

# A program for SU( $N$ ) color structure decomposition into multiplet bases using Wigner 3j and 6j coefficients and birdtrack techniques

**Master's thesis**

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Supervisor: *Malin Sjödahl*

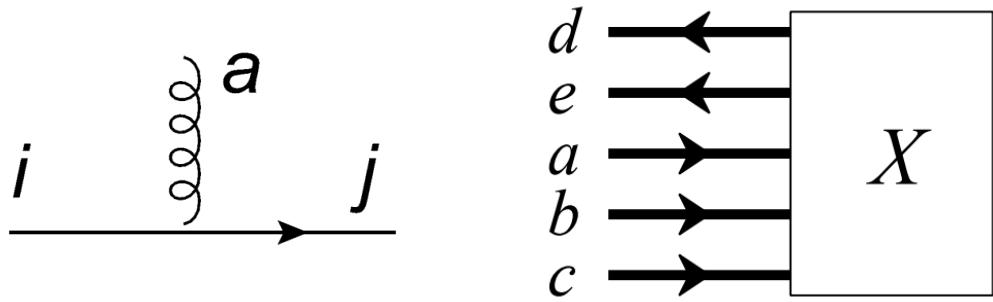


**LUND**  
UNIVERSITY

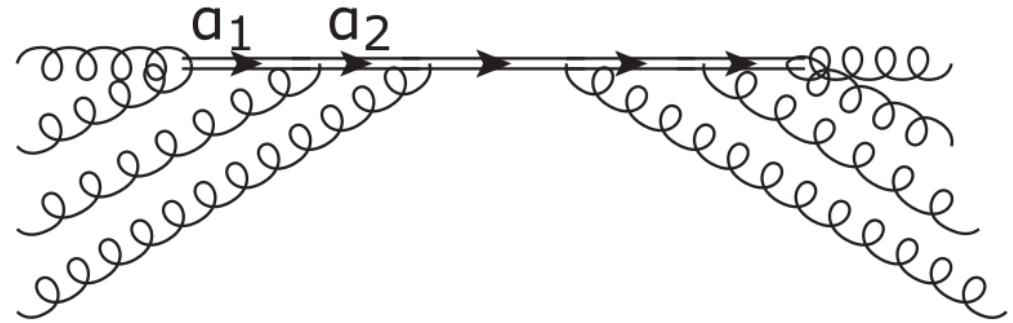
Department of Astronomy and Theoretical Physics,  
Lund University

# Outline of the Presentation

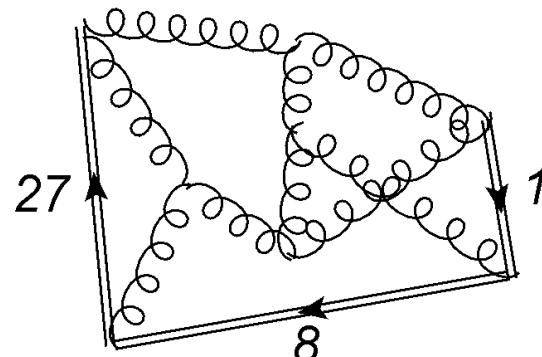
## 1. QCD and Birdtracks



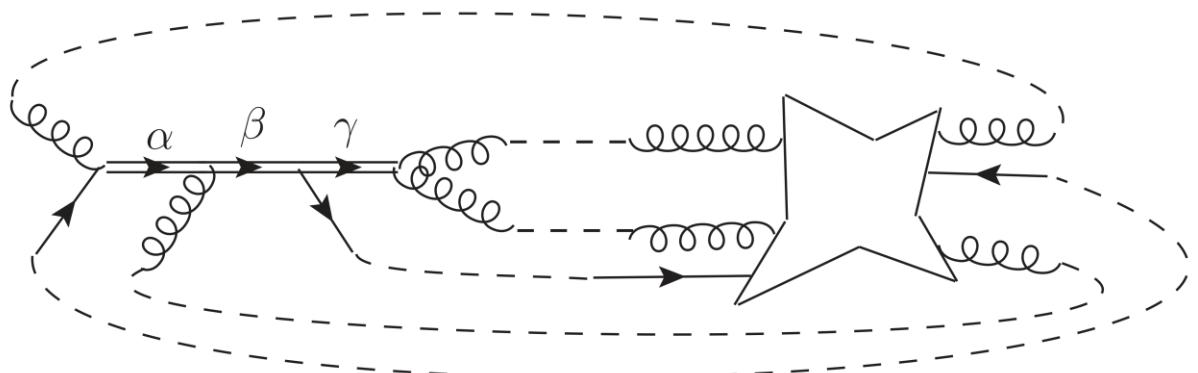
## 2. Trace vs multiplet bases



## 3. Color structure decomposition



## 4. Preliminary results



# Strong force Lagrangian

$$q^i \in \mathbb{C}^3 \equiv 3$$

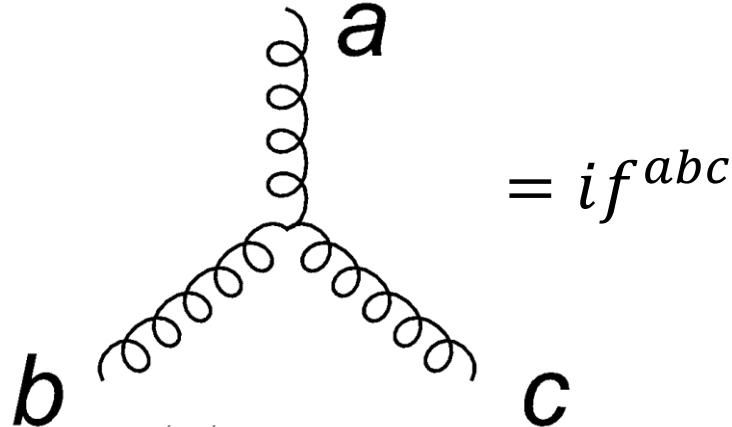
$$q_i \in \bar{3}$$

$$G_\mu^a \in \text{Adj}(SU(3)) \equiv 8$$

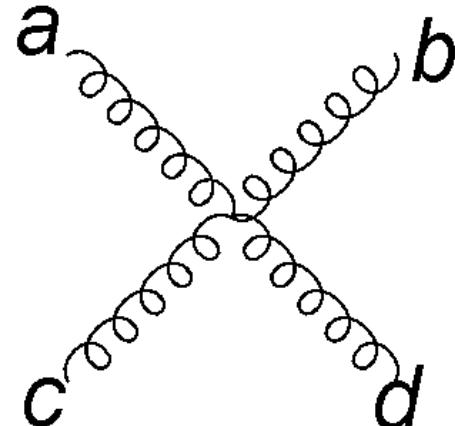
$$\mathcal{L} = \sum_{q=u,d,s,c,b,t} \bar{q}_i (i\partial_\mu \gamma^\mu - m) q^i - \frac{1}{4} (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)^2 + g_s G_\mu^a \bar{q}_i \gamma^\mu (t^a)_j^i q^j$$

$+ g_s f^{abc} (\partial_\mu G_\nu^a) G^{\mu a} G^{\nu c} - \frac{1}{4} g_s^2 (f^{eab} G_\mu^a G_\nu^b) (f^{ecd} G^{\mu c} G^{\nu d})$

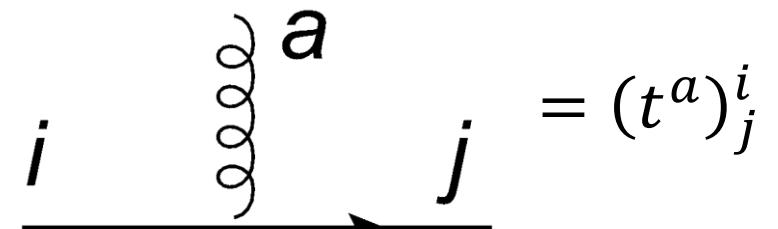
Three gluon vertex



Four gluon vertex



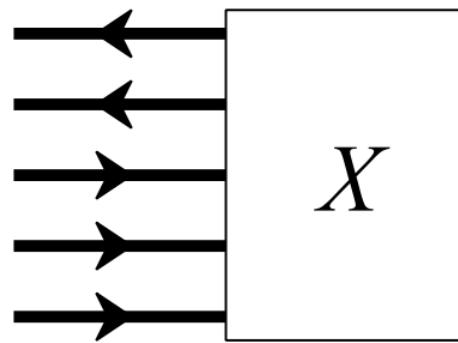
Quark – gluon vertex



# Birdtracks

$$\delta^j{}_k \equiv \begin{array}{c} j \\ \longrightarrow \\ k \end{array}$$

$$X_{de}^{abc} = 3^{\otimes 3} \otimes \bar{3}^{\otimes 2} \equiv$$



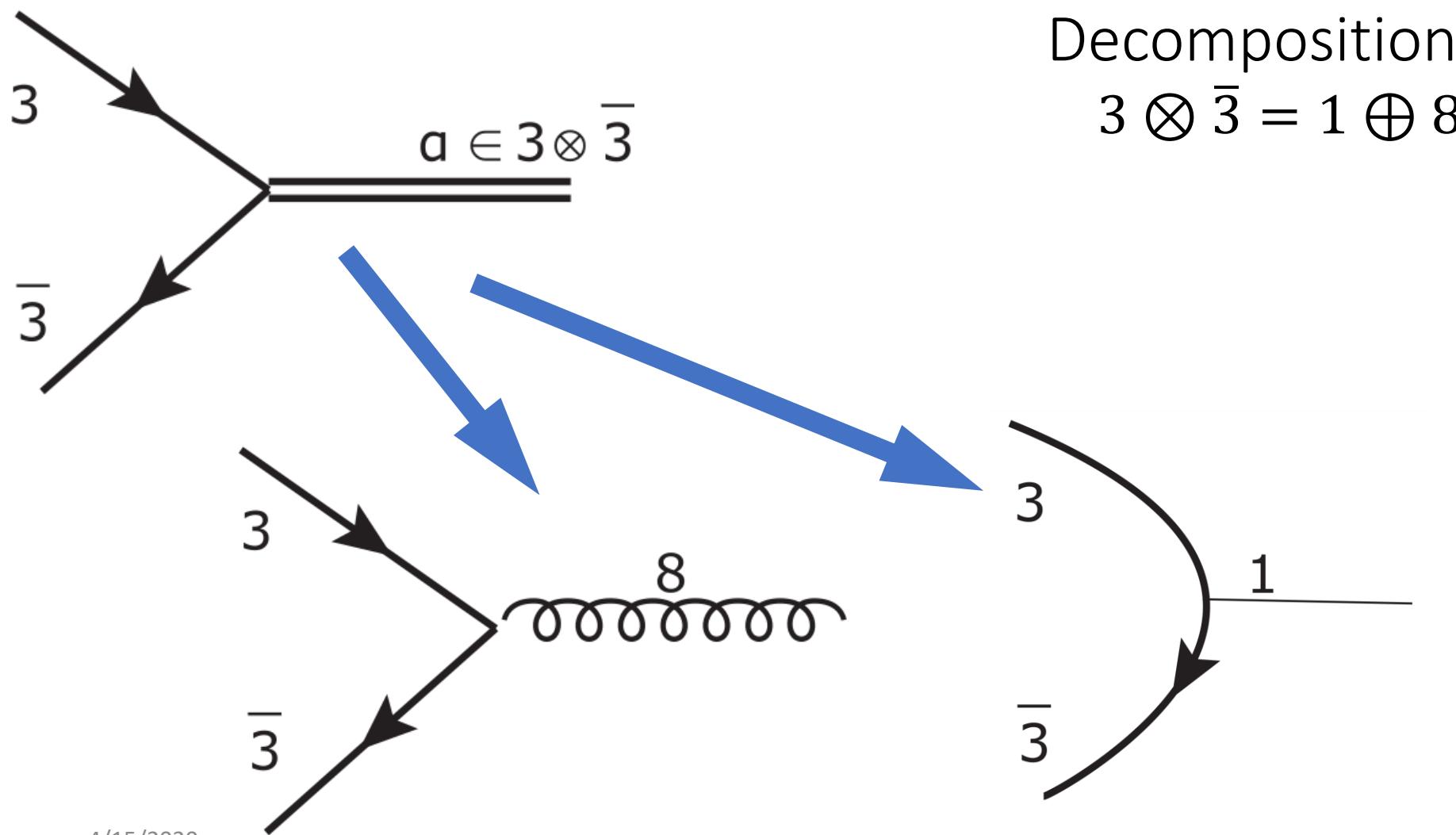
$$(t^a)^i{}_j \equiv \begin{array}{ccccc} & & a & & \\ & i & \downarrow & j & \\ & \hline & & & \end{array}$$

# Trace bases

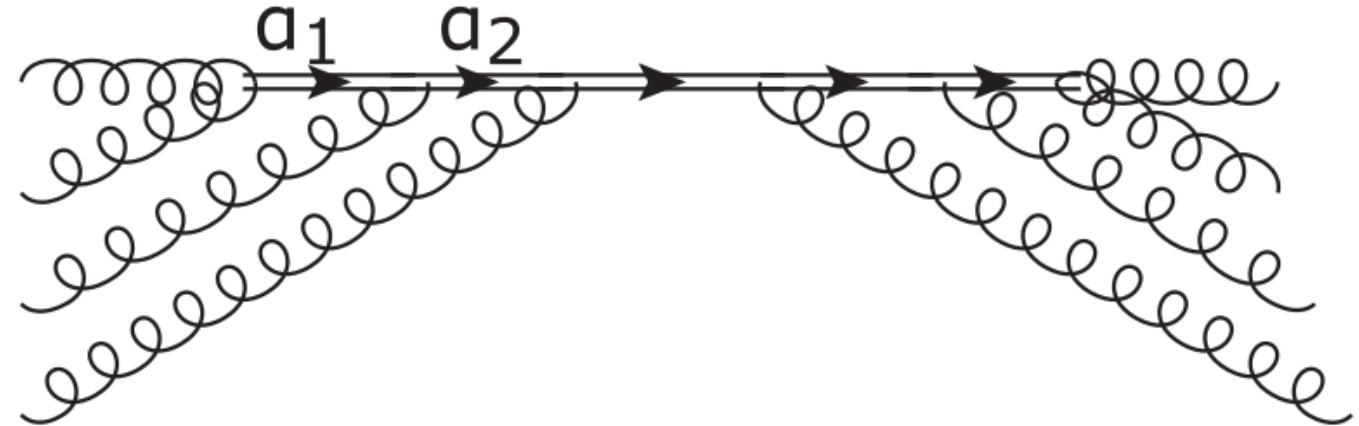
$$\begin{array}{c}
 \text{Diagram: } c \text{ (star-like vertex with three legs)} = A + B + C \\
 \text{Diagram: } c \in \bar{3} \otimes 3 \otimes 8 \otimes 8 = (t^a)_j^i (t^b)_k^j = t^a \cdot t^b \\
 \text{Diagram: } \left( \text{Diagram with two loops} \right) \times \text{Diagram with loop} \neq 0
 \end{array}$$

$= (t^a)_j^i (t^b)_l^j \delta_l^k$   
 $= Tr[t^a \cdot t^b] \delta$

# Multiplet bases

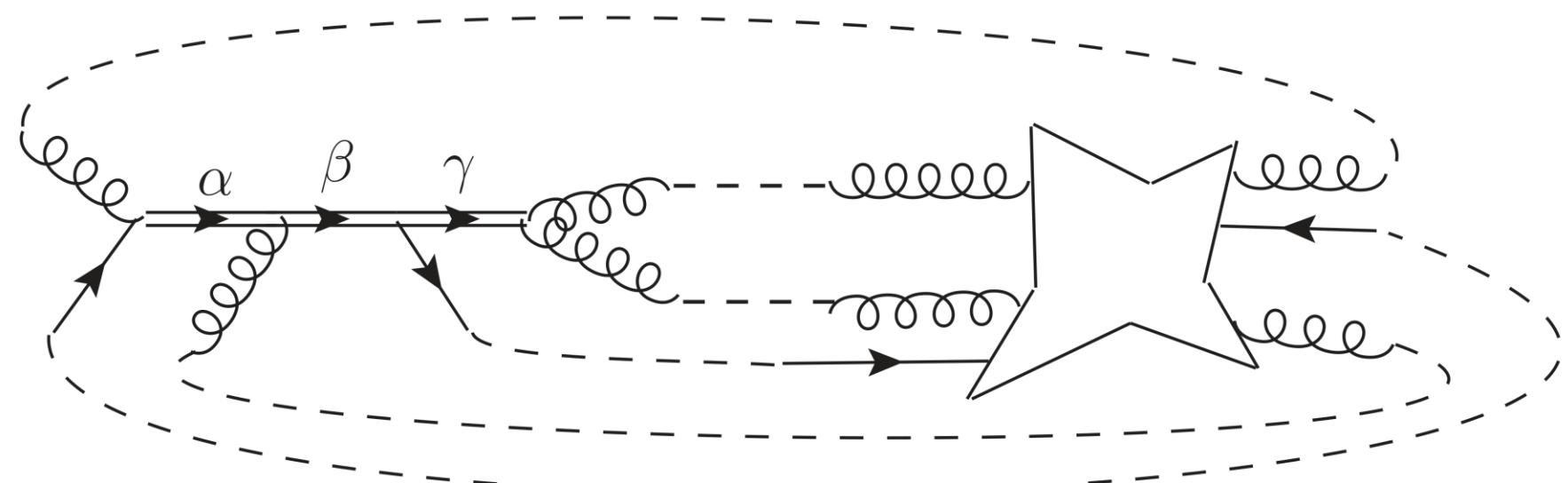


# Multiplet bases

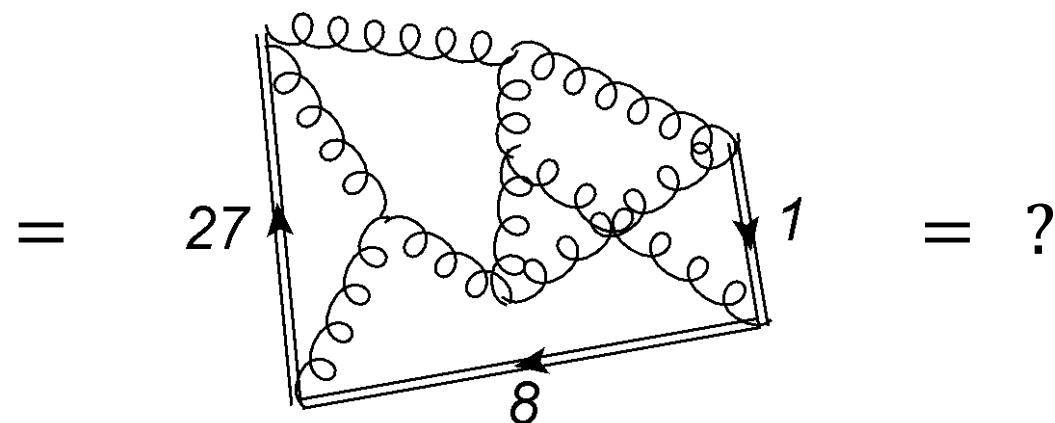
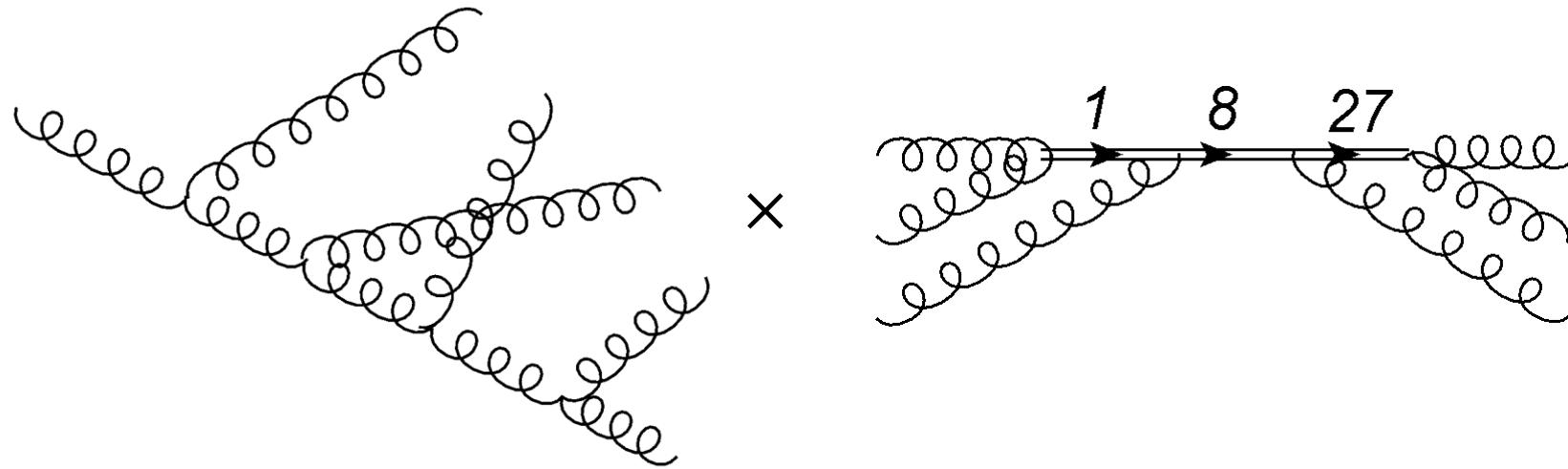


$$\alpha_1 \in 8 \otimes 8 = 0 \oplus 1 \oplus 3 \oplus \bar{3} \oplus 8 \oplus 8 \oplus 11 \oplus 27$$

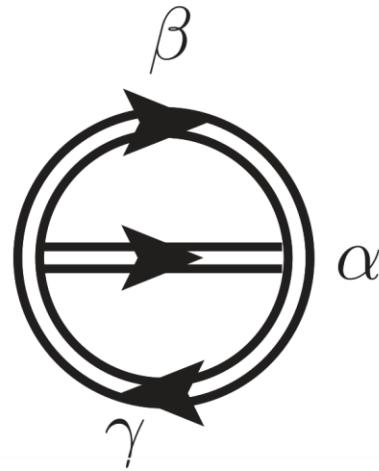
$$\alpha_2 \in 8 \otimes 8 \otimes 8$$



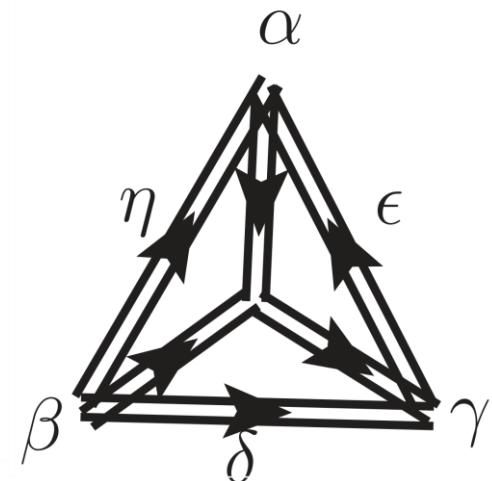
# Multiplet bases



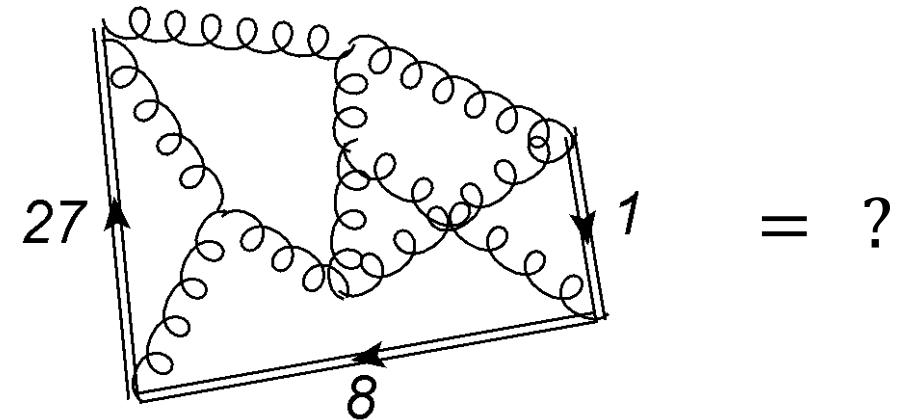
# Wigner 3j and 6j coefficients



3j



6j



"Any group-theoretical invariant quantity can be expressed in terms of Wigner 3- and 6-j coefficients"

Predrag Cvitanović. Group Theory. Princeton University Press, feb 2008.

M. Sjodahl and J. Thorén, *Decomposing color structure into multiplet bases*, [JHEP 09 \(2015\) 055](#)

# Important relations

$$\alpha = d_\alpha \quad (1)$$

$$\alpha = \frac{\beta}{d_\alpha} \quad (2)$$

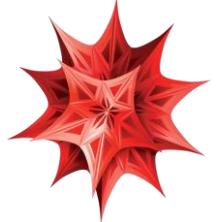
$$= ?$$

$$= \sum_a \frac{\text{Diagram with } \alpha, \beta, \gamma, \delta, \epsilon, \zeta}{\text{Diagram with } \alpha, \beta, \gamma, \delta}$$

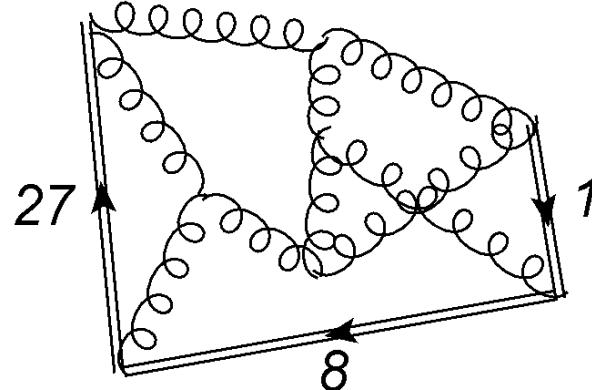
(3)

$$= \sum_\alpha \frac{d_\alpha}{\text{Diagram with } \mu, \nu, \alpha, \beta}$$

(4)



# Expressing amplitudes in multiplet bases



All the functions  
from the previous  
slide



$$-\frac{1}{2N_C(N_C^2-1)^3}$$

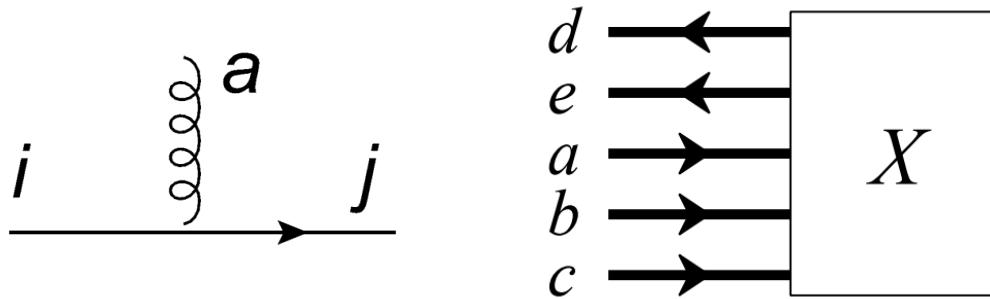
M. Sjodahl and J. Thorén, *Decomposing color structure into multiplet bases*, *JHEP* **09** (2015) 055

# Challenges

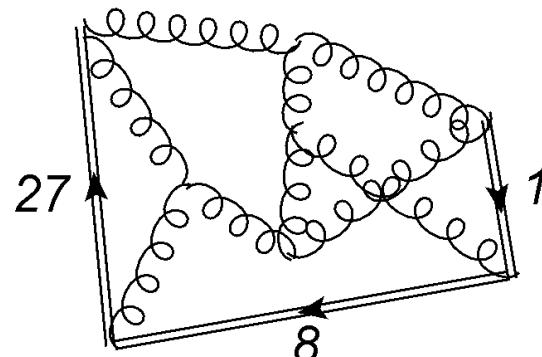
- Arbitrary basis  
 $\{1,8,27\} \rightarrow \{\alpha, \beta, \gamma\}$
- Automatization
- Tests

# Summary

## 1. QCD and Birdtracks

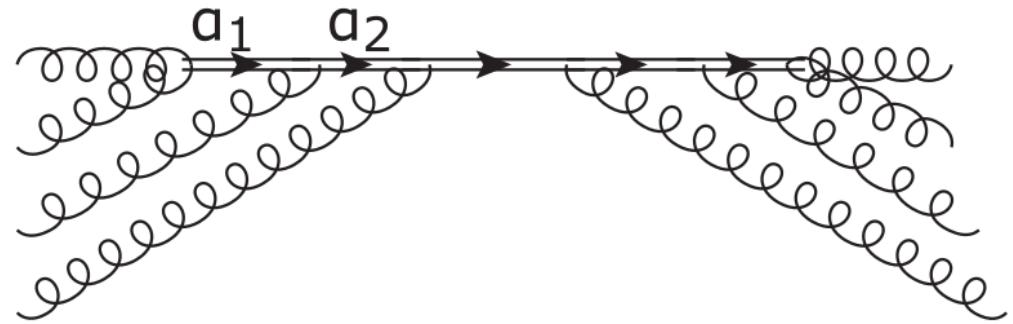


## 3. Color structure decomposition

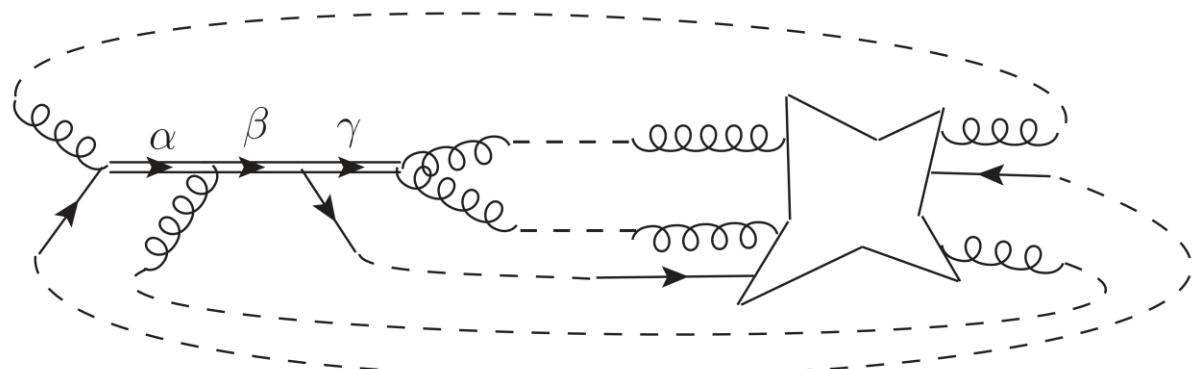


4/15/2020

## 2. Trace vs multiplet bases



## 4. Preliminary results

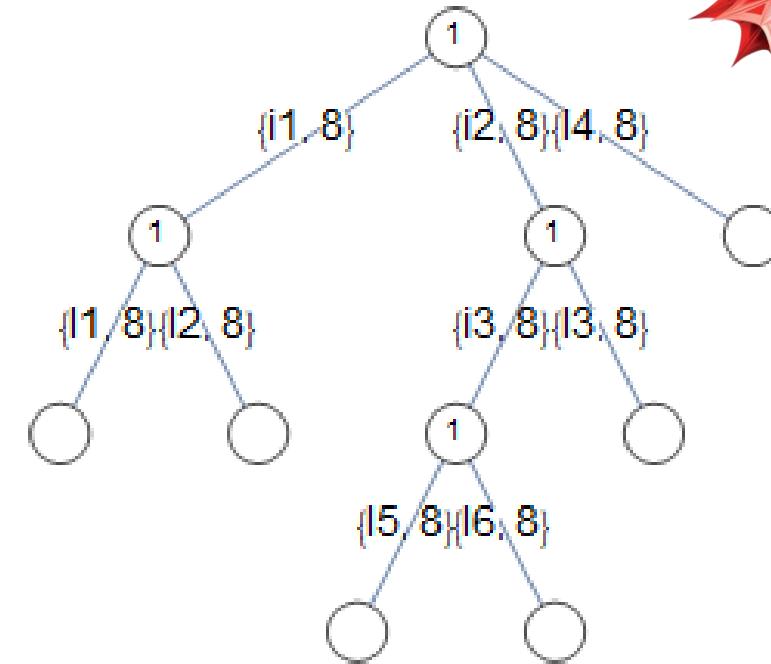


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# Expressing amplitudes in multiplet bases

A black and white line drawing of a complex, branching structure composed of many small, interconnected loops, resembling a stylized plant or a network. The structure is highly detailed, with numerous fine lines creating a dense, organic appearance.

# Graph[]



# Wolfram Mathematica®



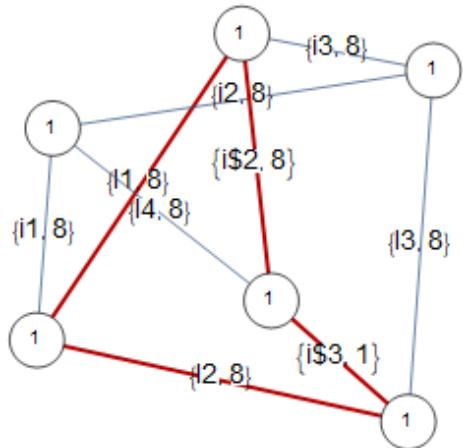
# Expressing amplitudes in multiplet bases

$$\begin{array}{c}
 \left( \text{Diagram 1} \right) * \times \text{Diagram 2} \\
 = \text{Diagram 3} \\
 \text{/.VertexCorrectionRel} = \frac{1}{N_C - N_C^3} \\
 \text{Diagram 4}
 \end{array}$$

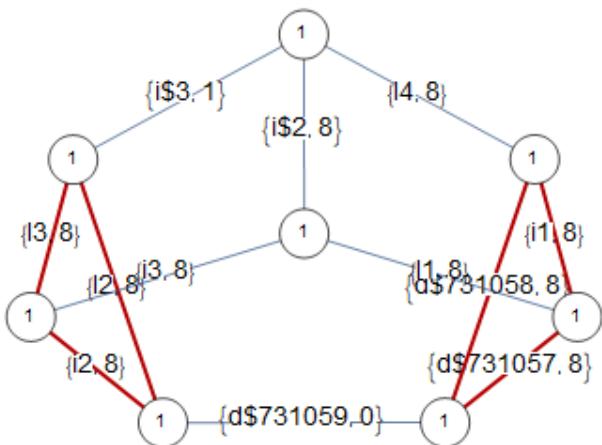
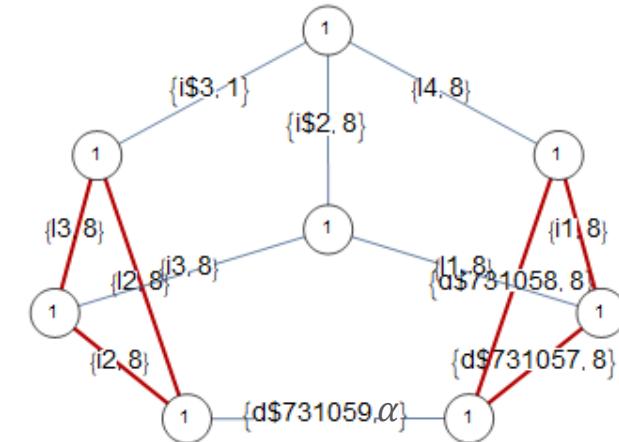
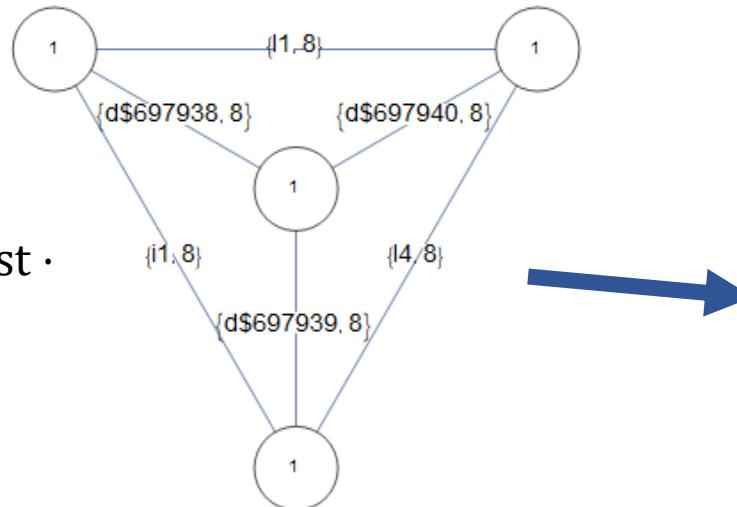
The diagrams are Feynman-like graphs representing amplitudes in multiplet bases. They consist of nodes (circles) and edges (lines). Nodes are labeled with '1' or numerical values in curly braces. Edges are labeled with pairs of numerical values in curly braces.

- Diagram 1:** A tree-like structure with 6 nodes. Root node: {1}. Level 1: {1}, {1}. Level 2: {1}, {1}, {1}. Level 3: {1}, {1}, {1}, {1}. Edges: (1,2){i1,8}, (1,3){i2,8}, (1,4){i4,8}, (2,3){i1,8}, (2,5){i5,8}, (2,6){i6,8}, (3,4){i3,8}, (3,6){i3,8}, (4,5){i5,8}, (5,6){i6,8}.
- Diagram 2:** A chain-like structure with 8 nodes. Edges: (1,2){i3,8}, (2,3){i2,8}, (3,4){i\$3,1}, (4,5){i\$2,8}, (5,6){i\$1,27}, (6,7){i5,8}, (7,8){i6,8}.
- Diagram 3:** A complex graph formed by merging Diagram 1 and Diagram 2. It has 12 nodes. Edges: (1,2){i1,8}, (1,3){i1,8}, (1,4){i2,8}, (1,5){i6,8}, (1,6){i3,8}, (1,7){i2,8}, (1,8){i3,8}, (2,3){i1,8}, (2,4){i2,8}, (2,5){i5,8}, (2,6){i3,8}, (2,7){i1,27}, (2,8){i3,8}, (3,4){i2,8}, (3,5){i4,8}, (3,6){i3,8}, (3,7){i2,8}, (3,8){i3,1}, (4,5){i2,8}, (4,6){i3,8}, (4,7){i2,8}, (4,8){i3,1}, (5,6){i1,8}, (5,7){i2,8}, (5,8){i2,8}, (6,7){i1,8}, (6,8){i3,8}, (7,8){i3,1}.
- Diagram 4:** A simplified version of Diagram 3, showing only the edges and nodes. Edges: (1,2){i1,8}, (1,3){i1,8}, (1,4){i2,8}, (1,5){i6,8}, (1,6){i3,8}, (1,7){i2,8}, (1,8){i3,8}, (2,3){i1,8}, (2,4){i2,8}, (2,5){i5,8}, (2,6){i3,8}, (2,7){i1,27}, (2,8){i3,8}, (3,4){i2,8}, (3,5){i4,8}, (3,6){i3,8}, (3,7){i2,8}, (3,8){i3,1}, (4,5){i2,8}, (4,6){i3,8}, (4,7){i2,8}, (4,8){i3,1}, (5,6){i1,8}, (5,7){i2,8}, (5,8){i2,8}, (6,7){i1,8}, (6,8){i3,8}, (7,8){i3,1}.

# Expressing amplitudes in multiplet bases

.InsertCompleteness[i\$2, I2, Cycl] =

$$\sum_{\alpha} \text{Const}(\alpha)$$

.VertexCorrectionRel = Const ·


$$-\frac{1}{2N_C(N_C^2-1)^3}$$

M. Sjodahl and J. Thorén, *Decomposing color structure into multiplet bases*, [JHEP 09 \(2015\) 055](#)

