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# Wilson is not anomalous: on gauge anomalies in SMEFT

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## The puzzle to solve

Can chiral interactions at dim.-6 generate gauge anomalies? There were doubts about the operators on the right, which enter in triangle diagrams:

$$\mathcal{O}_{\varphi\psi_R} = i\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\overline{\psi}^i_R\gamma^\mu\psi^j_R$$

$$\mathcal{O}^{(1)}_{\varphi\psi_L} = i\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\overline{\psi}^i_L\gamma^\mu\psi^j_L$$

$$\mathcal{O}^{(3)}_{\varphi\psi_L} = i\left(H^\dagger \overleftrightarrow{D}^a_\mu H\right)\overline{\psi}^i_L\tau^a\gamma^\mu\psi^j_L$$

## Current arguments

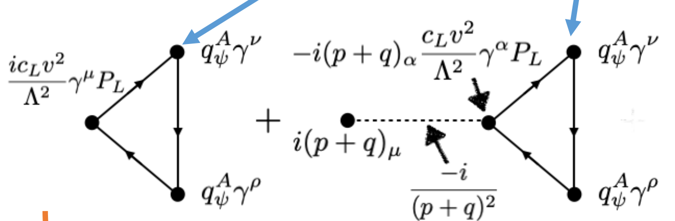
The following toy model:

$$\mathcal{L} = -\frac{F^2_{A,\mu\nu}}{4g^2_A} + |\partial\varphi|^2 - V(|\varphi|) + i\overline{\psi}^k\left(\partial\!\!\!/ + iq^A_k A\!\!\!/\right)\psi^k$$

$$+\sum_{l=L,R}\sum_{k=1}^{2} i\frac{c_{l,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi\right)\overline{\psi}^k_l\gamma^\mu\psi^k_l \qquad \left(\varphi = \frac{v+h}{\sqrt{2}}e^{i\theta/v}\right)$$

has a chiral global $U(1)_B$ symmetry with this conserved current:

$$J^B_\mu = \underbrace{q^B_k\overline{\psi}^k\gamma_\mu\gamma_5\psi^k}_{\text{Leads to usual gauge anomaly cancellation conditions}} + q^B_\varphi v\left(\sum_{l=L,R}\frac{vc_{l,k}}{\Lambda^2}\overline{\psi}^k_l\gamma_\mu\psi^k_l - \partial_\mu\theta\right) + \underbrace{\mathcal{O}(h)}_{\text{No role here}}$$




They cancel each other. No dim-6 contribution!

## Reasoning with Effective Field Theories (EFTs)

Toy model + gauged chiral $U(1)_B$ + Mass for the fermions.
Integrate the fermions out, keep only bosons.
The anomalies will be carried by Wess-Zumino terms:

Axionic: $\theta F_i\tilde{F}_j$ Generalized Chern-Simons: $A_i\wedge A_j\wedge F_k$

In our toy-model, they are:

$$\mathcal{L}_{EFT} \supset -\frac{\left(q^A_k\right)^2}{16\pi^2}\frac{\theta}{v}F_A\tilde{F}_A - \frac{\left(q^B_k\right)^2}{24\pi^2}\frac{\theta}{v}F_B\tilde{F}_B - \frac{\left(q^A_k\right)^2 q^B_k}{6\pi^2}A_\mu B_\nu\tilde{F}^{\mu\nu}_A$$

Usual gauge anomaly cancellation conditions.
No dim-6 contribution!

## EFT argument for SMEFT

Gauge anomalies are independent of Yukawa and gauge coupling values. Then, suppose all SM fermions much heavier than W, Z and integrate them all out of SMEFT!

$$\mathcal{L}_{EFT} \supset -\frac{C_{F_0W^3}}{32\pi^2}v\pi^3\, B^{\mu\nu}\tilde{W}^3_{\mu\nu} + \frac{v^2E_{A_0BB}}{16\pi^2\Lambda^2}W^3_\mu B_\nu\tilde{B}^{\mu\nu} + \ldots$$

$$\delta\left(\mathcal{L}_{EFT}\right) = 0\,\frac{c_i}{\Lambda^2} + \left(\sum_{\psi_L,\psi_R} q^i_\psi q^j_\psi q^k_\psi\right)\epsilon^i A^j\tilde{A}^k$$

No dim-6 contribution — Usual conditions

**SMEFT at dim-6 is gauge anomaly (from triangle diagrams) free**

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# Appendix A

## On the conditions derived by Catà et al[1],…

They say that to make SMEFT gauge anomaly free, the following relations among dim-6 Wilson Coefficients (WCs) must be satisified:

$$c_{\varphi q}^{(3)} = c_{\varphi l}^{(3)} \qquad \frac{c_{\varphi q}^{(1)}}{y_q} = \frac{c_{\varphi l}^{(1)}}{y_l} = \frac{c_{\varphi u}}{y_u} = \frac{c_{\varphi d}}{y_d} = \frac{c_{\varphi e}}{y_e}$$

## …their clear counterexample,…

Consider the SM and add a singlet Weyl fermion with a Majorana mass and interaction with the neutrinos:

$$\mathcal{L}_{BSM}^{\text{Int}} = -\left(\lambda_N\right)_i \bar{N}\tilde{H}^\dagger \ell_{L,i}$$

integrate it out and the only non-vanishing WCs are:

$$\frac{c_{\varphi l,ij}^{(3)}}{\Lambda^2} = -\frac{\lambda_{N,i}^*\lambda_{N,j}}{4M_N^2} \quad \frac{c_{\varphi l,ij}^{(1)}}{\Lambda^2} = \frac{\lambda_{N,i}^*\lambda_{N,j}}{4M_N^2} \quad \frac{c_{5,ij}}{\Lambda} = \frac{\lambda_{N,i}\lambda_{N,j}}{2M_N}$$

Both the SM and the NP sector are gauge anomaly free on their own.

## …and what they missed

What they computed is equivalent to (in our toy model) have considered the classically non-conserved “current”:

$$\tilde{J}_\mu^B = 2q_\varphi^B\left(\frac{c_{L,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + \frac{c_{R,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q_k^B\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

Which misses the Goldstone contribution.

Notice that the constraints they found are trivially satisfied by models in the class of Universal Theories.

[1]: O. Catà, W. Killian, N. Kreher. *Gauge Anomalies in the Standard-Model Effective Field Theory*. arXiv:2011.09976

## Details on our toy model and the current reasoning

The $U(1)_A$ is a vector-like gauge symm. Under which the scalar is neutral. The U(1)B transformation law is:

$$\varphi \to e^{iq_\varphi^B\epsilon_B}\varphi, \ \ \psi_k \to e^{iq_k^B\gamma_5\epsilon_B}\psi_k$$

And its Noether current in the unbroken phase is:

$$J_\mu^B = -iq_\varphi^B\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi + 2i\frac{c_{L,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,L}\gamma_\mu\psi_{k,L} + 2i\frac{c_{R,k}}{\Lambda^2}\left|\varphi\right|^2\overline{\psi}_{k,R}\gamma_\mu\psi_{k,R}\right) + q_k^B\overline{\psi}_k\gamma_\mu\gamma_5\psi_k$$

If we want to gauge $U(1)_B$, we should cancel these anomalies:

$$U(1)_A^2\times U(1)_B : \ \left(q_k^A\right)^2 q_k^B = 0\,, \quad U(1)_B^3 : \ \left(q_k^B\right)^3 = 0$$

The Lagrangian with both gauge symmetries is:

$$\mathcal{L} = -\frac{1}{4g_A^2}F_{A,\mu\nu}^2 - \frac{1}{4g_B^2}F_{B,\mu\nu}^2 - \left|D\varphi\right|^2 - V(|\varphi|) + i\overline{\psi}_k\not{D}\psi_k + i\frac{c_{L,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{D}_\mu\varphi\right)\overline{\psi}_{k,L}\gamma^\mu\psi_{k,L} + i\frac{c_{R,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{D}_\mu\varphi\right)\overline{\psi}_{k,R}\gamma^\mu\psi_{k,R}$$

In the broken phase, the GB-fermion couplings are:

$$\mathcal{L} \supset -\frac{vc_{L,k}}{\Lambda^2}\partial_\mu\theta\overline{\psi}_{k,L}\gamma^\mu\psi_{k,L} - \frac{vc_{R,k}}{\Lambda^2}\partial_\mu\theta\overline{\psi}_{k,R}\gamma^\mu\psi_{k,R}$$

And to compute the anomalies, we compute:

$$\partial^\mu\langle 0|J_\mu^B(x)J_\nu^A(y)J_\rho^A(z)|0\rangle$$

The Feynman diagrams cancel only after taking the derivative

For completeness, the EOMs:

$$\Box\varphi + V'(|\varphi|) + i\frac{c_{L,k}}{\Lambda^2}\partial\varphi\overline{\psi}_{k,L}\gamma\psi_{k,L} + i\frac{c_{L,k}}{\Lambda^2}\partial\left(\varphi\overline{\psi}_{k,L}\gamma\psi_{k,L}\right) + (L\leftrightarrow R) = 0$$

$$\not{D}\psi_{k,L/R} + \frac{c_{L/R,k}}{\Lambda^2}\left(\varphi^\dagger\overleftrightarrow{\partial}_\mu\varphi\right)\gamma^\mu\psi_{k,L/R} = 0\ ,$$

# Appendix B

## More on the EFT reasoning for our toy model

The Yukawa term that we add to the model is:

$$\delta\mathcal{L} = -y_k \varphi \overline{\psi}_{k,L} \psi_{k,R} + h.c. \qquad \left( m_k = \frac{y_k v}{\sqrt{2}} \right)$$

And gauge invariance requires:

$$2\, q_k^B = q_\varphi^B$$

The anomalous Ward identities do not change.

The anomalous gauge variation of the EFT is:

$$\delta\mathcal{L}_{\text{eff}} = -\frac{\epsilon_A}{24\pi^2} 2 \left(q_k^A\right)^2 q_k^B F_A \tilde{F}_B - \frac{\epsilon_B}{24\pi^2} \left[ \left(q_k^A\right)^2 q_k^B F_A \tilde{F}_A + 2 \left(q_k^B\right)^3 F_B \tilde{F}_B \right]$$

And the gauge transformation rules are:

$$\delta A_\mu = -\partial_\mu \epsilon_A, \quad \delta B_\mu = -\partial_\mu \epsilon_B, \quad \delta\theta = v q_\varphi^B \epsilon_B$$

## And the details on our SMEFT computation

We parametrize the Higgs as:

$$\varphi = e^{i \frac{\pi^a}{v} \sigma^a} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \delta\pi^a = \frac{v}{2}\epsilon^a - v\delta^{a3} y_\varphi \epsilon_Y$$

Modifications to the LH gauge couplings:

$$-\overline{\psi}_{L,i} \Bigg( \delta_{ij} \left[ y_\psi \not{B} + T^a \not{W}^a \right] + \left[ v^2 \left( y_\varphi \not{B} - \frac{\not{W}^3}{2} \right) - v \not{\partial}\pi^3 \right] \left[ \frac{c^{(1)}_{\varphi\psi,ij}}{\Lambda^2} - 2T^3 \frac{c^{(3)}_{\varphi\psi,ij}}{\Lambda^2} \right] + 2T^{a=1,2} \frac{c^{(3)}_{\varphi\psi,ij}}{\Lambda^2} \left[ \frac{v^2}{2} \not{W}^a + v \not{\partial}\pi^a \right] \Bigg) \psi_{L,j}$$

Yukawa couplings:

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{\sqrt{2}} \left( (v + i\pi^3)\overline{u}_L + i(\pi^1 + i\pi^2)\overline{d}_L \right) Y_u u_R \\ & -\frac{1}{\sqrt{2}} \left( (v - i\pi^3)\overline{d}_L + i(\pi^1 - i\pi^2)\overline{u}_L \right) Y_d d_R \\ & -\frac{1}{\sqrt{2}} \left( (v - i\pi^3)\overline{e}_L + i(\pi^1 - i\pi^2)\overline{\nu}_L \right) Y_e e_R + h.c. \end{aligned}$$

EFT from dim-4 couplings:

$$\begin{aligned} \mathcal{L}_{\text{EFT}} \supset & -\frac{1}{16\pi^2} \frac{\pi^3}{v} B\tilde{B} \left( 3\left[ y_u^2 + y_Q y_u - y_d^2 - y_Q y_d \right] + y_\nu^2 + y_L y_\nu - y_e^2 - y_e y_L \right) \\ & -\frac{1}{16\pi^2} \frac{\pi^3}{v} B\tilde{W}^3 \left( \frac{3(y_d + 4y_Q + y_u)}{2} + \frac{y_e + 4y_L + y_\nu}{2} \right) \\ & -\frac{1}{8\pi^2} B_\mu W_\nu^3 \tilde{B}^{\mu\nu} \frac{(y_\nu + y_e)(y_e + y_L + y_\nu) + 3(y_u - y_d)(y_d + y_Q + y_u)}{2} \\ & -\frac{1}{8\pi^2} B_\mu W_\nu^3 \tilde{W}^{3,\mu\nu} \frac{3(y_u + y_d) + y_e + y_\nu}{4} \end{aligned}$$

Gauge variation:

$$\delta\mathcal{L}_{\text{EFT}} = -\frac{\epsilon_Y}{16\pi^2} \left[ \left( 6y_Q^3 + 2y_L^3 - 3y_u^3 - 3y_d^3 - y_e^3 - y_\nu^3 \right) B\tilde{B} + \frac{3y_Q + y_L}{2} W^3 \tilde{W}^3 \right] - \frac{\epsilon_3}{16\pi^2} \left( 3y_Q + y_L \right) B\tilde{W}^3$$

WCs of the bosonic EFT:

$$\mathcal{C}_{F_0 W^3} = \frac{1}{6\Lambda^2} \left[ 3c^{(1)}_{\varphi d} + 3c^{(1)}_{\varphi u} + 12c^{(1)}_{\varphi Q} + c^{(1)}_{\varphi e} + 4c^{(1)}_{\varphi L} \right]$$

$$\begin{aligned} E_{A_0 BB} = & c^{(1)}_{\varphi u}(y_Q - y_u)(y_Q + 2y_u) - c^{(1)}_{\varphi d}(y_d - y_Q)(2y_d + y_Q) \\ & - c^{(1)}_{\varphi Q} \left( y_d^2 + y_d y_Q - 4y_Q^2 + y_Q y_u + y_u^2 \right) \\ & - c^{(3)}_{\varphi Q}(y_d - y_u)(y_d + y_Q + y_u) - \frac{1}{3} c^{(1)}_{\varphi e}(y_e - y_L)(2y_e + y_L) \\ & - \frac{1}{3} c^{(1)}_{\varphi L} \left( y_e^2 + y_e y_L - 4y_L^2 + y_L y_\nu + y_\nu^2 \right) \\ & - \frac{1}{3} c^{(3)}_{\varphi L} \left( y_e^2 + y_L y_e + y_\nu^2 + y_\nu y_e \right) \end{aligned}$$

## Further reading:


J. Preskill, *Gauge anomalies in an effective field theory*, Annals Phys. 210 (1991) 323–379

F. Feruglio, *A Note on Gauge Anomaly Cancellation in Effective Field Theories*, arXiv: 2012.13989 [hep-ph]