

# ASP EXERCISE TO MODEL THE EVOLUTION OF COVID-19 IN AFRICA

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# The simplest model – SIR



$t$  is time, to be measured in days

$S = S(t)$  is the number of *susceptible* individuals,

$I = I(t)$  is the number of *infected* individuals, and

$R = R(t)$  is the number of *recovered* individuals.

$N$  is the total population

$s(t) = S(t)/N$ , the susceptible fraction of the population,

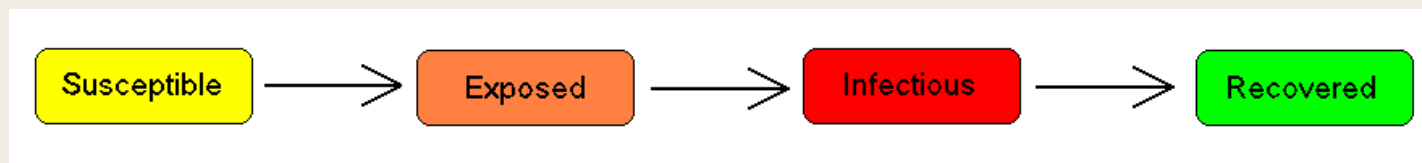
$i(t) = I(t)/N$ , the infected fraction of the population, and

$r(t) = R(t)/N$ , the recovered fraction of the population.

At each time  $t$ ,  $s(t) + i(t) + r(t) = 1$

# More complicated models (1)

- The SIS Model with vital dynamics and constant population
  - *Introduce birth and death rates*
- The MSIR model
  - *For some infections such as measles, consider passive immunity (for a few months after births, babies are immune to the disease)*
- Carrier State
  - *Some people who have had an infectious disease such as [tuberculosis](#) never completely recover and continue to [carry](#) the infection, whilst not suffering from the disease themselves.*
- The SEIR model
  - *For many important infections there is a significant incubation period during which individuals have been infected but are not yet infectious themselves. During this period the individual is in compartment E (for exposed).*



# More Complicated Models (2)

## ■ The SEIS model

- *The SEIS model is like the SEIR model (above) except that no immunity is acquired at the end*

## ■ The MSEIR model

- *For the case of a disease, with the factors of passive immunity, and a latency period there is the MSEIR model.*

## ■ The MSEIRS model

- *An MSEIRS model is similar to the MSEIR, but the immunity in the R class would be temporary, so that individuals would regain their susceptibility when the temporary immunity ended.*

## ■ Variable contact rates and pluriannual or chaotic epidemics

- *It is well known that the probability of getting a disease is not constant in time. Some diseases are seasonal, such as the [common cold viruses](#), which are more prevalent during winter. With childhood diseases, such as measles, mumps, and rubella, there is a strong correlation with the school calendar, so that during the school holidays the probability of getting such a disease dramatically decreases.*

**Which model is best suited for Covid-19?**

# Start with the SIR Model - Some Assumptions (I)

- **No addition to the susceptible group**

- *The population is quarantine (a realistic assumption)*
- *No new births (not so realistic)*
- *The only way a person leaves the susceptible group is by becoming infected*

- **$ds(t) / dt = f ( s(t), i(t), b)$**

- *$b$  = quantifies the amount of contacts between susceptible and infected people*
- *Each individual has a fixed number  $b$  of contacts per day to spread the disease*
- *Not all these contacts are with susceptible people*

- **The population is homogeneously mixed**

- *The fraction of the contacts with susceptible people is  $s(t)$*
- *On average, each infected person generates  $b s(t)$  new infected persons*

- **Assume large population with small infected persons**

- *Probability that a given susceptible persons encounters more than 1 infected person a day in very low*

# Some Assumptions (II)

- A fraction  $k$  of the infected population will recover on a any given day
- By “infected”, we mean “infectious”, i.e. capable of spreading the disease to a susceptible person
- A recovered person may still be miserable, and might die later of something else
  - *Recovered = recovered + dead*

# The SIR ...

- The rate of change in the susceptible population

1)  $\frac{dS}{dt} = -bs(t)I(t)$

2)  $\frac{ds}{dt} = -bs(t)i(t)$

- The rate of change in the recovered population

3)  $\frac{dr}{dt} = ki(t)$

- The rate of change in the infected population

4)  $\frac{ds}{dt} + \frac{dr}{dt} + \frac{di}{dt} = 0$

5)  $\frac{di}{dt} = bs(t)i(t) - ki(t)$

- Finally, complete the equations by setting the initial conditions. These may / will differ for each country

- $S(0) = \text{number of people at time } t=0$        $\rightarrow$        $s(0) = 1$

- $I(0) = \text{number of infected people a } t=0$        $\rightarrow$        $i(0) = ?$

- $R(0) = \text{number of recovered people at } t=0;$        $\rightarrow$        $0.$

- The parameters k and b might also be difference for different countries

# The SIR in a specific example

- Assume that the population is 8 million. Further, hardly anyone was immune at the beginning of the epidemic, so almost everyone was susceptible. We will assume that there was a trace level of infection in the population, say, 10 people. Thus, our initial values for the population variables are:

- $S(0) = 8.0 e^6$

- $I(0) = 10$

- $R(0) = 0$

- $s(0) = 1$

- $i(0) = 1.25 e^{-6}$

- $r(0) = 0$

**(Note: The sum of our starting populations is not exactly  $N$ , nor is the sum of our fractions exactly 1. The trace level of infection is so small that this won't make any difference.)**



# The complete set of equations specific example

$$\frac{ds}{dt} = -b s(t) i(t), \quad s(0) = 1,$$

$$\frac{di}{dt} = b s(t) i(t) - k i(t), \quad i(0) = 1.25 \times 10^{-6},$$

$$\frac{dr}{dt} = k i(t), \quad r(0) = 0.$$

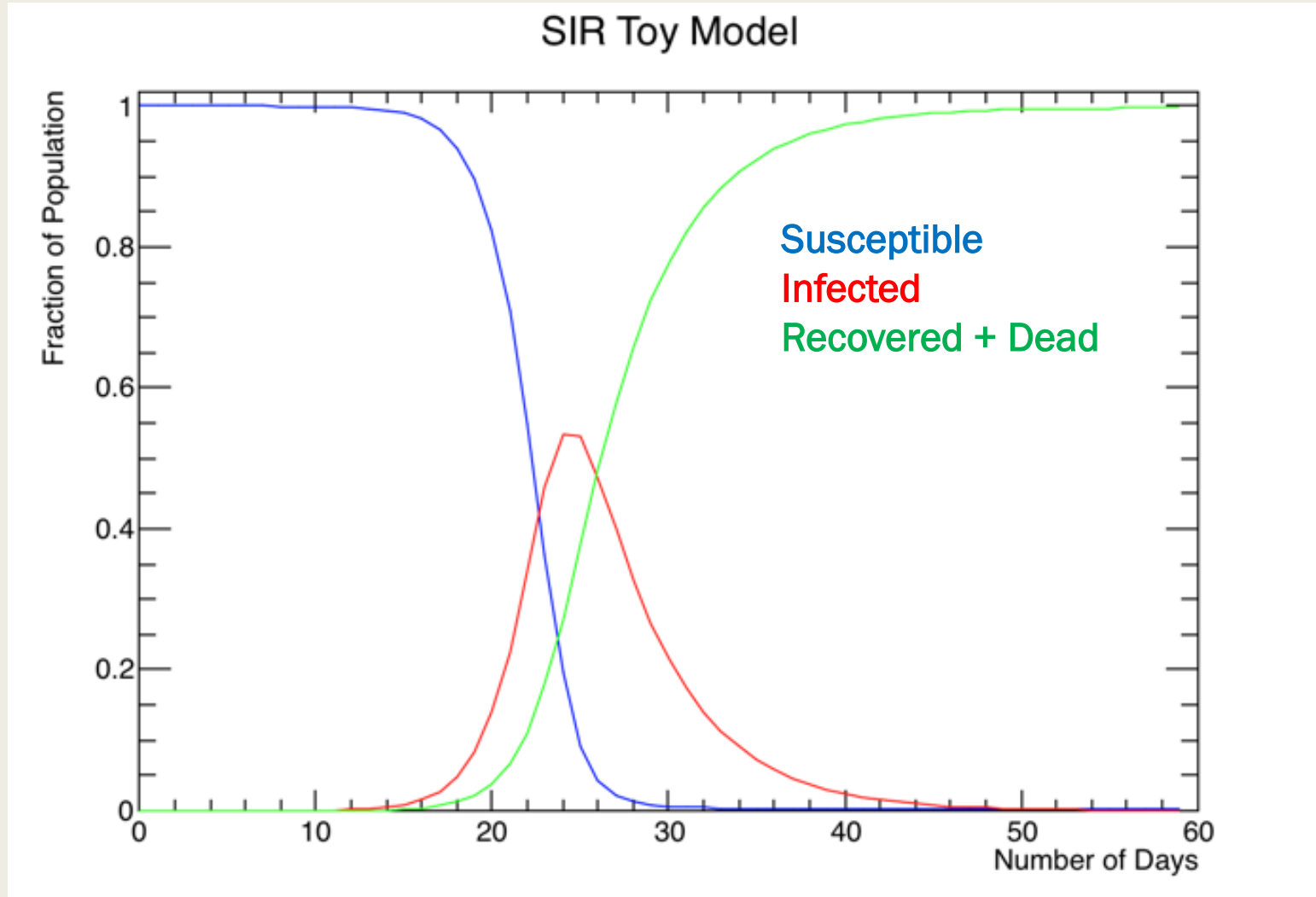
**We do not know what b and k are but they can be estimated / guessed for this particular example**

- Assume each infected would make a possibly infectious contact every day, then **b** would be **1.0**
- Assume the average period of infectiousness at 2 days, so that would suggest **k = 1/2**

**Exercise:**

**solve this set of equations and produce the plots of s(t), i(t) and r(t) as a function of t (days) on the same graphs (but with different colors)**

# My numerical solutions using Euler's method—my code is available in c++



# To a more complicated model

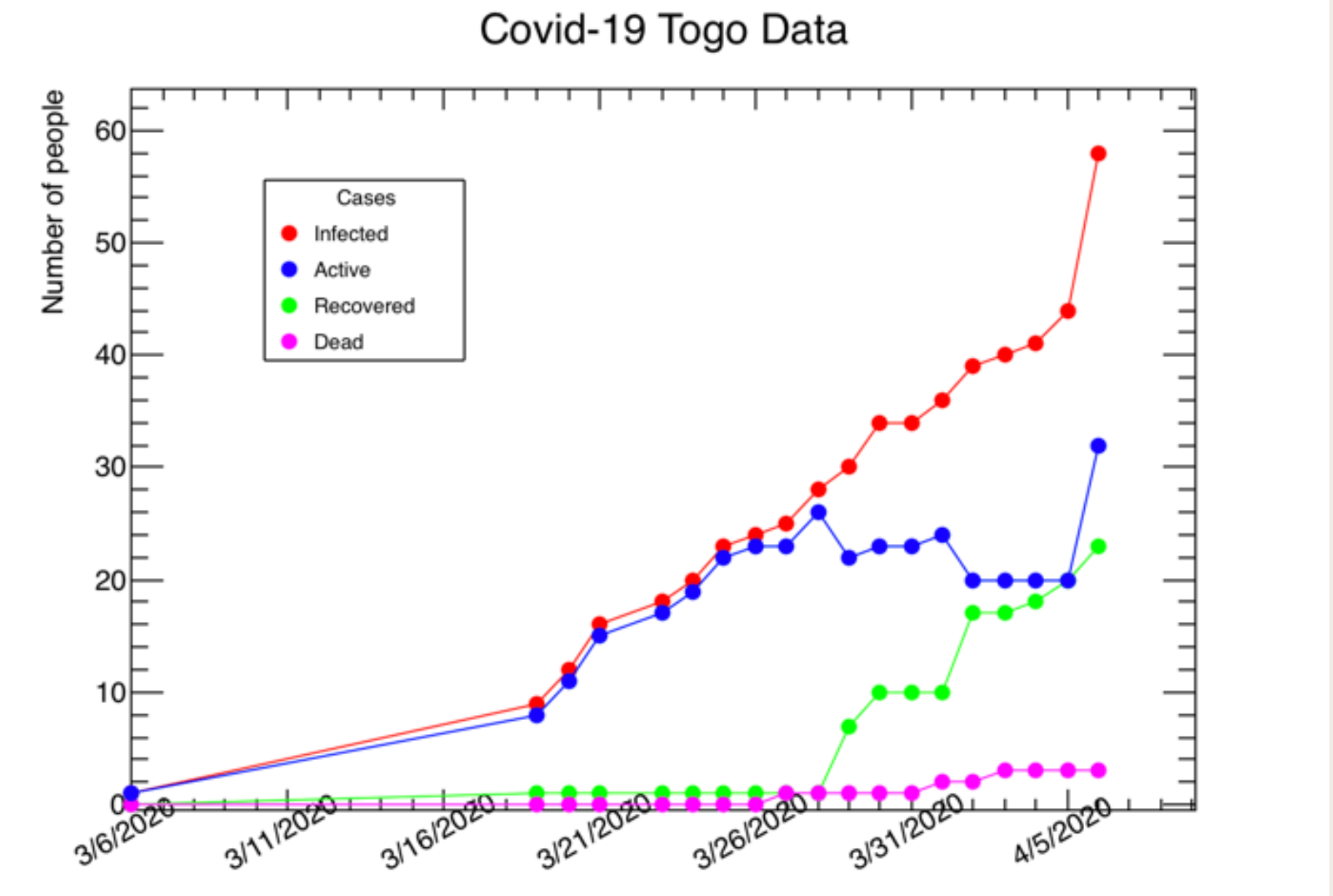
- When we finish with the SIR model, we will introduce a more complicated model
  - *Do something simple first; understand it; compare it to data; understand the discrepancies*
  - *The SIR may be easily extended by adding vital dynamics – birth rate and death rate in the given population*
  - *Then move to a more sophisticated model*

# Exercise

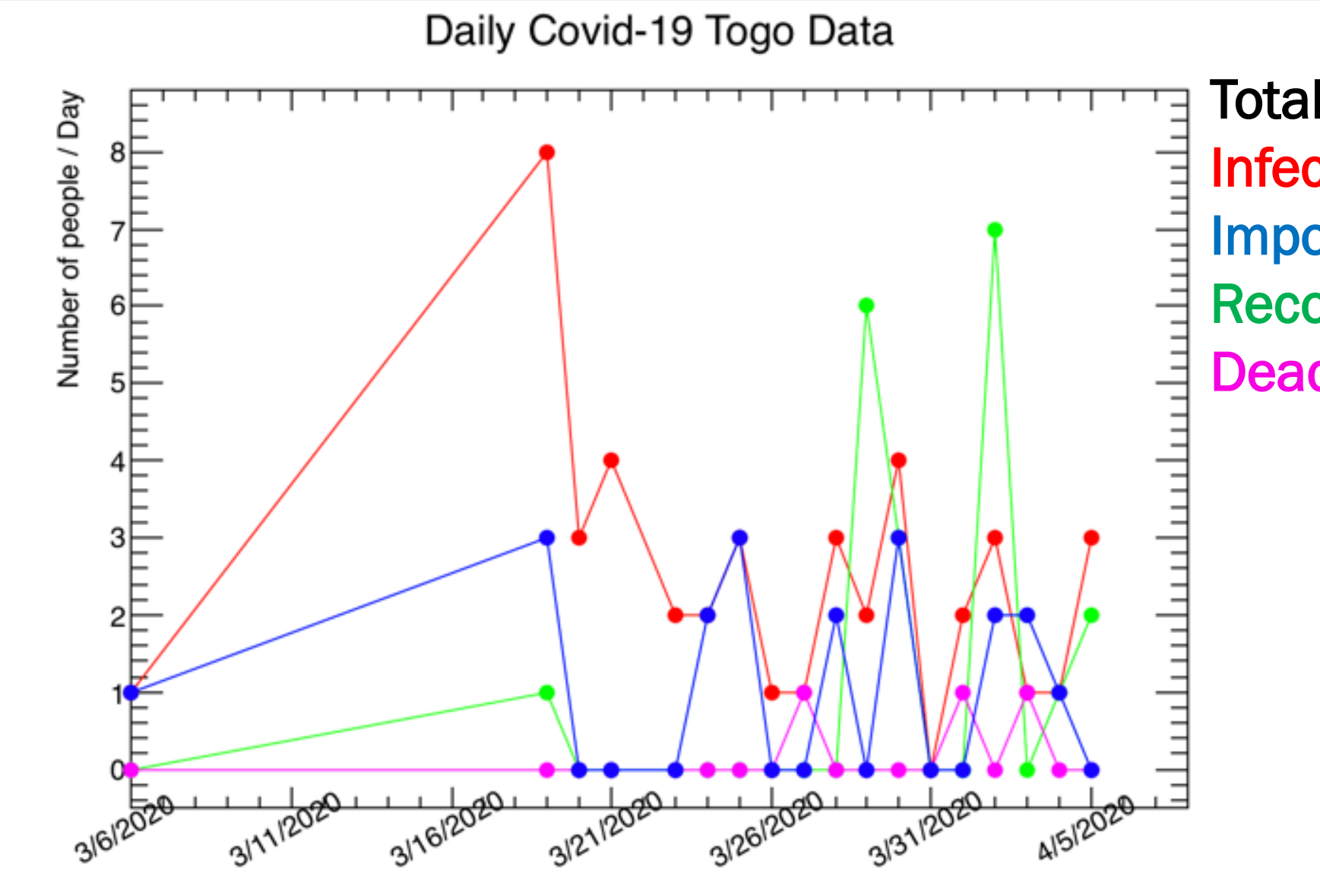
- **Start with the SIR model**
  - *Implement the solution of the equations (analytical or numerical) in page 9 using any software tools that you are familiar with*
  - *Reproduce the plot on page 10*
- **Collect the Covid-19 data from your country**
- **Plot the data**
  - *Number of infectious as a function time in days*
  - *Number of (Recovered + Dead) on the same graph*
- **Adjust the model parameters (fitting),  $b$ ,  $k$  and  $i(0)$  to fit the data**
  - *Plot the fitted functions on the same graph*
  - *Discuss if the resulting parameters are realistic*
  - *Make a projection / prediction about the pandemic will end in your country*
- **Uncertainties in the data**
  - *Discuss any uncertainties in the data*
- **Next: More sophisticated models**

**Next Meeting on Thursday at 14:30 GMT**

# Togo (1) - Cumulative Cases



# Togo (2) - daily data



**Totals:**  
Infected (44)  
Imported (18)  
Recovered (20)  
Dead (3)