Spinning the x tale

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BLACK HOLES <=> QUANTUM PARTICLES

- No hair theorem: BH is characterized by (M, Q, |S|), with no reference to the origin of its makeup.

the same as elementary particles

To which extent do BHs behaves as particles ? Aren't all objects essentially point particles at long distances?

From a world-line description, the compact object are differentiated by the distinct ways it sources the back ground Goldberger and Rothstein, Phys. Rev. D 73, 104029 (2006)

$$S = \int d\sigma \left\{ -m\sqrt{u^2} + c_E E^2 + c_B B^2 + \cdots \right\}$$

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} u^\alpha u^\beta$$

$$B_{\mu\nu} := \frac{1}{2}\epsilon_{\alpha\beta\gamma\mu} R^{\alpha\beta}{}_{\delta\nu} u^\gamma u^\delta$$

Tidal Love numbers: vanishing for BHs but not Neutron stars

BLACK HOLES <=> QUANTUM PARTICLES

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Porto, PRD 73, 104031 (2006)
Porto and Rothstein PRD 78 (2008) 044013
Levi and Steinhoff JHEP 1509, 219 (2015)

This is more prominent when spins are included

$$S = \int d\sigma \left\{ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{SI}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}(y^\mu)\right] \right\}$$

$$L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n}} S^{\mu_{2n+1}}$$

C#=1 for Kerr BHs !

Levi and Steinhoff JHEP 1509, 219 (2015)

Vines, Class. Quant. Grav. 35, no. 8, 084002 (2018)

C#=1 for Kerr BHs !

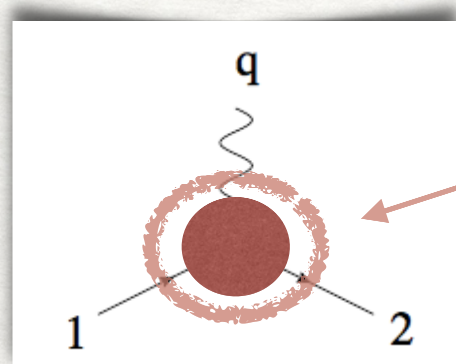
- What is the on-shell avatar of the no-hair theorem?
- How do they encode/differentiate distinct black hole (like) solutions?
- What are the physical principles that selects these coefficients ?

$$\mathcal{O}^{phys_a}{}_{phys_b} \longrightarrow \epsilon_2^{(s),\,\mu\nu\cdots}\,\mathcal{O}\,\epsilon^{(s)}_{1,\,\mu\nu\cdots} \longrightarrow M_3(q^{+2},\mathbf{1}^s,\mathbf{2}^s)$$

-

Begin with a worldline operator

End with a scattering amplitude



The distinct three point amplitudes encode the distinct Wilson Coefficients

GENERAL MASSIVE AMPLITUDES

N. Arkani-Hamed, Tzu-Chen Huang, Y-t H 1709.04891

Consider an amplitude for massive states. Since it is a scalar function that carries the quantum number of the physical state (Little group)

$$\langle in\ t \to +\infty | out\ t \to -\infty \rangle \to M_n^{\{I_1 I_2 \cdots I_{2s_1}\},\{J_1 J_2 \cdots J_{2s_2}\}\cdots}$$

I =1,2 are doublets of SU(2) Little group.

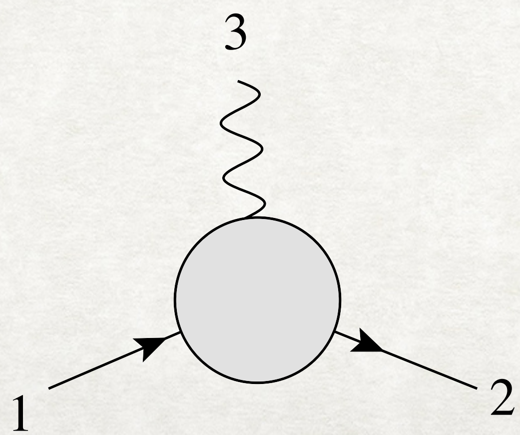
We introduce spinor-helicity formalism: a 4-vector is equivalent to a 2x2 matrix

$$Det(p^{\alpha\dot{\alpha}}) = m^2 \qquad p^{\alpha\dot{\alpha}} = \lambda_1^\alpha \tilde{\lambda}_1^{\dot{\alpha}} + \lambda_2^\alpha \tilde{\lambda}_2^{\dot{\alpha}} = \lambda^{I\alpha} \tilde{\lambda}_I^{\dot{\alpha}}$$

This naturally introduces the requisite SU(2) indices

$$M_n^{\{I_1 I_2 \cdots I_{2s_1}\},\cdots} = \lambda_1^{I_1\alpha_1} \lambda_1^{I_2\alpha_2} \cdots \lambda_1^{I_{2s_1}\alpha_{2s_1}} M_{n,\ \{\alpha_1\alpha_2\cdots\alpha_{2s_1}\}\cdots}$$

Let us consider three-point amplitude with one massless and two equal mass



$$M^h{}_{\{\alpha_1\alpha_2\cdots\alpha_{2S_1}\},\{\beta_1\beta_2\cdots\beta_{2S_2}\}}$$

GENERAL MASSIVE AMPLITUDES

We need two vectors to span the 2-d space: we have the massless spinor of the massless leg

$$Det(p^{\alpha\dot{\alpha}}) = 0 \qquad p^{\alpha\dot{\alpha}} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$$

$$(u_{\alpha}, v_{\alpha}) = (\lambda_{3,\alpha}, \epsilon_{\alpha\beta}\lambda_3^{\beta})$$

There is another variable that carry the opposite helicity weight of the massless leg

$$2p_2 \cdot p_3 = \langle 3|p_2|3] = 0 \longrightarrow x\lambda_3^{\alpha} = \frac{p_2^{\alpha\dot{\alpha}}\tilde{\lambda}_{3\dot{\alpha}}}{m}$$

This allows us to define the x factor which carries positive helicity

$$M^h{}_{\{\alpha_1\alpha_2\cdots\alpha_{2S_1}\},\{\beta_1\beta_2\cdots\beta_{2S_2}\}}$$

Three point amplitude is constructed from (x, **λ**, **ε**)

We have a parameterization of the three-point coupling that is purely kinematic in nature. For example for photon (h=1)

g-2 for electron and W

$$s = \frac{1}{2}: \qquad x\left(\epsilon_{\alpha\beta} + g_1\, x\frac{\lambda_\alpha\lambda_\beta}{m}\right)$$

$$s = 1: \qquad x\left(\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2} + g_1\epsilon_{\alpha_2\beta_2}\, x\frac{\lambda_{\alpha_1}\lambda_{\beta_1}}{m} + g_2\, x^2\frac{\lambda_{\alpha_1}\lambda_{\beta_1}\lambda_{\alpha_2}\lambda_{\beta_2}}{m^2}\right)$$

$$\vdots$$

$$s = 2: \qquad x\left(\epsilon^4 + g_1\epsilon^3\, x\frac{\lambda^2}{m} + g_2\,\epsilon^2 x^2\frac{\lambda^4}{m^2} + g_3\epsilon\frac{\lambda^6}{m^3} + g_4\frac{\lambda^8}{m^3}\right)$$

$$\vdots$$

THE SIMPLEST MASSIVE-AMPLITUDE

Let's consider simplest possible amplitude is given by a pure x term

$$M_3(q^{+1}, \mathbf{1}^s, \mathbf{2}^s) = x\, m\epsilon^{2s}, \quad M_3(q^{+2}, \mathbf{1}^s, \mathbf{2}^s) = x^2\, m\epsilon^{2s}$$

Or after putting back the external polarization (spinors, vectors, tensors ..)

$$M_3(q^{+1}, \mathbf{1}^s, \mathbf{2}^s) = x\, m \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}, \quad M_3(q^{+2}, \mathbf{1}^s, \mathbf{2}^s) = x^2\, m \left(\frac{\langle \mathbf{12} \rangle}{m}\right)^{2s}$$

Note that beyond spin-2 there are no Lagrangian for consistent fundamental higher-spin particles, but we have its interaction!

THE SIMPLEST MASSIVE-AMPLITUDE

To see what kind of interaction this describes, we compare this with amplitude from the 1-particle EFT

$$L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n}} S^{\mu_{2n+1}}$$

we find that

Guevara, Ochirov and Vines JHEP 1909, 056 (2019)
Chung, Huang, Kim and Lee, JHEP 1904, 156 (2019)

$$C_{\mathrm{S}^n} = 1 + \frac{n(n-1)}{4s} + \frac{n(n-1)(n^2-5n+10)}{32s^2} + \frac{n(n-1)(n^4-14n^3+71n^2-154n+144)}{384s^3} + \mathcal{O}(s^{-4})$$

It matches to C=1 in the large s limit!

The classical spin-limit corresponds to $\boxed{s \to \infty, \quad \hbar \to 0}$ with $\boxed{s\hbar}$ fix

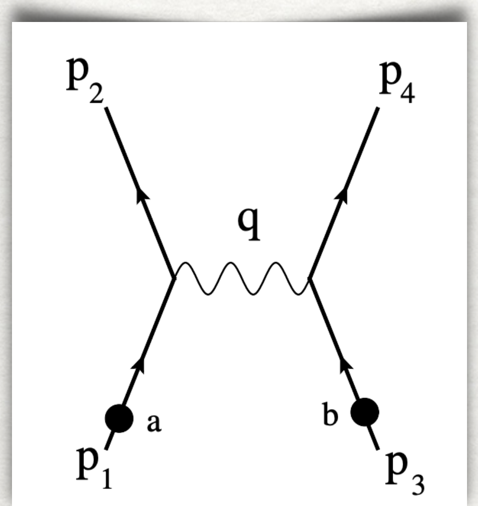
THE GENERAL 1 PM POTENTIAL

With the on-shell form the 1 PM spinning potential is straight forward:

Chung, Huang, Kim, Lee 2003.06600

$$M_s^{2\eta} = \frac{\kappa m x^{2\eta}}{2}\varepsilon_2^* \left[\sum_{n=0}^{2s} \frac{C_n}{n!}\left(-\eta \frac{q\cdot S}{m}\right)^n\right]\varepsilon_1 \quad \longrightarrow \quad W(\tau) = \sum_{n=0}^{\infty} \frac{C_n}{n!}\tau^n$$

The on-shell approach leads to a compact form for general spinning objects



$$V_{\text{1PM}}^{(\text{general})} = -\frac{4\pi G m_a^2 m_b^2}{E_a E_b}\int \frac{d^3\vec{q}}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{q^2}\left[\frac{1}{2}\sum_{s=\pm 1} e^{2s\theta} W_a(s\tau_a) W_b(s\tau_b)\right] U^{(a)} U^{(b)}$$

Hilbert space matching (Thomas-Wigner rotation)

$$\epsilon^\mu(p) = G(p;p_0)\epsilon(p_0)$$

$$\epsilon^{*\mu}(p_{\text{out}})\epsilon_\mu(p_{\text{in}}) = \epsilon^{*\mu}(p_{\text{in}})\left[G(p_{\text{out}};p_0)G(p_0;p_{\text{in}})\epsilon(p_{\text{in}})\right]_\mu$$

$$\longrightarrow \quad U_{\text{rotation}}^{(a)} = \exp\left[-i\left(\frac{m_b}{r_a E}\right)\varepsilon(q,u_a,u_b,a_a)\right], \qquad r_a \equiv 1 + \frac{E_a}{m_a}$$

	LO	NLO	NNLO	N^{3}LO
S^1	[22]	[22]	[28]	?
$S_a S_b$	[22]	[26]	[26]	-
S_a^2	[22]	[22]	[29]	-
S^3	[27]	[25]*	-	-
S^4	[27]	-	-	-

[22] M. Levi, Phys. Rev. D82 (2010) 104004
[25] R. Aoude, K. Haddad and A. Helset,2001.09164
[26] V. Vaidya, Phys. Rev. D91 (2015) 024017
[27] B. R. Holstein and A. Ross, 0802.0716
[28] R. A. Porto and I. Z. Rothstein Phys. Rev. Lett. 97 (2006) 021101
[29] R. A. Porto and I. Z. Rothstein, Phys. Rev. D78 (2008) 044012

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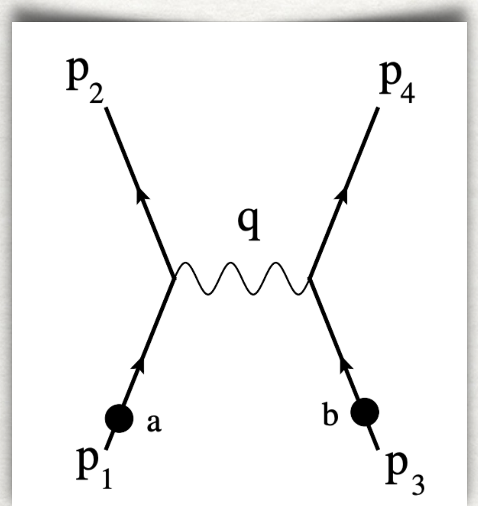


$$V_{1\mathrm{PM}}^{(\mathrm{general})} = -\frac{4\pi G m_a^2 m_b^2}{E_a E_b} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2} \left[\frac{1}{2} \sum_{s=\pm 1} e^{2s\theta} W_a(s\tau_a) W_b(s\tau_b) \right] U^{(a)} U^{(b)}$$

Thomas-Wigner rotation

$$\epsilon^\mu(p) = G(p; p_0)\epsilon(p_0) \qquad \epsilon^{*\mu}(p_{\mathrm{out}})\epsilon_\mu(p_{\mathrm{in}}) = \epsilon^{*\mu}(p_{\mathrm{in}}) \left[G(p_{\mathrm{out}}; p_0) G(p_0; p_{\mathrm{in}}) \epsilon(p_{\mathrm{in}}) \right]_\mu$$

$$U_{\mathrm{rotation}}^{(a)} = \exp\left[-i \left(\frac{m_b}{r_a E} \right) \varepsilon(q, u_a, u_b, a_a) \right] , \quad r_a \equiv 1 + \frac{E_a}{m_a}$$

Setting C_n=1 we find the complete 1 PM Kerr potential

Vines, Class. Quant. Grav. 35 (2018) no.8, 084002

$$V_{1\mathrm{PM}}^{(\mathrm{BH})} = -\frac{G m_a^2 m_b^2}{2 E_a E_b} \sum_{s=\pm 1} e^{2s\theta} \left| \vec{r} + s \frac{E(\vec{p} \times \vec{a}_0)}{m_a m_b \sinh\theta} - \frac{\vec{p} \times \vec{a}_a}{m_a r_a} - \frac{\vec{p} \times \vec{a}_b}{m_b r_b} \right|^{-1}$$

THE SIMPLEST MASSIVE-AMPLITUDE

Coming back to the Wilson coefficients

$$L_{SI} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \ldots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \ldots S^{\mu_{2n}} S^{\mu_{2n+1}}$$

The matching should be done with the inclusion of HM

$$\varepsilon_I^*(\mathbf{2}) \cdot \varepsilon_J(\mathbf{1}) = \varepsilon_I^*(\mathbf{1}) \left[e^{i\frac{\vec{q}}{m}\cdot\vec{K}} \right] \varepsilon_J(\mathbf{1}) = \varepsilon_I^*(\mathbf{1}) \left[\sum_{n=0}^{s} \frac{(-1)^n}{(2n)!} \left(\frac{\vec{q}\cdot\vec{K}}{m} \right)^{2n} \right] \varepsilon_J(\mathbf{1})$$
$$= \varepsilon_I^*(\mathbf{1}) \left[\sum_{n=0}^{s} \frac{(-1)^n (2s-2n)!}{(2s)!} \binom{s}{n} \left(\frac{q\cdot S}{m} \right)^{2n} \right] \varepsilon_J(\mathbf{1})$$

The ``physical" Wilson coefficient is that measured at infinity:

$$M_s^{2\eta} = \frac{\kappa m x^{2\eta}}{2} \varepsilon_I^*(\mathbf{1}) \left[\sum_{i=0}^{s} \frac{(-1)^i (2s-2i)!}{(2s)!} \binom{s}{i} \left(-\eta \frac{q\cdot S}{m} \right)^{2i} \right] \left[\sum_{j=0}^{2s} \frac{C_{\mathrm{S}^j}}{j!} \left(-\eta \frac{q\cdot S}{m} \right)^{j} \right] \varepsilon_J(\mathbf{1})$$
$$= \frac{\kappa m x^{2\eta}}{2} \varepsilon_I^*(\mathbf{1}) \left[\sum_{n=0}^{2s} \frac{C_{\mathrm{S}^n_{eff}}}{n!} \left(-\eta \frac{q\cdot S}{m} \right)^{n} \right] \varepsilon_J(\mathbf{1}),$$

$$\frac{C_{\mathrm{S}^m_{eff}}}{m!} = \sum_{i=0}^{\lfloor m/2 \rfloor} \frac{(-1)^i (2s-2i)!}{(2s)!} \binom{s}{i} \frac{C_{\mathrm{S}^{m-2i}}}{(m-2i)!}$$

$\rightarrow$ Effective Wilson coefficient for minimal coupling is 1

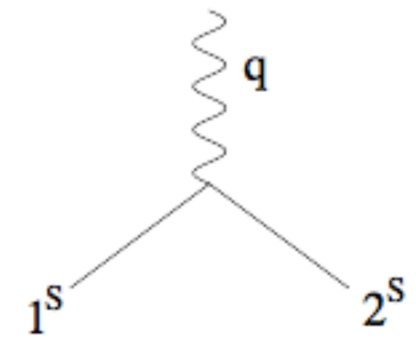
Chung, Huang, Kim, 1908.08463
Aoude, Haddad, Helset, 2001.09164
Bern, Luna, Roiban, Shen, Zeng, 2005.03071

Minimally coupled higher-spin particles do exists, they are Kerr black holes

- What is minimal coupling telling us about Kerr?

THE ON-SHELL VIEW POINT (DOUBLE COPY)

First, we see that gravitational minimally coupling, is simply a double copy of electric-magnetic minimal coupling !



$$= g(xm)^h \frac{\langle \mathbf{12} \rangle^{2S}}{m^{2S}}$$

where $h = (1, 2)$ and $g = (\frac{\kappa}{2}, \frac{e}{\sqrt{2}})$

This imply that black holes are a double copy of some electrically charged object. Indeed such a relation was shown for the Kerr-Schild form of the metric:

$$g_{\mu\nu} = g^0_{\mu\nu} + k_\mu k_\nu \phi(r), \quad A^{\mu a} = c^a k^\mu \phi(r)$$

Monteiro, O'Connell, White, JHEP 1412 (2014) 056

Schwarzschild sol <-> Coloumb potential

Kerr sol <-> rotating charged disk with radius a

THE ON-SHELL VIEW POINT(EXPONENTIALS)

Minimal coupling is really a reflection that the spin is completely "intrinsic"

Arkani-Hamed, Huang, O'Connell, JHEP 01 (2020) 046

$$h=+1: \quad \sqrt{2}ie_2 x\frac{\langle \mathbf{2}\mathbf{2}'\rangle^{2S}}{m^{2S-1}}, \quad h=-1: \quad \sqrt{2}ie_2\frac{1}{x}\frac{[\mathbf{2}\mathbf{2}']^{2S}}{m^{2S-1}}$$

The only spin-pieces is contained in the external wave functions (the λs). Indeed 2′ is related to 2 by a boost

.

$$|\mathbf{2}'\rangle = |\mathbf{2}\rangle + \frac{1}{8}\omega_{\mu\nu}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)|\mathbf{2}\rangle$$

$$\omega_{\mu\nu} = -\frac{1}{m_2^2}(p_{2\mu}q_\nu - p_{2\nu}q_\mu)$$

Each spinor bracket can be written in terms of spin operators

$$\frac{1}{m_2}\langle \mathbf{2}\mathbf{2}'\rangle = \mathbb{I} + \frac{1}{2m_2^2}\hbar\langle \mathbf{2}|\not{q}|\mathbf{2}] = \mathbb{I} + \frac{1}{2Sm_2}\bar{q}\cdot s$$

$$s^\mu = -\frac{1}{2m_2}\epsilon^{\mu\nu\rho\sigma}p_{2\nu}J_{\rho\sigma}$$

Thus the spinor brackets becomes operators on the Hilbert-space with

$$h=\pm 1: \quad \lim_{S\to\infty} ie_2\sqrt{2}m\, x^{\pm 1}\left(\mathbb{I}\pm\frac{\bar{q}\cdot s}{2Sm}\right)^{2S} = ie_2\sqrt{2}m x^{\pm 1}e^{\pm \bar{q}\cdot a}$$

$$a = \frac{s}{m}$$

Taking S-> infinity

The spin factor exponentiates!!

THE ON-SHELL VIEW POINT (COMPLEX SHIFT)

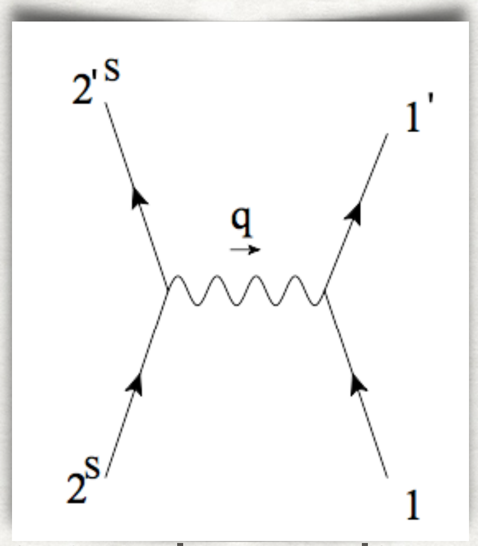
For minimal coupling the spin-dependence exponentiates in the large spin-limit

$$h=+1: \quad \frac{e_2}{\sqrt{2}} x \frac{\langle \mathbf{2 2'}\rangle^S}{m^{S-1}}, \quad h=-1: \quad \frac{e_2}{\sqrt{2}} \frac{1}{x} \frac{[\mathbf{2 2'}]^S}{m^{S-1}}$$

$$\lim_{S\to\infty} \frac{e_2}{\sqrt{2}} m\, x \left(\mathbb{I} \pm \frac{1}{Sm} \bar{q}\cdot s\right)^S = \frac{e_2}{\sqrt{2}} m x e^{\pm \bar{q}\cdot a}$$

Note that q is the transverse momenta, and hence after Fourier transform, relates to impact parameter. So the difference between s=0 and spinning case

This implies a complex-shift relating Kerr to Schwarzschild in the context of physical observables. For exp the impulse imparted on a probe



$$\Delta p_1^\mu = \frac{1}{4m_1 m_2} \int \hat{d}^4 q\, \hat{\delta}(q\cdot u_1)\hat{\delta}(q\cdot u_2) e^{-iq\cdot b}\, iq^\mu\, M_4(1,2\to 1',2')\,|_{q^2\to 0}$$

Kosower, Maybee, O'Connell JHEP 02 (2019) 137

Once again in the q^2=0 limit the amplitude factorizes to our minimal coupling!

$$M_4(1,2\to 1',2')\,|_{q^2\to 0} = -\frac{e_1 e_2}{2\bar{q}^2}\left(\frac{x_{11'}}{x_{22'}} e^{-\bar{q}\cdot a} + \frac{x_{22'}}{x_{11'}} e^{\bar{q}\cdot a}\right)$$

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$$\lim_{S\to\infty}\frac{e_2}{\sqrt{2}}m\,x\left(\mathbb{I}\pm\frac{1}{Sm}\bar{q}\cdot s\right)^S=\frac{e_2}{\sqrt{2}}mxe^{\pm\bar{q}\cdot a}$$

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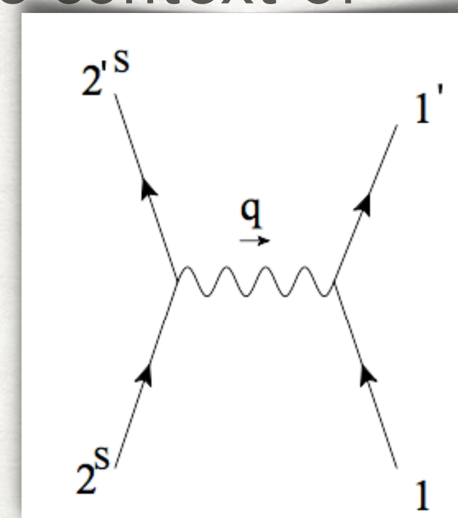
$$\Delta p_1^\mu=\frac{1}{4m_1m_2}\int \hat{d}^4q\,\hat{\delta}(q\cdot u_1)\hat{\delta}(q\cdot u_2)e^{-iq\cdot b}iq^\mu\,M_4\left(1,2\to 1',2'\right)|_{q^2\to 0}$$



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The Kerr sol is a complex shift of the Schwarzschild sol!

THE JANIS NEWMAN SHIFT

This is simply the mysterious Janis Newman shift!

Once again consider BH solutions in the Kerr-Schild form

$$g_{\mu\nu} = g^0_{\mu\nu} + k_\mu k_\nu \phi(r),$$

$$\text{Schwarzschild}: \quad \phi_{\text{Sch}}(r) = \frac{r_0}{r}, \quad k_\mu = (1, \hat{r})$$

$$\text{Kerr}: \quad \phi_{\text{Kerr}}(r) = \frac{r_0 r}{r^2 + a^2 \cos^2\theta}, \quad k_\mu = (1, \hat{r})$$

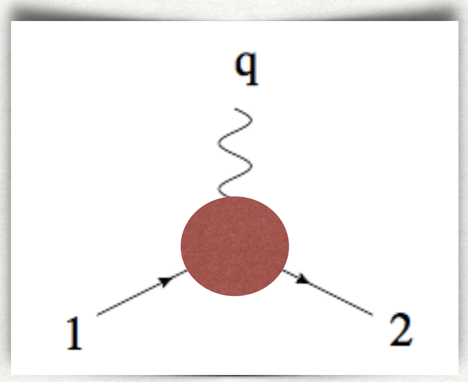
The Kerr solution is simply a complex shift of Schwaz-Schild!

Newman, Janis
J. Math. Phys. 6, 915 (1965)

$$\begin{aligned}\phi_{\text{Sch}}(r)|_{r\to r+ia\cos\theta} &= \frac{r_0}{2}\left(\frac{1}{r} + \frac{1}{\bar{r}}\right)\Big|_{r\to r+ia\cos\theta} \\ &= \frac{r_0 r}{r^2 + a^2\cos^2\theta} = \phi_{\text{Kerr}}(r)\,.\end{aligned}$$

This is precisely the shift induced by the exponentiation!

THE ON-SHELL VIEW POINT (DUALITIES)



Is there other ways of ``exponentiating" minimal coupling ?

$$x \rightarrow x f(q^\mu, s_\mu, p_\mu)$$

Since $p \cdot q = p \cdot s = 0$ already have the spin-shift $x \rightarrow x\, e^{\frac{q \cdot s}{m}}$ the only other possibility is a complex phase shift! $x \rightarrow x e^{i\theta}$

For E&M, a complex charge describes the coupling of a dyon (objects with both magnetic and electric charge)

A complex charge describes the coupling of a dyon (see Donal's talk)

$$x \to x e^{i\theta}$$

By double copy, there must be a corresponding gravitation solution:

$$x^2 \to x^2 e^{i2\theta}$$

The double copy of a dyon is Taub-Nut!

TAUB-NUT FROM DYON

The dyon impulse

$$\Delta p_1^\mu = -i\frac{q_e|C|}{2}\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}$$
$$\left((\cosh w+\sinh w)e^{-i\theta}+(\cosh w-\sinh w)e^{+i\theta}\right)$$
$$= -i\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}q_e\left(Q\cosh w - iG\sinh w\right)$$

Double copying then yields

$$i\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}\left(Q_G\cosh 2w - iG_G\sinh 2w\right)$$

Indeed this matches with the impulse computed in Taub-Nut space-time at 1 PM

$$ds^2 = -\frac{A}{B}(dt+2n\cos\theta d\phi)^2 + B(d^2\theta+\sin^2\theta d^2\phi)+\frac{B}{A}d^2r, \qquad A = r^2-2mr-\ell^2,\ B = r^2+\ell^2$$

At 1 PM, we are considering the linear approximation,

$$h_{00} = \frac{2m}{r},$$
$$h_{0i} = 4\ell\cos\theta d\varphi = 4\ell\frac{\cos\theta}{x^2+y^2}\left(-ydx+xdy\right),$$
$$h_{ij} = \frac{2mx_ix_j}{r^3}.$$

$\longrightarrow$

$$\Gamma^i_{00} = -\frac{1}{2}\partial^i h_{00},$$
$$\Gamma^i_{0j} = \frac{1}{2}\eta^{ik}\left(\partial_k h_{0j} - \partial_j h_{0k}\right),$$
$$\Gamma^i_{jk} = \frac{2mx^i}{r^3}\delta_{jk} - \frac{3mx^ix_jx_k}{r^5}.$$

TAUB-NUT FROM DYON

The dyon impulse

See Donal's Talk

$$\Delta p_1^\mu = -i\frac{q_e|C|}{2}\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}$$
$$\left((\cosh w + \sinh w)e^{-i\theta} + (\cosh w - \sinh w)e^{+i\theta}\right)$$
$$= -i\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}q_e\left(Q\cosh w - iG\sinh w\right)$$

Double copy then yields

$$i\int d^4\bar{q}\,\delta(\bar{q}\cdot u_1)\delta(\bar{q}\cdot u_2)e^{-i\bar{q}\cdot\Pi b}\frac{\bar{q}^\mu}{\bar{q}^2}\left(Q_G\cosh 2w - iG_G\sinh 2w\right)$$

Indeed this matches with the impulse computed in Taub-Nut space-time at 1 PM

At 1 PM, we are considering the linear approximation,

$$\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau}.$$

$$\rightarrow$$

$$-\frac{d\vec{v}}{d\tau} = \left(\frac{2\gamma^2-1}{\gamma}\right)\vec{\nabla}\phi - 4\gamma\,\vec{v}\times(\vec{\nabla}\times\vec{A})$$
$$= \left(\frac{2\gamma^2-1}{\gamma}\right)\vec{E} - 4\gamma\,\vec{v}\times\vec{B}.$$

$$\phi \equiv -\frac{1}{2}h_{00}$$
$$A_i = -\frac{1}{4}h_{0i}$$

$$\rightarrow$$

$$\frac{d\vec{v}}{d\tau} = -\left(\frac{\cosh 2w}{\cosh w}\right)\vec{E} + 2\left(\frac{\sinh 2w}{\sinh w}\right)\vec{v}\times\vec{B}.$$

TAUB-NUT FROM SCHWARZSCHILD

The double copy of a dyon is Taub-Nut!

$$x^2 \to x^2 e^{i2\theta}$$

The phase transformation indicates that
1. The Taub-Nut metric is again some complex shift acting on the Schwarzschild metric
2. The shift has an interpretation as a electric-magnetic duality transformation.

TAUB-NUT FROM E&M DUALITY

Not long after Janis and Newman's realization of Kerr/Schwarzschild correspondence, Talbot generalized the complex shift to obtain Taub-NUT

$$u = u' - ia\cos\theta + 2i\ell\log\sin\theta, \qquad r = r' + ia\cos\theta - i\ell,$$

$$m = m' - i\ell.$$

The complex shift introduced by Talbot can be generated by a BMS super translation:

The expansion of asymptotically flat metrics around future null infinity

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\ &\quad + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 - 2U_z dudz - 2U_{\bar{z}}dud\bar{z} \\ &\quad + \ldots, \end{aligned}$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$U_z = -\frac{1}{2}D^z C_{zz}$$

The metric is invariant under BMS translation

$$\xi_f = f\partial_u - \frac{1}{r}\left(D^z f\partial_z + D^{\bar{z}} f\partial_{\bar{z}}\right) + D^z D_z f\partial_r + \mathcal{O}\left(r^{-2}\right), \qquad f = f(z,\bar{z}).$$

For real f BMS translation is a symmetry of the metric

The associated super translation charge is given as

$$T(f) = \frac{1}{4\pi G}\int_{\mathcal{I}^+_-} d^2z\,\gamma_{z\bar{z}} f(z,\bar{z}) m_B.$$

TAUB-NUT FROM E&M DUALITY

The expansion of asymptotically flat metrics around future null infinity

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 - 2U_z dudz - 2U_{\bar{z}}dud\bar{z} + \ldots,$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

$$U_z = -\frac{1}{2}D^z C_{zz}$$

The Taub-NUT metric is Uri Kol, Massimo Porrati 1907.00990

$$C_{zz} = i\ell\gamma_{z\bar{z}}\frac{1+|z|^4}{z^2}$$

This is generated from Schwarzschild via an imaginary super translation

$$f(z,\bar{z}) = -2i\ell\log\frac{2(1+|z|^2)}{|z|} = 2i\ell\log\sin\theta. \longrightarrow (2i\ell\log\sin\theta)\partial_u - (i\ell)\partial_r - \frac{4i\ell}{r}\cot\theta d\theta + \mathcal{O}\left(r^{-2}\right)$$

TAUB-NUT FROM E&M DUALITY

2. The shift has an interpretation as a electric-magnetic duality transformation.

Huang, Kol, O'Connell, 1911.06318

This is generated from an imaginary super translation

$$f(z,\bar{z}) = -2i\ell\log\frac{2(1+|z|^2)}{|z|} = 2i\ell\log\sin\theta.$$ $\longrightarrow$ $$(2i\ell\log\sin\theta)\partial_u - (i\ell)\partial_r - \frac{4i\ell}{r}\cot\theta d\theta + \mathcal{O}\left(r^{-2}\right)$$

Let us see what happens to the charges. It was shown by Kol and Porrati that the super translation charge is accompanied by a dual charge

$$\mathcal{M}(\varepsilon) = \frac{i}{16\pi G}\int_{\mathcal{I}^+_-} d^2z\,\varepsilon(z,\bar{z})\gamma^{z\bar{z}}\left(D_{\bar{z}}^2 C_{zz} - D_z^2 C_{\bar{z}\bar{z}}\right)$$

Under the super translation, the T charge deforms as

$$T(\varepsilon) \to T(\varepsilon) + \frac{1}{4\pi G}\int_{\mathcal{I}^+_-} d^2z\,\varepsilon(z,\bar{z})\gamma^{z\bar{z}} D_z^2 D_{\bar{z}}^2 f(z,\bar{z}),$$

using $D_z^2 D_{\bar{z}}^2 f = -i\ell\gamma_{z\bar{z}}^2$ we find $T(\varepsilon) \to T(\varepsilon) - i\frac{\ell}{4\pi G}\int_{\mathcal{I}^+_-} d^2z\,\gamma_{z\bar{z}}\varepsilon(z,\bar{z}).$

or

$$T(\varepsilon) \to T(\varepsilon) - i\mathcal{M}_{\mathrm{NUT}}(\varepsilon).$$

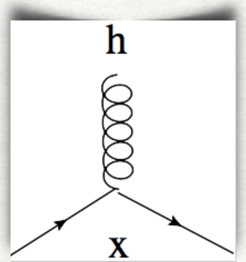
Thus we see that the imaginary shift rotates the super-translation charge with the dual supertranslation

SUMMARY

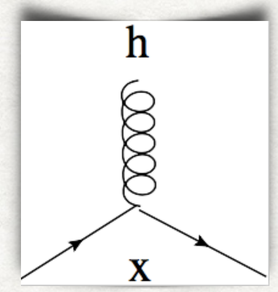
We have seen that properties of BH solutions can now be cleanly cast into on-shell elements providing convenient basis to manifest the simplicity of BHs.

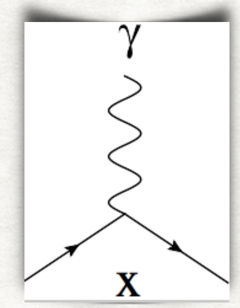
See Donal's Talk

Schwarzschild

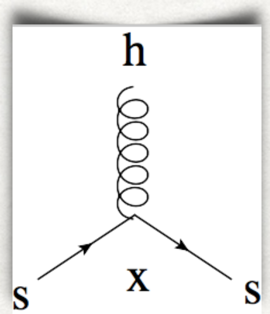


Reissner-Nordstorm





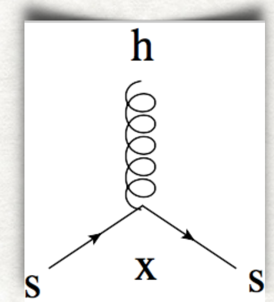
Kerr

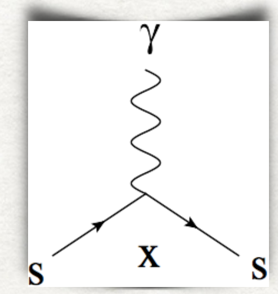


Guevara, Ochirov, Vines, JHEP 1909 (2019) 056

Arkani-Hamed, Huang, O'Connell, JHEP 01 (2020) 046

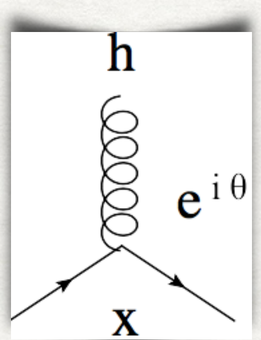
Kerr Newman





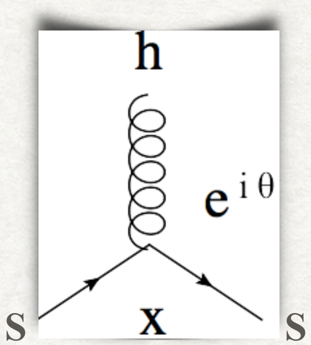
Moynihan, JHEP 01 (2020) 014
Chung, Huang, Kim, 1911.12775

Taub-NUT



Huang, Kol, O'Connell, 1911.06318

Kerr Taub-NUT



Emond, Kol, Moynihan ,O'Connell, in progress

Quantum Corrections to Minimal coupling

Recall that the non-minimal couplings corresponds to anomalous moments

$$s = \frac{1}{2}: \qquad x\left(\epsilon_{\alpha\beta} + g_1\, x\frac{\lambda_\alpha\lambda_\beta}{m}\right)$$

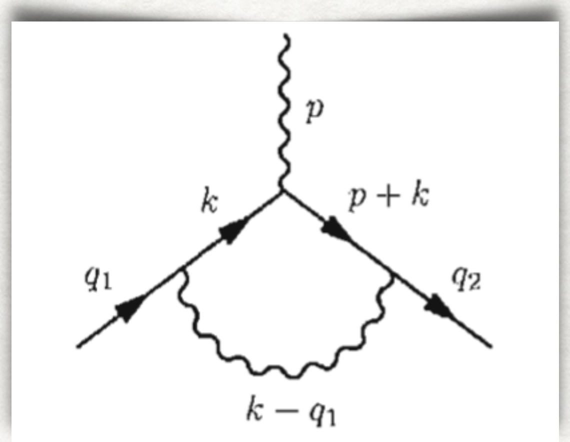
$$s = 1: \qquad x\left(\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2} + g_1\epsilon_{\alpha_2\beta_2}\, x\frac{\lambda_{\alpha_1}\lambda_{\beta_1}}{m} + g_2\, x^2\frac{\lambda_{\alpha_1}\lambda_{\beta_1}\lambda_{\alpha_2}\lambda_{\beta_2}}{m^2}\right)$$

$$\vdots$$

$$s = 2: \qquad x\left(\epsilon^4 + g_1\epsilon^3\, x\frac{\lambda^2}{m} + g_2\,\epsilon^2 x^2\frac{\lambda^4}{m^2} + g_3\epsilon\frac{\lambda^6}{m^3} + g_4\frac{\lambda^8}{m^3}\right)$$

$$\vdots$$

g-2 for electron and W

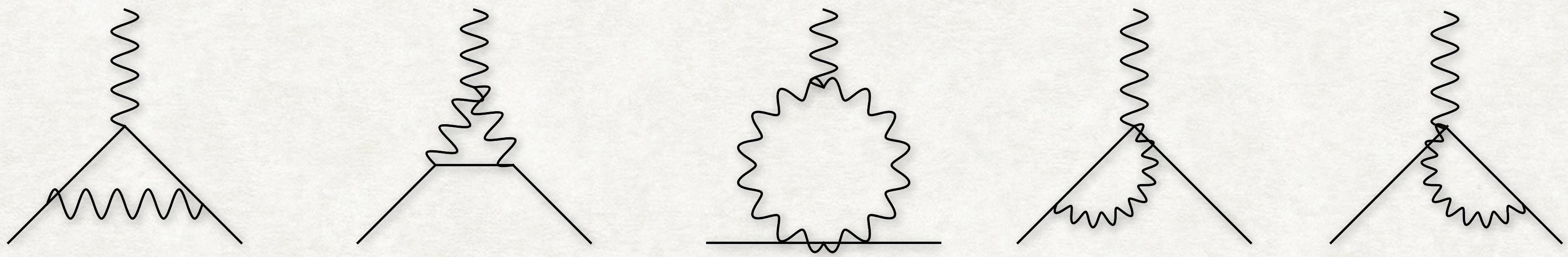


$g_1 = 0.$

is there anomalous moments for gravity?

Chen, Chung, Huang, Kim, in progress

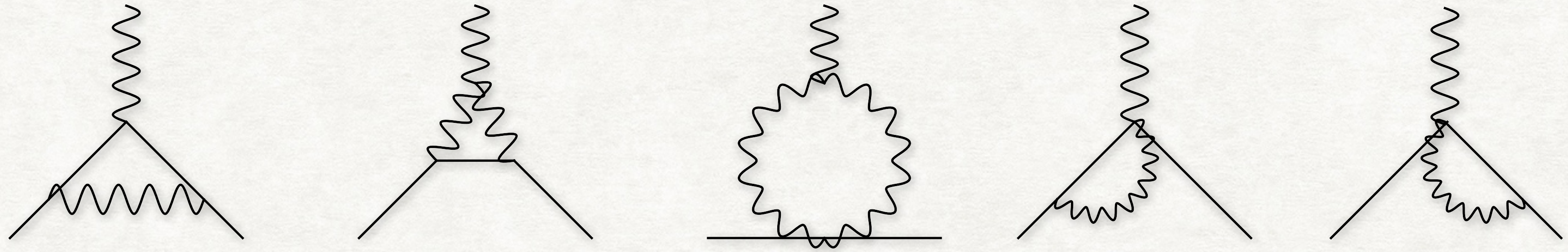
Spin-1/2



g_1 $-\frac{1}{1536\epsilon}-\frac{145}{9216}$ $\frac{5}{192\epsilon}+\frac{83}{1152}$ 0 $-\frac{13}{1024\epsilon}-\frac{173}{6144}$ $=0$

Quantum Corrections to Minimal coupling

Spin-1

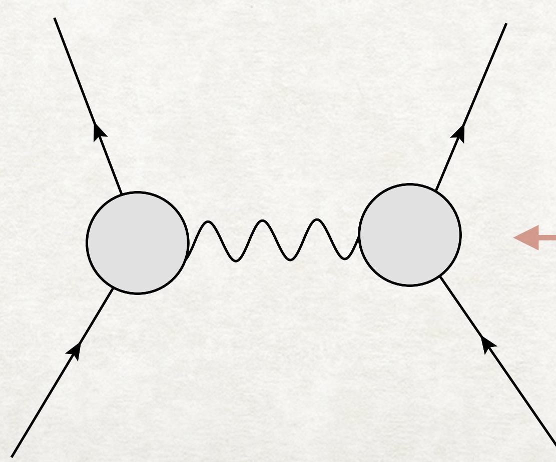


g_1	$-\frac{1}{768\epsilon}-\frac{11}{576}$	$\frac{5}{256\epsilon}+\frac{9}{128}$	0	$-\frac{7}{384\epsilon}-\frac{59}{1152}$	=0
g_2	$\frac{1}{96\epsilon}+\frac{203}{4608}$	$\frac{3}{256}$	0	$-\frac{1}{96\epsilon}-\frac{11}{288}$	= 9/512 finite

There are UV divergences for minimal coupling

$$m^2 x \to m^2 x \left[1+\frac{m^2}{M_{pl}^2}\left(\frac{1}{\epsilon}+\log+\delta\right)\right]$$

$$0 \to \frac{9}{64} m^2 x^3 \frac{\lambda^4}{m^2}\frac{m^2}{M_{pl}^2}$$



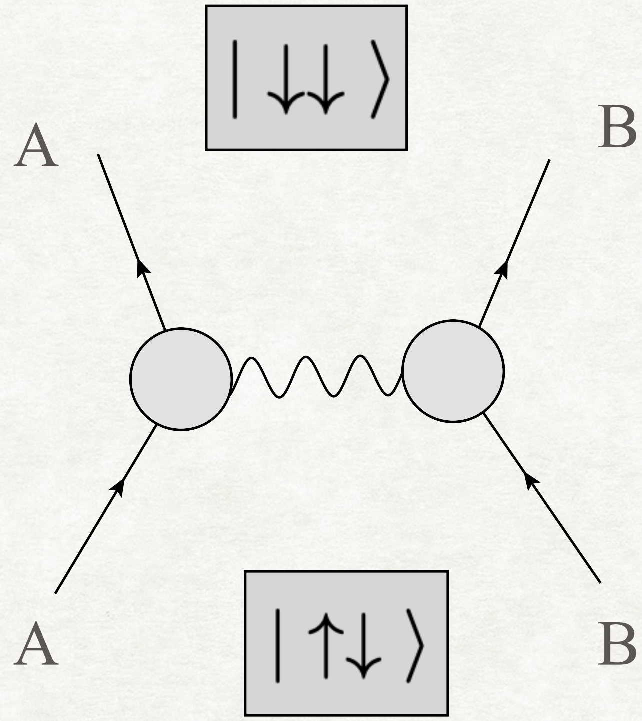
$$\boxed{\left(1+\frac{m^2}{M_{pl}^2}\frac{9}{64}\right)\frac{(q\cdot S)^2}{m^2}}$$

Black hole pairs as maximal entangled state

What is the principle that selected minimal coupling ?

The scattering amplitude also represents a transition in spin space

Cervera-Lierta, Latorre, Rojo, Rottoli, *SciPost Phys*. 3 (2017) 5, 036

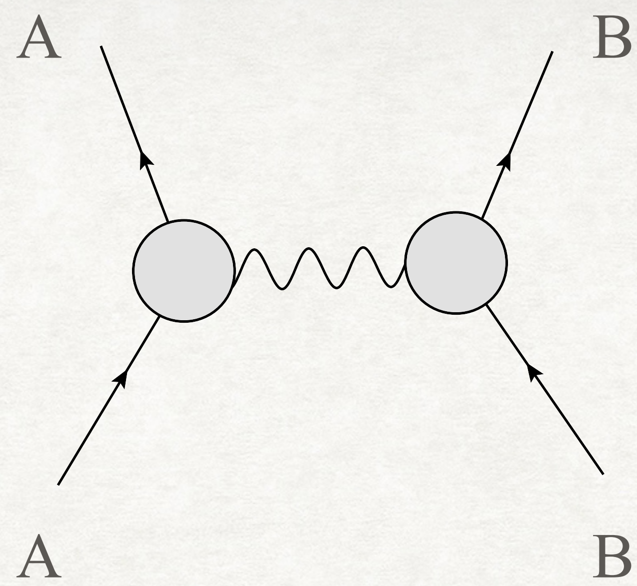


Take the spin 1/2 interaction,

$$|in\rangle = |\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

The scattering amplitude is the interaction part of the S-matrix, S=1+ i T , which generates the entangled outstate

$$\begin{aligned}|out\rangle = \mathcal{S}|in\rangle = |in\rangle &+ M(A^aB^bA^{+\frac{1}{2}}B^{+\frac{1}{2}})|\uparrow\uparrow\rangle + M(A^aB^bA^{+\frac{1}{2}}B^{-\frac{1}{2}})|\uparrow\downarrow\rangle \\ &+ M(A^aB^bA^{-\frac{1}{2}}B^{+\frac{1}{2}})|\downarrow\uparrow\rangle + M(A^aB^bA^{-\frac{1}{2}}B^{-\frac{1}{2}})|\downarrow\downarrow\rangle\end{aligned}$$



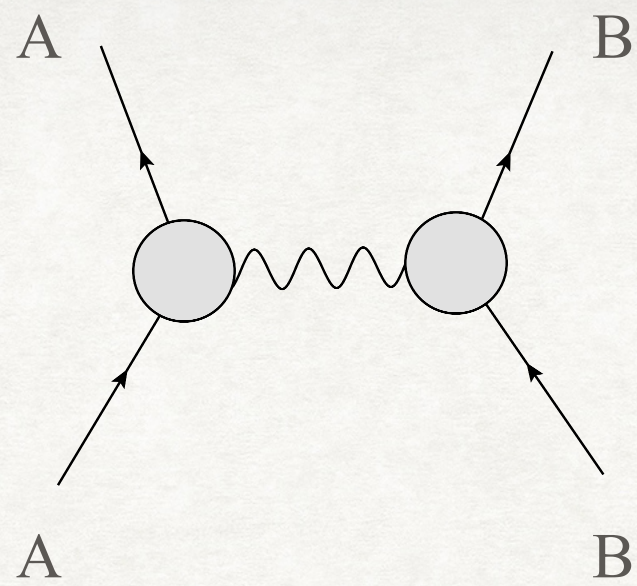
The scattering amplitude is the interaction part of the S-matrix, S=1+ i T , which generates the entangled outstate

$$\begin{aligned}|out\rangle = \mathcal{S}|in\rangle = |in\rangle + M(A^a B^b A^{+\frac{1}{2}} B^{+\frac{1}{2}})|\uparrow\uparrow\rangle + M(A^a B^b A^{+\frac{1}{2}} B^{-\frac{1}{2}})|\uparrow\downarrow\rangle \\ + M(A^a B^b A^{-\frac{1}{2}} B^{+\frac{1}{2}})|\downarrow\uparrow\rangle + M(A^a B^b A^{-\frac{1}{2}} B^{-\frac{1}{2}})|\downarrow\downarrow\rangle\end{aligned}$$

We can measure the magnitude of the entanglement, by computing the Von Neumann entropy. Begin with the reduced density matrix

$$\rho_A = \sum_B \langle B|out\rangle\langle out|B\rangle$$

The Von Neumann entropy is then $S = -tr(\rho_A \log \rho_A)$ which is maximally Entangled if evaluate to ln 2.



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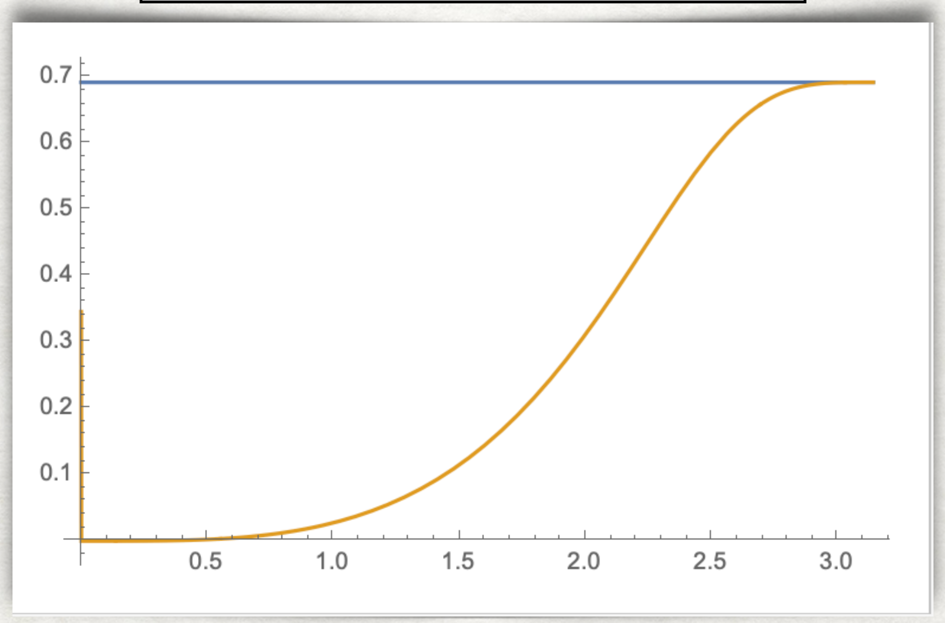

SUMMARY

- **We have seen that in terms of on-shell basis, properties of BH solutions are cleanly captured**

  $\longrightarrow$ **they are kinematically minimal**
- **The simplicity in the on-shell basis reflect hidden relations for the classical solutions:**

  double copy, complex shifts, duality transformations
- **These relations are in fact non-perturbative**