N-jettiness Subtraction With Subleading Power Corrections

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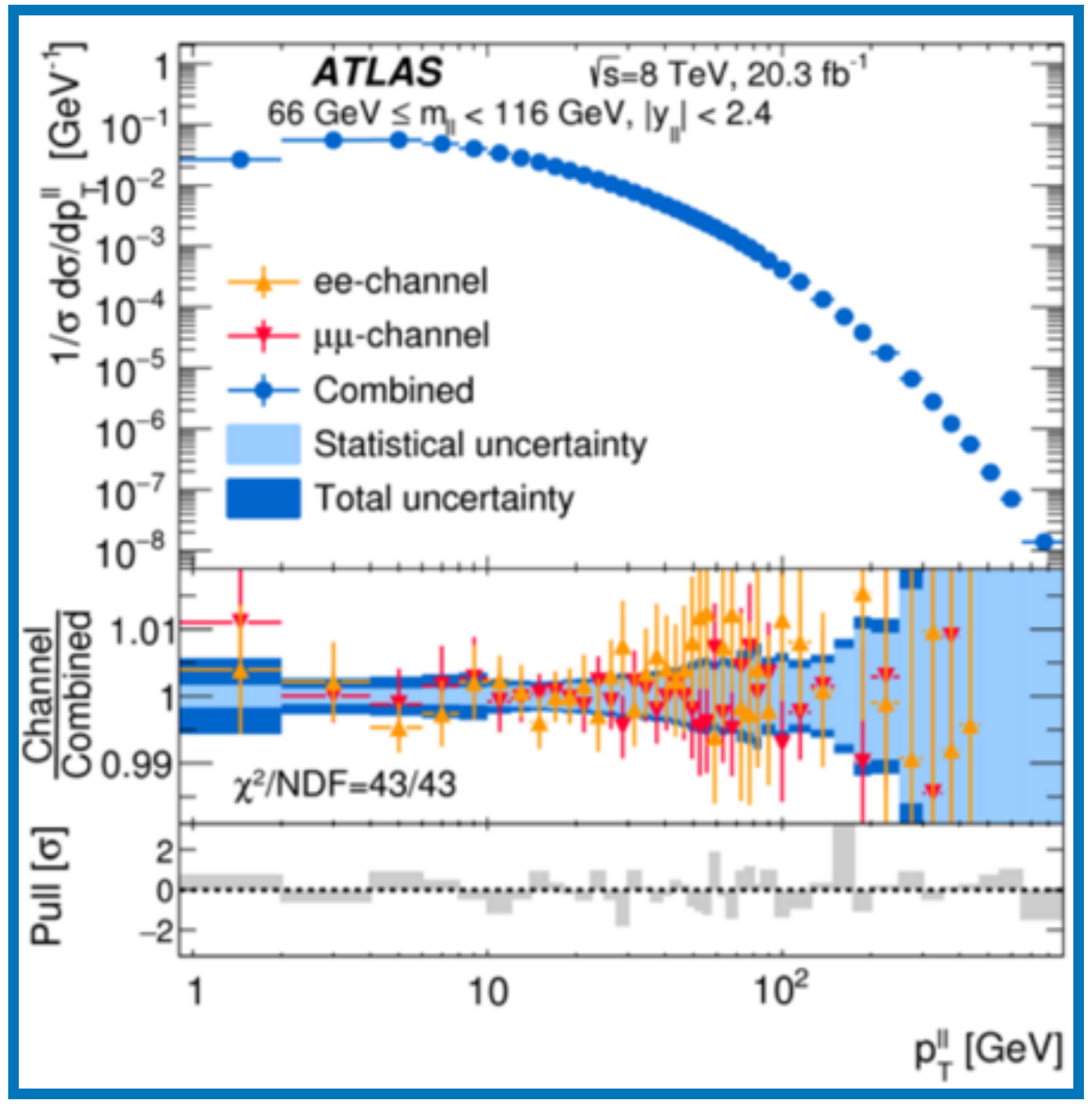
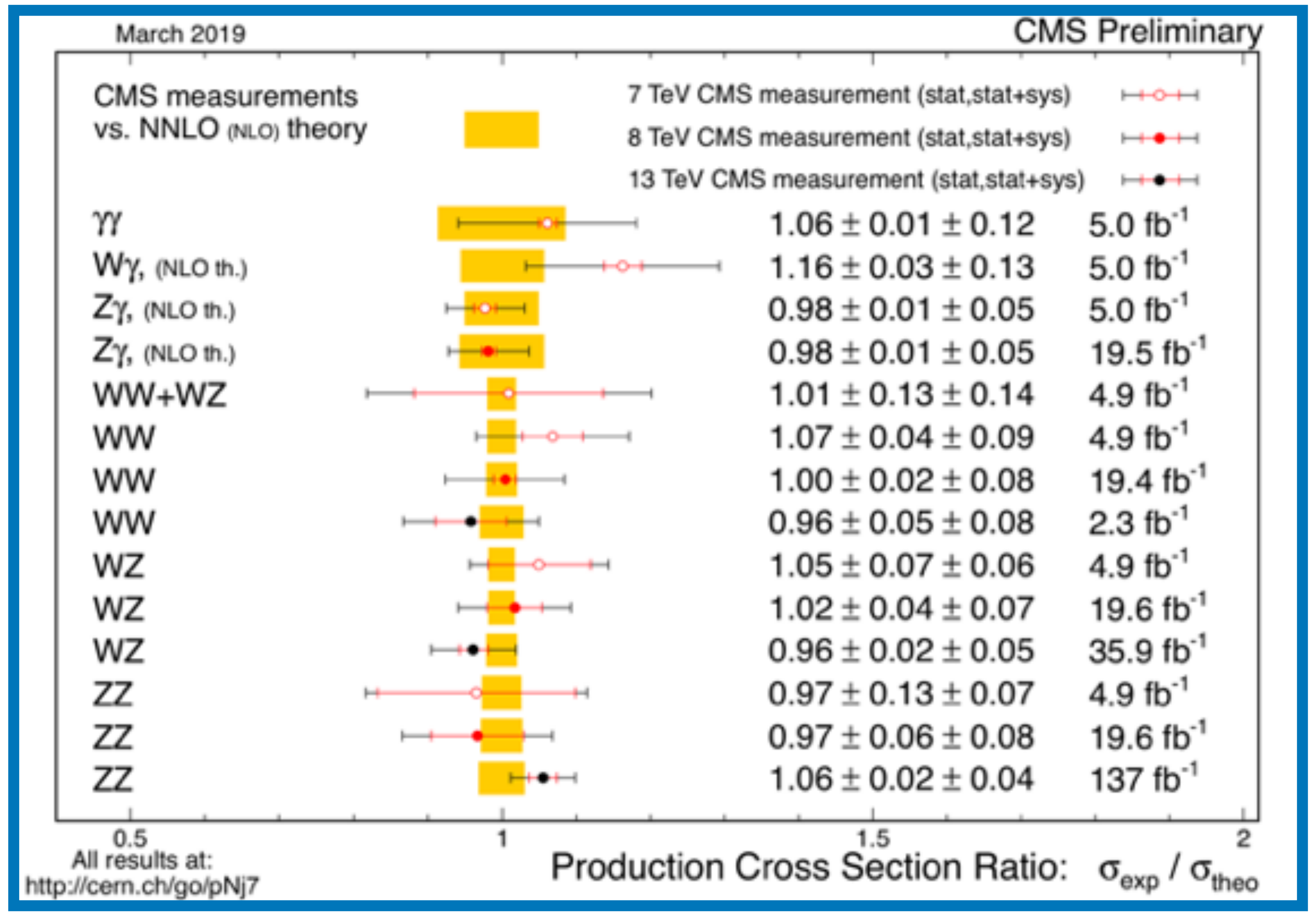
Why Precision?

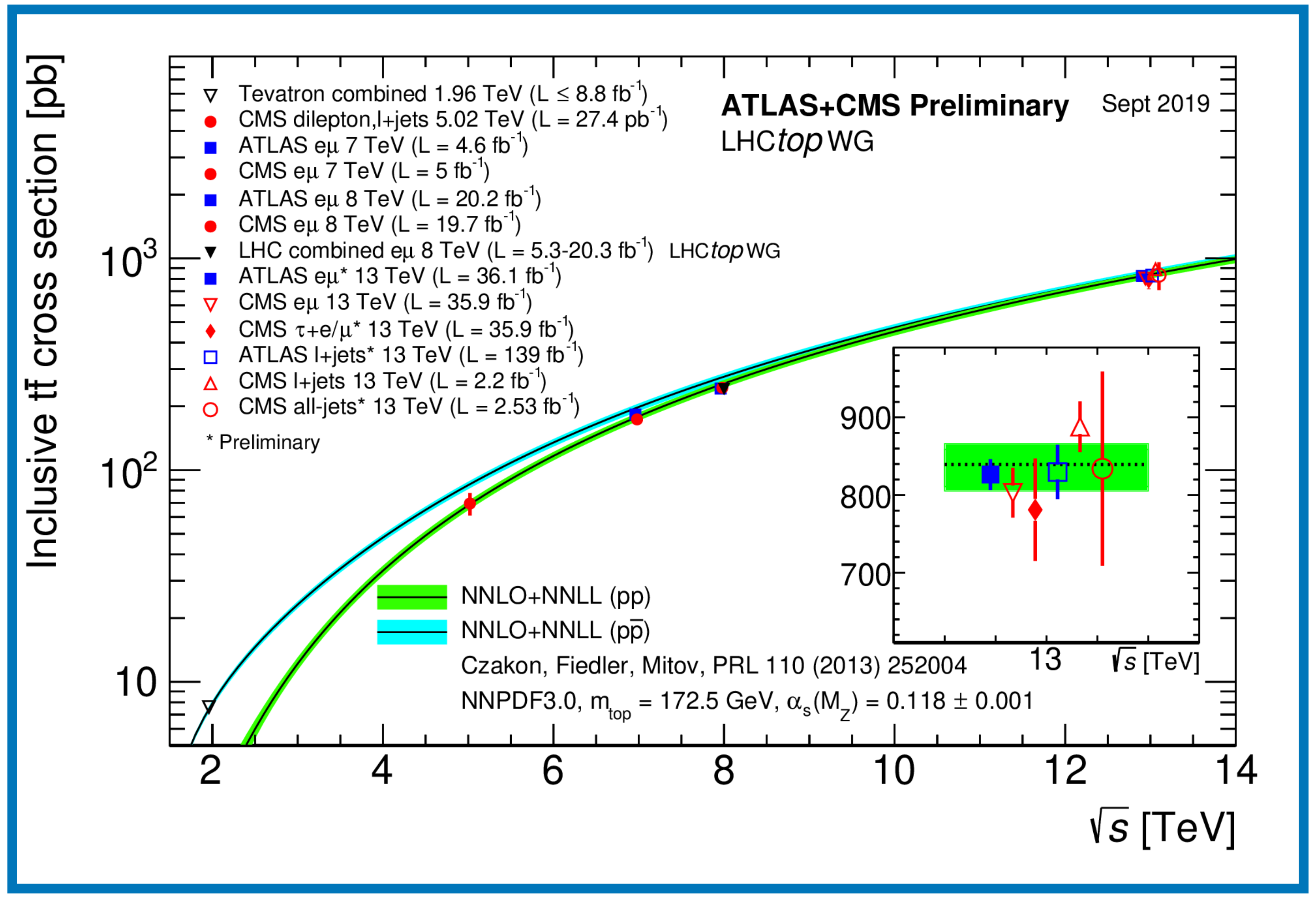
Practical reasons: we are facing the limitations now. The LHC has turned into a precision machine.



Z-pT measured to better than 1% up to 200 GeV



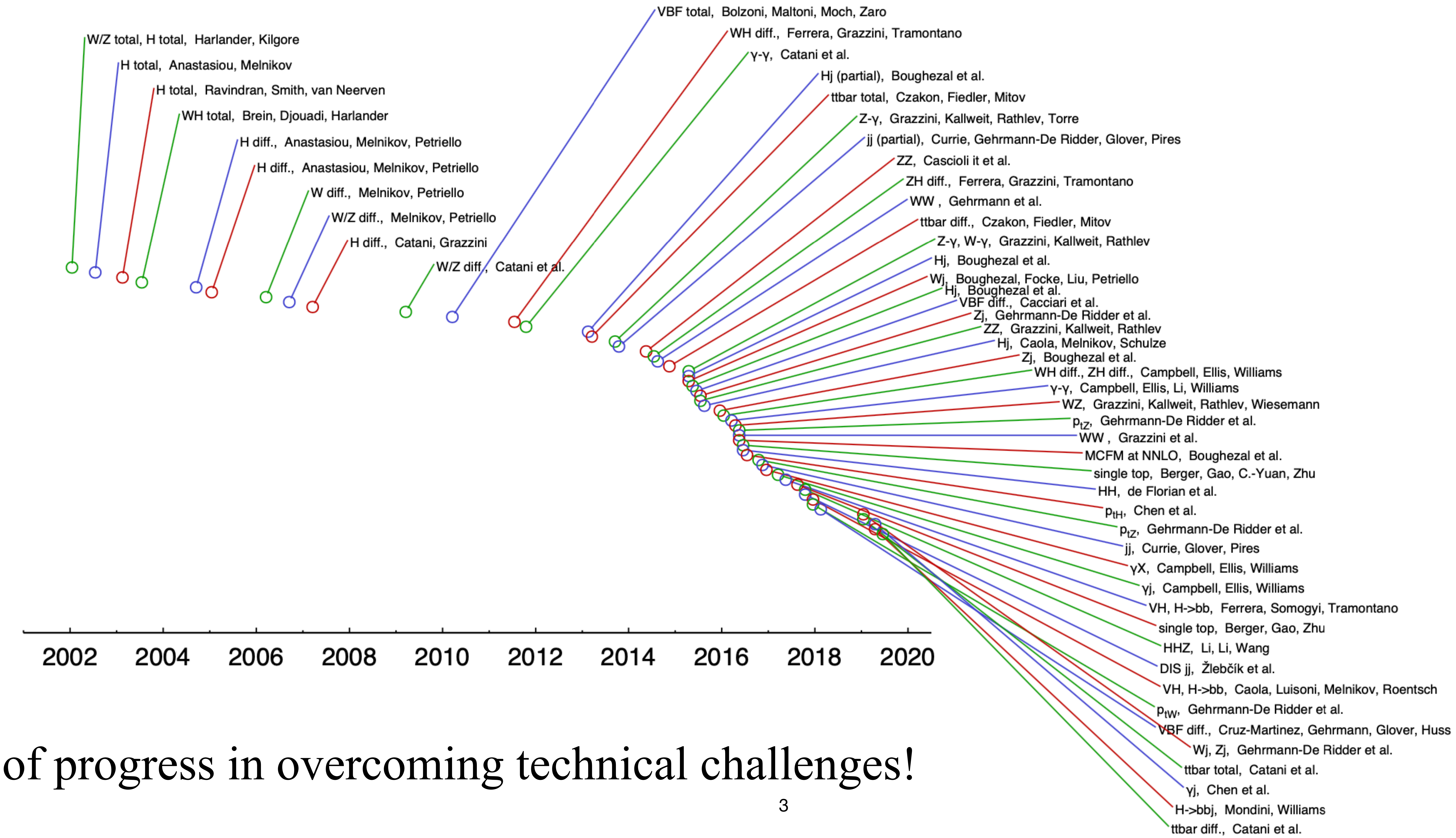
Experimental errors approaching ± 5% for a large class of di-boson measurements



Current top measurements are challenging even NNLO theory

Progress on the NNLO Front

From G. Salam (September 2019)



Lots of progress in overcoming technical challenges!

Challenges of NNLO

- Several ingredients are needed for NNLO predictions. My focus here will be on the real radiation.

  - In dim reg, virtual poles $1/\varepsilon^n$ need to cancel with real poles
  - Real poles (soft and collinear) are only explicit after integrating over the phase space
  - We need a way to extract the final-state singularities

$$\int \left[\frac{A}{\varepsilon^4} + \frac{B}{\varepsilon^3} + \frac{C}{\varepsilon^2} + \frac{D}{\varepsilon} + E\right] \mathrm{d}\Phi_2$$

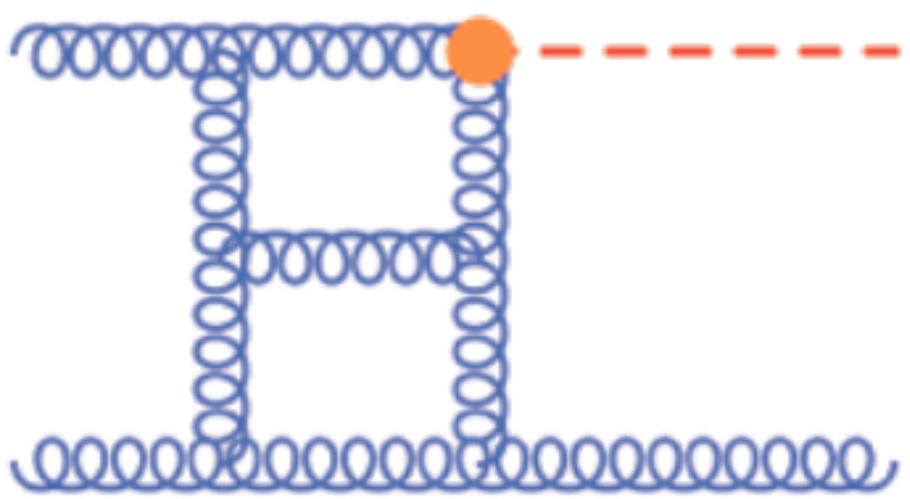
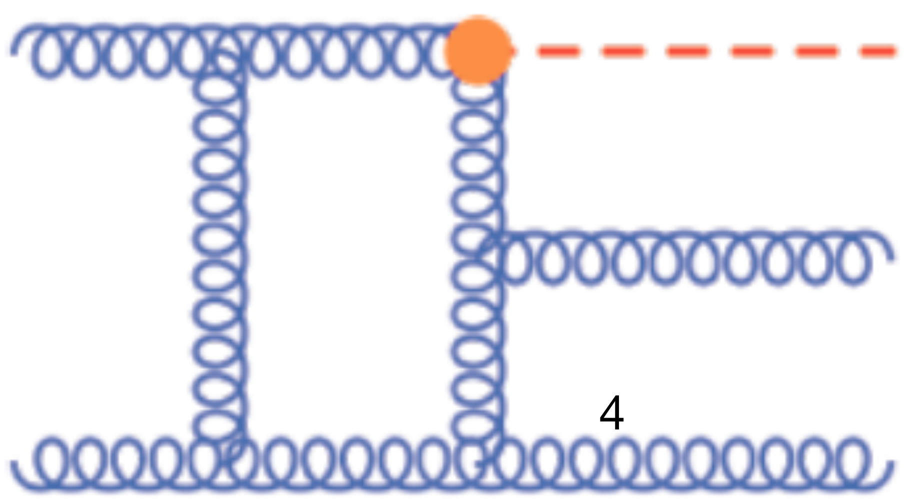
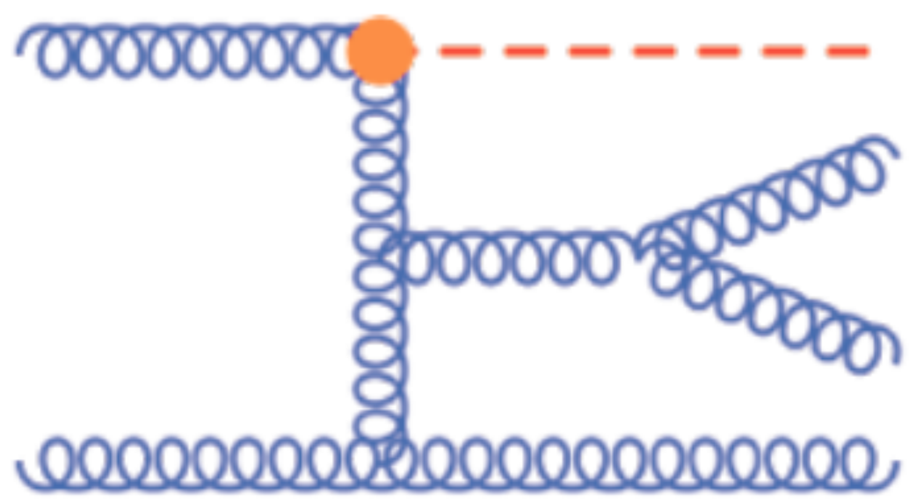
VV

$$\int \left[\frac{A'}{\varepsilon^2} + \frac{B'}{\varepsilon} + C'\right] \mathrm{d}\Phi_3$$

RV

$$\int A'' \mathrm{d}\Phi_4$$

RR

Dealing with IR Singularities

Two classes of solutions are used:

- Subtraction methods:

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \left[\mathrm{d}\sigma^R - \mathrm{d}\sigma^A\right] + \int_{m+1} \mathrm{d}\sigma^A + \int_m \mathrm{d}\sigma^V$$

Real minus counterterm: finite

Integrated counterterm: known analytically

Virtual

* The construction of the subtraction terms dσ^A @ NNLO is an art by itself, multiple methods are used:

- Sector decomposition Anastasiou, Melnikov, Petriello; Binoth, Heinrich
- Antenna subtraction Kosower; Gehrmann, Gehrmann De Ridder, Glover
- Sector Improved Residue Subtraction Czakon; RB, Melnikov, Petriello; Czakon, Heymes; Caola, Melnikov, Rontsch
- Colorful subtraction Del Duca, Duhr, Kardos, Somogyi, Trocsanyi
- Projection to Born Cacciari, Dreyer, Karlberg, Salam, Zanderighi

Dealing with IR Singularities

- EFT-assisted methods:

$$\boxed{\sigma = \int d\tau_N \frac{d\sigma}{d\tau_N}\theta(\tau^{cut} - \tau_N) + \int d\tau_N \frac{d\sigma}{d\tau_N}\theta(\tau_N - \tau^{cut})}$$

a simpler effective theory description is available for this region

have one more resolved jet than at Born level; **need one order lower in perturbation theory in this region!**

✱ Q_T-subtraction: Catani, Grazzini (2007); $\boldsymbol{\tau_N = Q_T}$ (transverse momentum of the final state color singlet)

✱ **N-jettiness subtraction:** RB, Focke, Liu,Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)

The N-jettiness Subtraction Scheme

- **N-jettiness τ_N**: an event-shape variable designed to veto final-state jets Stewart, Tackmann, Waalewijn 2009

N = Number of jets

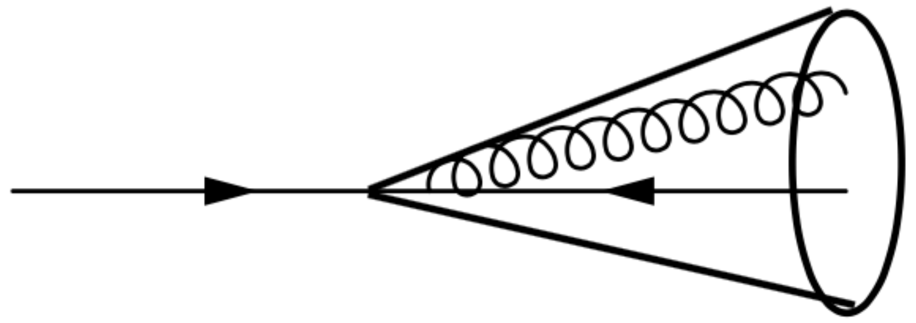
$$\mathcal{T}_N \equiv \sum_k \min_i \left\{ \frac{2 q_i \cdot p_k}{Q_i} \right\}$$

All final-state partons

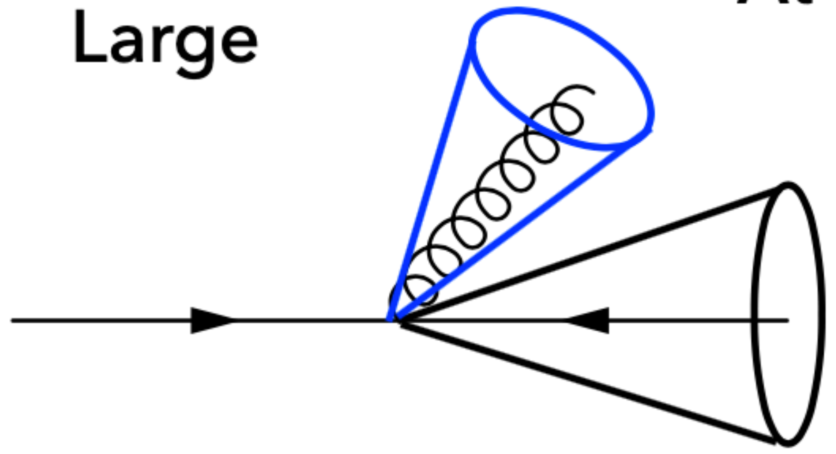
Momenta of the 2 beams and Born final-state jets

Normalization constants

Exactly 0 jets ← Small — $\mathcal{T}_0$ — Large → At least 1 jet



Radiation is soft or collinear to the beam/jet(s)



Radiation becomes a new jet

Results from N-jettiness Subtraction

- W + 1jet @ NNLO — RB, Focke, Liu, Petriello (2015)
- Higgs + 1jet @ NNLO — RB, Focke, Giele, Liu, Petriello (2015)
- Z + 1jet @ NNLO — RB, Campbell, Ellis, Focke, Giele, Liu, Petriello (2016)
- Color singlet processes @ NNLO — RB, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams (2016)
- Associated Higgs production @ NNLO — Campbell, Ellis, Williams (2016)
- Diphoton production @ NNLO — Campbell, Ellis, Williams (2016)
- DIS + 1jet @ NNLO — Abelof, RB, Liu, Petriello (2016)
- ZZ @ NNLO — Heinrich, Jahn, Jones, Kerner, Pires (2018)

...

N-jettiness Factorization Formula

$$\boxed{\sigma(\mathcal{T} < \mathcal{T}_{\text{cut}}) = \int H \otimes B_a \otimes B_b \otimes S \otimes \left[\sum_{n}^{N} J_n\right] + \cdots}$$

describes hard radiation

describes radiation collinear to initial-state beams; ***universal***

describes soft radiation; ***semi-universal;*** depends on number of jets

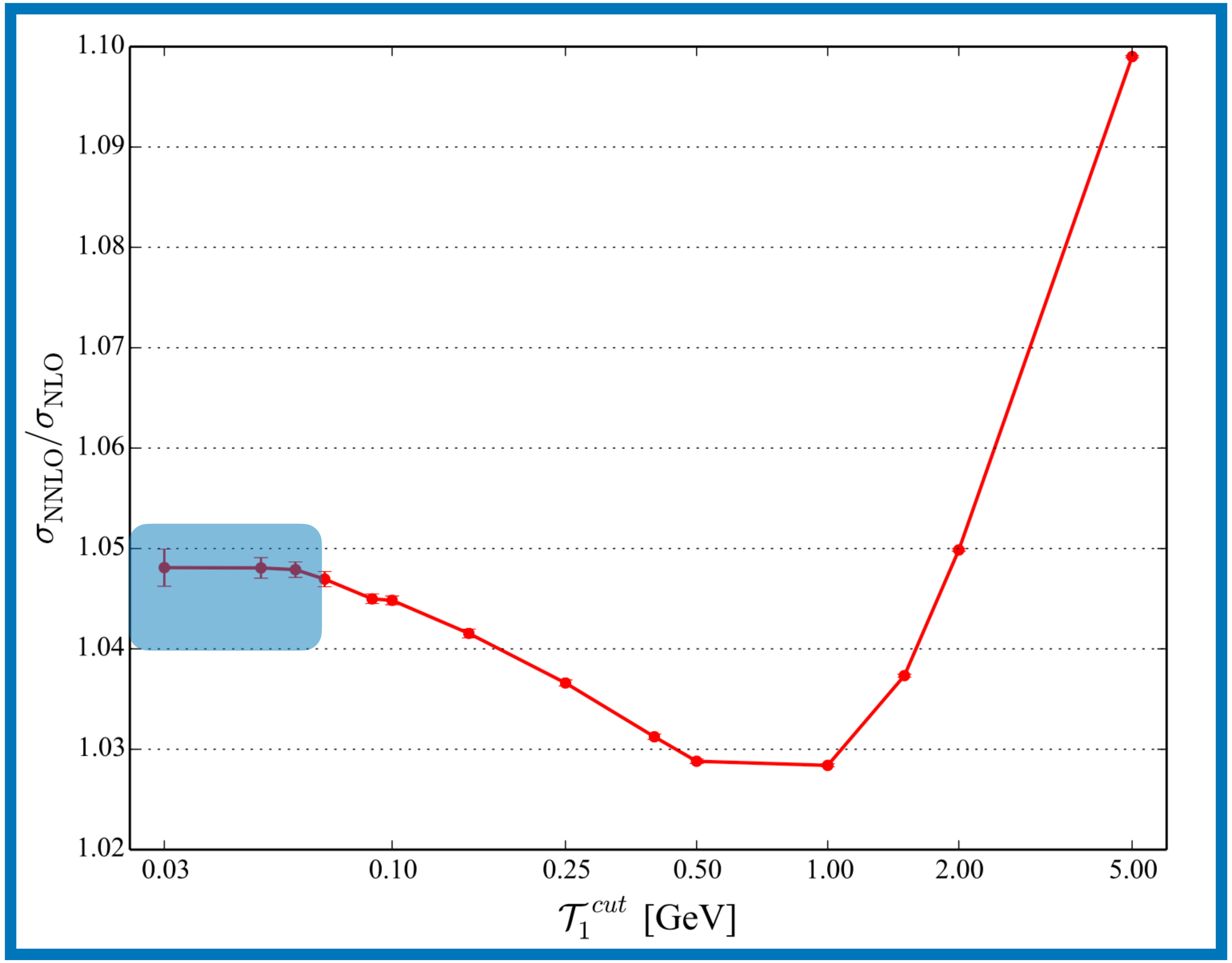
describes radiation collinear to final-state jets; ***universal***

- **Ellipses**: power corrections, they are small for low τ_{cut}. Leading power corrections go like $\tau_{\text{cut}} \log^{2n-1}(\tau_{\text{cut}})$ where n is the order of perturbation theory.

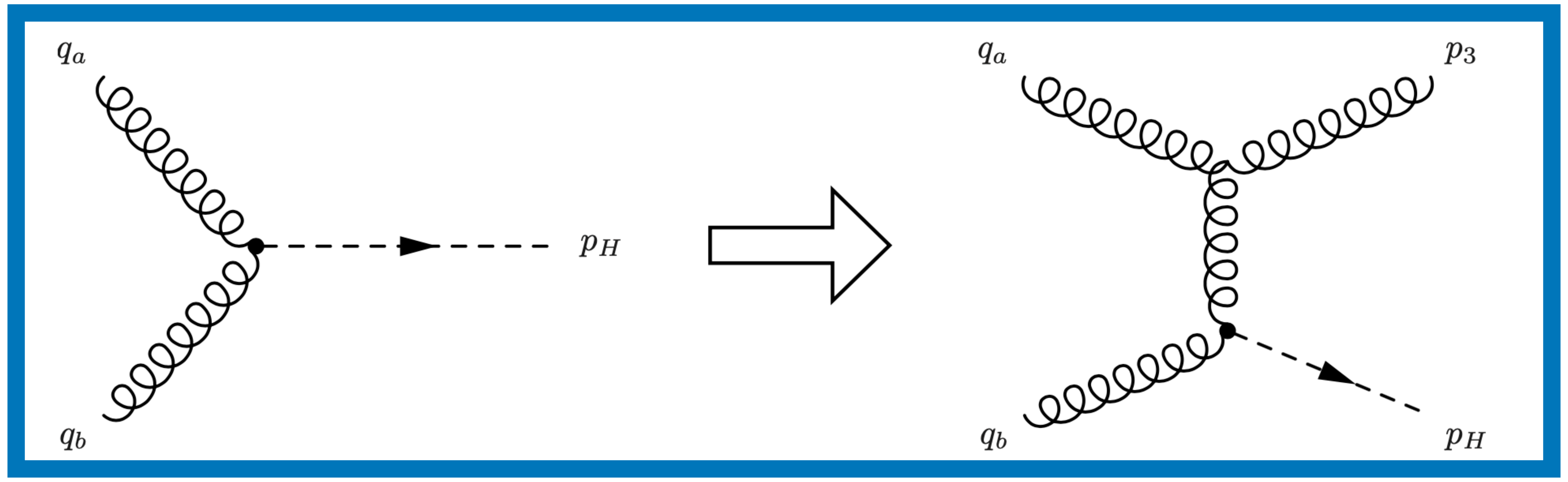
Without the Power Corrections

- Power corrections force us to low τ_{cut} where numerical integration errors for the above the cut cross section become large.
- Leaves us with a limited range of τ_{cut} to use for predictions
- Can we understand power corrections analytically to lessen the dependence on τ_{cut}?

Z+1j



Power Corrections: Color Singlet Case



- Compute the full Next-to-Leading Power (LL+NLL) analytically. Focus on NLO for simplicity.

- Study the numerical impact of different τ definitions (different normalizations)

$$\mathcal{T}_N \equiv \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\} = \sum_k \min_i \left\{ \frac{n_i \cdot p_k}{\rho_i} \right\}$$

($\boldsymbol{\rho}_i = Q_i/2E_i$)

n_i: light like vectors for the beam directions

Choices of Normalization for τ_0

- Power corrections depend strongly on the definition of tau, can we find a choice that minimizes them?

$$\mathcal{T} \equiv \min\left\{\frac{n_a \cdot p_3}{\rho_a}, \frac{n_b \cdot p_3}{\rho_b}\right\} = \min\{\mathcal{T}_a, \mathcal{T}_b\}$$

Hadronic:

$$\mathcal{T}^{\text{had}} \equiv \min\{n_a \cdot p_3, n_b \cdot p_3\}$$

$$n_a^\mu = (1,0,0,1) \qquad n_b^\mu = (1,0,0,-1)$$

Leptonic:

$$\mathcal{T}^{\text{lep}} \equiv \min\{e^Y n_a \cdot p_3, e^{-Y} n_b \cdot p_3\}$$

takes into account the rapidity of the color singlet

Strategy for the Computation

1. Define regions: beam a, beam b and soft

2. Expand the phase space in each region

$$\frac{\mathrm{dPS}_{\text{beam }a}}{\mathrm{d}Y\,\mathrm{d}Q^2} = \frac{\mathrm{d}\hat{\mathrm{PS}}_{\mathrm{Born}}}{\mathrm{d}Y\,\mathrm{d}Q^2}\int_{x_a}^{1}\mathrm{d}z_a\int_0^{\mathcal{T}_{\mathrm{cut}}}\mathrm{d}\mathcal{T}\left[\Phi^{(0,0)}_{\text{beam }a} + \Phi^{(1,0)}_{\text{beam }a}\mathcal{T} + \Phi^{(0,1)}_{\text{beam }a}(1-z_a) + \dots\right]$$

$$\frac{\mathrm{dPS}_{\mathrm{soft}}}{\mathrm{d}Y\,\mathrm{d}Q^2} = \frac{\mathrm{d}\hat{\mathrm{PS}}_{\mathrm{Born}}}{\mathrm{d}Y\,\mathrm{d}Q^2}\int\mathrm{d}\mathcal{T}_a\int\mathrm{d}\mathcal{T}_b\left[\Phi^{(0,0)}_{\mathrm{soft}} + \Phi^{(1,0)}_{\mathrm{soft}}\mathcal{T}_a + \Phi^{(0,1)}_{\mathrm{soft}}\mathcal{T}_b + \dots\right]$$

($z_a \to 1$ is the soft limit)

PROCESS INDEPENDENT COEFFICIENTS

3. Expand the matrix element (amplitude²) in each region

$$\mathcal{M}_{\text{beam }a} = \mathcal{M}^{(-1,-1)}_{\text{beam }a}\,\mathcal{T}^{-1}\,(1-z_a)^{-1} + \mathcal{M}^{(-1,0)}_{\text{beam }a}\,\mathcal{T}^{-1} + \mathcal{M}^{(0,-1)}_{\text{beam }a}\,(1-z_a)^{-1}$$

$$\mathcal{M}_{\mathrm{soft}} = \mathcal{M}^{(-1,-1)}_{\mathrm{soft}}\,\mathcal{T}_a^{-1}\,\mathcal{T}_b^{-1} + \mathcal{M}^{(-1,0)}_{\mathrm{soft}}\,\mathcal{T}_a^{-1} + \mathcal{M}^{(0,-1)}_{\mathrm{soft}}\,\mathcal{T}_b^{-1}$$

PROCESS DEPENDENT IN PRINCIPLE

Power Counting at NLO

$\frac{d\sigma_{\mathrm{NLO}}}{d\mathcal{T}}$	LL	NLL	NNLL
LP	$\left[\frac{\log \mathcal{T}}{\mathcal{T}}\right]_+$	$\left[\frac{1}{\mathcal{T}}\right]_+$	$\delta(\mathcal{T})$
NLP	$\boxed{\log \mathcal{T}}$	$\boxed{1}$	

- RB, Liu, Petriello, 2016
- Moult, Rothen, Stewart, Tackmann, Zhu, 2016

- RB, Isgro, Petriello, 2018
- Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2018

Matrix Elements Expansion

- In this very simple case, the full NLO matrix element is known and already factorized, so it is easiest to just expand it

$$|\mathcal{M}(gg \to Hg)|^2 = \frac{8C_A\pi\alpha_s}{m_H^4(1-\varepsilon)}|\mathcal{M}(gg\to H)|^2 \left[\frac{m_H^8+s_{12}^4+s_{13}^4+s_{23}^4}{s_{12}s_{13}s_{23}}(1-2\varepsilon) + \frac{\varepsilon}{2}\frac{\left(m_H^4+s_{12}^2+s_{13}^2+s_{23}^2\right)^2}{s_{12}s_{13}s_{23}}\right]$$

S. Dawson (1990)

- In the beam (a) region, for example

$$|\mathcal{M}(gg\to Hg)|^2 = |\mathcal{M}(gg\to H)|^2\,(16C_A\alpha_s\pi)\left\{\boxed{\frac{1}{Q_a\mathcal{T}}\frac{\left(1-z_a+z_a^2\right)^2}{(1-z_a)z_a}} + \boxed{\frac{1}{m_H^2}\left[5-\frac{1}{1-z_a}+\frac{1}{z_a^2}-\frac{1}{z_a}+z_a-\frac{2}{1-\varepsilon}\right]} + \mathcal{O}(\mathcal{T})\right\}$$

LEADING POWER: PREDICTABLE FROM THE LP COLLINEAR LIMIT. THE Z_A->1 LIMIT IS PREDICTABLE FROM THE LP SOFT LIMIT (LP - LL)

NEXT TO LEADING POWER, IN GENERAL NOT WRITTEN IN A UNIVERSAL FORM

NLP-LL: PREDICTABLE FROM THE NEXT-TO-LEADING SOFT LIMIT

Next to Leading Power, Leading Log

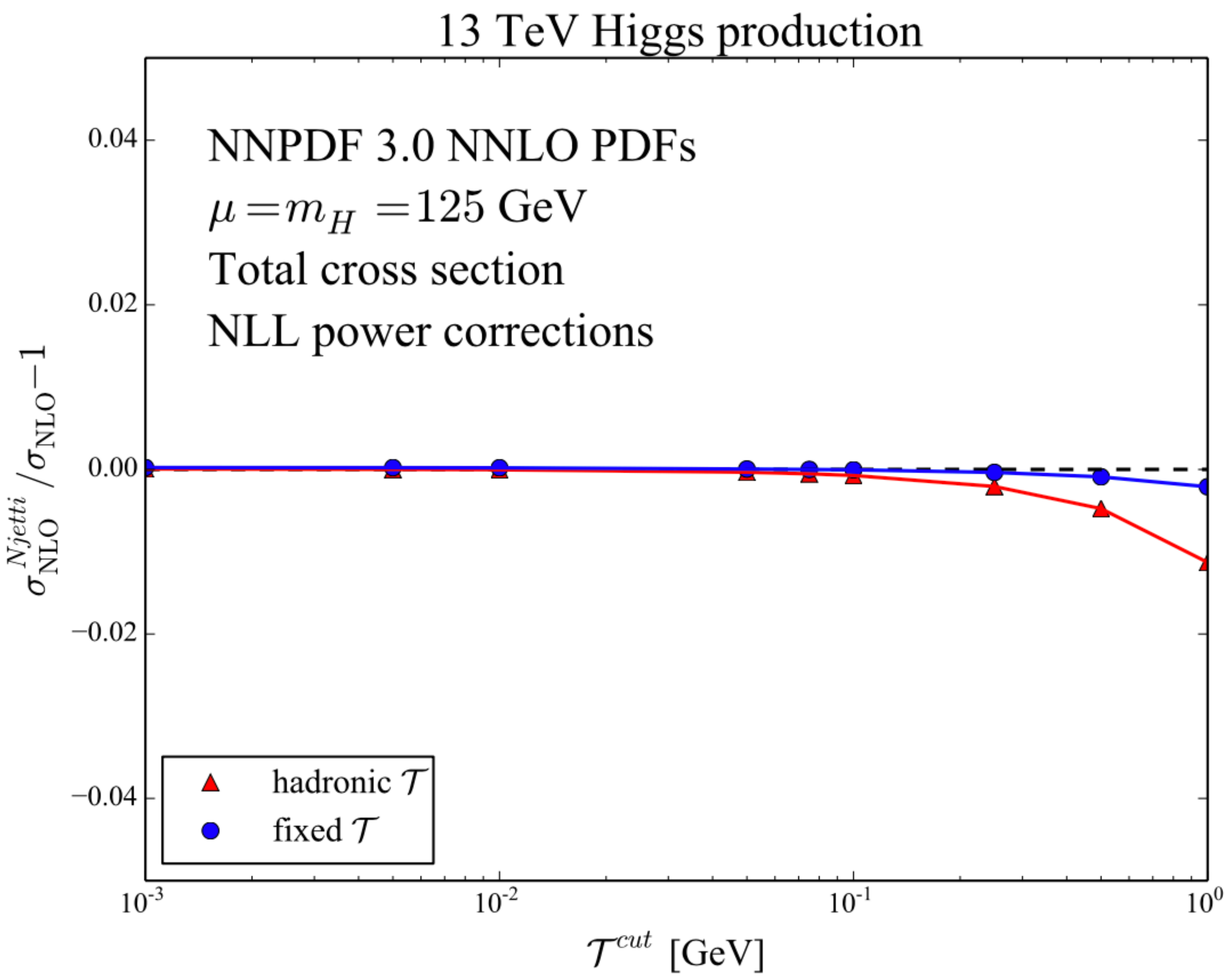
- NLO, NLP-LL cross section below the cut for gg → H for the hadronic definition:

$$\frac{\mathrm{d}\sigma_{\text{NLP-LL}}}{\mathrm{d}Y\mathrm{d}Q^2\mathrm{d}\mathcal{T}} = \hat{\sigma}_{\text{Born}}\left(\frac{C_A\alpha_s}{\pi}\right)\left\{\left[\frac{e^Y\rho_a}{Q}\log\left(\frac{\mathcal{T}}{Qe^Y\rho_a}\right) + \frac{\rho_b e^{-Y}}{Q}\log\left(\frac{\mathcal{T}}{Q\rho_b e^{-Y}}\right)\right] f_a(x_a)f_b(x_b)\right.$$
$$\left. - \frac{x_a}{Qe^Y\rho_a}\log\left(\frac{\mathcal{T}e^Y\rho_a}{Q}\right) f_a'(x_a)f_b(x_b) - \frac{x_b}{Q\rho_b e^{-Y}}\log\left(\frac{\mathcal{T}\rho_b e^{-Y}}{Q}\right) f_a(x_a)f_b'(x_b)\right\}$$

- Derivatives of the PDFs appear

- There is a rapidity term multiplying the log, which suggests that for the hadronic definition, power corrections will be large for large rapidities.

Numerical Results



Dashed: exact NLO

Colored: N-jettiness NLO

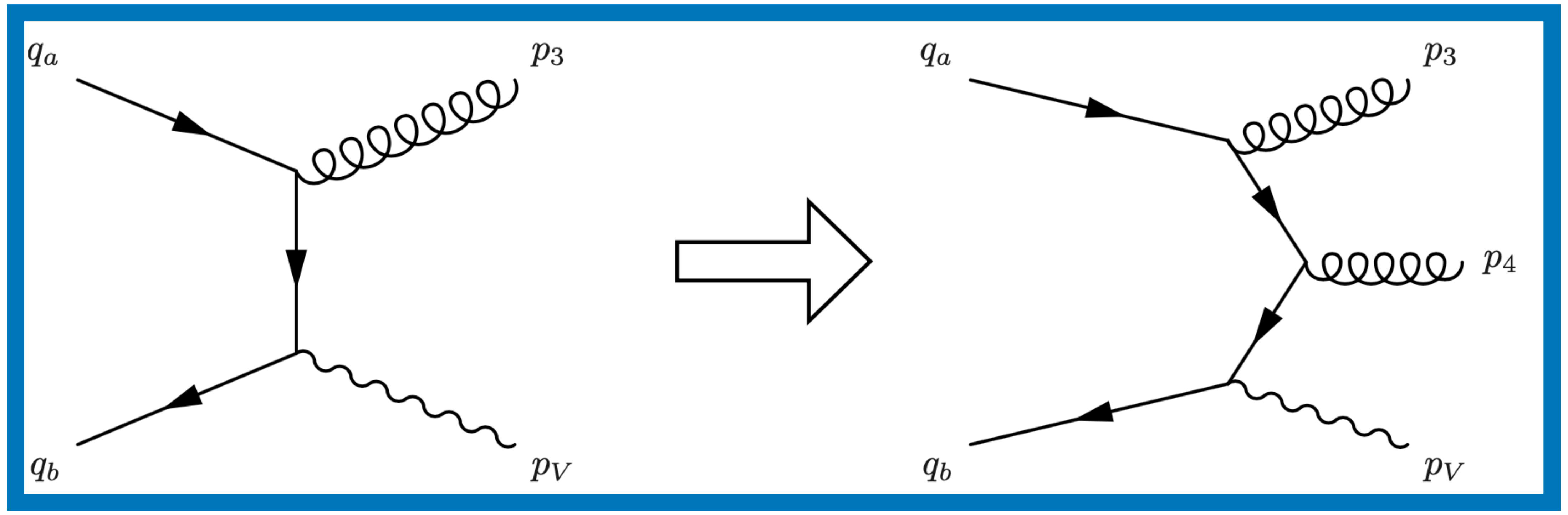
Hadronic LL power corrections go like: $\dfrac{e^Y}{Q} \log\left(\dfrac{\mathcal{T}_{\rm had}}{Qe^Y}\right)$

Leptonic LL power corrections go like: $\dfrac{1}{Q} \log\left(\dfrac{\mathcal{T}_{\rm lep}}{Q}\right)$

$Q = m_H$

- With the inclusion of power corrections, deviations from dipole subtraction are better than 0.1% with the leptonic definition.
- Leptonic definition at leading power is an optimal choice for N-jettiness definition

Power Corrections for V + 1jet



- Compared to the color singlet case, we have one more collinear direction in the definition:

$$\mathcal{T}_0 \equiv \min\left\{\frac{n_a \cdot p_3}{\rho_a}, \frac{n_b \cdot p_3}{\rho_b}\right\}$$

2 possibilities

$$\begin{aligned}\mathcal{T}_1 \equiv \quad & \min\left\{\frac{n_a \cdot p_3}{\rho_a}, \frac{n_b \cdot p_3}{\rho_b}, \frac{n_J \cdot p_3}{\rho_J}\right\} \\ + \quad & \min\left\{\frac{n_a \cdot p_4}{\rho_a}, \frac{n_b \cdot p_4}{\rho_b}, \frac{n_J \cdot p_4}{\rho_J}\right\}\end{aligned}$$

in principle 3x3=9 possibilities

Strategy: What is New With 1 jet

- Pre-clustering jet algorithm to determine the jet-direction
  Use N-jettiness to pre-cluster rather than the usual anti-kT
- Soft function divided into hemisphere and non-hemisphere
  Does this framework also hold at NLP? Does it introduce new poles?
- The matrix elements are more complicated
  What can we say about their factorization properties at NLP?
- Is there an equivalent to the leptonic definition?

N-jettiness as a Pre-clustering Jet Algorithm

- Previous N-jettiness implementations had the jet direction determined via a pre-clustering jet algorithm

$$\Delta_{34}^2 \equiv (\eta_3 - \eta_4)^2 + (\phi_3 - \phi_4)^2$$

Jet cone radius

$\Delta_{34} > R$ Partons are separated: the hardest momentum is the jet

$\Delta_{34} < R$ Partons are close: the sum of the momenta is the jet

- This procedure would complicate the computation of the NLP due to the tough phase space constraints imposed by the anti-kT jet algorithm. We will define as the parton distance the scalar product that appears in the measurement function.

First assume p_3 is hardest (p_3<->p_4 symmetry)

$$\mathcal{T}_1 \equiv \min\left\{\frac{n_a \cdot p_3}{\rho_a}, \frac{n_b \cdot p_3}{\rho_b}, \frac{n_J \cdot p_3}{\rho_J}\right\} + \min\left\{\frac{n_a \cdot p_4}{\rho_a}, \frac{n_b \cdot p_4}{\rho_b}, \frac{n_J \cdot p_4}{\rho_J}\right\}$$

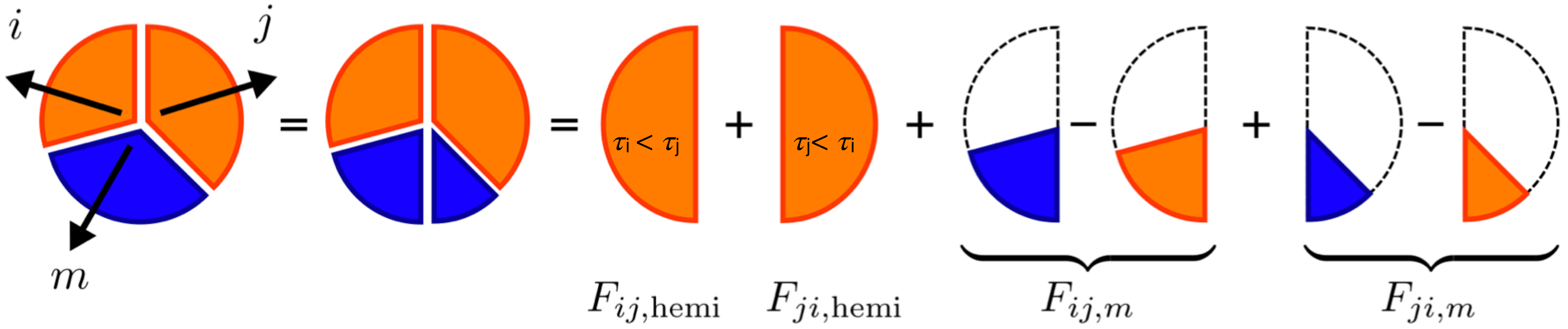
If p_4 is closest to n_a or n_b: p_3 is the jet

If p_4 is closest to p_3: p_3+p_4 is the jet

Now only 3 possibilities

The Soft Hemisphere Function

From 1102.4344



- At LP: hemisphere soft function → pole, LL
  $F_{ij,m}$ and $F_{ji,m}$ non-hemisphere soft functions → no pole therefore not LL
- This is a useful decomposition because the divergent integrals do not depend on the m direction: less scales, easier integrals.
- At NLP, we found that the hemisphere decomposition can be applied, but now the non-hemisphere terms also have poles, and therefore contribute to the LL.
- Poles cancel only after summing beam, jet, soft hemisphere and soft non-hemisphere contributions, and is therefore a non-trivial check of this result

Subleading Factorization of the Matrix Elements

- Burnett, Kroll, Low (1968) derived the subleading soft factorization of amplitudes squared in QED. It also holds in QCD upon adding the appropriate color factors.

LP eikonal factor

$$\mathcal{M}_{\rm soft}^{\rm NLO,LP+NLP} = (4\pi\alpha_s)\sum_{i,j}\left\{\boxed{-\frac{p_i\cdot p_j}{(k\cdot p_i)\,(k\cdot p_j)}}+\frac{p_i\cdot p_j}{(k\cdot p_i)\,(k\cdot p_j)}k\cdot\frac{\partial}{\partial p_j}-\frac{1}{k\cdot p_i}p_i\cdot\frac{\partial}{\partial p_j}\right\}$$
$$\left\langle \mathcal{A}_{\rm Born}\middle|\vec{T}_i\cdot\vec{T}_j\middle|\mathcal{A}_{\rm Born}\right\rangle$$

- We expand the matrix elements in each region:

$$\mathcal{M}_{\rm soft}(\mathcal{T}_a,\mathcal{T}_b,\mathcal{T}_J)=\sum_{n,m,l}\mathcal{M}_{\rm soft}^{(n,m,l)}\mathcal{T}_a^n\mathcal{T}_b^m\mathcal{T}_J^l$$

$$\mathcal{M}_{\rm beam}(\mathcal{T},z_a,\phi)=\sum_{n,m}\mathcal{M}_{\rm beam}^{(n,m)}(\phi)\mathcal{T}^n(1-z_a)^m$$

- The theorem can predict ALL the expansion coefficients needed for the NLP-LL. This means that the LL power corrections are process independent!

 (we still need the full NLO matrix elements at NLL)

Final Form of Power Corrections

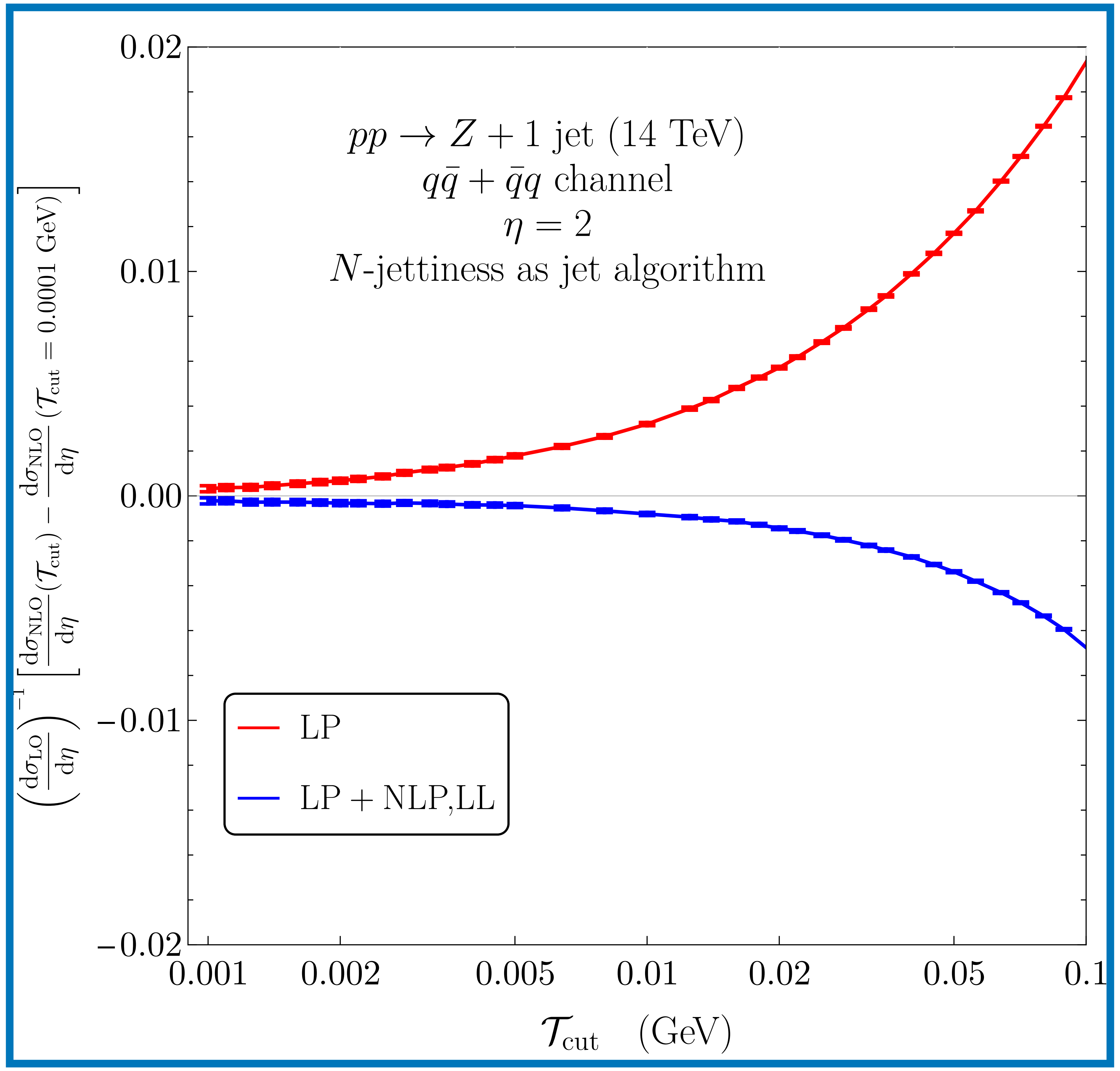
$$\boxed{\frac{d\sigma^{\mathrm{NLP}}}{dQ^2\, dY\, dp_T\, d\eta\, d\mathcal{T}} = \frac{d\hat{\mathrm{PS}}_{\mathrm{Born}}}{dQ^2\, dY\, dp_T\, d\eta} \left(\frac{\alpha_s}{4\pi}\right) \sum_\alpha \left\{ C_\alpha^{\mathrm{LL}} \log \frac{\mathcal{T}}{Q_\alpha} + C_\alpha^{\mathrm{NLL}} \right\}}$$

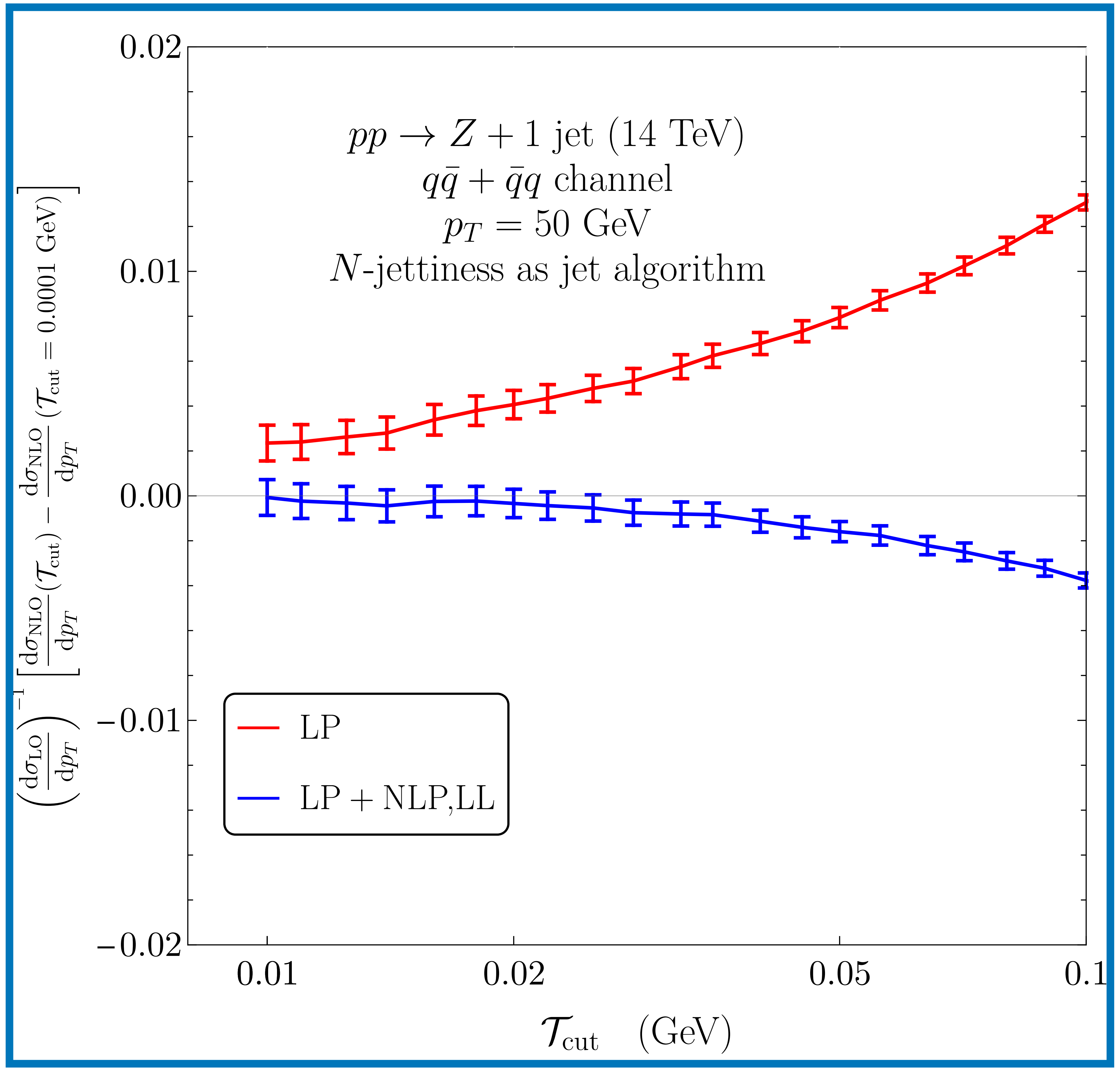
- The sum over α runs over all the regions: beam a, beam b, jet, soft hemisphere, soft non-hemisphere
- The coefficients C^{LL}_α are products of process independent phase space coefficients (which contain PDF derivatives) and matrix element coefficients.
- The arguments Q_α have a quite simple form: $\hat{s}_{ij} = \frac{n_i \cdot n_j}{2\rho_i \rho_j}$ $\rho_{i,j}$ are hardness measures

$$Q_{\mathrm{soft}\ ij\ \mathrm{hemi}} = \mu\sqrt{\hat{s}_{ij}} \qquad Q_{\mathrm{soft}\ ij\ \mathrm{non\text{-}hemi}} = \frac{\mu \hat{s}_{im}}{\sqrt{\hat{s}_{ij}}} \qquad Q_{\mathrm{beam}\ a} = \frac{\mu^2}{\sqrt{s x_a \rho_a}} \qquad Q_{\mathrm{beam}\ b} = \frac{\mu^2}{\sqrt{s x_b \rho_b}} \qquad Q_{\mathrm{jet}} = \frac{\mu^2}{2 p_T \rho_J \cosh\eta}$$

- Without a subleading collinear theorem, there is no general way to express the NLL coefficients.

Results in the QQBar Channel





- We have analytically checked the cancellation of poles.
- Inclusion of NLP-LL corrections reduces the cross section dependence on $\boldsymbol{\tau}_{\text{cut}}$ and allows for higher values of $\boldsymbol{\tau}_{\text{cut}}$.

Different Normalizations

- So far we performed numerical studies for the hadronic definition, where $\rho_a = \rho_b = \rho_J = 1$

- Recent work in the literature (1906.01020) revisited the NNLO Higgs+1 jet calculation (1505.03893) by defining N-jettiness in a boosted frame where the system H+1jet is at rest, finding good improvement with respect to the hadronic case.

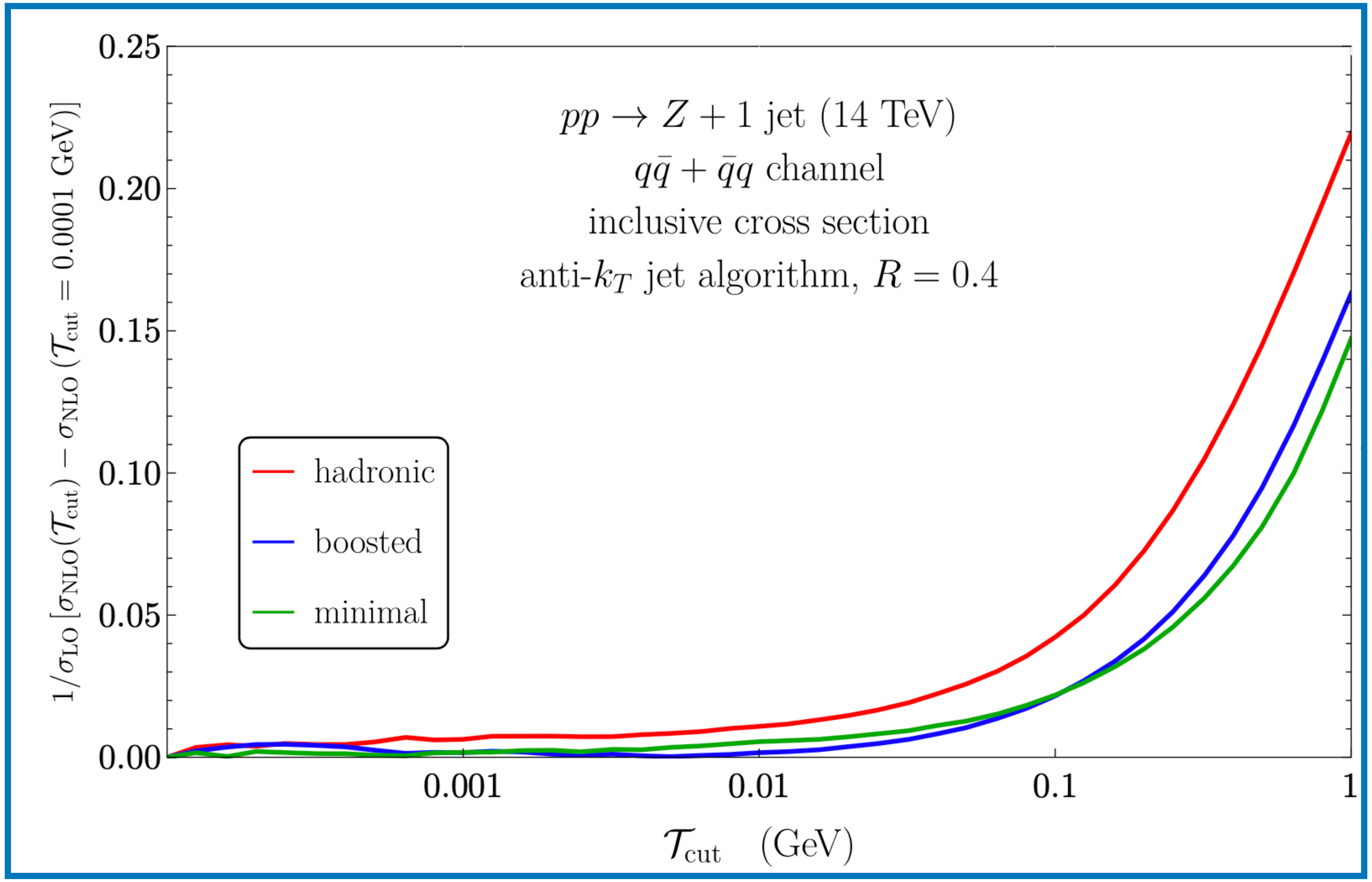
- Boosted definition: $\hat{Q}^2 \equiv (p_V + q_J)^2 = s x_a x_b$ $\quad \hat{Y} \equiv \frac{1}{2} \log \left[\frac{(p_V + q_J) \cdot n_b}{(p_V + q_J) \cdot n_a} \right] = \frac{1}{2} \log \frac{x_a}{x_b}$

$$\boxed{\rho_a^{\text{boosted}} = e^{-\hat{Y}} \qquad \rho_b^{\text{boosted}} = e^{\hat{Y}} \qquad \rho_J^{\text{boosted}} = \frac{\cosh(\eta - \hat{Y})}{\cosh \eta}}$$

- This resembles the leptonic definition in the choice of ρ_a and ρ_b. It minimizes $Q_{\text{beam a}}$ and $Q_{\text{beam b}}$ but Q_{jet} is not minimized. We therefore define:

$$\boxed{\rho_a^{\text{minimal}} = e^{-\hat{Y}} \qquad \rho_b^{\text{minimal}} = e^{\hat{Y}} \qquad \rho_J^{\text{minimal}} = \frac{1}{\cosh \eta}}$$

Comparison of the Three Normalizations



- Comparison done with anti-kT pre-clustering
- We see an improvement from hadronic → boosted → minimal

at 1 GeV: 22% → 16% → 15%

Summary

- The simplicity of the N-jettiness subtraction method made it an attractive approach in deriving jet cross sections at NNLO
- In order to achieve an accuracy that matches the current experimental precision for many processes it is helpful to include NLP corrections for a better numerical stability
- Our analytical derivation of these terms for V+jet at NLO revealed many interesting results:

➡ Using the N-jettiness as a clustering jet algorithm simplifies the calculation of the NLP-LL and NLP-NLL

➡ The NLP-LL result can be written in a universal form thanks to the subleading soft factorization theorem

➡ In order to achieve a universal form for the NLP-NLL, a subleading collinear factorization theorem is needed

➡ The minimal definition of the N-jettiness minimizes the impact of the power corrections. Good agreement was found with the boosted definition. Both boosted and minimal definitions should be used at NNLO for a better numerical stability.