ASP EXERCISE TO MODEL THE EVOLUTION OF COVID-19 IN AFRICA

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The simplest model – SIR

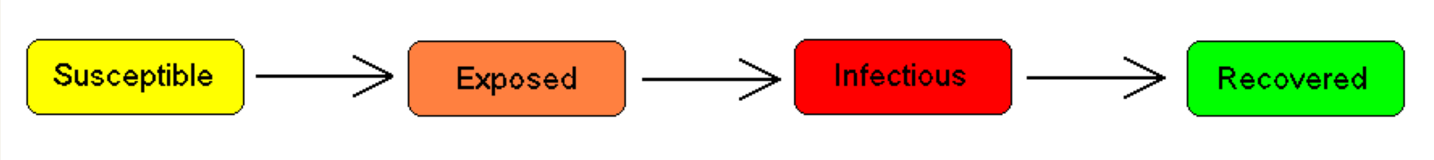
t	is time, to be measured in days
S = S(t)	is the number of *susceptible* individuals,
I = I(t)	is the number of *infected* individuals, and
R = R(t)	is the number of *recovered* individuals.
N	is the total population

s(t) = S(t)/N,	the susceptible fraction of the population,
i(t) = I(t)/N,	the infected fraction of the population, and
r(t) = R(t)/N,	the recovered fraction of the population.

At each time **t**, **s(t) + i(t) + r(t) = 1**

More complicated models (1)

- **The SIS Model with vital dynamics and constant population**
  - *Introduce birth and death rates*
- **The MSIR model**
  - *For some infections such as measles, consider passive immunity (for a few months after births, babies are immune to the disease)*
- **Carrier State**
  - *Some people who have had an infectious disease such as tuberculosis never completely recover and continue to carry the infection, whilst not suffering from the disease themselves.*
- **The SEIR model**
  - *For many important infections there is a significant incubation period during which individuals have been infected but are not yet infectious themselves. During this period the individual is in compartment E (for exposed).*



More Complicated Models (2)

- **The SEIS model**
  - *The SEIS model is like the SEIR model (above) except that no immunity is acquired at the end*
- **The MSEIR model**
  - *For the case of a disease, with the factors of passive immunity, and a latency period there is the MSEIR model.*
- **The MSEIRS model**
  - *An MSEIRS model is similar to the MSEIR, but the immunity in the R class would be temporary, so that individuals would regain their susceptibility when the temporary immunity ended.*
- **Variable contact rates and pluriannual or chaotic epidemics**
  - *It is well known that the probability of getting a disease is not constant in time. Some diseases are seasonal, such as the common cold viruses, which are more prevalent during winter. With childhood diseases, such as measles, mumps, and rubella, there is a strong correlation with the school calendar, so that during the school holidays the probability of getting such a disease dramatically decreases.*

Which model is best suited for Covid-19?

Start with the SIR Model - Some Assumptions (I)

- No addition to the susceptible group
  - *The population is quarantine (a realistic assumption)*
  - *No new births (not so realistic)*
  - *The only way a person leaves the susceptible group is by becoming infected*
- ds(t) / dt = f (s(t), i(t), b)
  - ***b*** *= quantifies the amount of contacts between susceptible and infected people*
  - *Each individual has a fixed number* ***b*** *of contacts per day to spread the disease*
  - *Not all these contacts are with susceptible people*
- The population is homogeneously mixed
  - *The fraction of the contacts with susceptible people is* ***s(t)***
  - *On average, each infected person generates* ***b s(t)*** *new infected persons*
- Assume large population with small infected persons
  - *Probability that a given susceptible persons encounters more than 1 infected person a day in very low*

Some Assumptions (II)

- A fraction k of the infected population will recover on a any given day
- By “infected”, we mean “infectious’, i.e. capable of spreading the disease to a susceptible person
- A recovered person may still be miserable, and might die later of something else
  - *Recovered = recovered + dead*

The SIR …

- The rate of change in the susceptible population
  1) $\frac{dS}{dt} = -bs(t)I(t)$
  2) $\frac{ds}{dt} = -bs(t)i(t)$
- The rate of change in the recovered population
  3) $\frac{dr}{dt} = ki(t)$
- The rate of change in the infected population
  4) $\frac{ds}{dt} + \frac{dr}{dt} + \frac{di}{dt} = 0$

  5) $\frac{di}{dt} = bs(t)i(t) - ki(t)$
- Finally, complete the equations by setting the initial conditions. These may / will differ for each country
  - *S(0) = number of people at time t=0* → *s(0) = 1*
  - *I(0) = number of infected people a t=0* → *i(0) = ?*
  - *R(0) = number of recovered people at t=0;* → *0.*
- The parameters k and b might also be difference for different countries

The SIR in a specific example

- Assume that the population is 8 million. Further, hardly anyone was immune at the beginning of the epidemic, so almost everyone was susceptible. We will assume that there was a trace level of infection in the population, say, 10 people. Thus, our initial values for the population variables are:
  - $S(0) = 8.0\ e^{6}$
  - $I(0) = 10$
  - $R(0) = 0$
  - $s(0) = 1$
  - $i(0) = 1.25\ e^{-6}$
  - $r(0) = 0$

(Note: The sum of our starting populations is not exactly N, nor is the sum of our fractions exactly 1. The trace level of infection is so small that this won't make any difference.)

The complete set of equations specific example

$$\frac{ds}{dt} = -b\, s(t)\, i(t), \qquad s(0) = 1,$$

$$\frac{di}{dt} = b\, s(t)\, i(t) - k\, i(t), \qquad i(0) = 1.25 \times 10^{-6},$$

$$\frac{dr}{dt} = k\, i(t), \qquad r(0) = 0.$$

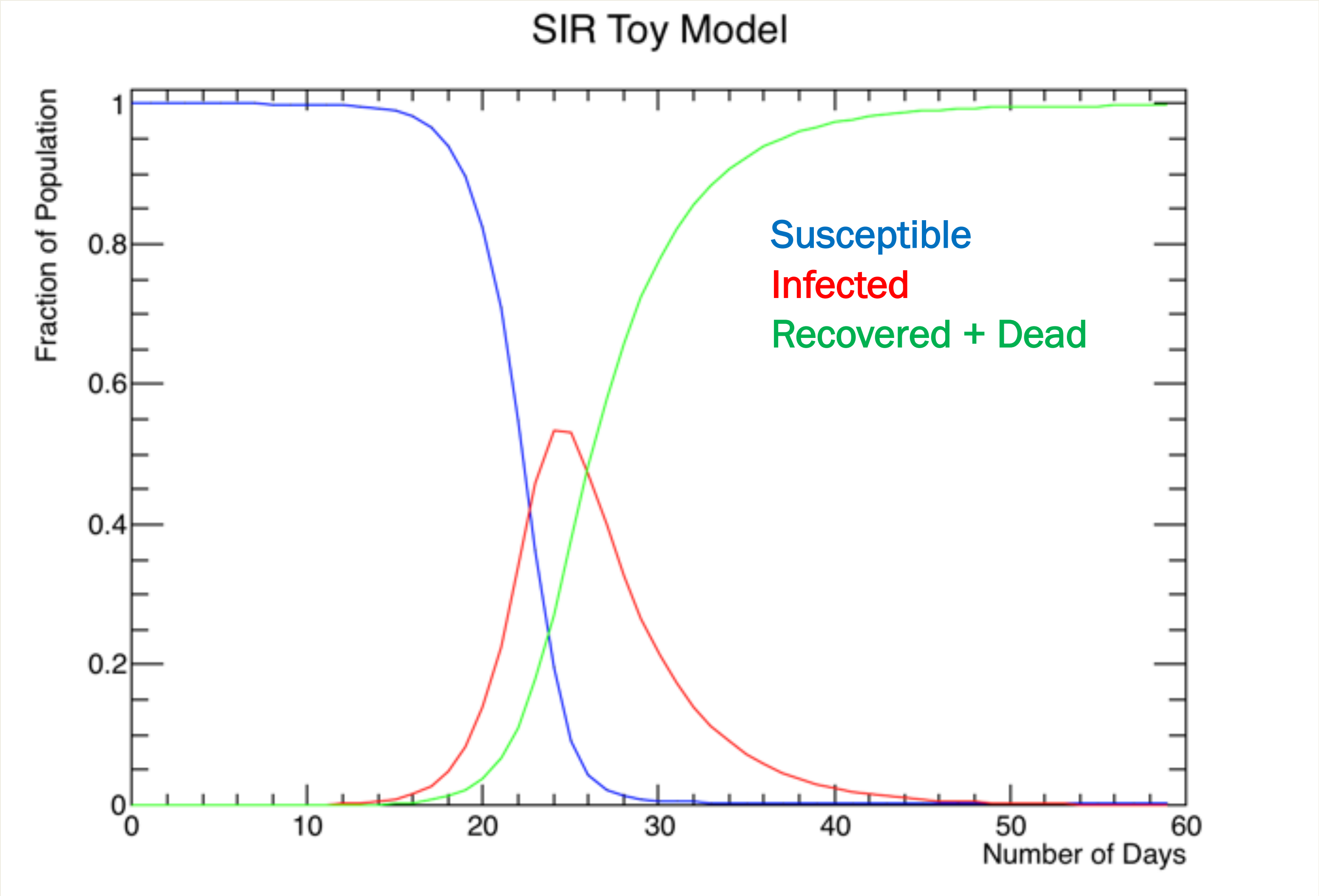
We do not know what b and k are but they can be estimated / guessed for this particular example

- Assume each infected would make a possibly infectious contact every day, then **b** would be **1.0**
- Assume the average period of infectiousness at 2 days, so that would suggest **k = 1/2**

Exercise:

solve this set of equations and produce the plots of s(t), i(t) and r(t) as a function of t (days) on the same graphs (but with different colors)

My numerical solutions using Euler’s method—my code is available in c++



To a more complicated model

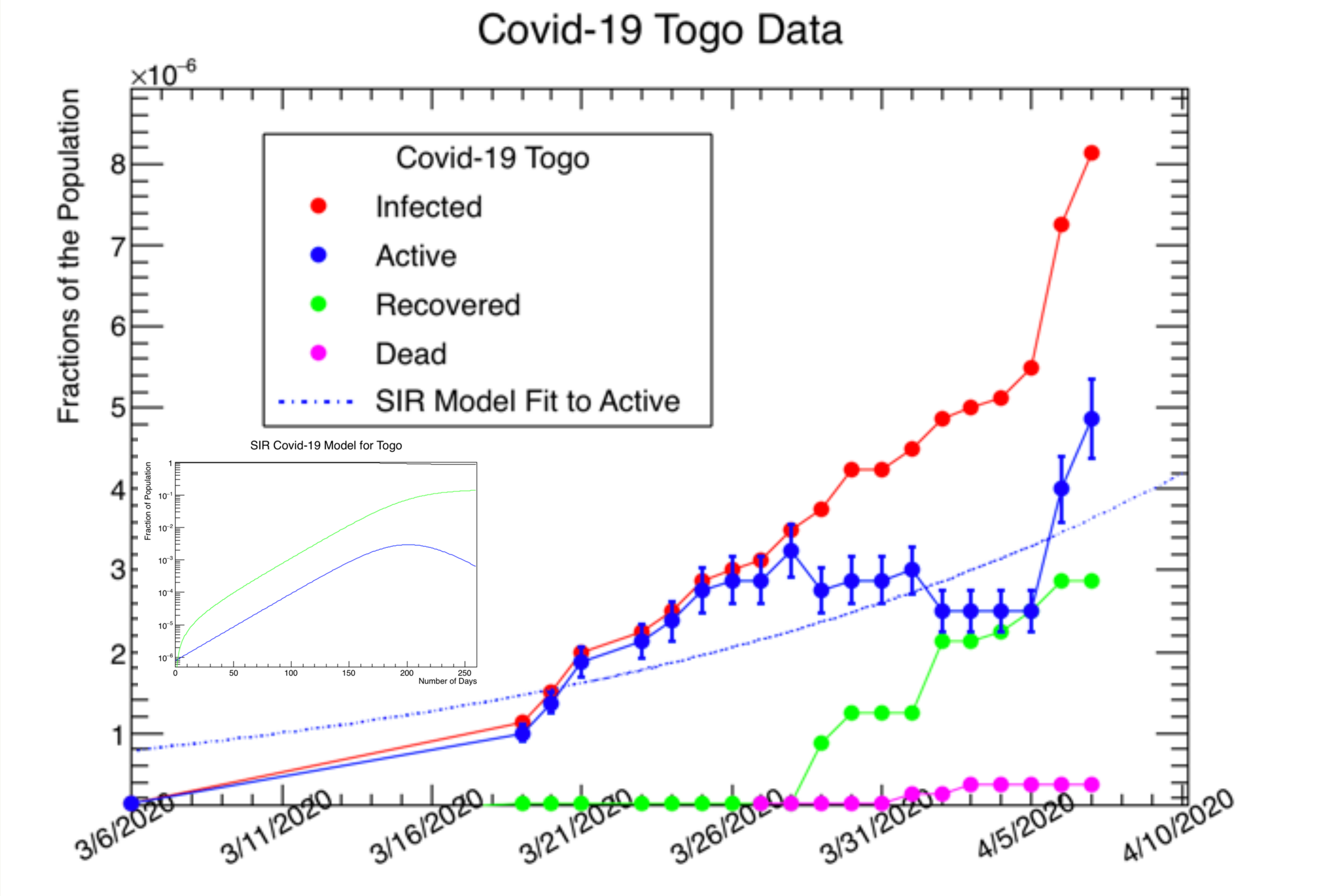
- When we finish with the SIR model, we will introduce a more complicated model
  - *Do something simple first; understand it; compare it to data; understand the discrepancies*
  - *The SIR may be easily extended by adding vital dynamics — birth rate and death rate in the given population*
  - *Then move to a more sophistical model*

Exercise

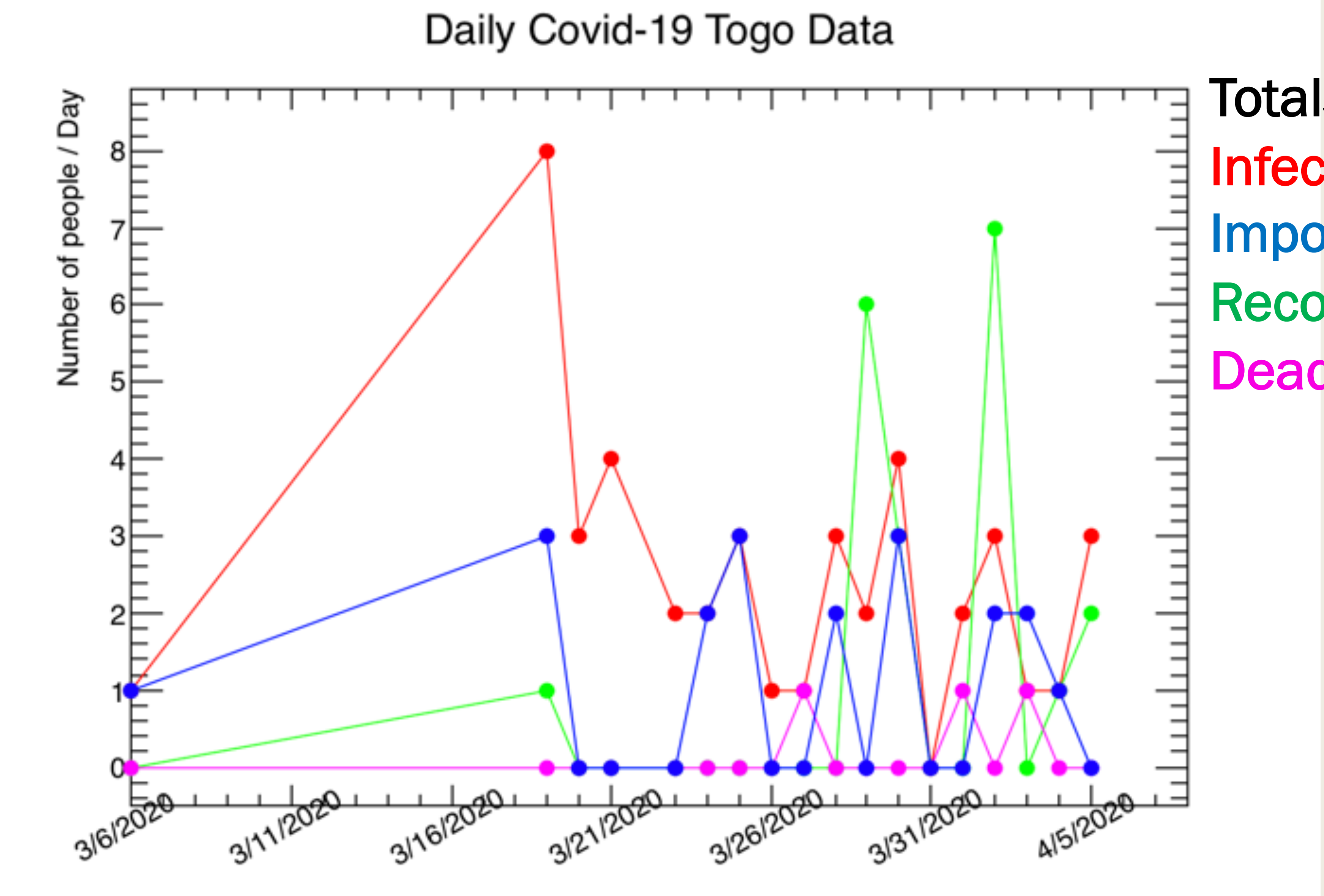
- **Start with the SIR model**
  - *Implement the solution of the equations (analytical or numerical) in page 9 using any software tools that you are familiar with*
  - *Reproduce the plot on page 10*
- **Collect the Covid-19 data from your country**
- **Plot the data**
  - *Number of infectious as a function time in days*
  - *Number of (Recovered + Dead) on the same graph*
- **Adjust the model parameters (fitting), b, k and i(0) to fit the data**
  - *Plot the fitted functions on the same graph*
  - *Discuss if the resulting parameters are realistic*
  - *Make a projection / prediction about the pandemic will end in your country*
- **Uncertainties in the data**
  - *Discuss any uncertainties in the data*
- **Next: More sophisticated models**

Next Meeting on Thursday April 16 at 14:30 GMT

Togo (1) – Cumulative Cases



Togo (2) – daily data



Totals:
Infected (44)
Imported (18)
Recovered (20)
Dead (3)