \frac{\Delta\Gamma^i_e}{\Gamma^i_e} \right)$

Results

- Correction factor for the electron is about three times larger than that for the muons (both are negative) and this difference is due to smallness of the electron mass compared to muon mass.

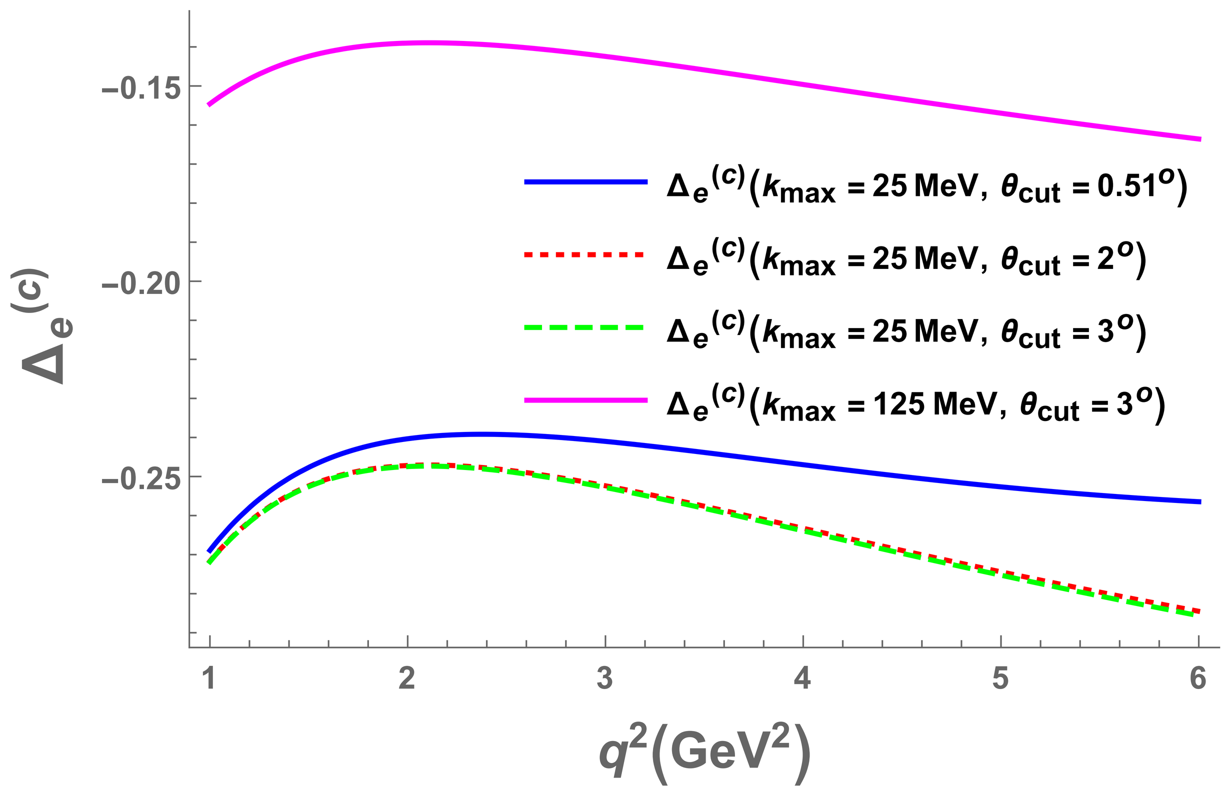

Figure 1: $\mathcal{O}(\alpha)$ corrections to charged $B \to K e^+ e^-$.

- The QED corrections impact more massive charged particles significantly less compared to lighter particles.
- There is a mild dependence on the photon energy cut k_{max}.

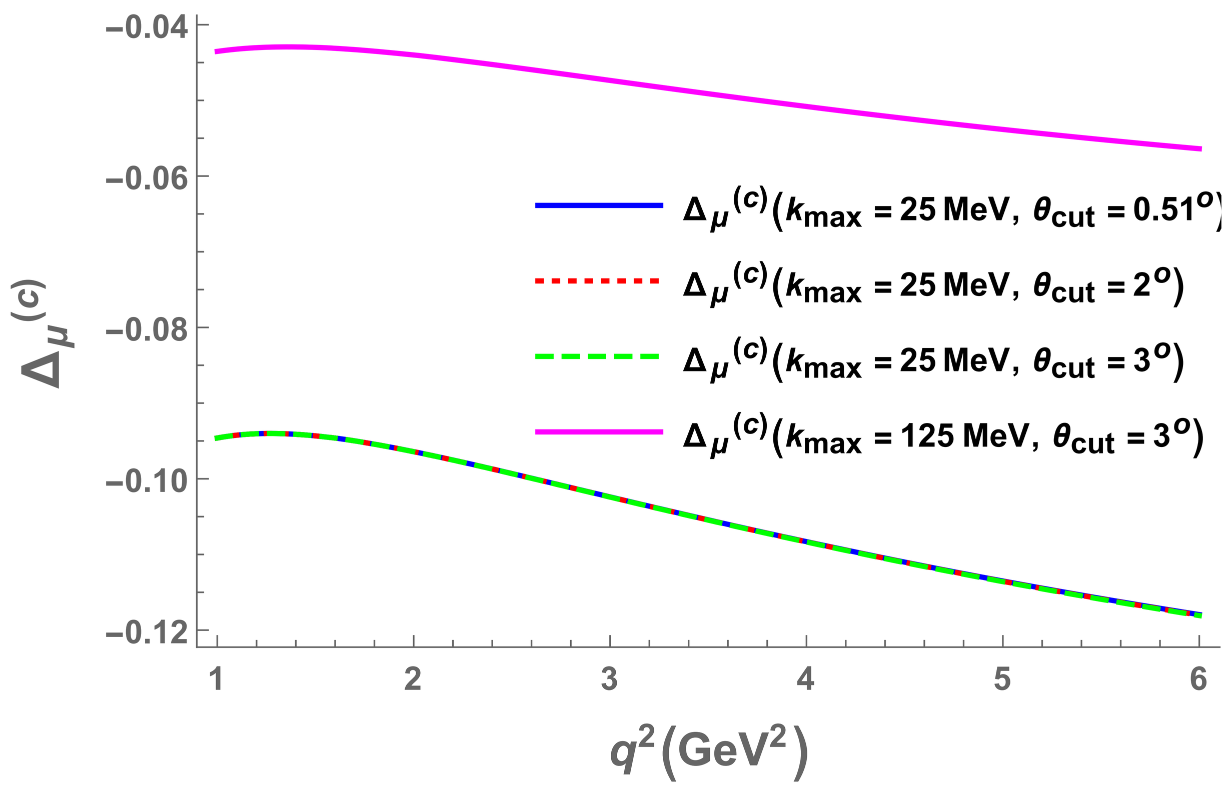

Figure 2: $\mathcal{O}(\alpha)$ corrections to charged $B \to K \mu^+ \mu^-$.

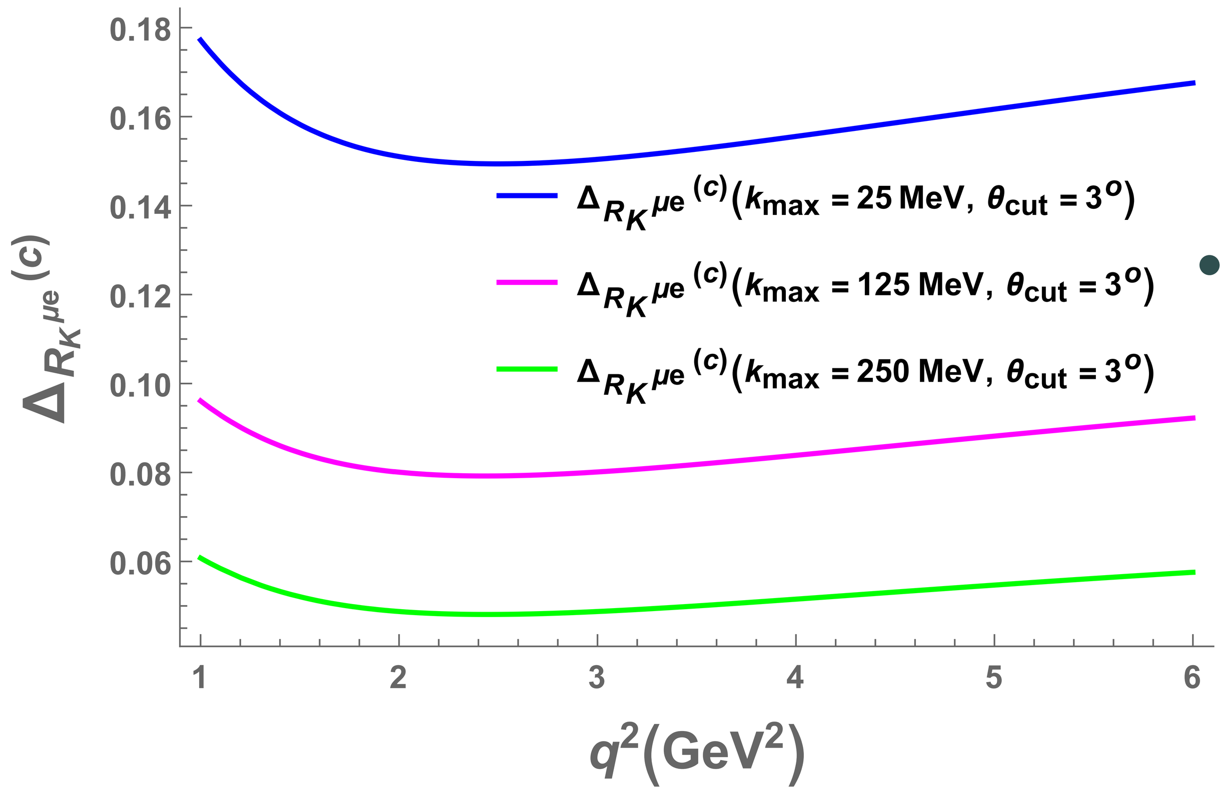

Figure 3: $\mathcal{O}(\alpha)$ corrections to charged $R_k^{\mu e}$.

- It is sensitive to θ_{cut}, particularly for the case of electrons. Choosing $\theta_{cut} \sim$ few degrees, this sensitivity essentially disappears. For muon, it is not that sensitive.

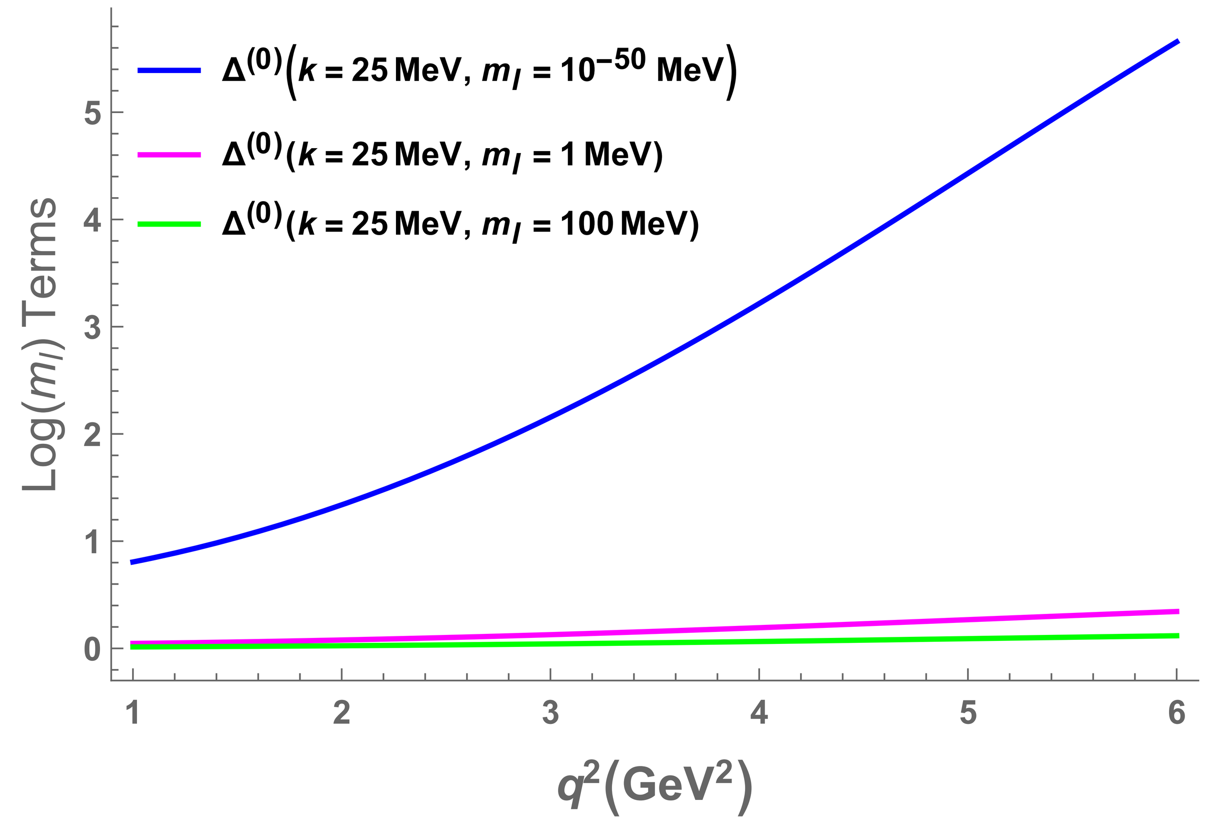

Figure 4: log m_ℓ terms

- log m_ℓ terms: log m_ℓ terms correspond to hard collinear logs.
- We can see the explicit cancellation by choosing a different set of kinematical variables, $t = (p_B - p_k)^2$, $s = (p_k + p_2)^2$, $x = (p_k + k)^2$ and $q^2 = (p_2 + p_3)^2$ and E_k in the rest frame of $(q + k)^2$. With a different method results match with G. Isidori et. al.[JHEP 12 (2020) 104].

Summary and Conclusions

1. We have fixed the contact term demanding the gauge invariance of the matrix amplitude.

Observ.	k_{max} ($\theta_{cut} = 3°$)	**Correction (in %)**
Δ^c_e	125 MeV	~ 14
Δ^c_μ	125 MeV	~ 5
$R_{k^{\mu e}}$	125 MeV	~ 8

- The differential decay rate is found to be independent of IR regulator used and thereby showing the cancellation of soft divergences.

2. The effect of collinear divergence is taken care by choosing $\theta_{cut} \sim$ few degrees. We have also shown the cancellation of hard collinear divergences with the proper choice of kinematical variables.
3. The corrections are found to be negative.