Color Decompositions from Unitarity

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based on 1908.02695 with Ben Page

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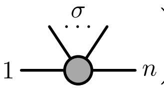
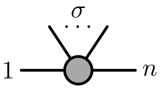
Aim of this talk

"Proper" color decompositions of the form

$$\mathcal{A}(1,X,n) = \sum_{\sigma\in\mathcal{B}_X^{1,n}} C(1,\sigma,n)A(1,\sigma,n)$$

$$= \sum_{\sigma\in\mathcal{B}_X^{1,n}} C\Big($$


$$\Big)\, A\Big($$


$$\Big)$$

"stretched" by 2 arb. chosen labels
$1 \longleftarrow$ and $\longrightarrow n$

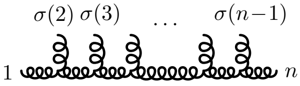
$\mathcal{B}_X^{1,n}$ — basis of $\leq (n-2)!$ permutations
indep. under KK relations Kleiss, Kuijf '88

$$A(1,\alpha,n,\beta) = (-1)^{|\beta|} \sum_{\sigma\in\alpha\sqcup\!\sqcup\beta^{\mathsf{T}}} A(1,\sigma,n)$$

DDM decomposition

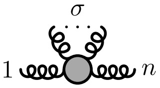
Del Duca, Dixon, Maltoni '99

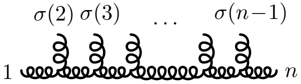
$$\mathcal{A}_n = \sum_{\sigma \in S_{n-2}} \tilde{f}^{\,a_1 a_{\sigma(2)} b_1} \tilde{f}^{\,b_1 a_{\sigma(3)} b_2} \dots \tilde{f}^{\,b_{n-4} a_{\sigma(n-2)} b_{n-3}} \tilde{f}^{\,b_{n-3} a_{\sigma(n-1)} a_n} \times A(1, \sigma(2), \dots, \sigma(n-1), n)$$

$$= \sum_{\sigma \in S_{n-2}} C\Bigg($$ 

$$\Bigg) A(1, \sigma(2), \dots, \sigma(n-1), n)$$

I.e. for pure gluons

$$C\Bigg($$ 

$$\Bigg) = C\Bigg($$ 

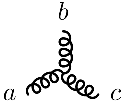
$$\Bigg)$$ non-trivial!

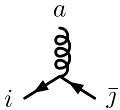
C.f. standard SU(N) trace decomposition

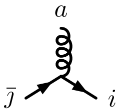
$$\mathcal{A}_n = \sum_{\sigma \in S_{n-1}} \left\{ \mathrm{Tr}[T^{a_1} T^{a_{\sigma(2)}} \dots T^{a_{\sigma(n)}}] + (-1)^n \text{c.c.} \right\} A(1, \sigma(2), \dots, \sigma(n))$$

Color Feynman rules

$C(\dots)$ replaces exposed vertices by

$\tilde{f}^{abc} =$ 

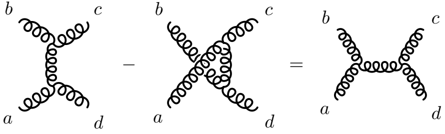
$,\qquad T^a_{i\bar{\jmath}} =$ 

$,\qquad T^a_{\bar{\jmath}i} =$ 

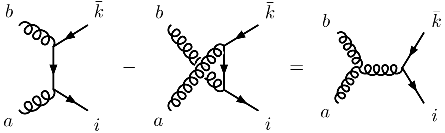
$= -T^a_{i\bar{\jmath}}$

Basic color algebra:

$$\tilde{f}^{dae}\tilde{f}^{ebc} - \tilde{f}^{dbe}\tilde{f}^{eac} = \tilde{f}^{abe}\tilde{f}^{dec}$$



$$T^a_{i\bar{\jmath}}T^b_{j\bar{k}} - T^b_{i\bar{\jmath}}T^a_{j\bar{k}} = \tilde{f}^{abe}\,T^e_{i\bar{k}}$$



SU(N) : $\quad T^a_{i\bar{\jmath}}T^a_{k\bar{l}} = \delta_{i\bar{l}}\delta_{k\bar{\jmath}} - \frac{1}{N_c}\delta_{i\bar{\jmath}}\delta_{k\bar{l}}$ — not used here directly

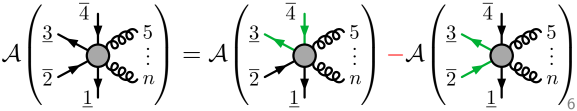
underlie color ordering for $A(\dots)$

Outline

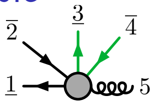
1. Preliminaries
2. Color decompositions
   with flavored matter
3. Loops at full color
4. Summary & outlook

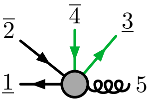
Color Decompositions with Flavored Matter*

*NB! Arbitrary matter but in distinct-flavor pairs, fermionic signs separate; e.g. 4 quarks at any loop:

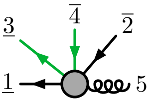


5-pt qg-stretch example





$$\mathcal{A}_{5,2} = C(\underline{1},\overline{2},\underline{3},\overline{4},5)A(\underline{1},\overline{2},\underline{3},\overline{4},5) + C(\underline{1},\overline{2},\overline{4},\underline{3},5)A(\underline{1},\overline{2},\overline{4},\underline{3},5)$$
$$+ C(\underline{1},\underline{3},\overline{4},\overline{2},5)A(\underline{1},\underline{3},\overline{4},\overline{2},5)$$



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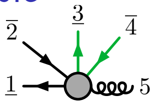

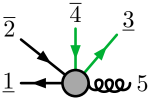

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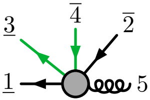
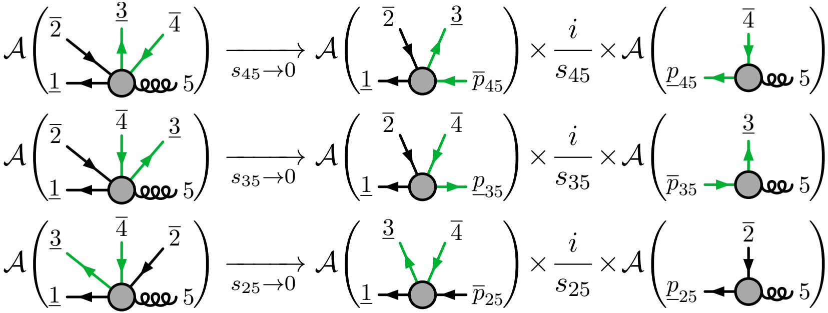
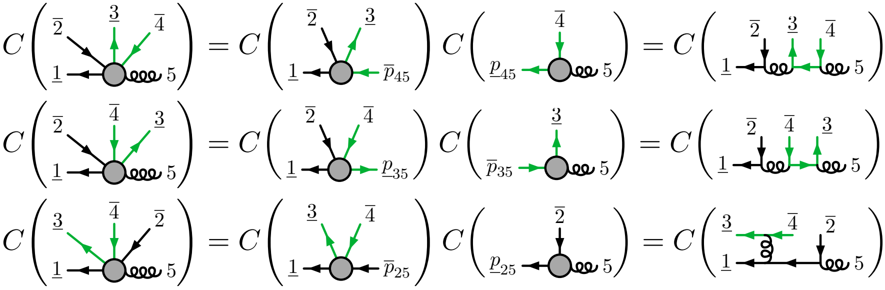


Factorization limits fix basis above:



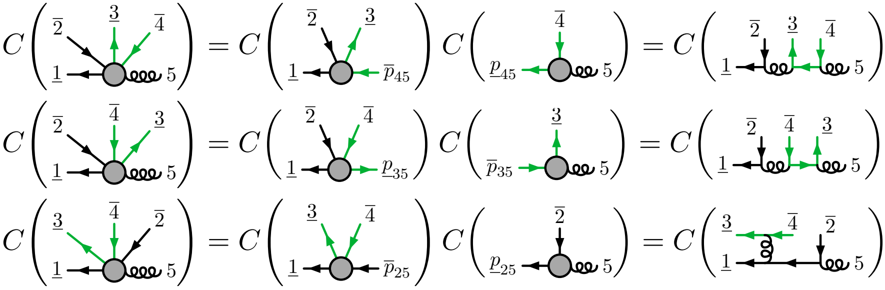
5-pt qg-stretch example

Factorization limits fix color factors:

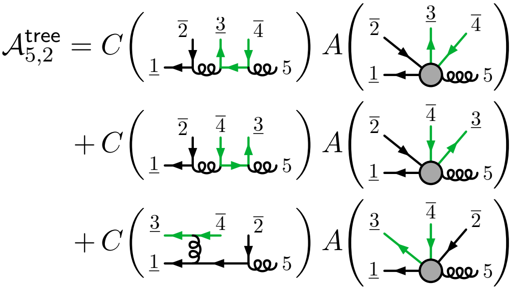


5-pt qg-stretch example

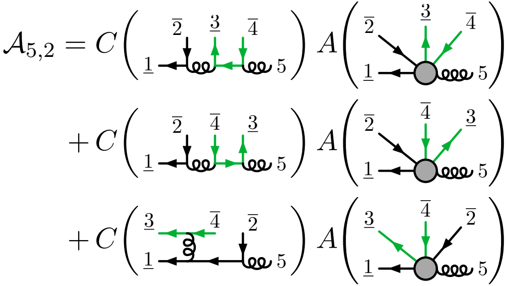
Factorization limits fix color factors:



$\Rightarrow$ Decomposition:



Observations from 5-pt example



- Presence of fact. channels in $\mathcal{A}_{n,k}$ constrain basis of $A(\dots)$
- Factorization allows color recursion if lower-pt known
- Comb-like structures unless stretch by quarks of same flavor
- Luckily, $q\bar{q}$ stretch most studied (apart from pure gluons)

Like-flavor $q\bar{q}$ stretch

Basis: Melia '13

Consider $A(\underline{1},\overline{2},\sigma) = A(\overline{2},\sigma,\underline{1})$: stretch by $\overline{2} \longleftarrow$ and $\longrightarrow \underline{1}$

$$\mathcal{B}^{2,1}_{n,k} = \Big\{\sigma \in [\text{bracket structures}]_{k-1} \times [\text{gluon insertions}]_{n-2k}\Big\}$$

- e.g. pure-quark permutation $A(\overset{[}{\overline{2}},\overset{\{}{\underline{3}},\overset{\{}{\underline{5}},\overset{\}}{\overline{6}},\overset{\{}{\underline{7}},\overset{\}}{\overline{8}},\overset{\}}{\overline{4}},\overset{]}{\underline{1}})$
- full Melia basis for $n=6$, $k=3$:
  $A(\overset{[}{\overline{2}},\overset{\{}{\underline{3}},\overset{\}}{\overline{4}},\overset{\{}{\underline{5}},\overset{\}}{\overline{6}},\overset{]}{\underline{1}})$, $A(\overset{[}{\overline{2}},\overset{\{}{\underline{5}},\overset{\}}{\overline{6}},\overset{\{}{\underline{3}},\overset{\}}{\overline{4}},\overset{]}{\underline{1}})$, $A(\overset{[}{\overline{2}},\overset{\{}{\underline{3}},\overset{\{}{\underline{5}},\overset{\}}{\overline{6}},\overset{\}}{\overline{4}},\overset{]}{\underline{1}})$, $A(\overset{[}{\overline{2}},\overset{\{}{\underline{5}},\overset{\{}{\underline{3}},\overset{\}}{\overline{4}},\overset{\}}{\overline{6}},\overset{]}{\underline{1}})$
- quark-bracket orientations chosen but fixed
- gluon are allowed everywhere (except between $\underline{1}$ and $\overline{2}$)

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Decomposition: Johansson, AO '15
proven by Melia '15

$$\mathcal{A}_{n,k} = \sum_{\sigma \in \mathcal{B}^{2,1}_{n,k}} C(\underline{1},\overline{2},\sigma)\, A(\underline{1},\overline{2},\sigma)$$

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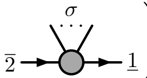
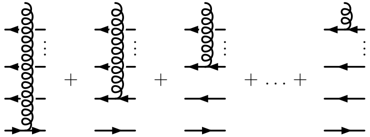
$C\Big($  $\Big)$

$$= (-1)^{k-1}\,[2|\,\sigma\,|1]\Bigg|\begin{array}{l} g \;\to\; \Xi^{a_g}_{l(g)} \\ q \;\to\; \{q|\,T^b \otimes \Xi^b_{l(q)-1} \\ \bar{q} \;\to\; |q\} \end{array}$$

$$\{q|T^a|\bar{q}\} = T^a_{i_q\bar{\imath}_{\bar{q}}}, \qquad [2|T^a|1] = T^a_{\bar{\imath}_2 i_1} = -T^a_{i_1\bar{\imath}_2}$$

$$\Xi^a_l = \sum_{r=1}^{l} \underbrace{\mathbb{I}\otimes\cdots\otimes\mathbb{I}\otimes\overbrace{T^a_r\otimes\mathbb{I}\otimes\cdots\otimes\mathbb{I}\otimes\overline{\mathbb{I}}}^{r}}_{l} =$$



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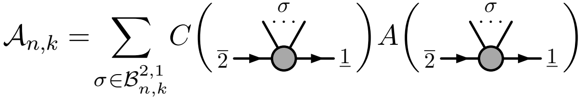
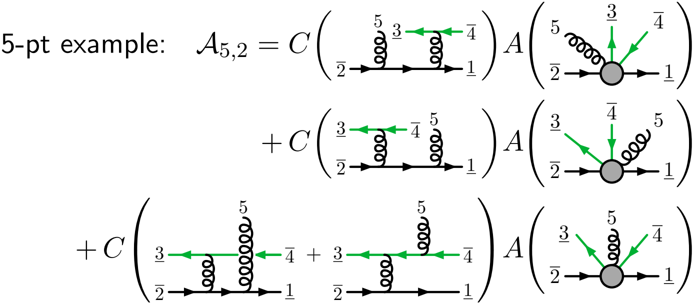
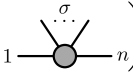


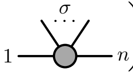


Towards arbitrary stretches

So far: gg stretch for pure YM [DDM]
$q\bar{q}$ stretch for QCD [Melia+JO]

Want generic stretch:

$\mathcal{A}(1, X, n) = \sum_{\sigma \in \mathcal{B}^{1,n}_X} C\Big($ 

$\Big)\, A\Big($ 

$\Big)$

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$$\mathcal{A}(1,X,n) = \sum_{\sigma\in\mathcal{B}_X^{1,n}} C\left(1 \overset{\sigma}{\cdots} n \right) A\left(1 \overset{\sigma}{\cdots} n \right)$$

$$\underset{s_{1P}=0}{\text{Res}}\, \mathcal{A}\left(1 \overset{P\cup R}{\cdots} n \right) = \mathcal{A}\left(1 \overset{P}{\cdots} \bar{p} \right) \times \mathcal{A}\left(p \overset{R}{\cdots} n \right)$$

$$= \sum_{\pi\in\mathcal{B}_P^{1,\bar{p}}} \sum_{\rho\in\mathcal{B}_R^{p,n}} C\left(1 \overset{\pi}{\cdots} \bar{p} \right) C\left(p \overset{\rho}{\cdots} n \right) A\left(1 \overset{\pi}{\cdots} \bar{p} \right) A\left(p \overset{\rho}{\cdots} n \right)$$

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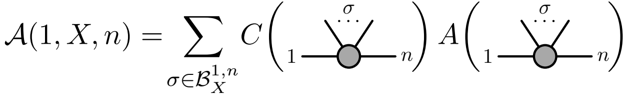
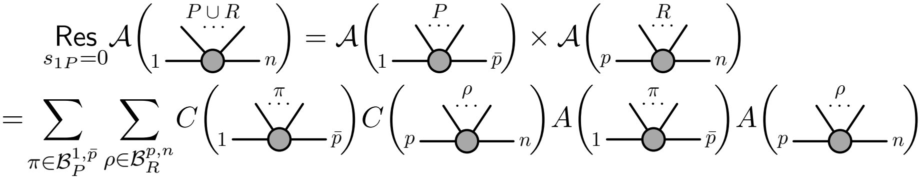
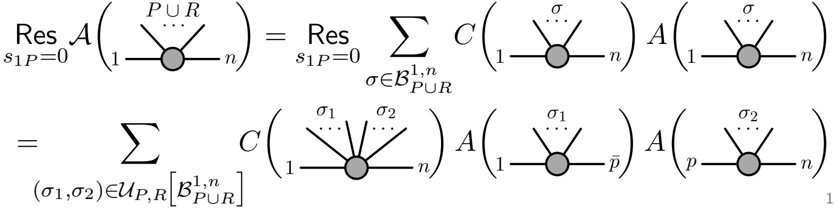
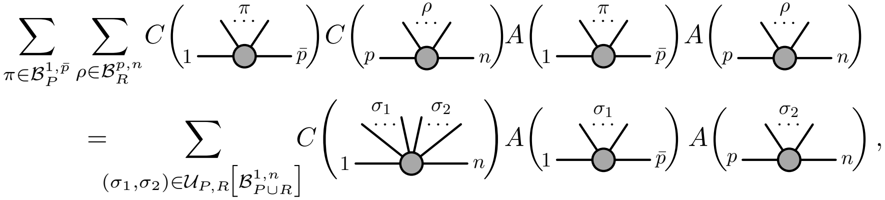




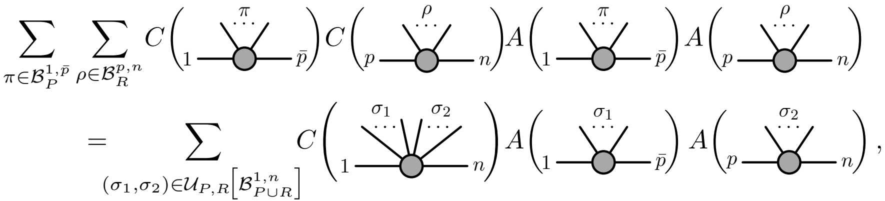


Towards arbitrary stretches



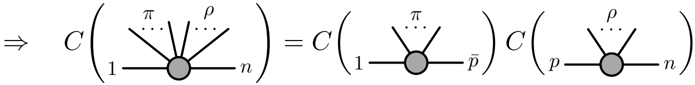
$$\mathcal{U}_{P,R}\left[\mathcal{B}^{1,n}_{P\cup R}\right] = \left\{ (\pi,\rho) \in \mathrm{S}_P \times \mathrm{S}_R \;\middle|\; \pi \oplus \rho \in \mathcal{B}^{1,n}_{P\cup R},\ \operatorname*{Res}_{s_{1P}=0} A(1,\pi,\rho,n) \neq 0 \right\}$$

Towards arbitrary stretches

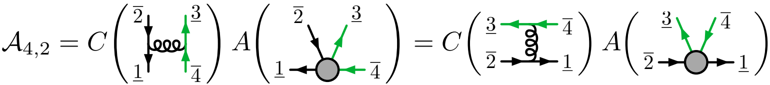


$$\mathcal{U}_{P,R}\big[\mathcal{B}^{1,n}_{P\cup R}\big] = \Big\{(\pi,\rho) \in \mathrm{S}_P \times \mathrm{S}_R \;\Big|\; \pi \oplus \rho \in \mathcal{B}^{1,n}_{P\cup R},\; \underset{s_{1P}=0}{\mathrm{Res}}\, A(1,\pi,\rho,n) \neq 0\Big\}$$

Provided "co-unitarity" $\mathcal{U}_{P,R}\big[\mathcal{B}^{1,n}_{P\cup R}\big] = \mathcal{B}^{1,\bar{p}}_{P} \times \mathcal{B}^{p,n}_{R}$



Minimal example:



OK for qQ failure for $q\bar{q}$: no fact. channel

Arbitary stretches

Observation: length of all bases' must be Melia's $\boxed{\dfrac{(n-2)!}{k!}}$

Skip to result: allow flips for unclosed brackets

$$\mathcal{Q}_F = \bigcup_{f \in F} \bigcup_{E \in \mathbb{P}(F \setminus f)} \left\{ \overset{\{}{(f)} \oplus \pi \oplus \overset{\}}{(\bar{f})} \oplus \rho \;\middle|\; (\pi, \rho) \in \mathcal{Q}_E \times \mathcal{Q}_{(F \setminus f) \setminus E} \right\}$$

$$\overline{\mathcal{Q}}_F = \bigcup_{f \in F} \bigcup_{E \in \mathbb{P}(F \setminus f)} \left\{ \overset{\{}{(f)} \oplus \pi \oplus \overset{\}}{(\bar{f})} \oplus \rho \;\middle|\; (\pi, \rho) \in \mathcal{Q}_E \times \overline{\mathcal{Q}}_{(F \setminus f) \setminus E} \right\}$$
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$$\mathcal{G}_{n-2k} = \left\{ \sigma \in \mathrm{S}_G \;\middle|\; G = \{g_{2k+1}, \ldots, g_n\} \right\}$$

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$$\cup \Big\{ \overset{[}{(\bar{f})} \oplus \pi \oplus \overset{]}{(f)} \oplus \rho \;\Big|\; (\pi,\rho) \in \mathcal{Q}_E \times \overline{\mathcal{Q}}_{(F\backslash f)\backslash E} \Big\}$$

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$$q\bar{q}: \quad \mathcal{B}^{2,1}_{n,k} = \big\{ A(\overline{2}, \sigma, \underline{1}) \;\big|\; \sigma \in \mathcal{Q}_{2(k-1)} \sqcup \mathcal{G}_{n-2k} \big\}$$

Melia '13

$$qQ: \quad \mathcal{B}^{1,4}_{n,k} = \big\{ A(\underline{1}, \sigma, \overline{4}) \;\big|\; (\underline{1}) \oplus \sigma \oplus (\overline{4}) \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k} \big\}$$

$$qg: \quad \mathcal{B}^{1,n}_{n,k} = \big\{ A(\underline{1}, \sigma, n) \;\big|\; (\underline{1}) \oplus \sigma \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k} \big\}$$

$$gg: \quad \mathcal{B}^{n-1,n}_{n,k} = \big\{ A(n-1, \sigma, n) \;\big|\; \sigma \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k-2} \big\}$$

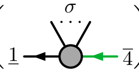
NEW (qQ, qg, gg)

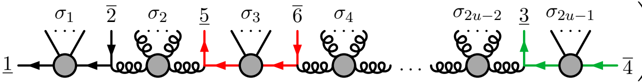
Formal construction and proof of co-unitarity in 1908.02695

Distinct-flavor qQ stretch

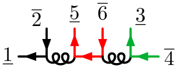
$$\mathcal{B}^{1,4}_{n,k} = \left\{ A(\underline{1},\sigma,\overline{4}) \;\middle|\; (\underline{1})\oplus\sigma\oplus(\overline{4}) \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k} \right\}$$

For perm. $(\underline{1},\sigma,\overline{4}) = \big(\{1,\sigma_1,2\},\sigma_2,\{5,\sigma_3,6\},\sigma_4,\ldots,\sigma_{2u-2},\{3,\sigma_{2u-1},4\}\big) \quad \Rightarrow$

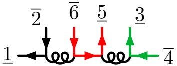
$C\Big($ 

$\Big) = C\Big($ 

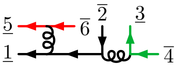
$\Big)$

e.g. $\mathcal{A}_{6,3} = C\Big($ 

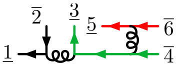
$\Big)$ $\{\ \}\ \{\ \}\ \{\ \}$ $A(\underline{1},\overline{2},\underline{5},\overline{6},\underline{3},\overline{4})$

$+\, C\Big($ 

$\Big)$ $\{\ \}\ [\ ]\ \{\ \}$ $A(\underline{1},\overline{2},\overline{6},\underline{5},\underline{3},\overline{4})$

$+\, C\Big($ 

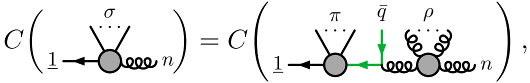
$\Big)$ $\{\ \{\ \}\ \}\ \{\ \}$ $A(\underline{1},\underline{5},\overline{6},\overline{2},\underline{3},\overline{4})$

$+\, C\Big($ 

$\Big)$ $\{\ \}\ \{\ \{\ \}\ \}$ $A(\underline{1},\overline{2},\underline{3},\underline{5},\overline{6},\overline{4})$

Similarly: qg and gg stretches

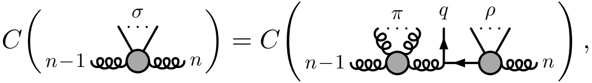
$$\mathcal{B}^{1,n}_{n,k} = \left\{ A(\underline{1},\sigma,n) \;\middle|\; (\underline{1}) \oplus \sigma \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k} \right\}$$



where $\sigma = \big(\pi, \bar{q}\}, \rho\big)$

implicitly used @1-loop in Kälin '17

$$\mathcal{B}^{n-1,n}_{n,k} = \left\{ A(n{-}1,\sigma,n) \;\middle|\; \sigma \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k-2} \right\}$$



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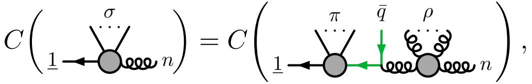
Similarly: qg and gg stretches

$$\mathcal{B}_{n,k}^{1,n} = \left\{ A(\underline{1}, \sigma, n) \;\middle|\; (\underline{1}) \oplus \sigma \in \overline{\mathcal{Q}}_{2k} \sqcup \mathcal{G}_{n-2k} \right\}$$



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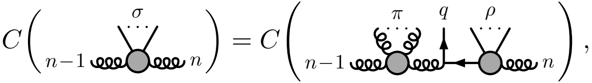


where $\sigma = \big(\pi, \{q, \rho\big)$

NB! Induction via other valid fact. channel possible

Tree-level summary:

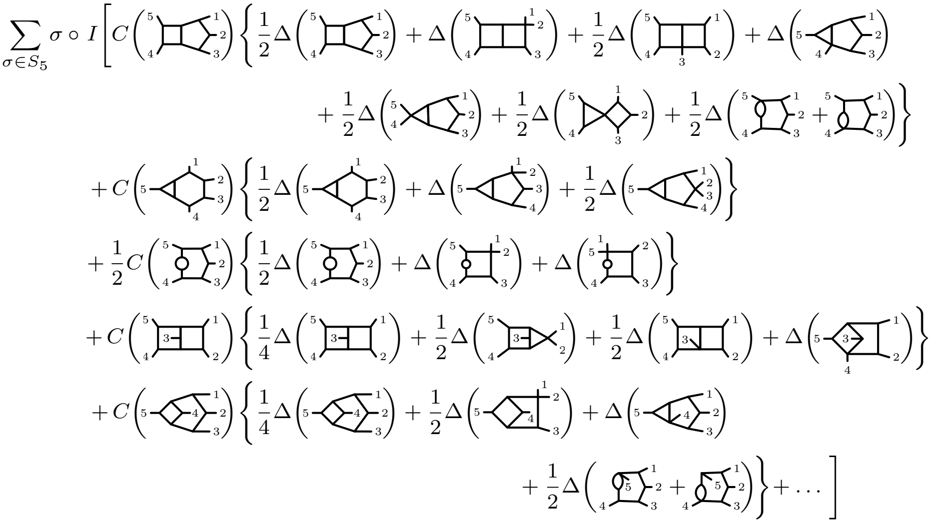
- All ($1 \leftarrow$ stretches $\rightarrow n$) intertwined by mutual factorization
- All decompositions but $q\bar{q}$ implied by factorization dividing 1 and n — for free once amp. bases are chosen co-unitary
- Fortunately, $q\bar{q}$ and pure glue previously known [DDM, Melia + JO]

Loops at Full Color

Full-color 2-loop amplitude in pure YM

Badger, Mogull, AO, O'Connell '15

$$\mathcal{A}_5^{(2)} =$$



Integrated by Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia '19

General color construction

Badger, Mogull, AO, O'Connell '15

Methodology in AO, Page '16

Full color within unitarity/integrand reduction:*

$$\mathcal{A}_n^{(L)} = \sum_{i\,\in\,\text{KK-indep. 1PI graphs}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{C_i\,\Delta_i}{S_i \prod_{l\in i} D_l}$$

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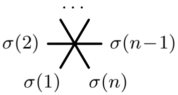
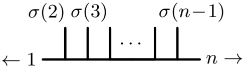
e.g. for pure YM:

num./cut vertices:

$\sum_{\sigma\in S_n/D_n}$ 

$\longrightarrow$

color factors:

$\sum_{\sigma\in S_{n-2}}$ 

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* Bern, Dixon, Dunbar, Kosower '94; Britto, Cachazo, Feng '04; Ossola, Papadopoulos, Pittau '06; Mastrolia, Mirabella, Ossola, Peraro '12; Badger, Frellesvig, Zhang '12; Bourjaily, Herrmann, Trnka '17 [Enrico's talk]

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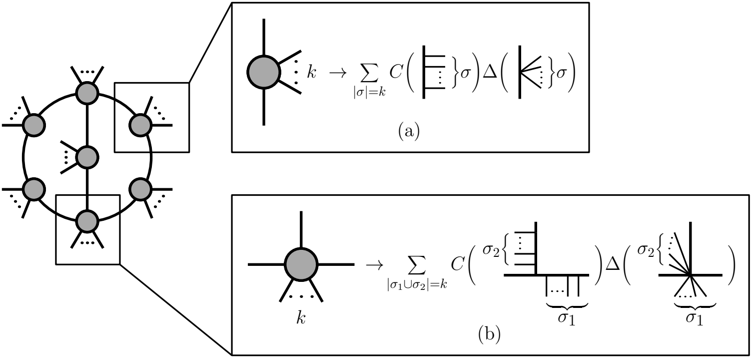
e.g. for pure YM at 2 loops:



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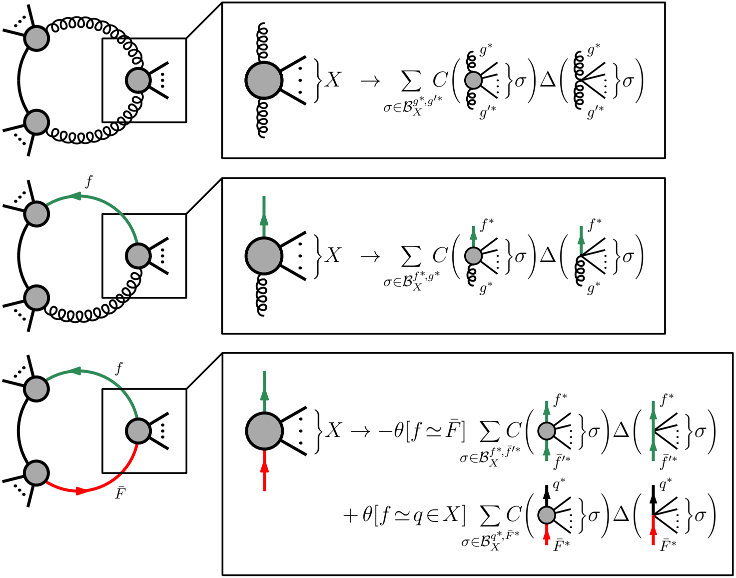
1 loop with matter

3 types of vertices:

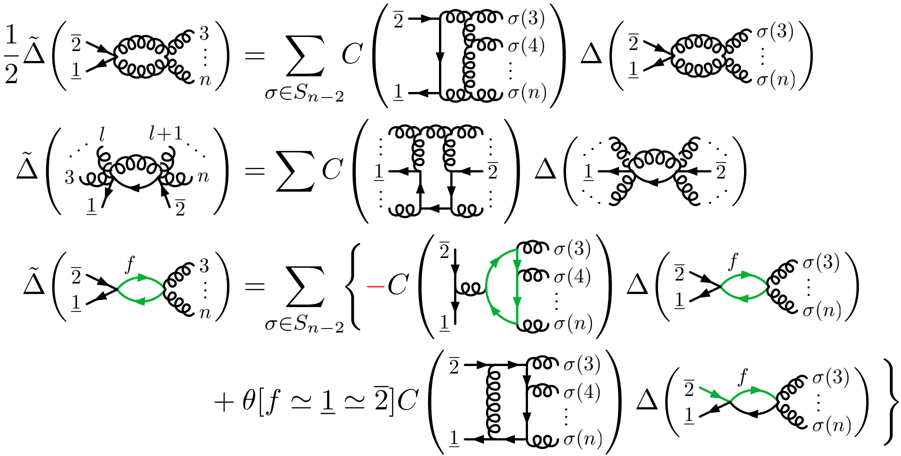


fermionic − may be replaced by + for scalars

n-pt 1-loop example

$\mathcal{A}_{n,1}^{(1)}$ in QCD: 2 quarks & $(n-2)$ gluons;   NB! arrows are fermionic

$\Rightarrow$ bubble color-dressed numerators decompose as



consistent with Kälin '17

Summary & outlook

- Subject: flexible Kleiss-Kuijf-reduced color representations
- New bases and decompositions for qQ, qg and gg stretches
- Previous results reused via factorization
  Del Duca, Dixon, Maltoni '99
  Melia '13
  Johansson, AO '15
- Applicable to graviton-matter amplitudes via color-kinematics
  Plefka, Wormsbecher '18

Summary & outlook

- Subject: flexible Kleiss-Kuijf-reduced color representations
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- Applicable to loops via gen. unitarity via method of

  Badger, Mogull, AO, O'Connell '15
  AO, Page '16
  also used in Bourjaily, Herrmann, Langer, McLeod, Trnka '19
  [Enrico's talk]
- Implemented in numerical unitary framework

  Abreu, Febres Cordero, Ita, Page, Sotnikov '18
  Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov '19
- Orthogonal/complementary to SU(N) trace methods

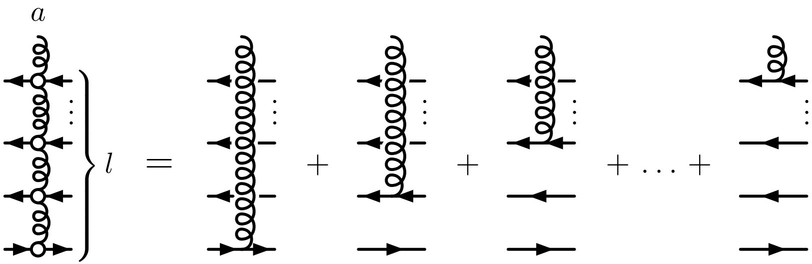
  Bern, Kosower '90
  Bern, Dixon, Kosower '94
  Edison, Naculich '11
  Ita, Ozeren '11
  Reuschle, Weinzierl '13
  Schuster '13
- Hopefully helpful for future calculations beyond leading color!

Thank you, and stay safe!

Backup slides

Tensor in JO color factors

$$\Xi^a_l = \sum_{s=1}^{l} \underbrace{1\otimes\cdots\otimes 1\otimes \overbrace{T^a\otimes 1\otimes\cdots\otimes 1\otimes \bar{1}}^{s}}_{l}$$



$$\left[\Xi^a_l,\, \Xi^b_l\right] = \tilde{f}^{abc}\,\Xi^c_l$$

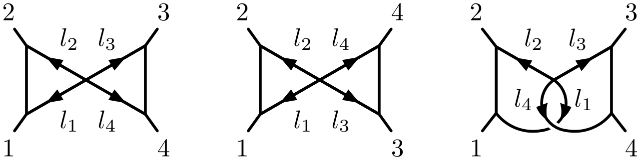
Loop-level KK relations

Badger, Mogull, AO, O'Connell '15
AO, Page '16

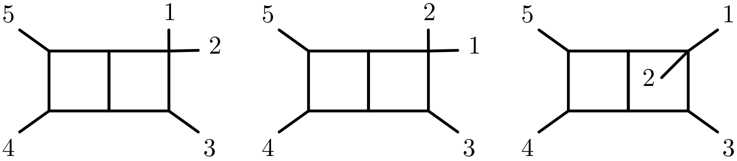
Question: do irred. numerators Δ_i satisfy extra relations?
Answer: **yes**, they inherit KK relations from cuts.

$$A(1,2,3,4) + A(1,2,4,3) + A(1,4,2,3) = 0$$

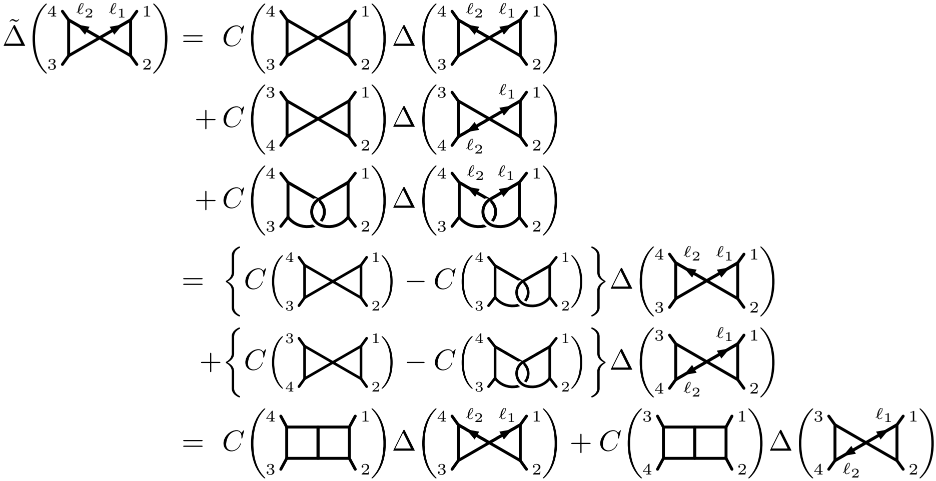
4 points, 2 loops:



5 points, 2 loops:



2-loop example in detail

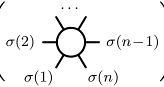


DDM-based 1-loop decomposition

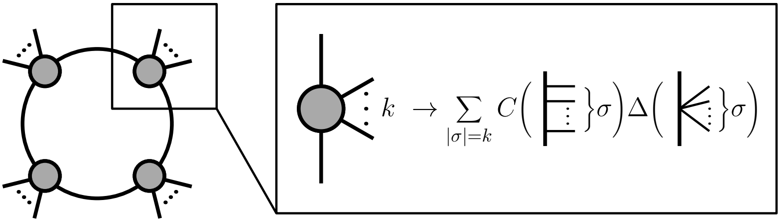
Del Duca, Dixon, Maltoni '99

$$\mathcal{A}_n^{(1)} = \sum_{\sigma \in S_n/D_n} \tilde{f}^{\,b_1 a_{\sigma(1)} b_2} \tilde{f}^{\,b_2 a_{\sigma(2)} b_3} \ldots \tilde{f}^{\,b_n a_{\sigma(n)} b_1} \, A^{(1)}(\sigma(1), \sigma(2), \ldots, \sigma(n))$$

$$= \sum_{\sigma \in S_n/D_n} C\Big($$



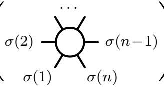
$$\Big) \, A^{(1)}(\sigma(1), \sigma(2), \ldots, \sigma(n))$$

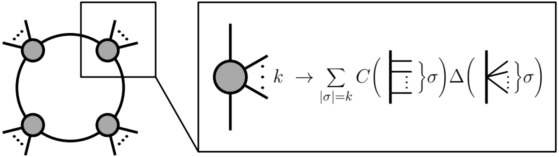


DDM-based 1-loop decomposition

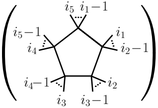
Del Duca, Dixon, Maltoni '99
1-loop KK relations by Bern, Kosower '90

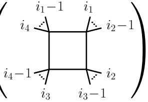
$$\mathcal{A}_n^{(1)} = \sum_{\sigma\in S_n/D_n} C\left(\cdots \right) A^{(1)}(\sigma(1),\sigma(2),\ldots,\sigma(n)),$$



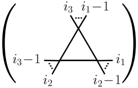


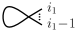
$$A^{(1)}(1,2,\ldots,n) = I\Bigg[\sum_{\substack{1\le i_1<i_2<i_3\\<i_4<i_5\le n}} \Delta\left(\cdots\right) + \sum_{\substack{1\le i_1<i_2\\<i_3<i_4\le n}} \Delta\left(\cdots\right)$$





$$+ \sum_{1\le i_1<i_2<i_3\le n} \Delta\left(\cdots\right) + \sum_{1\le i_1<i_2\le n} \Delta\left(\cdots\right) + \sum_{1\le i_1\le n} \Delta\left(\cdots\right) \Bigg]$$

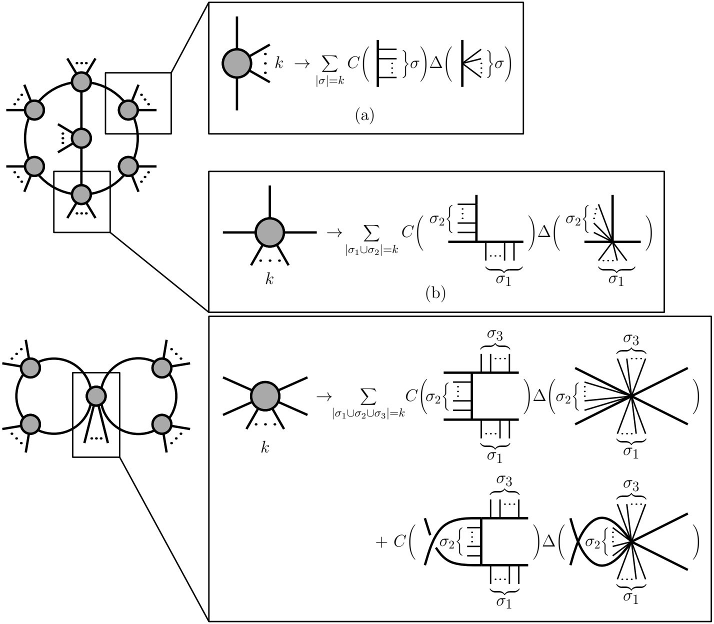




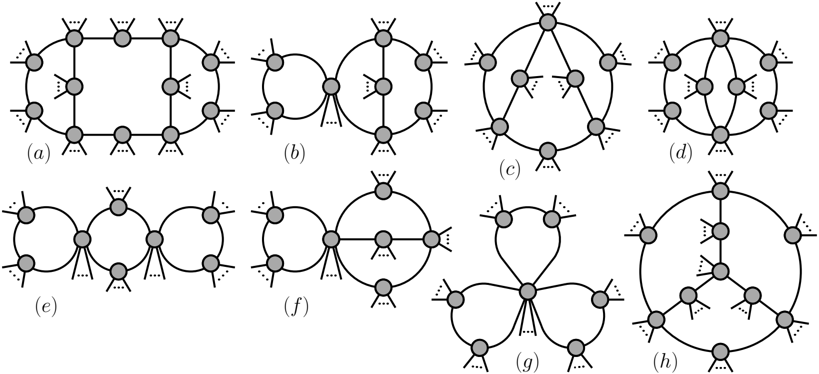
DDM stretches at 2 loops

Badger, Mogull, AO, O'Connell '15

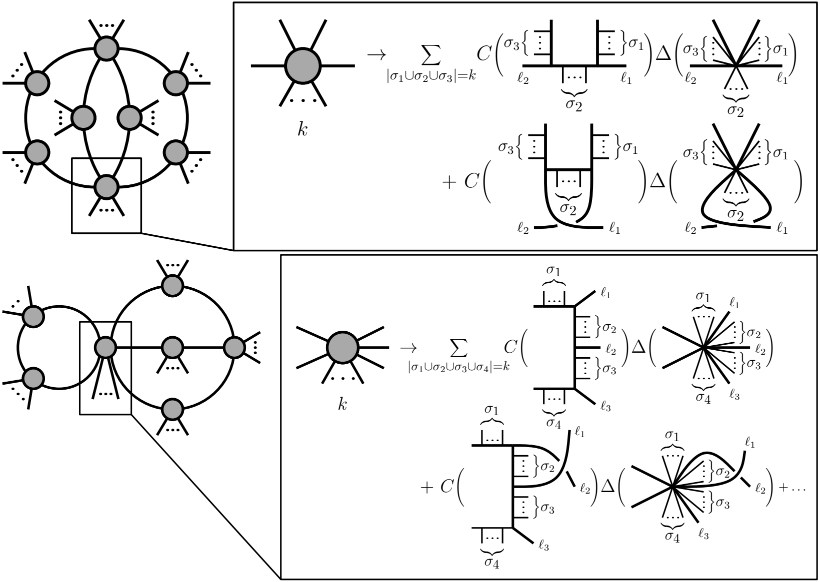
Generalization and subtleties in AO, Page '16



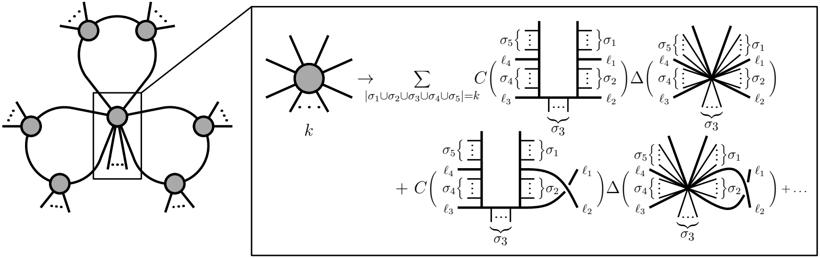
3-loop topologies

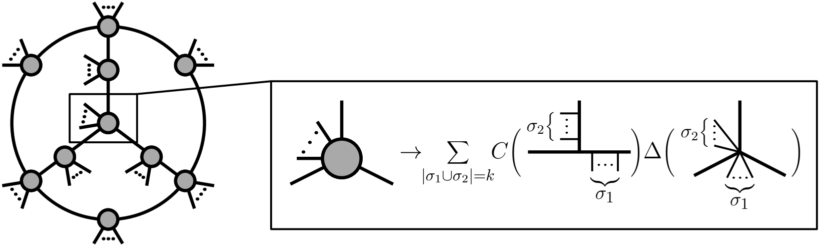


3-loop topologies (d) and (f)



3-loop topologies (g) and (h)





3-loop $\mathcal{N} = 4$ example

AO, Page '16

integrand by Bern, Carrasco, Dixon, Johansson, Kosower, Roiban '07

integrated by Henn, Mistlberger '16