LCSR application to radiative tau decay

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Abstract

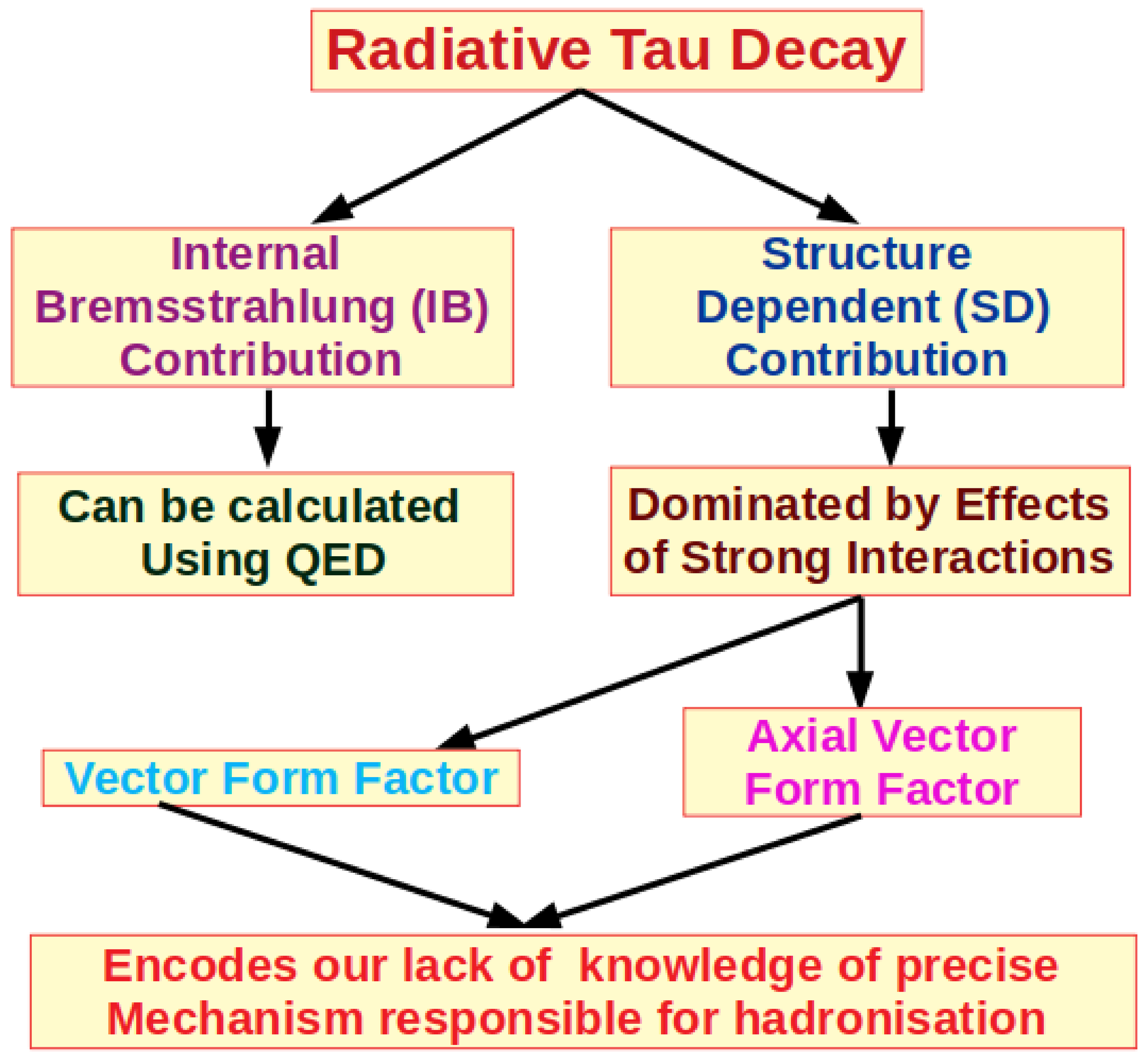
The non-perturbative QCD effects involved in radiative tau decay ($\tau^- \to \pi^- \nu_\tau \gamma$) are encoded in two form factors (FFs); the vector (F_V) and the axial vector (F_A) FFs. These FFs are same as involved in radiative pion decay with the crucial difference that the momentum transfer squared, t, between the $\pi - \gamma$ system is positive, which makes them timelike and also as t can now take values upto m_τ^2, it can produce real hadronic resonances. We calculate the analytical form for these form factors using the method of light cone sum rules (LCSR) and present the decay width and the invariant mass spectrum in the $\pi - \gamma$ system. We found that the structure dependent parameter, γ, (i.e. $\frac{F_A}{F_V}$ at zero momentum transfer) is in good agreement with the experimental determination.

Introduction

- τ is the heaviest lepton ($m_\tau = 1776.86 \pm 0.12 MeV$ [1]) and has enormous decay channels including hadronic channels.
- The hadronic decays of tau are helpful in our understanding of the dynamics of strong interaction involved in the hadronization of QCD currents.
- The branching ratio of non-radiative decay ($\tau^- \to \pi^- \nu_\tau$) is $(10.82 \pm 0.05)\%$ [1].

The amplitude of radiative tau deacy ($\tau \to \pi \nu_\tau \gamma$) is divided into:

1. ***Internal bremmtrahlung (IB):*** *Emission from either the incoming or the outgoing particles and can be calculated using QED.*
2. ***Structure Dependent (SD):*** *Captures the long disctance dynamics of strong interaction and can be parametrized by vector and axial-vector form factors (FFs) $F_V^{(\pi)}$ and $F_A^{(\pi)}$, respectively using Lorentz symmetry. It also includes a contact term (CT), which emerges as a consequence of gauge invariance. CT contribution turns out to be equal and opposite to the m_τ independent IB contribution and hence they cancel out each other in the total amplitude.*



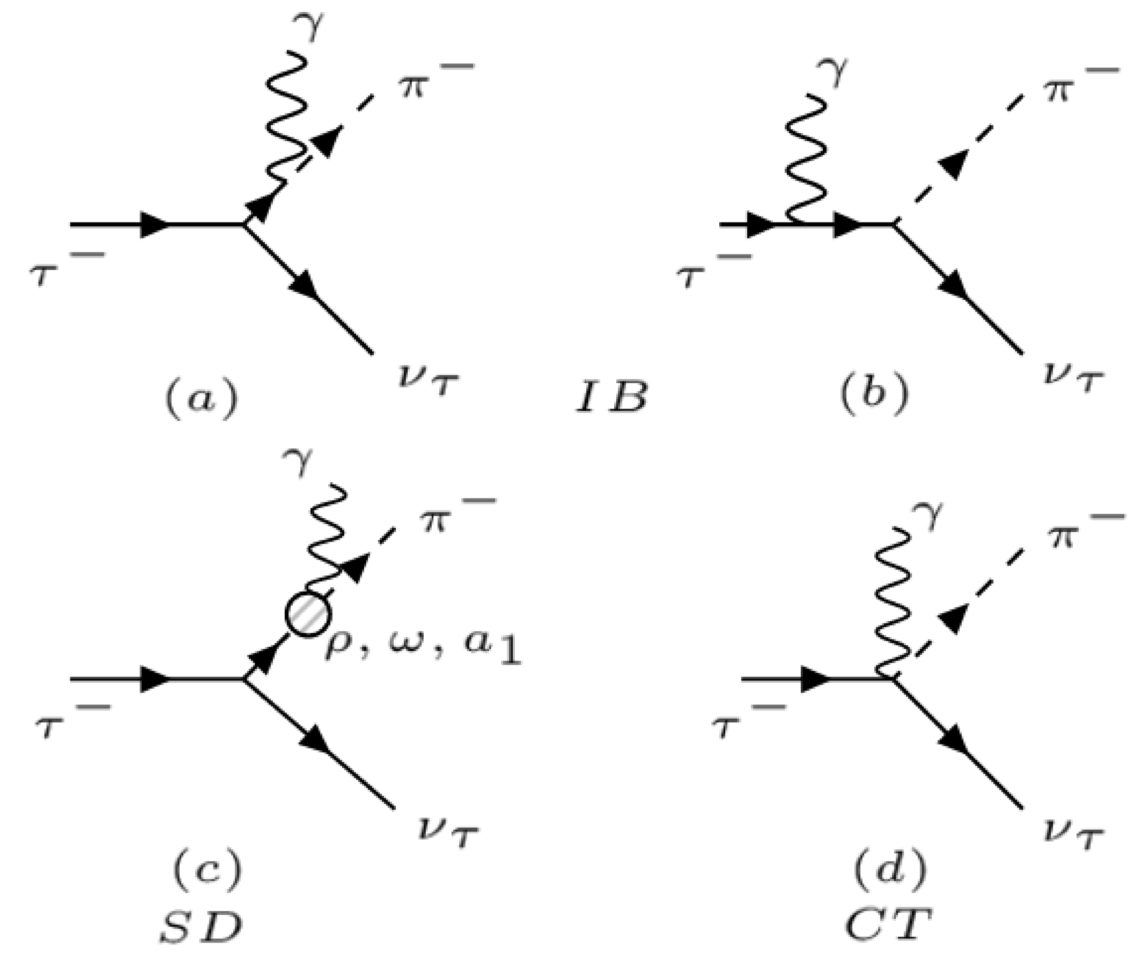


Figure 1: Feynman diagrams showing different possibilities of photon emission.

- Using a naive calculation, the radiative decay branching ratio is expected to be $\mathcal{B}(\tau^- \to \pi^- \nu_\tau \gamma) \approx \mathcal{B}(\tau \to \rho \nu_\tau) * \mathcal{B}(\rho \to \pi\gamma) \approx \mathcal{O}(10^{-3})$.
- Even though this branching ratio is not very small, radiative decay is not observed experimentally yet which makes the study of this mode important.
- Non-perturbative method (eg. χPT, Lattice QCD, LCSR) is required to calculate FFs.

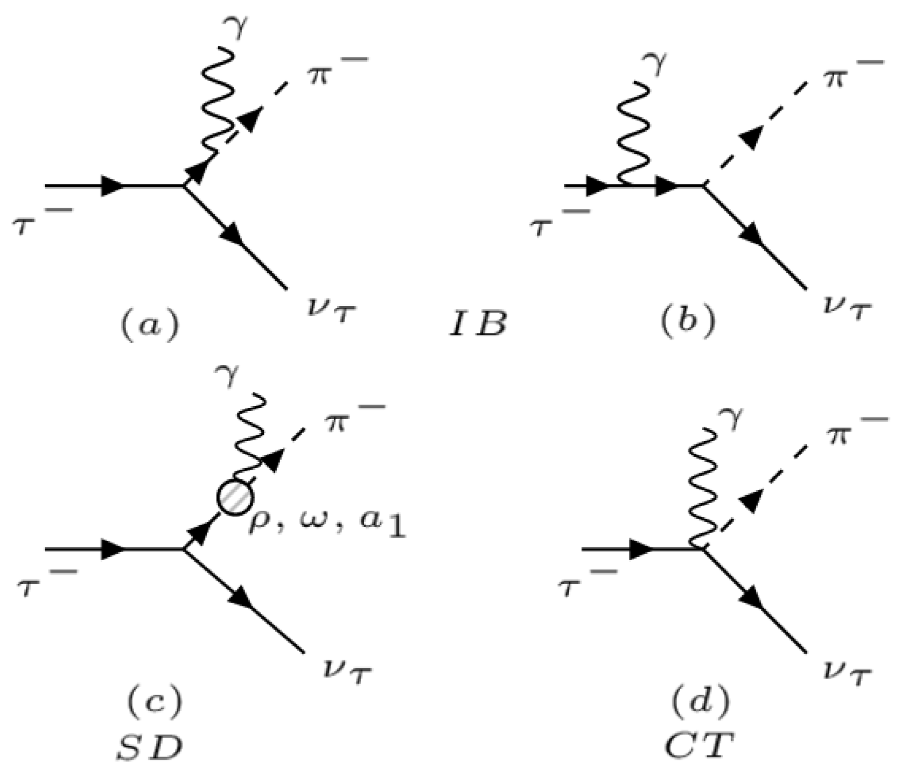
Amplitude Calculation

- The amplitude of radiative tau decay $(\tau^-(p_1) \to \pi^-(p_2)\nu_\tau(p_3)\gamma(k))$ is,

$$\mathcal{A}(\tau^- \to \pi^-\nu_\tau\gamma) = \frac{G_F}{\sqrt{2}} V_{ud} \langle \pi^-\nu_\tau\gamma|(\bar{\nu}_\tau\Gamma^\mu\tau)(\bar{d}\Gamma_\mu u)|\tau^-\rangle \qquad (1)$$

where, $\Gamma^\mu = \gamma^\mu(1-\gamma_5)$, G_F is Fermi constant and V_{ud} is the CKM element .

- The photon can be emitted from either of the charged particles.
- The photon emission from tau can be trivially calculated using QED.
- The photon emission from pion is dictated by the hadronic matrix element:



$$T^{\alpha\mu}(p_2,k) = i\int d^4x e^{ikx}\langle \pi^-|T\{j^\alpha_{em}(x)\bar{d}\Gamma^\mu u(0)\}|0\rangle \qquad (2)$$

where, $j^\alpha_{em}(x) = Q_\psi\bar{\psi}(x)\gamma^\alpha\psi(x) = -\bar{\tau}\gamma^\alpha\tau + Q_u\bar{u}\gamma^\alpha u + Q_d\bar{d}\gamma^\alpha$.

- It can be parametrized in terms of two gauge invariant scalar functions of $t = P^2 = (p_2+k)^2$ as,

$$T^{\alpha\mu}(p_2,k) = F_A^{(\pi)}[g^{\alpha\mu}(P.k) - P^\alpha k^\mu] + iF_V^{(\pi)}\epsilon^{\alpha\mu\beta\nu}P_\beta k_\nu - if_\pi g^{\alpha\mu} + if_\pi\frac{P^\alpha P^\mu}{P.k}. \qquad (3)$$

The final form of amplitude turns out to be,

$$\mathcal{A}(\tau^- \to \pi\nu_\tau\gamma) = \mathcal{A}_{IB} + \mathcal{A}_A + \mathcal{A}_V \qquad (4)$$

with,

$$\mathcal{A}_{IB} = \frac{G_F}{\sqrt{2}}V_{ud}\left[ief_\pi m_\tau\bar{u}_\nu\left\{\frac{\epsilon^*.p_1}{p_1.k} - \frac{\not{k}\not{\epsilon}^*}{2p_1.k} - \frac{\epsilon^*.p_2}{p_2.k}\right\}(1+\gamma_5)u_\tau\right]$$

$$\mathcal{A}_A = \frac{G_F}{\sqrt{2}}V_{ud}\left[ie\epsilon^{*\alpha}(\bar{u}_\nu\Gamma^\mu u_\tau)\left(iF_A^{(\pi)}\left[g_{\alpha\mu}(P.k) - P_\mu k_\alpha\right]\right)\right]$$

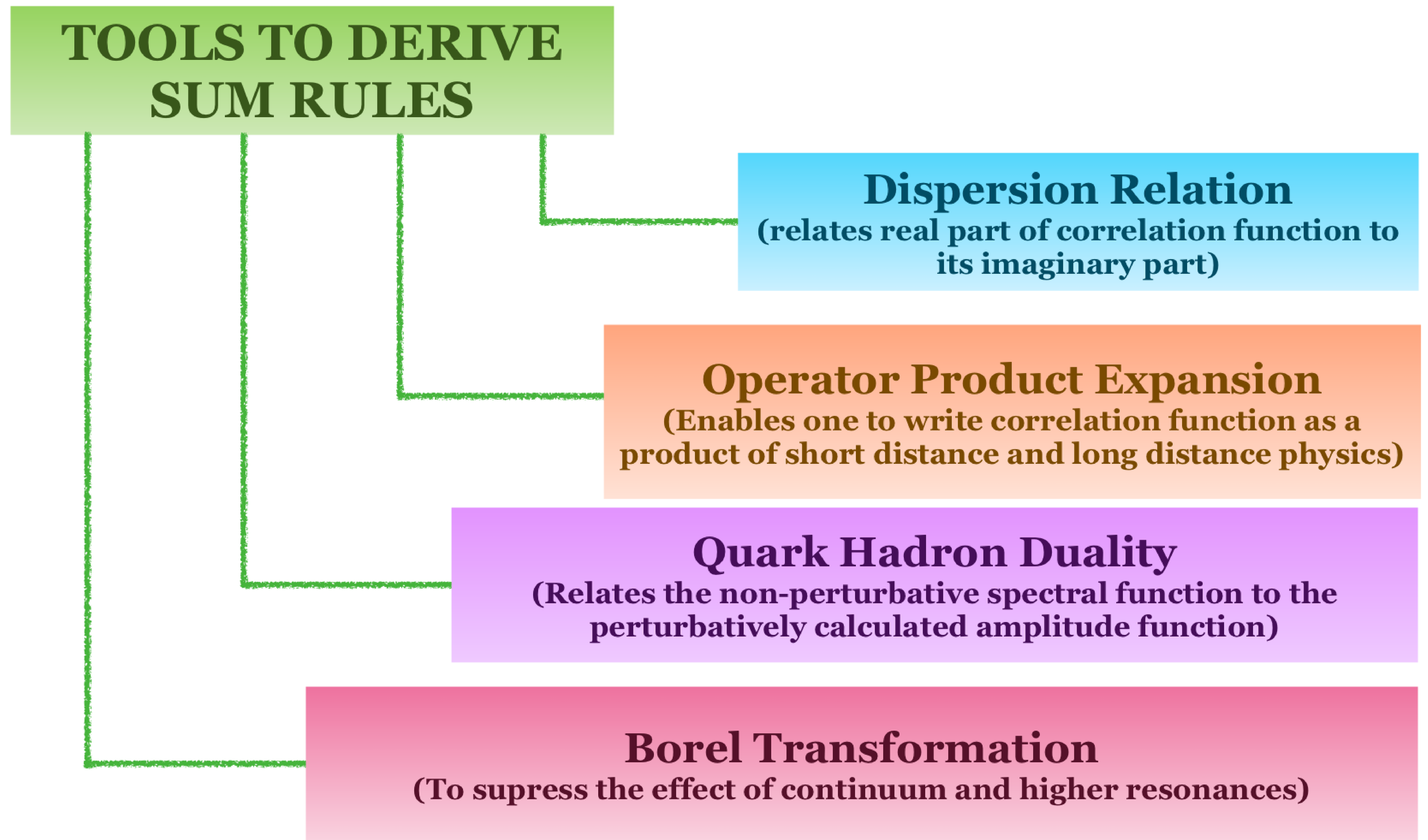
$$\mathcal{A}_V = -\frac{G_F}{\sqrt{2}}V_{ud}\left[ie\epsilon^{*\alpha}(\bar{u}_\nu\Gamma^\mu u_\tau)\left(F_V^{(\pi)}\epsilon_{\alpha\mu\beta\nu}P^\beta k^\nu\right)\right]$$

Main Objectives

1. To calculate $F_V^{(\pi)}(t)$ and $F_A^{(\pi)}(t)$ using light cone sum rules.
2. To calculate the structure dependent parameter, $\gamma = \frac{F_A^{(\pi)}(0)}{F_V^{(\pi)}(0)}$.
3. To compute the radiative tau decay width and the invariant mass spectrum of the $\pi - \gamma$ system.

Light cone sum rules (LCSR)

- The method of QCD sum rules was developed by Shifman, Vainstein and Zakharov (SVZ) in 1979.
- **Basic idea:** To derive the hadronic parameters using the analytical properties of the correlation function.



- In the lightcone limit, the bi-local operator sandwiched between the pion state and vacuum is expressed as,

$$\left\langle \pi^0(p)|\bar{u}(y)\gamma_\mu\gamma_5 u(x)|0\right\rangle_{x^2=0} = -if_\pi p_\mu\int_0^1 du\, e^{i(up_2.y+\bar{u}p.x)}\phi(u,\mu) \qquad (5)$$

where, $\bar{u} = 1-u$ and $\phi(u,\mu)$ is leading twist-2 distribution amplitude.

Form factor calculation using LCSR

- Firstly, we compute $T^{\alpha\mu}(P,k)$ in perturbative QCD (pQCD):

$$F_V^{QCD}(t) = \frac{if_\pi}{3}\int_0^1 du \frac{\phi(u,\mu)}{t\bar{u}+k^2u} \tag{6}$$

$$F_A^{QCD}(t) = -if_\pi \int_0^1 du\phi(u,\mu)\left(\frac{1-2\bar{u}}{t\bar{u}+k^2u}\right) \tag{7}$$

- Then, we compute the dispersion relation using the analytic properties of $T^{\alpha\mu}(P,k)$;

$$T^{\alpha\mu}(p_2,k) = \frac{2im_\rho f_\rho \epsilon^{\alpha\lambda\beta\nu} g_\lambda^\mu p_{2\beta} k_\nu F_{\rho\pi}(k^2)}{m_\rho^2-(p_2+k)^2-im_\rho\Gamma_\rho} + \frac{im_{a_1}f_{a_1}\left[2p_2.kg^{\alpha\mu}-2p_2^\alpha k^\mu\right]G_{a_1\pi}(k^2)}{m_{a_1}^2-(p_2+k)^2-im_{a_1}\Gamma_{a_1}} + \frac{1}{\pi}\int_{s_0^h}^\infty ds\frac{Im\{T^{\alpha\mu}(s,k)\}}{s-k^2-i\epsilon}. \tag{8}$$

  Here, s_0^h is the threshold of the lowest continuum state and Γ_ρ and Γ_{a_1} are the decay widths of ρ and a_1 mesons, respectively. $F_{\rho\pi}$ ($G_{a_1\pi}$) captures the physics of transition of ρ (a_1)-meson to the π-meson.

- The contribution from the lowest resonance states i.e. ρ,ω and a_1-mesons is considered.
- The factor of 2 in the first term corresponds to $\rho+\omega$ contribution.
- $F_V^{(\pi)}(t)$ ($F_A^{(\pi)}(t)$) gets contribution from the ρ and ω (a_1) mesons.
- Using duality approximation, the sum rules turns out to be;

$$\text{For } F_V^{(\pi)}(t): \quad \frac{2m_\rho f_\rho F_{\rho\pi}(k^2)}{m_\rho^2-t} = \frac{if_\pi}{3}\int_0^{u_0} du\frac{\phi(u)}{t\bar{u}+k^2u}. \tag{9}$$

$$\text{For } F_A^{(\pi)}(t): \quad \frac{2im_{a_1}f_{a_1}G_{a_1\pi}(k^2)}{m_{a_1}^2-t} = -if_\pi\int_0^{u_0}\phi(u)\left(\frac{1-2\bar{u}}{t\bar{u}+k^2u}\right). \tag{10}$$

with $u_0 = \frac{s_0}{k^2+s_0} = 1$ (as $k^2=0$).

Result 1 : Form Factors

- On applying Borel transformation on the sum rules, the analytical form for FFs is,

$$F_V^{(\pi)}(t) = -i\frac{f_\pi}{3(m_\rho^2-t-im_\rho\Gamma_\rho)}\int_0^1 du\frac{\phi(u)}{\bar{u}}e^{\frac{m_\rho^2}{M^2}}, \tag{11}$$

$$F_A^{(\pi)}(t) = -i\frac{f_\pi}{m_{a_1}^2-t-im_{a_1}\Gamma_{a_1}}\int_0^1 \frac{\phi(u)}{\bar{u}}(1-2\bar{u})e^{\frac{m_{a_1}^2}{M^2}}. \tag{12}$$

  where, M is the Borel parameter.

- For the following computation, we have used the asymptotic and Chernyak-Zhitnisky form of the pion distribution amplitude,

$$\phi_\pi^{asym}(u,\mu) = 6u\bar{u} \quad \text{and} \quad \phi_\pi^{CZ}(u,\mu) = 6u\bar{u}\left[1+\frac{3a_2(\mu)}{2}\{5(u-\bar{u})^2-1\}\right], \tag{13}$$

  where, $a_2(\mu) = 0.12\left(\frac{\alpha_s(\mu)}{\alpha_s(1GeV)}\right)^{\frac{50}{9\beta_0}}$ with α_s and β_0 being the QCD coupling strength and leading order beta function, respectively.

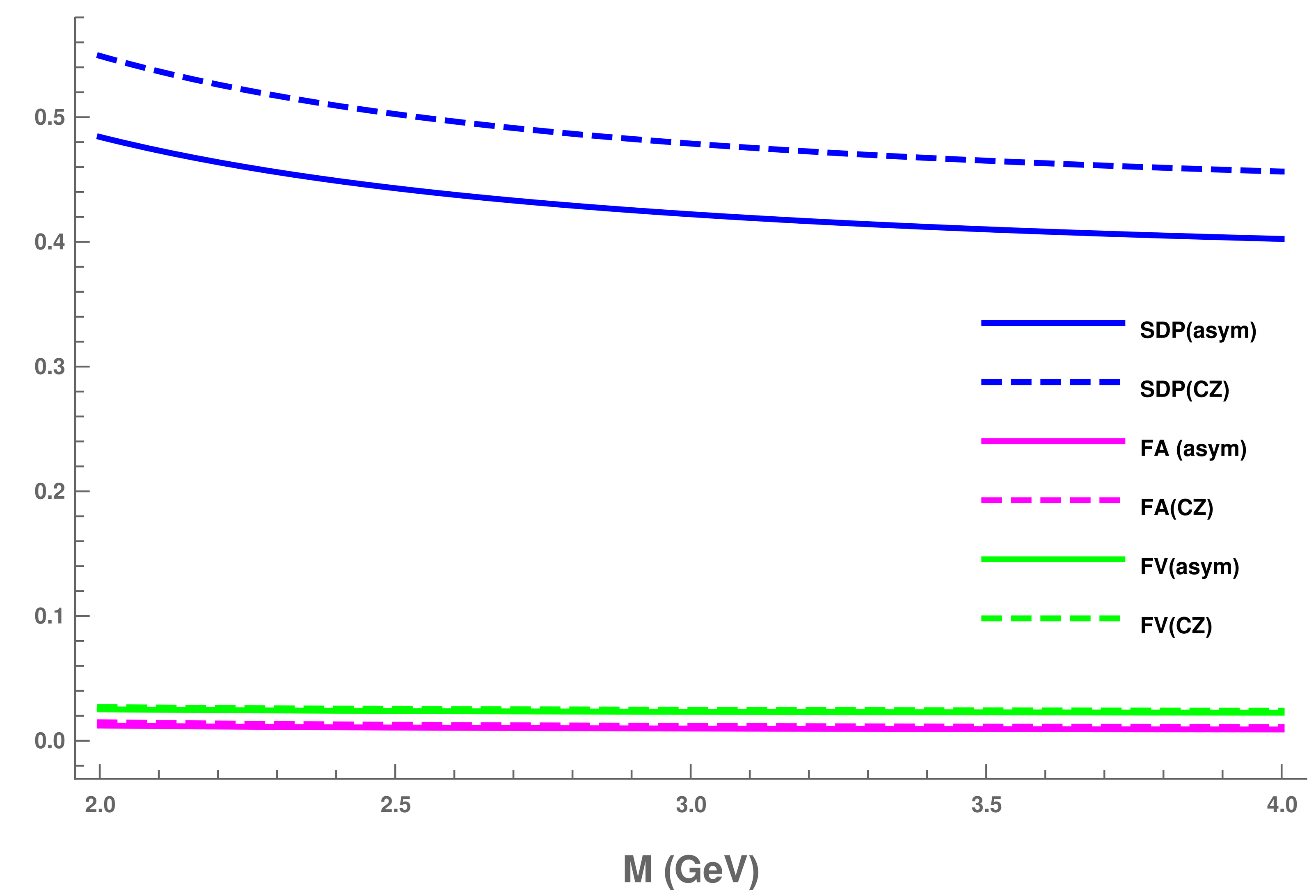


Figure 2: The variation of $F_V^{(\pi)}(0)$(Green), $F_A^{(\pi)}(0)$(Magenta) and structure dependent parameter (Blue) vs the Borel parameter M (in GeV).

Result 2: Decay width and Spectrum

- The IB contribution suffers from infrared(IR) divergences which can be taken care by putting a threshold on the photon energy (50 MeV here).
- Structure dependent parameter, $\gamma|_{CZ}(M = 3.35GeV)$ is 0.469.
- Radiative tau decay width is found to decrease with an increase in Γ_{a_1}. The effect of t dependence of decay widths is also studied.
- The value of normalised decay width i.e. $\bar{\Gamma} = \Gamma(\tau \to \pi\nu\gamma)/\Gamma(\tau \to \pi\nu)$ is found to be,

	Using ϕ^{asym}	Using ϕ^{CZ}
$\bar{\Gamma}_{IB}$	1.36×10^{-2}	1.36×10^{-2}
$\bar{\Gamma}_{SD}$	$(1.87 \pm 0.30) \times 10^{-3}$	$(2.29 \pm 0.43) \times 10^{-3}$
$\bar{\Gamma}_{int}$	$(3.82 \pm 2.14) \times 10^{-4}$	$(4.90 \pm 2.60) \times 10^{-4}$
$\bar{\Gamma}_{all}$	$(1.56 \pm 0.04) \times 10^{-2}$	$(1.61 \pm 0.06) \times 10^{-2}$

- The uncertainties are found to be about 10% and are dominated by SD contribution.
- A rough estimate for kaon in the final state gives an approximate normalised decay width to be roughly half of that for the pion.

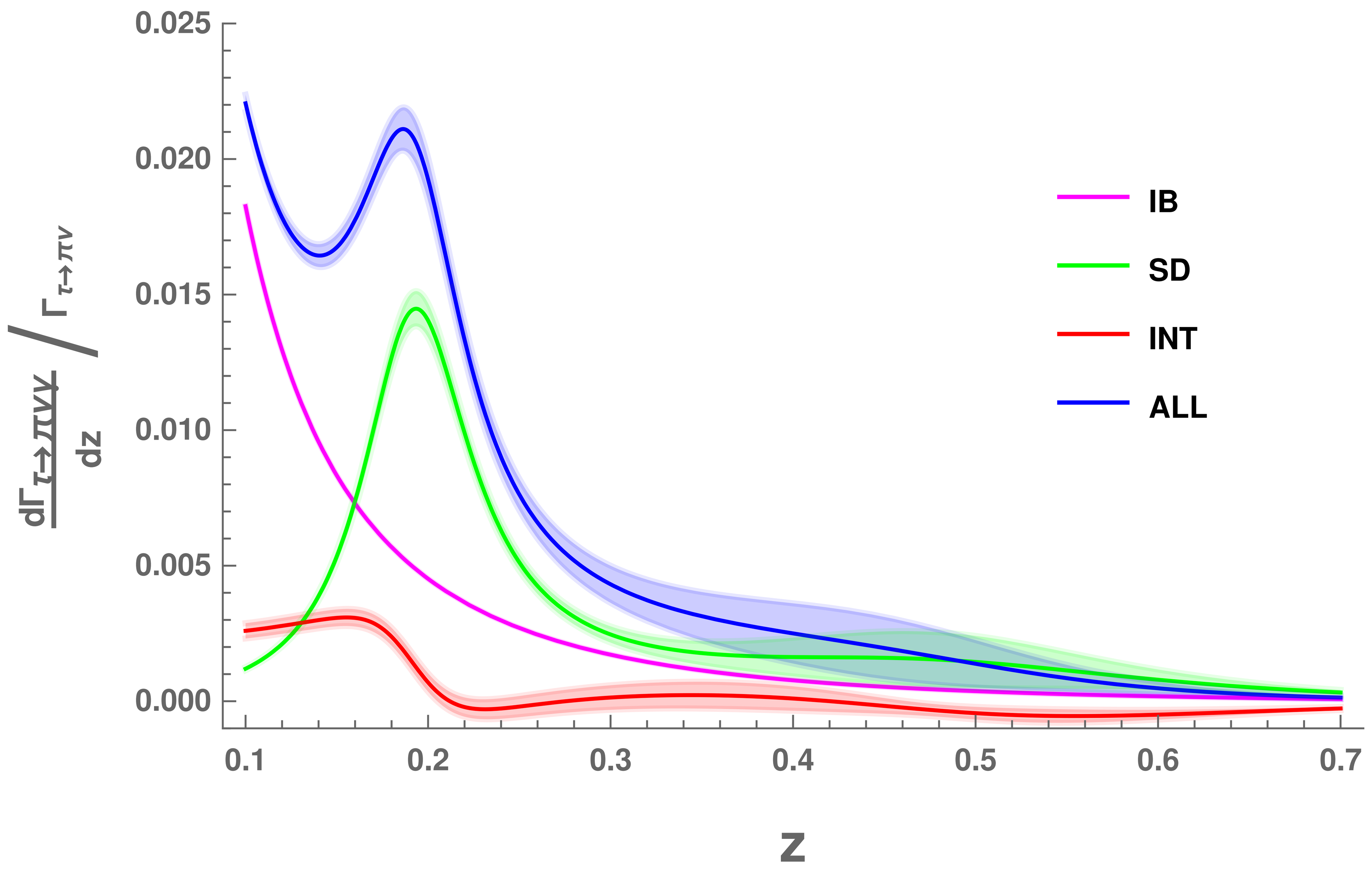


Figure 3: The invariant mass spectrum of the $\pi - \gamma$ system considering asymptotic distribution amplitude. z is $\frac{t}{m_\tau^2}$. Shaded region shows the uncertainities.

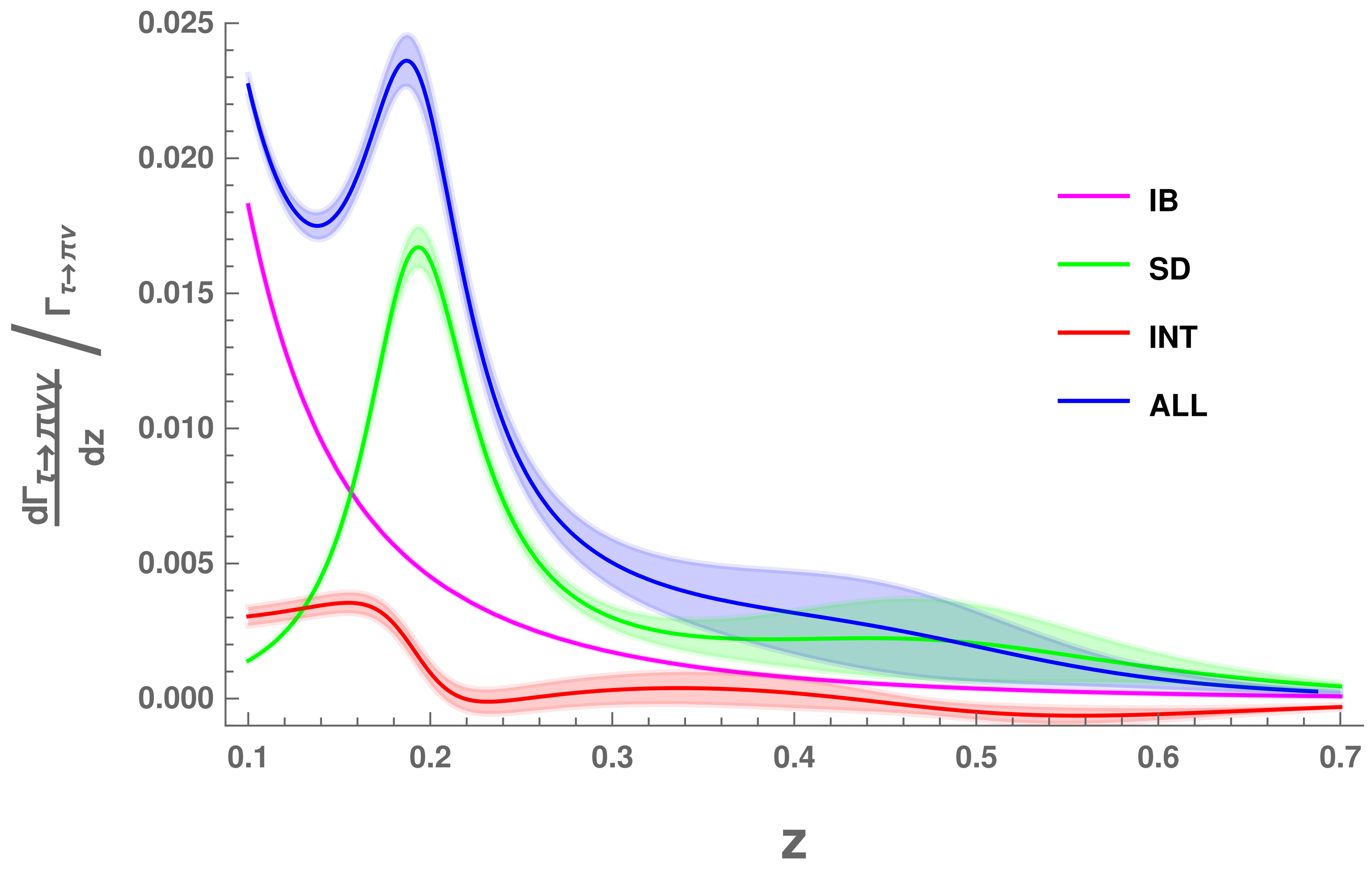


Figure 4: The invariant mass spectrum of the $\pi - \gamma$ system considering CZ distribution amplitude. z is $\frac{t}{m_\tau^2}$. Shaded region shows the uncertainities.

Conclusions and Discussions

- We present a detailed prediction for the rate and photon spectrum for radiative tau decay.
- A contact term appears as a consequence of gauge invariance.
- The decay includes two timelike FFs which are calculated in the framework of LCSR upto twist-2 accuracy.
- The structure dependent parameter is been calculated and is found to be in good agreement with experimental value (including sign) obtained from radiative pion decay.
- The normalised rate values are consistent with the study done using resonance χPT by Roig et. al [2].
- This is the first application of LCSR to such a mode and a study of higher twist contribution and radiative corrections will be interesting.

References

[1] M. Tanabashi et al. (Particle Data Group), Review of Particle Physics, Phys. Rev. D 98, 030001 (2018).)

[2] Z. H. Guo and P. Roig, One meson radiative tau decays, Phys. Rev. D 82, 113016 (2010).