

Matching & Merging of *High Energy Jets* with PYTHIA

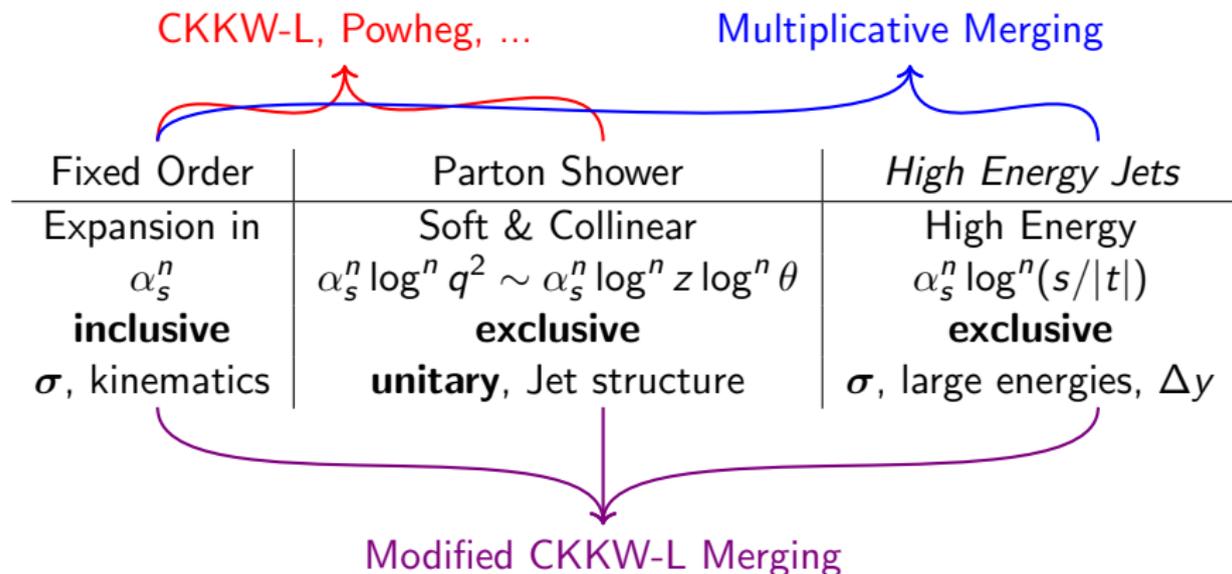
Combining high energy and collinear logarithms

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HEJ provides an **all order resummation** of logarithms $\log(s/|t|)$. All approximations are made on a Matrix Element level, while keeping the full Phase Space dependency. This allows easy **matching & merging** of HEJ with Fixed-Order Matrix Elements, giving both leading-order and leading logarithm accuracy.

⇒ Stabilises observables which depend on large $\Delta y \sim \log(s/|t|)$

$$d\sigma_{resum}^{\text{HEJ}} = \sum_{n=2}^{\infty} d\Omega_n f(x_1, x_2, \mu_f) \left| \mathcal{M}(p, \mu_r)_n^{\text{HEJ}} \right|^2$$

Matching & Merging HEJ with FO

- HEJ is fully differential, all approximates on Amplitude/ME
- ⇒ Replace HEJ with full ME for first M jets (k -Factor)

$$\left| \mathcal{M}(p, \mu_r)_n^{\text{HEJ}} \right|^2 \rightarrow k_n(p) \left| \mathcal{M}(p, \mu_r)_n^{\text{HEJ}} \right|^2$$

$$k_n(p) = \sum_{m=2}^M \int d\Omega'_m \delta(p' - j_m(p)) \frac{\left| \mathcal{M}(p', \mu_r)_m^{\text{LO}} \right|^2}{\left| \mathcal{M}(p', \mu_r)_m^{\text{HEJ,LO}} \right|^2}$$

- ⇒ Mapping $j_m(p)$ of n parton to m (Born)-jets

Matching & Merging HEJ with FO

$$\begin{aligned}
 \sigma_{2j}^{\text{resum,match}} = & \sum_{f_1, f_2} \sum_m \prod_{j=1}^m \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{d^2 \mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{dy_j^B}{2} \right) (2\pi)^4 \delta^{(2)} \left(\sum_{k=1}^m \mathbf{p}_{k\perp}^B \right) && \text{Fixed Order} \\
 & \cdot x_a^B f_a(x_a^B, Q_a^B) x_b^B f_b(x_b^B, Q_b^B) \frac{|\overline{\mathcal{M}}^B|^2}{(\hat{s}^B)^2} \\
 & \cdot \frac{|\overline{\mathcal{M}}_{\text{HEJ}}^{\text{tree}}|^{-2} (2\pi)^{-4+3m} 2^m (\hat{s}^B)^2}{x_a^B f_{a,f_1}(x_a^B, Q_a^B) x_b^B f_{b,f_2}(x_b^B, Q_b^B)} && \text{Overlap HEJ} \\
 & \cdot \sum_{n=2}^{\infty} \int_{p_{1\perp}=\cdot 9p_j, \perp}^{p_{1\perp}=\infty} \frac{d^2 \mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=\cdot 9p_j, \perp}^{p_{n\perp}=\infty} \frac{d^2 \mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{d^2 \mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \delta^{(2)} \left(\sum_{k=1}^n \mathbf{p}_{k\perp} \right) \\
 & \cdot \mathbf{T}_y \prod_{i=1}^n \left(\int \frac{dy_i}{2} \right) \mathcal{O}_{mj}^e \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^m \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\}) \\
 & \cdot x_a f_{a,f_1}(x_a, Q_a) x_b f_{b,f_2}(x_b, Q_b) \frac{|\overline{\mathcal{M}}_{\text{HEJ}}^{f_1 f_2 \rightarrow f_1 g \dots g f_2}(\{p_i\})|^2}{\hat{s}^2} .
 \end{aligned}$$

Parton Showers

Parton Shower resum **soft and collinear** logarithms $\log(q^2)$ through consecutive evaluation of **splitting probabilities** P^P . The total energy of an process is distributed over many particles, without changing the cross-section (**unitary**).

⇒ No-splitting probability = Sudakov Form factor

$$\Delta_{\alpha}^P(x, \rho_i, \rho_j) = \exp \left(- \int_{\rho_j}^{\rho_i} d\rho \int dz P_{\alpha}^P(\rho, z, x) \right)$$

$$d\sigma^P = C_0^P \prod_{i=0}^{\infty} \left(P^P(\rho_i) \Delta^P(\rho_i, \rho_{i+1}) dz_i d\rho_i \right)$$

⇒ Distributing momenta over many partons

Goal Add different multiplicity ME together

- ⇒ Transform FO from *inclusive* to *exclusive* states
- Add Sudakov

Goal Generate N *hardest* emissions through ME

- ⇒ Cut phase space at (arbitrary) t_{MS} (*Merging Scale*)
- $t(\rho) > t_{MS} \Rightarrow$ ME region
- $t(\rho) < t_{MS} \Rightarrow$ PS region
- ⇒ Shower freely from $\rho(t_{MS})$

$$d\sigma^{\text{CKKW-L}} = \sum_{n=2}^N d\sigma_n^{\text{FO}} \Delta^P(t_n, t_{MS}) \prod_{i=1}^n \Delta^P(\rho_{i-1}, \rho_i)$$

- Construct states S_0, \dots, S_n with scales ρ_0, \dots, ρ_n from FO event
- How would have the shower generated a FO state?
- ⇒ Undo splittings (*Shower History*)
- Select most likely history $\sim \prod P^P(\rho_i)$

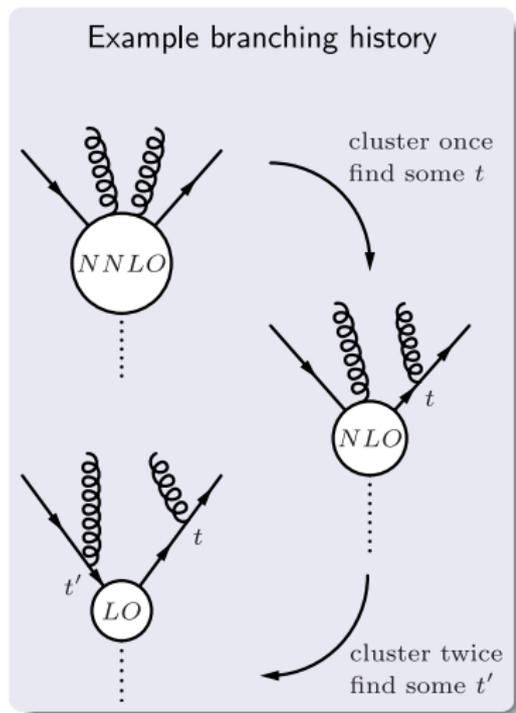
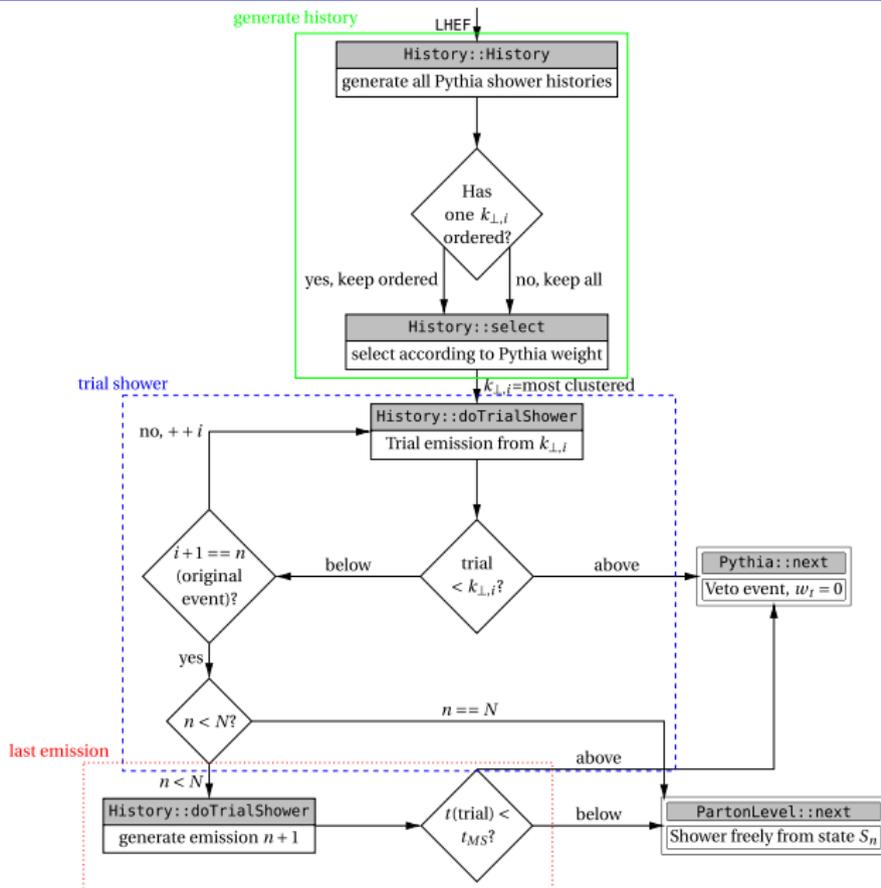
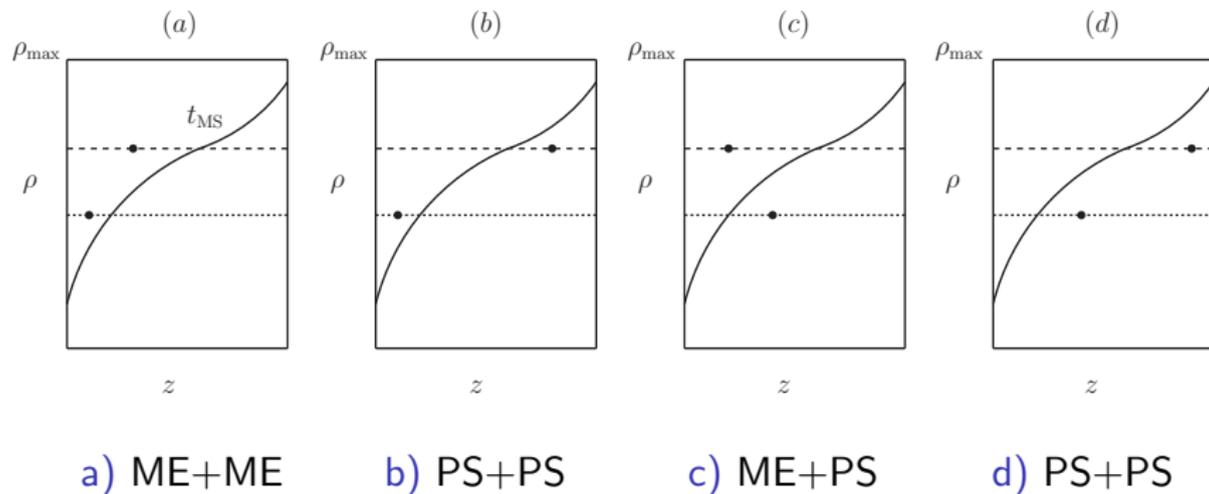


Illustration by S. Schumann

CKKW-L Algorithm





Merging HEJ with PS

- HEJ is already exclusive
- ⇒ Cross-section includes *unitary* Δ^H and *non-unitary* R term
- Exact form of Δ^H and R unknown (and do not matter)

$$d\sigma_{resum}^{\text{HEJ}} = \sum_{n=2}^{\infty} d\Omega_n R(p) \prod_{i=1}^n \left[P^H(\rho_i) \Delta^H(\rho_{i-1}, \rho_i) \right]$$

Goal m hardest emissions from HEJ, *softer* from PS

- Generate history of HEJ states (similar to CKKW-L)
- ⇒ Switch to (unrestricted) shower when PS becomes dominant

$$d\sigma^{\text{HEJ+PS}} = \sum_{n=2}^{\infty} d\Omega_n R(p) \prod_{i=1}^{m-1} \left(P_i^H \Delta_{i,i-1}^H \right) \\ \cdot \Delta_{m,m-1}^H P_m^{P+!H} \prod_{i=m+1}^n \left(P_i^P \Delta_{i,i-1}^P \right)$$

Merging HEJ with PS

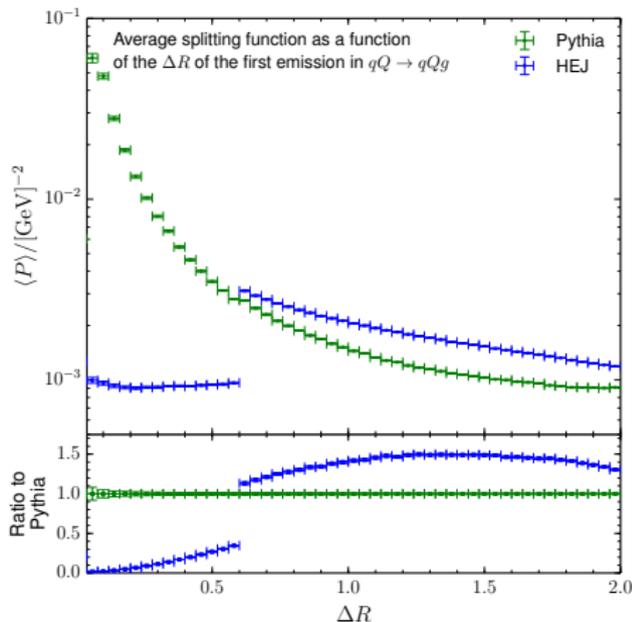
Cutting the phase space

HEJ splitting probability

Similar to derivation of P^P

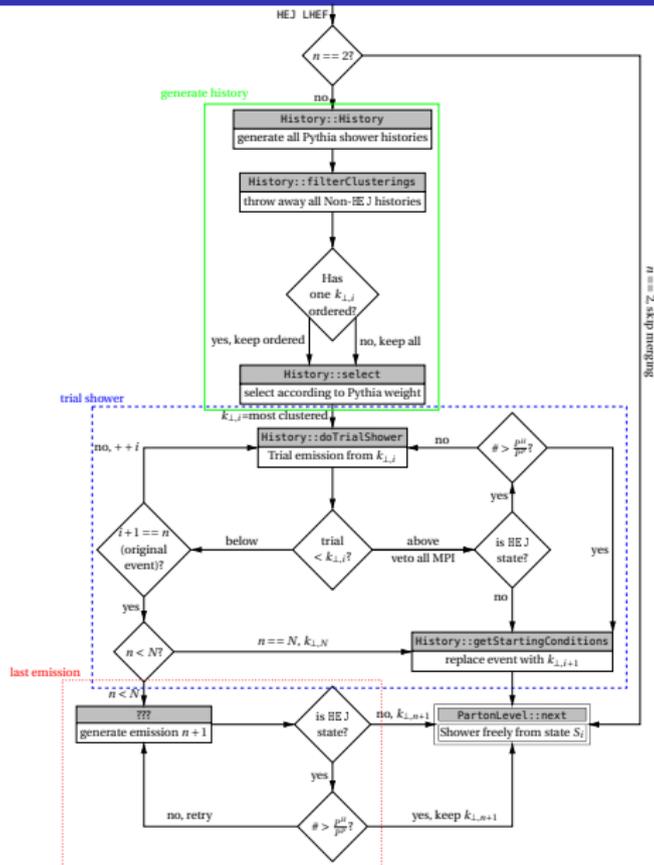
$$P^H := \frac{1}{2} \frac{1}{16\pi^2} \frac{|\mathcal{M}_{\text{HEJ}}^{n+1}|^2}{|\mathcal{M}_{\text{HEJ}}^n|^2}$$

- Use P^H for cut
- $P^P > P^H \Rightarrow$ PS region
- $P^P < P^H \Rightarrow$ HEJ region
- Overlap $P^{P+!H} = \frac{P^P}{P^H}$
- Use $P^{P+!H}$ in trial shower
- ⇒ Extra Sudakov $\Delta^{P+!H}$



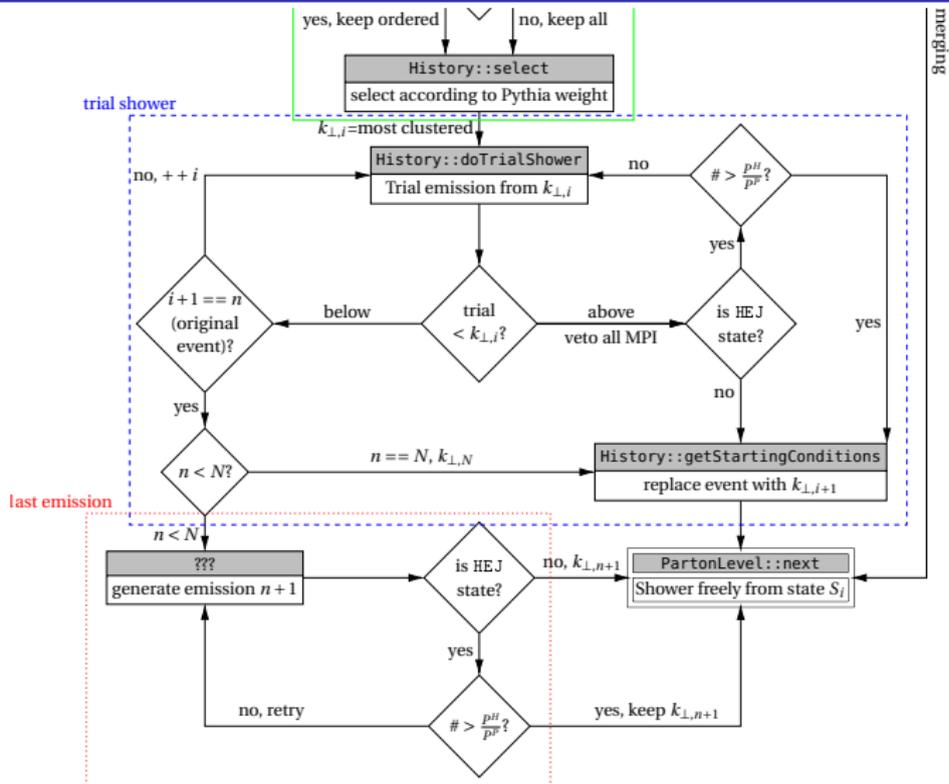
arxiv:1712.00178

HEJ+PYTHIA merging algorithm



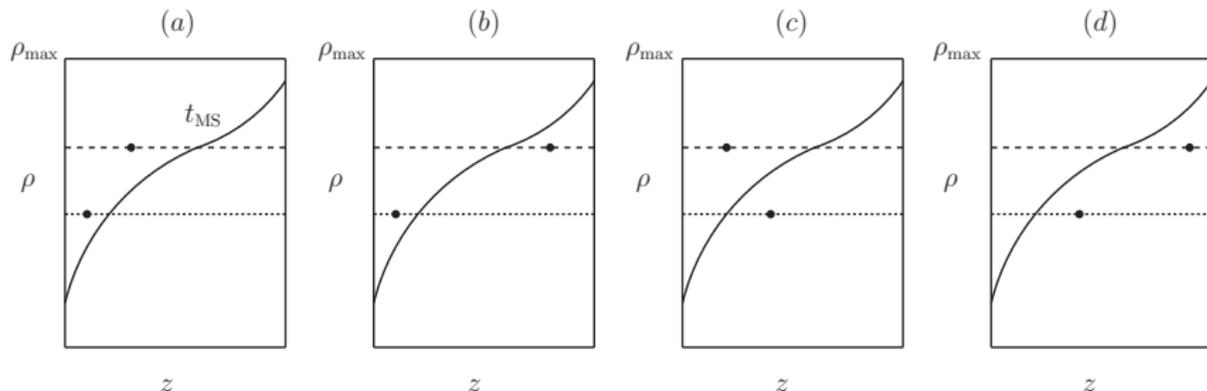
HEJ+PYTHIA merging algorithm

Trial shower



HEJ+PYTHIA merging

Phase space filling



a) HEJ+HEJ

b) PS+PS

c) HEJ+PS

d) PS+PS

⇒ Completely equivalent to CKKW-L

⇒ Main difference: t_{MS} is not a fixed scale, but *fuzzy*

- There is no single *perfect* calculation
- ⇒ FO (inclusive), PS (soft & collinear), HEJ (large rapidities)
 - (Almost) always more than one effect for each observable
- ⇒ Combine (merge) different predictions
- FO+PS: e.g. CKKW-L
- FO+HEJ: Multiply matrix elements
- ⇒ FO+PS+HEJ: **Modified CKKW-L merging**

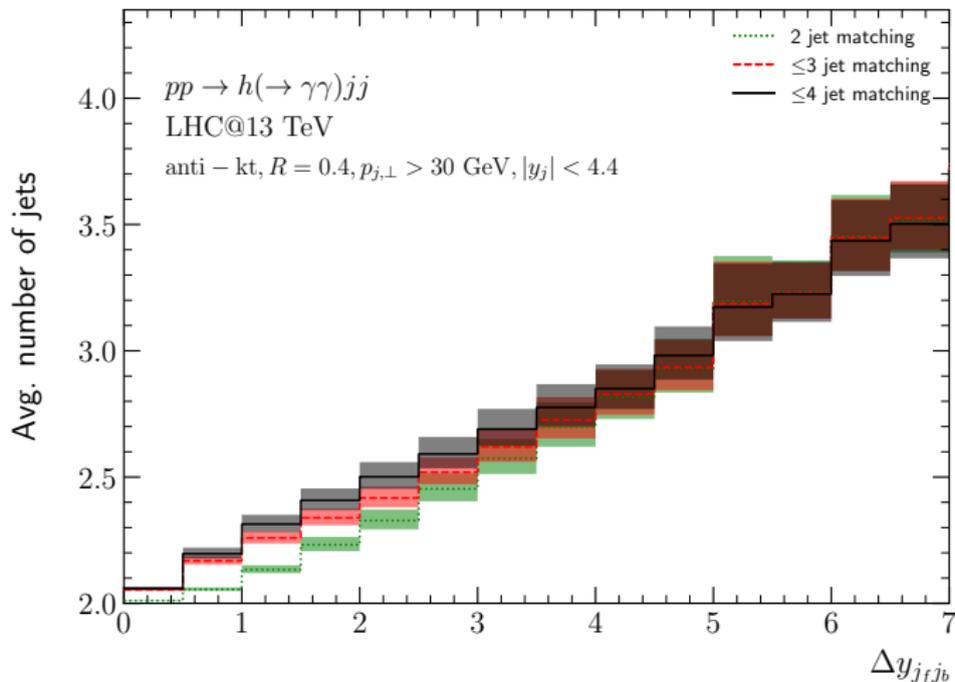
Backup slides

Sudakov veto algorithm

- Generate $\Delta^P(\rho_{i-1}, \rho_i)$ for each $i \in [1, n]$
- \Rightarrow *Sudakov veto algorithm*
- \Rightarrow Generate emission from $S' = S_0$ (*Trial Shower*)
- if $\rho' < \rho_{i+1}$: trial shower from next state S_{i+1}
- else $\rho' > \rho_{i+1}$ veto whole event
- \Rightarrow keeps

$$1 - \int_{\rho_{i+1}}^{\rho_i} dt \int dz P^P(t) = \Delta^P(\rho_i, \rho_{i+1})$$

Matching & Merging HEJ with FO



⇒ Small Δy through FO matching

⇒ Large Δy HEJ approximation already accurate

arxiv:1805.04446