Higher-Loop Zeros in EFT Anomalous Dimension Matrices

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Zoomplitudes 2020

Z. B., J. Parra-Martinez and E. Sawyer, arXiv:1910.05831 and
arXiv:2005.xxxxx

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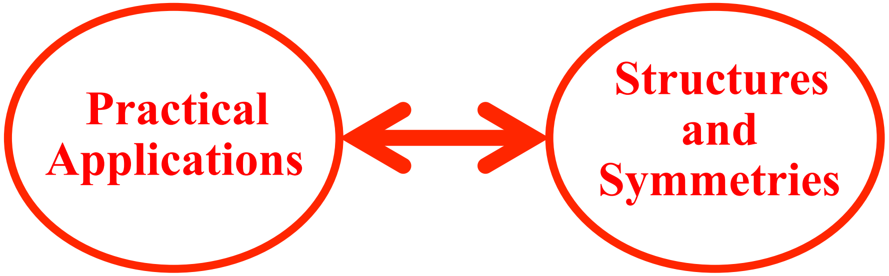
Outline

The two pillars of our field:

1. **Finding new structures and symmetries in scattering amplitudes.**
2. **Helping our friends in other fields solve nontrivial problems of physical interest.**



We are hearing a lot about both of these sides of amplitudes.

A good example of practical applications are EFT's for BSM physics. Also structures and symmetries.

See talks from Y. Shadmi and N. Craig

Here I will discuss I want to talk about EFT non-renormalization theorems.

Standard Model EFT

Buchmuller and Wyler; Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek

- **As we are finding out, it is difficult to find BSM physics at the LHC.**
- **The most popular top down models for new physics haven't been found.**
- **Need open mind: quantified by constructing EFT's for BSM physics.**
- **For good reason SMEFT and its cousin HEFT are becoming more popular.**

$$\mathcal{L}_{\mathrm{SM}} + \Delta\mathcal{L} \qquad \Delta\mathcal{L} = \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)}$$

For example dimension 6 operators

At dimension 6: 59 independent operators, not including flavor indices

$F^3, \quad \phi^2 F^2, \quad D^2\phi^4, \quad \psi^4,$ etc

Basic idea is simple: Parametrize new physics using EFTs

Here I will discuss a simplified version of SMEFT to discuss some generic properties.

Anomalous Dimensions

$$\mathcal{L}_{\rm SM} + \Delta\mathcal{L} \qquad \Delta\mathcal{L} = \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)}$$

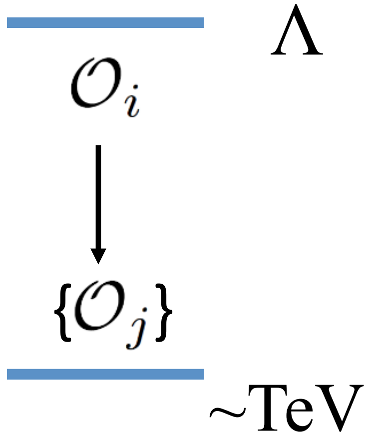
dimension 6 operators

- **At dimension 6: 59 independent operators, not including flavor indices**
- **Operator mixing controlled by the anomalous dimension matrix, 3481 entries**
- **In impressive work, one-loop dim 6 anomalous dimension matrix computed by Alonso, Jenkins, Manohar and Trott**

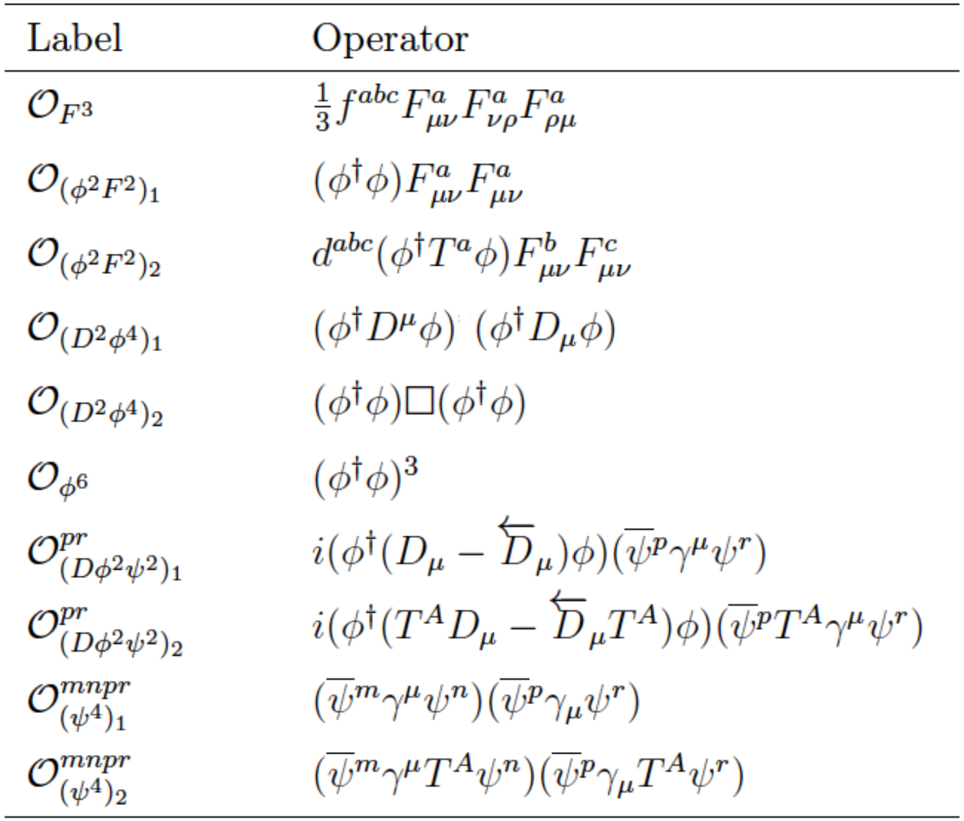
determines operator mixing

$$16\pi^2\frac{\partial c_i}{\partial\log\mu} = \gamma_{ij}^{\rm UV} c_j$$

Gives insight into how experimental constraints from one operator inform us about coefficients of other operators.



Dimension Six Operators



Label	Operator
$\mathcal{O}_{F^3}$	$\frac{1}{3} f^{abc} F^a_{\mu\nu} F^a_{\nu\rho} F^a_{\rho\mu}$
$\mathcal{O}_{(\phi^2 F^2)_1}$	$(\phi^\dagger \phi) F^a_{\mu\nu} F^a_{\mu\nu}$
$\mathcal{O}_{(\phi^2 F^2)_2}$	$d^{abc} (\phi^\dagger T^a \phi) F^b_{\mu\nu} F^c_{\mu\nu}$
$\mathcal{O}_{(D^2 \phi^4)_1}$	$(\phi^\dagger D^\mu \phi) (\phi^\dagger D_\mu \phi)$
$\mathcal{O}_{(D^2 \phi^4)_2}$	$(\phi^\dagger \phi) \Box (\phi^\dagger \phi)$
$\mathcal{O}_{\phi^6}$	$(\phi^\dagger \phi)^3$
$\mathcal{O}^{pr}_{(D\phi^2\psi^2)_1}$	$i(\phi^\dagger (D_\mu - \overleftarrow{D}_\mu) \phi)(\overline{\psi}^p \gamma^\mu \psi^r)$
$\mathcal{O}^{pr}_{(D\phi^2\psi^2)_2}$	$i(\phi^\dagger (T^A D_\mu - \overleftarrow{D}_\mu T^A) \phi)(\overline{\psi}^p T^A \gamma^\mu \psi^r)$
$\mathcal{O}^{mnpr}_{(\psi^4)_1}$	$(\overline{\psi}^m \gamma^\mu \psi^n)(\overline{\psi}^p \gamma_\mu \psi^r)$
$\mathcal{O}^{mnpr}_{(\psi^4)_2}$	$(\overline{\psi}^m \gamma^\mu T^A \psi^n)(\overline{\psi}^p \gamma_\mu T^A \psi^r)$

List of operators we consider here

Closely related to operators of SMEFT.

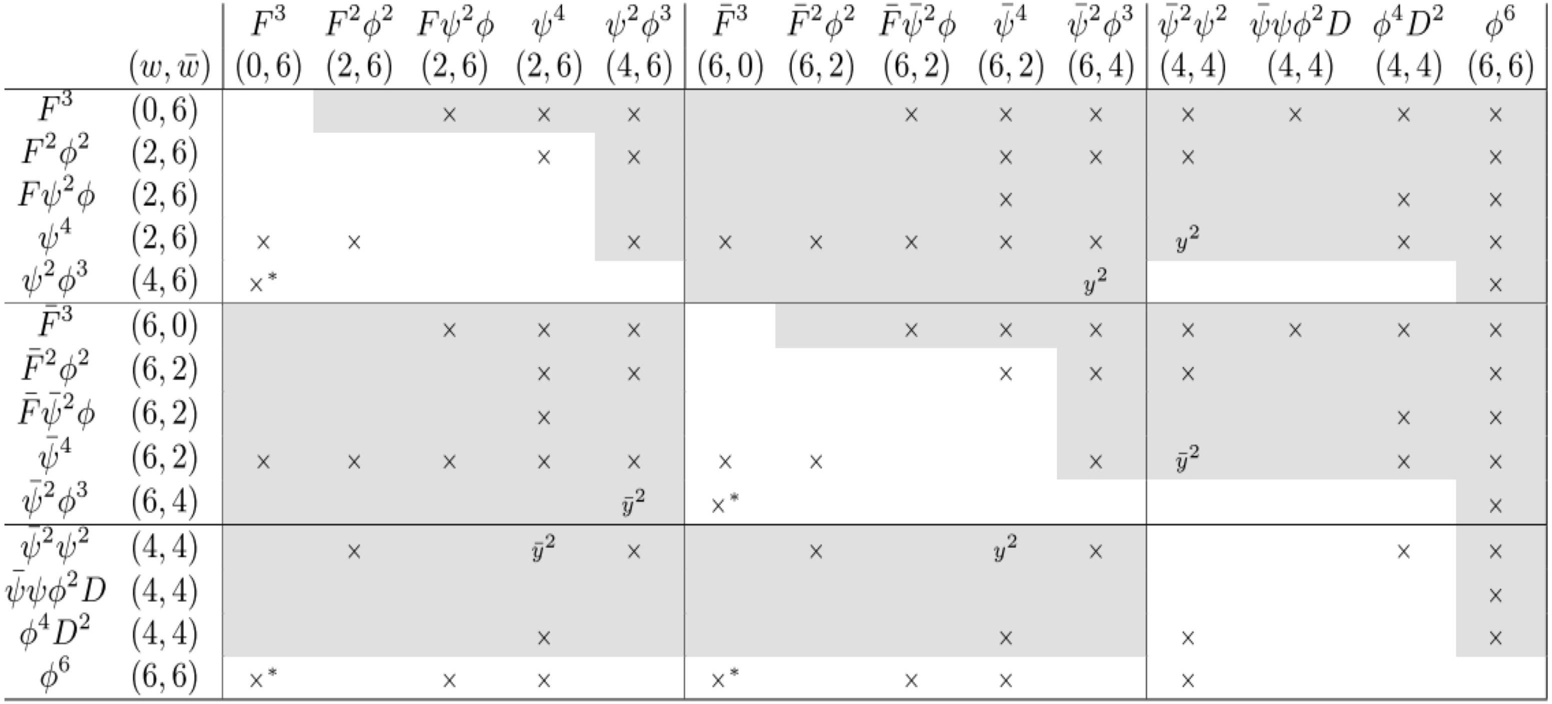
For our study we use simplified version of SMEFT. Dirac Fermions, no masses, no Yukawas. Same ideas apply to any EFT. Verified our calculations again Jenkins, Manohar et al. Significant overlap.

Zeros in SMEFT

Unexpected zeros in one-loop anomalous dimension matrix
Followed a pattern analogous to susy.
Explanations from helicity and angular momentum

Alonso, Jenkins, Manohar, Trott;
Elias-Miro, Espinoza, Pamarol

Cheung and Shen
Jaing, Shu, Xiao, Zheng



	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$		▨	▨ ×	▨ ×	▨ ×	▨	▨	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×
$F^2\phi^2$	$(2,6)$				×	▨ ×	▨	▨	▨	▨ ×	▨ ×	▨ ×	▨	▨	▨ ×
$F\psi^2\phi$	$(2,6)$					▨	▨	▨	▨	▨ ×	▨	▨	▨	▨ ×	▨ ×
ψ^4	$(2,6)$	×	×			▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ y^2	▨	▨ ×	▨ ×
$\psi^2\phi^3$	$(4,6)$	$\times^*$					▨	▨	▨	▨	▨ y^2				▨ ×
$\bar{F}^3$	$(6,0)$	▨	▨	▨ ×	▨ ×	▨ ×		▨	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×
$\bar{F}^2\phi^2$	$(6,2)$	▨	▨	▨	▨ ×	▨ ×				×	▨ ×	▨ ×	▨	▨	▨ ×
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$	▨	▨	▨	▨ ×	▨					▨	▨	▨	▨ ×	▨ ×
$\bar{\psi}^4$	$(6,2)$	▨ ×	▨ ×	▨ ×	▨ ×	▨ ×	×	×			▨ ×	▨ $\bar{y}^2$	▨	▨ ×	▨ ×
$\bar{\psi}^2\phi^3$	$(6,4)$	▨	▨	▨	▨	▨ $\bar{y}^2$	$\times^*$								▨ ×
$\bar{\psi}^2\psi^2$	$(4,4)$	▨	▨ ×	▨	▨ $\bar{y}^2$	▨ ×	▨	▨ ×	▨	▨ y^2	▨ ×			×	▨ ×
$\bar{\psi}\psi\phi^2 D$	$(4,4)$	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨				▨ ×
$\phi^4 D^2$	$(4,4)$	▨	▨	▨	▨ ×	▨	▨	▨	▨	▨ ×	▨	×			▨ ×
ϕ^6	$(6,6)$	$\times^*$		×	×		$\times^*$		×	×		×			

Grey are zeros. x means trivial zero.

from Cheung and Shen 1505.01844

See Craig's talk

One-loop anomalous dimension matrix has a surprising number of zeros!

Nontrivial Zeros in Anomalous Dimension Matrix

A simple question: Do nontrivial zeros exist beyond 1 loop?

Nontrivial zero: you can write down Feynman diagrams but you still get zero

At first sight:

- **Helicity selection rules or susy embeddings only work at tree level.**
- **Sensible answer: "Not a snowball's chance in hell"**

But as we will see there are actually many new nontrivial zeros:

1. **Sometimes only integrals which can't generate the correct logs.**
2. **Sometimes we can adjust 1 loop scheme to make two-loop anomalous dimension vanish.**
3. **Sometime color selection rules kill contributions.**
4. **Sometimes helicity selection can become active if enough other cuts vanish.**

On Shell Methods

Two basic approaches:

1. Use unitarity method to calculate amplitudes and associated UV divergence.

ZB, Dixon, Dunbar, Kosower, etc

2. Slick form factor method. Anomalous dimensions directly in terms of cuts.

Caron-Huot and Wilhelm

- Method 1 needs no explanation here. Use our standard amplitude methods.
- Beyond 1 loop generally we need both because need to feed 1 loop amplitudes into cuts at 2 loops.
- Need good control of IR singularities. Dim. Reg. mixes IR and UV anomalous dimensions.

Reasons we like on-shell methods:

1. Efficient way to calculate.
2. Exposes new structures (like zeros).
3. Helps a lot with evanescent operator chaos.

Evanscent operator: vanishes in $D = 4$ but creates dim. reg. chaos beyond 1 loop

On Shell Methods

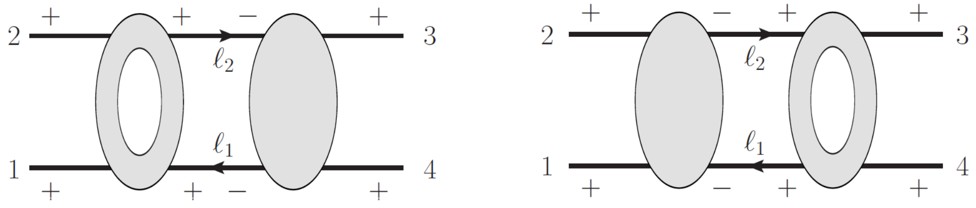
Simple observation helps a lot:

Don’t focus on UV infinities. Instead focus on renormalization scale.

Log(s_i) contains information we need. **Log(s_i/μ^2)**

- **Arguments of logs must be dimensionless.**
- **Imbalance gives renormalization scale dependence.**

Used in for example slick recalculation of two-loop R^3 divergence of gravity



ZB, Cheung, Chi, Davies, Dixon;
Z.B., Chi, Dixon and Edison

$N_b - N_f$:
bosons − # fermions

Subtract IR divergence in integrand and evaluate phase-space integrals.

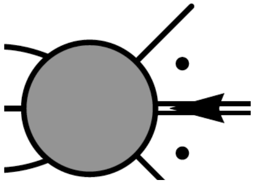
$$M^{2-\text{loop}}(++++)\Big|_{\ln\mu^2} = -\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} s_{12}s_{23}s_{13}\mathcal{T}^2 \frac{N_b - N_f}{8}\ln\mu^2 \qquad \mathcal{T} = \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}$$

- **Gives renormalization scale running of 2 loop R^3 counterterm.**
- **Avoids nontrivial chaos from evanescent operators and fields.**

Unitarity Cuts and Renormalization

Caron-Huot and Wilhelm

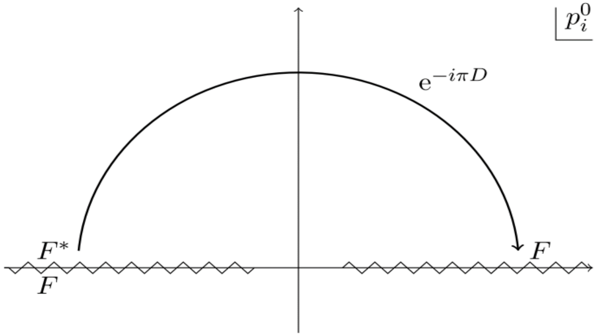
A very slick way to systematically deduce anomalous dimension from unitarity cuts



Form factor: Off-shell momentum injection from operator

$$F_j[p_1,...,p_n;q;\mu] = \langle p_1,...,p_n|\mathcal{O}_j(q)|0\rangle$$

$$F(p_1e^{i\alpha},\ldots,p_ne^{i\alpha}) = e^{i\alpha D}F(p_1,\ldots,p_n)$$



Dilation operator

Analyticity: $F = e^{-i\pi D}F^*$

Unitarity: $F = SF^*$

$$e^{-i\pi D}F_i^* = S\,F_i^*$$

Proven from $SS^\dagger = 1$ and treating operator insertion as perturbation

implied sum over unitarity cuts

Dilation operator (not including engineering dims):

$$DF_i = -\mu\partial_\mu F_i = [\Delta\gamma_{ij} + \delta_{ij}\beta\,\partial_g]\,F_j$$

$$\Delta\gamma = \gamma^{\text{UV}} - \gamma^{\text{IR}}$$

Slick way to extract anomalous dimensions directly from unitarity cuts.

One Loop Renormalization

$$e^{-i\pi D} F_i^* = S\, F_i^*$$

F_i : Form factor with operator i

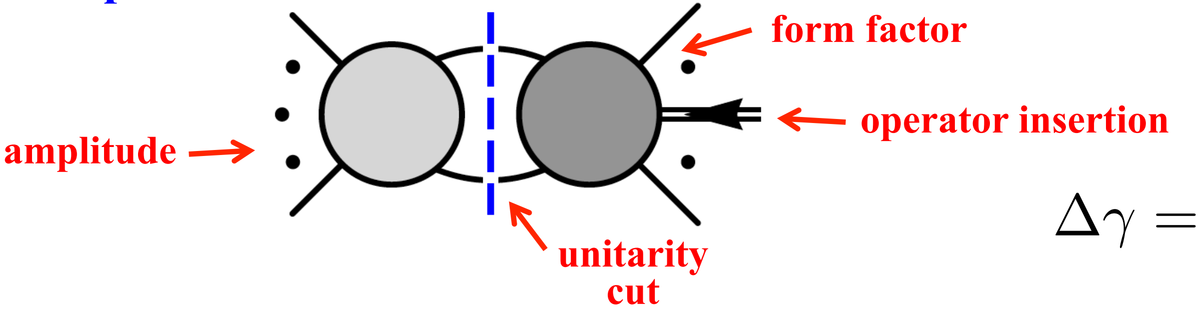
sum over unitarity cuts

Dilation operator

$$DF_i = -\mu \partial_\mu F_i = [\Delta\gamma_{ij} + \delta_{ij} \beta\, \partial_g]\, F_j$$

$$S = 1 + i\mathcal{M} \qquad (e^{-i\pi D} - 1) F_i^* = i\mathcal{M} F_i^*$$

Consider 1 loop



$$\Delta\gamma = \gamma^{\rm UV} - \gamma^{\rm IR}$$

$$\boxed{\left[\Delta\gamma_{ij}^{(1)} + \delta_{ij}\beta^{(1)}\partial_g\right] F_j^{(0)} = -\frac{1}{\pi}(\mathcal{M}F_i)^{(1)}}$$

Helicity selection rules:
If cut vanishes then
anomalous dim vanishes.

Up to IR can extract anomalous dimensions from unitarity cuts.

Dealing with IR

Dim. reg. does not distinguish between IR and UV

$$\Delta\gamma = \gamma^{\mathrm{UV}} - \gamma^{\mathrm{IR}}$$

To get UV, need to subtract IR from cut integrands or after integration

IR well understood. E.g at one loop:

Magnea and Sterman
Catani

$$A_i^{(1)} = \boldsymbol{I}^{(1)} A_i^{(0)} + A_i^{(1)\mathrm{fin}}$$

$$\boldsymbol{I}^{(1)} = \widetilde{g}^2 \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_{p=1}^{n} \sum_{q\neq p} \boldsymbol{T}_p \cdot \boldsymbol{T}_q \left[\frac{1}{\epsilon^2} + \frac{\gamma_p^c}{\boldsymbol{T}_p^2} \frac{1}{\epsilon} \right] \left(\frac{-\mu^2}{2k_p{\cdot}k_q} \right)^{\epsilon}$$

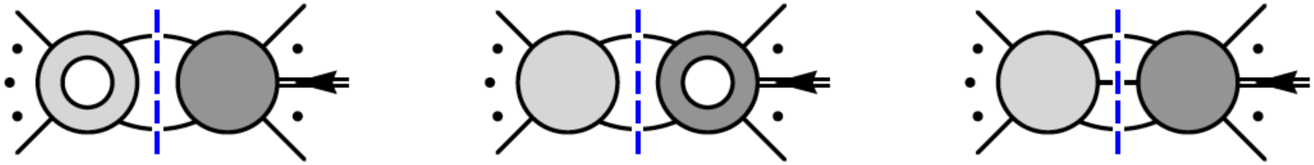
Similar formulas at higher loops.

Alternatively location of IR singularities in integrand understood and can subtract, especially at 1 loop.

Some useful IR information:

- **The IR is diagonal in the amplitudes and form factors (up to color)**
- **No IR singularity at 1^{st} loop order where diagrams exist.**

Two Loop Renormalization



Get 2 loop anomalous dimension directly in terms of unitarity cuts:

Two loop anomalous dimension

$$\Delta\gamma = \gamma^{\mathrm{UV}} - \gamma^{\mathrm{IR}}$$

$$\left[\Delta\gamma^{(1)}_{ij} + \delta_{ij}\beta^{(1)}\partial_g\right] F^{(1)}_j + \left[\Delta\gamma^{(2)}_{ij} + \delta_{ij}\beta^{(2)}\partial_g\right] F^{(0)}_j$$
$$- i\pi\,\frac{1}{2}\left[\Delta\gamma^{(1)}_{ik} + \delta_{ik}\beta^{(1)}\partial_g\right]\left[\Delta\gamma^{(1)}_{kj} + \delta_{kj}\beta^{(1)}\partial_g\right] F^{(0)}_j = -\frac{1}{\pi}(\mathcal{M}F_i)^{(2)}$$

sum over unitarity cuts

Can clean up by separating real and imaginary parts:

$$-i\pi\,\frac{1}{2}\left[\Delta\gamma^{(1)}_{ik} + \delta_{ik}\beta^{(1)}\partial_g\right]\left[\Delta\gamma^{(1)}_{kj} + \delta_{kj}\beta^{(1)}\partial_g\right] F^{(0)}_j = -\frac{1}{\pi}\mathrm{Im}(\mathcal{M}F_i)^{(2)} \quad = (\mathcal{M}\mathcal{M}F_i)^{(2)}$$

$$\boxed{\left[\Delta\gamma^{(1)}_{ij} + \delta_{ij}\beta^{(1)}\partial_g\right] \mathrm{Re}F^{(1)}_j + \left[\Delta\gamma^{(2)}_{ij} + \delta_{ij}\beta^{(2)}\partial_g\right] F^{(0)}_j = -\frac{1}{\pi}\left[\mathrm{Re}(\mathcal{M})\mathrm{Re}(F_i)\right]^{(2)}}$$

Key formula for 2 loop Δγ

unitarity cuts with imag. parts removed

ZB, Sawyer, Parra-Martinez

To get zero need vanishing not only in 2-particle cuts, but also in 3 particle cuts.

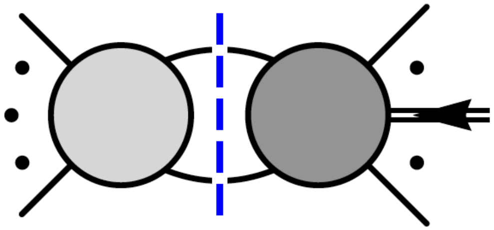
Minimal Form Factors

Caron-Huot and Wilhelm
ZB, Parra-Martinez, Sawyer

Another concept that helps a lot are minimal form factors:

Use minimal form factors e.g. *F*³ form factor look at 3 external gluon form factors

$$\left(\gamma_{ij}^{\mathrm{UV}} - \gamma_{ij}^{\mathrm{IR}} + \beta(g)\partial_g\right)^{(1)} \langle p_1, \ldots, p_n | \mathcal{O}_i | 0 \rangle^{(0)}$$
$$= -\frac{1}{\pi} \langle p_1, \ldots, p_n | \mathcal{M} \otimes \mathcal{O}_j | 0 \rangle$$



Minimal form factors

$\cancel{\beta(g)\partial_g}$

↓

First chance for operator mixing

$\cancel{\gamma_{ij}^{\mathrm{IR}}}$

Suppose that for trivial reasons an anomalous dimension vanishes up to *L* — 1 loops

At *L* loops for first potential contribution and minimal form factor:

$$\left(\gamma_{sl}^{\mathrm{UV}}\right)^{(L)} {}_s\langle p_1, \ldots, p_n | \mathcal{O}_s | 0 \rangle^{(0)} = -\frac{1}{\pi} {}_s\langle p_1, \ldots, p_n | \mathcal{M} \otimes \mathcal{O}_l | 0 \rangle$$

We use this to prove multi-loop nonrenormalization theorems.

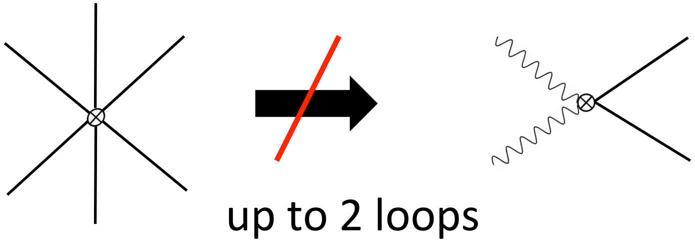
Nonrenormalization Theorem

In fact there are plenty of zeros at higher loops. Some of the zeros follow from a theorem

An L loop nonrenormalization theorem:

Consider operators $\mathcal{O}_s$ *and* $\mathcal{O}_l$ *such that* $l(\mathcal{O}_l) > l(\mathcal{O}_s)$.
$\mathcal{O}_l$ *can renormalize* $\mathcal{O}_s$ *at* L *loops only if* $L > l(\mathcal{O}_l) - l(\mathcal{O}_s)$.

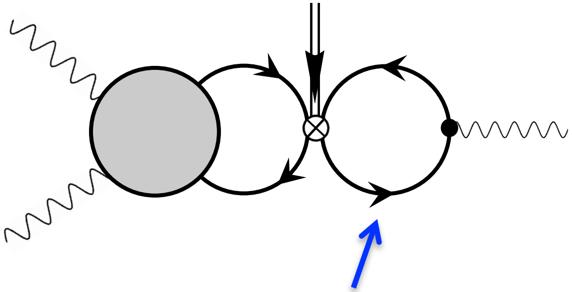
$l(\mathcal{O}_x)$: number of field insertions in operator
$\mathcal{O}_s$: short operator
$\mathcal{O}_l$: long operator



up to 2 loops

True even if you can write down Feynman diagrams

- **Important to focus on minimal form factors.**
- **Crucial to feed in IR understanding. If no lower loop diagrams won't get IR contamination.**
- **More generally, scaleless diagrams won't lead to UV renormalization.**
- **Easy to identify *even more* zeros than simple form of theorem given above.**

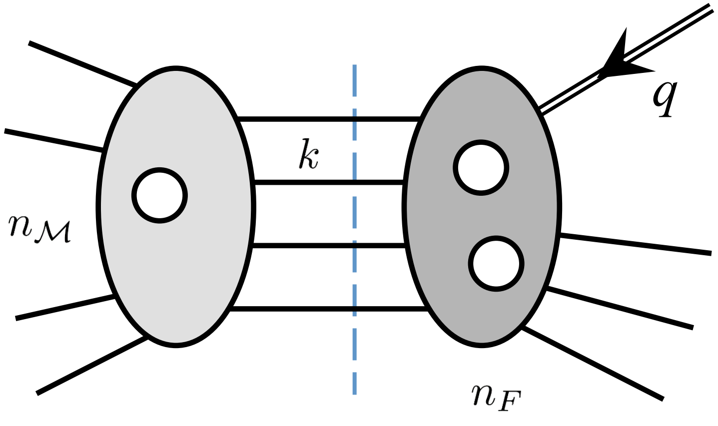


scaleless integral

Summary of Proof

Longer operators often cannot renormalize shorter operators, even when diagrams can be written down.

ZB, Parra-Martinez, Sawyer



n_F : # legs in form factor

$n_{\mathcal{M}}$: # legs in amplitude

L : total loop number

L_F : loops in form factor

$l(\mathcal{O}_l)$: number of field insertions in $\mathcal{O}_l$

Constraints:

Minimal form factor

$$n_{\mathcal{M}} + n_F - 2k = l(\mathcal{O}_s)$$

Required for nonzero result:

- ≥2 external legs on the left

  $$n_{\mathcal{M}} \geq k + 2$$

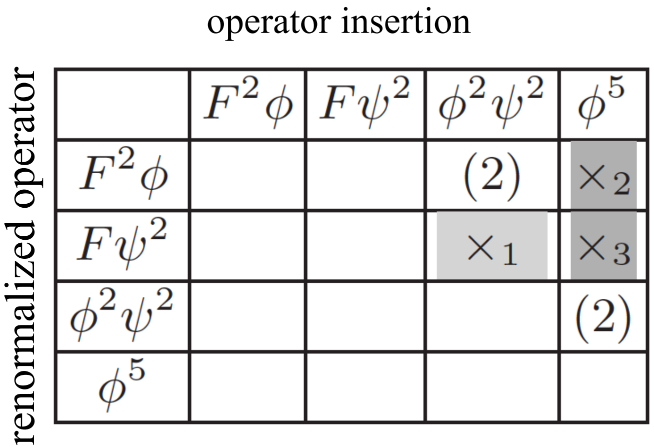
- No scaleless integrals on the right

  $$n_F \geq l(\mathcal{O}_l) - (L_F - 1) - \delta_{L_F,0}$$

Putting these constraints together shows:

$$\boxed{L > l(\mathcal{O}_l) - l(\mathcal{O}_s)}$$

Nonrenormalization Theorem Examples



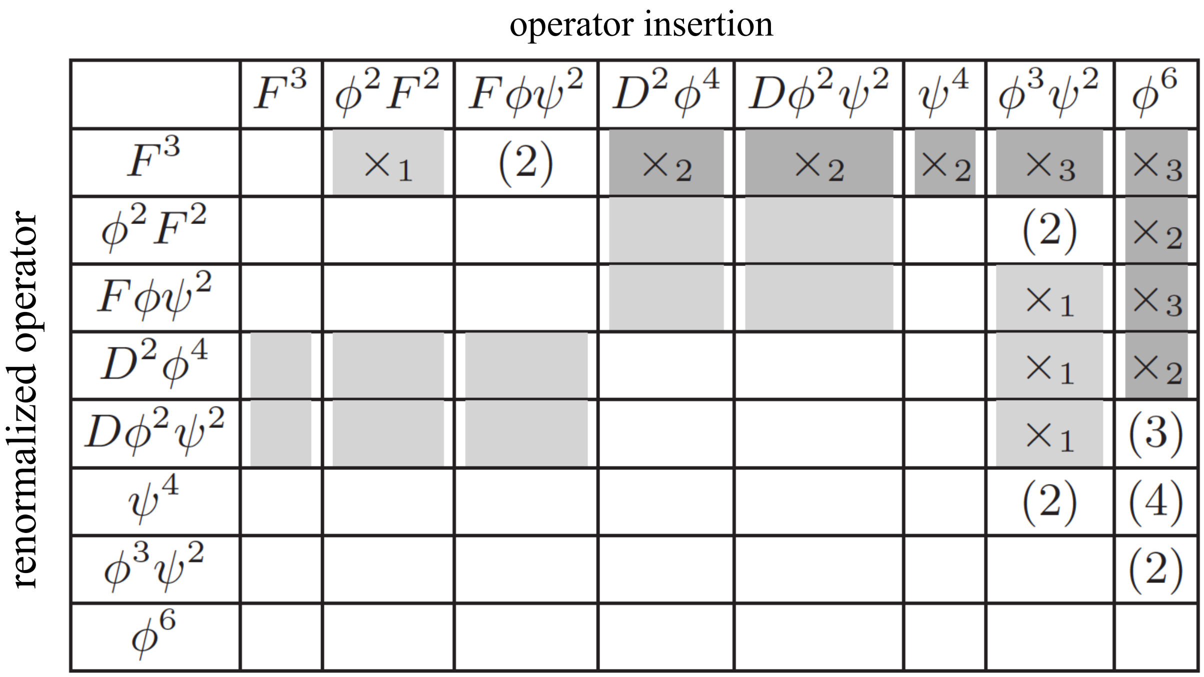
operator insertion

renormalized operator

	$F^2\phi$	$F\psi^2$	$\phi^2\psi^2$	ϕ^5
$F^2\phi$			(2)	▨ ×2
$F\psi^2$			▨ ×1	▨ ×3
$\phi^2\psi^2$				(2)
ϕ^5				

Dimension 5 operators

▨	One-loop Zero from helicity rules of Cheung and Shen
▨ ×1	Overlap of zeros of Cheung and Shen and our nonrenorm thems
×L	Theorems active at L loops
(L)	Delayed to L loops for trivial reasons



operator insertion

renormalized operator

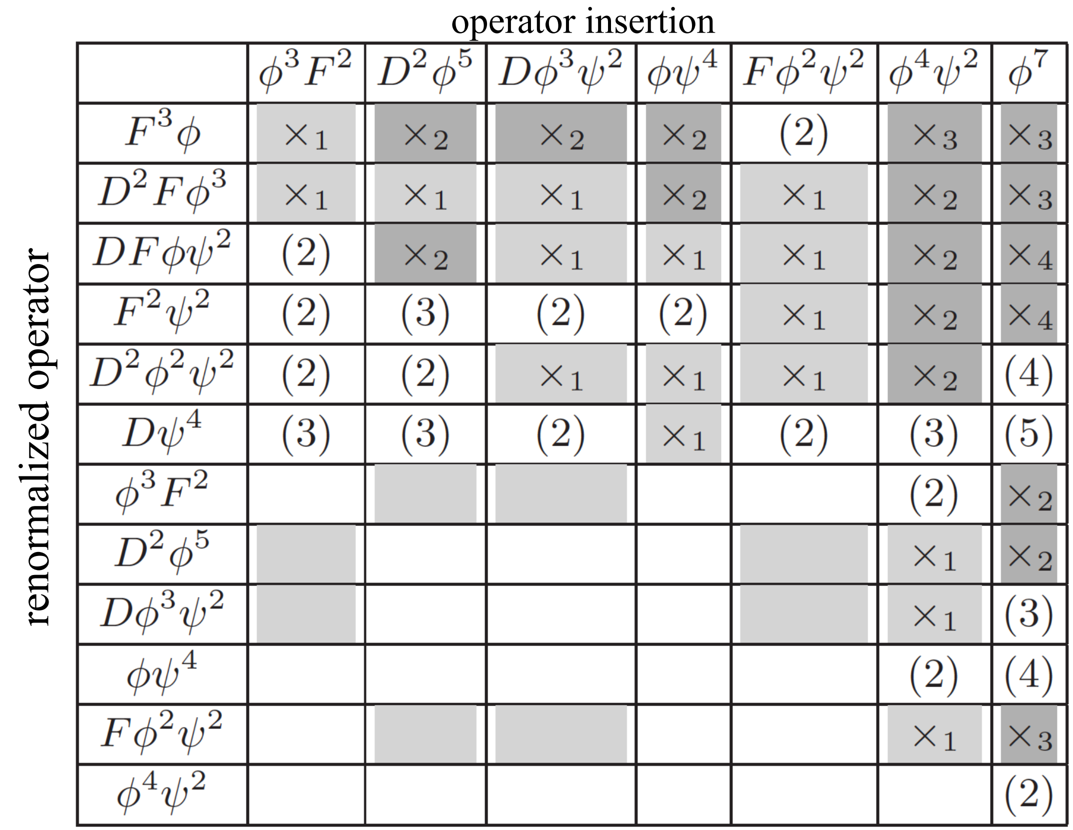
	F^3	ϕ^2F^2	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		▨ ×1	(2)	▨ ×2	▨ ×2	▨ ×2	▨ ×3	▨ ×3
ϕ^2F^2				▨	▨		(2)	▨ ×2
$F\phi\psi^2$				▨	▨		▨ ×1	▨ ×3
$D^2\phi^4$	▨	▨	▨				▨ ×1	▨ ×2
$D\phi^2\psi^2$	▨	▨	▨				▨ ×1	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								

Dimension 6 operators

Lots of new zeros at higher loops

Compared all the dim 6 one loop anomalous dims to Jenkins, Manohar,Trott and they agree! Done 2 ways.

Pattern of Zeros Dimension 7



operator insertion

renormalized operator

	ϕ^3F^2	$D^2\phi^5$	$D\phi^3\psi^2$	$\phi\psi^4$	$F\phi^2\psi^2$	$\phi^4\psi^2$	ϕ^7
$F^3\phi$	$\times_1$	$\times_2$	$\times_2$	$\times_2$	(2)	$\times_3$	$\times_3$
$D^2F\phi^3$	$\times_1$	$\times_1$	$\times_1$	$\times_2$	$\times_1$	$\times_2$	$\times_3$
$DF\phi\psi^2$	(2)	$\times_2$	$\times_1$	$\times_1$	$\times_1$	$\times_2$	$\times_4$
$F^2\psi^2$	(2)	(3)	(2)	(2)	$\times_1$	$\times_2$	$\times_4$
$D^2\phi^2\psi^2$	(2)	(2)	$\times_1$	$\times_1$	$\times_1$	$\times_2$	(4)
$D\psi^4$	(3)	(3)	(2)	$\times_1$	(2)	(3)	(5)
ϕ^3F^2						(2)	$\times_2$
$D^2\phi^5$						$\times_1$	$\times_2$
$D\phi^3\psi^2$						$\times_1$	(3)
$\phi\psi^4$						(2)	(4)
$F\phi^2\psi^2$						$\times_1$	$\times_3$
$\phi^4\psi^2$							(2)

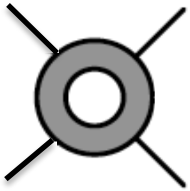
$\times_L$ Theorems active at L loops

Lots of new zeros at higher loops and for higher dimension operators

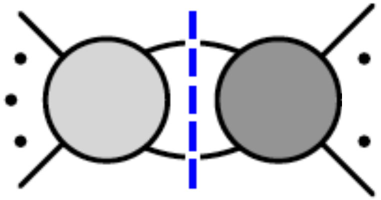
One-Loop Amplitudes

ZB, Parra-Martinez, Sawyer

Need 1 loop amplitudes to feed into 2-loop anomalous dimensions.



Use standard unitarity methods:



- Worked out all the 4 point amplitudes.
- *D* dimensional methods to keep an eye on evanescent issues and rational pieces.
- Use physical state projectors to simplify integrals.
- Overkill, but used FIRE

A.V. Smirnov

Glover and Tejeda-Yeomans

Many amplitudes look exactly like you might expect:

IR & UV subtracted

4 fermion, different flavors

$$A^{(1)}_{(\psi^4)_1}(1^+_{\psi_m} 2^-_{\bar{\psi}_n} 3^-_{\psi_p} 4^+_{\bar{\psi}_r})_{[1]} = -\frac{2}{9} g^2 (N_f t (3X_s + 2)(\delta_{mn} c^{rwwp}_{(\psi^4)_1} + \delta_{pr} c^{nwwm}_{(\psi^4)_1}) + 9(2s + t(5 - 3X_t)) c^{nmrp}_{(\psi^4)_1}) \,,$$

4 vector

$$A^{(1)}_{F^3}(1^+2^+3^+4^+)_{[1]} = g^3 c_{F^3}\big((4N(t-u) + 2u\beta_0)X_s + (4N(s-u) + 2u\beta_0)X_t - \frac{1}{2}(44N + 2N_f - N_s)u\big) \,,$$

$$X_s = \log\left(\frac{\mu^2}{s}\right)$$

$$X_t = \log\left(\frac{\mu^2}{t}\right)$$

But many are rational or zero:

$$A^{(1)}_{(D^2\phi^4)_1}(1^+2^+3_\phi 4_{\bar{\phi}})_{[1]} = -\frac{1}{2} g^2 c_{(D^2\phi^4)_1} N_s s \,,$$

$$A^{(1)}_{(D^2\phi^4)_1}(1^+2^-3_\phi 4_{\bar{\phi}})_{[1]} = 0 \,,$$

$$A^{(1)}_{(D\phi^2\psi^2)_1}(1^\pm 2^\pm 3_\phi 4_{\bar{\phi}})_{[1]} = A^{(1)}_{(D\phi^2\psi^2)_1}(1^\pm 2^\pm 3_\phi 4_{\bar{\phi}})_{[3]} = 0 \,,$$

$$A^{(1)}_{(D\phi^2\psi^2)_2}(1^+2^+3_\phi 4_{\bar{\phi}})_{[1]} = \frac{1}{3} g^2 c^{ww}_{(D\phi^2\psi^2)_2} N_f (s + 2t) \,,$$

$$A^{(1)}_{(D\phi^2\psi^2)_2}(1^+2^-3_\phi 4_{\bar{\phi}})_{[1]} = A^{(1)}_{(D\phi^2\psi^2)_2}(1^\pm 2^\pm 3_\phi 4_{\bar{\phi}})_{[3]} = 0 \,.$$

A surprising number of amplitudes are rational or vanish!

Craig, Jiang, Li, Sutherland.

One-Loop Amplitude Zeros

amplitude

operator	$F_+F_+F_+F_+$	$F_+F_+F_+F_-$	$F_+F_+F_-F_-$	$\bar\phi\phi F_+F_+$	$\bar\phi\phi F_+F_-$	$\bar\phi\phi\bar\phi\phi$	$\psi_-\bar\psi_+F_+F_+$	$\psi_-\bar\psi_+F_+F_-$	$\psi_-\bar\psi_+F_-F_+$	$\psi_+\bar\psi_-F_-F_-$	$\psi_+\bar\psi_-\phi\phi$	$\psi_+\bar\psi_-\psi_+\bar\psi_-$	$\psi_+\bar\psi_-\psi_-\bar\psi_+$	$\psi_+\bar\psi_+\psi_-\bar\psi_-$
$\mathcal{O}_{F^3}$	L	L	R	L	R	0	L	R	R	L	0	0	0	0
$\mathcal{O}_{(\phi^2F^2)_1}$	R	0	R	L	R	0	0	0	0	0	0	∅	∅	∅
$\mathcal{O}_{(\phi^2F^2)_2}$	R	0	R	L	L	0	0	0	0	0	0	∅	∅	∅
$\mathcal{O}_{(D^2\phi^4)_1}$	∅	∅	∅	0	0	L_0	∅	∅	∅	∅	0	∅	∅	∅
$\mathcal{O}_{(D^2\phi^4)_2}$	∅	∅	∅	R	0	L_0	∅	∅	∅	∅	L_0	∅	∅	∅
$\mathcal{O}_{(D\phi^2\psi^2)_1}$	∅	∅	∅	0	0	0	0	0	0	0	L_0	0	0	0
$\mathcal{O}_{(D\phi^2\psi^2)_2}$	∅	∅	∅	R	0	L_0	R	0	0	R	L_0	L_0	L_0	L_0
$\mathcal{O}_{(\psi^4)_1}$	∅	∅	∅	∅	∅	∅	R	0	0	R	L_0	L	L	L
$\mathcal{O}_{(\psi^4)_2}$	∅	∅	∅	∅	∅	∅	R	0	0	R	L_0	L	L	L

We calculated these one-loop amplitudes

Local, finite renormalization sets to zero!

Requires a detailed inspection

One loop amplitudes also have a remarkable number of zeros

L : contains Logs

R : rational

∅ : Trivial zeros. No diagrams.

0 : known zeros helicity, angular mom., explicit calculations

0 : New zero, requires inspection.

0 : Finite renormalization sets to zero.

I_0 : Finite renormalization removes rational piece.

See Craig's talk

Craig, Jiang, Li, Sutherland;
Jaing, Shu, Xiao, Zheng

Latest zeros

ZB, Parra-Martinez, Sawyer (to appear)

- **Zeros in one-loop matrix elements can imply new zeros at two loops.**
- **Expect more zeros in interference with trees.**

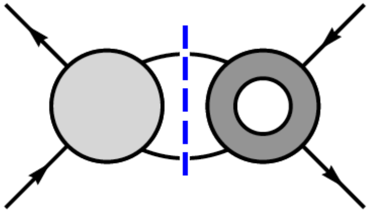
Azatov, Contino, Machado, Riva

More Two Loop Anomalous Dimension Zeros

$$\mathcal{O}_{(D^2\phi^4)_1} = (\phi^\dagger D^\mu \phi)(\phi^\dagger D_\mu \phi) \qquad \mathcal{O}_{(D^2\phi^4)_2} = (\phi^\dagger \phi)\Box(\phi^\dagger \phi)$$

$$\mathcal{O}_{(\psi^4)_1} = (\bar\psi^m \gamma^\mu \psi^n)(\bar\psi^p \gamma_\mu \psi^r) \qquad \mathcal{O}_{(\psi^4)_2} = (\bar\psi^m \gamma^\mu T^a \psi^n)(\bar\psi^p \gamma_\mu T^a \psi^r)$$

There is a nonvanishing cut and indeed we get nonzero:



$$\gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (D^2\phi^4)_1} = -8\tilde g^4 \frac{N_f^2(N-2)}{9N}$$

$$\gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (D^2\phi^4)_2} = 4\tilde g^4 \frac{N_f^2(2N-1)}{9N}$$

$$\tilde g^2 = \frac{g^2}{(4\pi)^2}$$

But easy to remove with a finite renormalizations (scheme choice):

$$c_{(D^2\phi^4)_1} \longrightarrow c_{(D^2\phi^4)_1} + \frac{10\tilde g^2(N-2)N_f}{9N} c_{(D\phi^2\psi^2)_2}\,,$$

$$c_{(D^2\phi^4)_2} \longrightarrow c_{(D^2\phi^4)_2} + \frac{5\tilde g^2(2N-1)N_f}{9N} c_{(D\phi^2\psi^2)_2}\,,$$

$$c_{(D\phi^2\psi^2)_2} \longrightarrow c_{(D\phi^2\psi^2)_2} - \frac{2\tilde g^2 N_f}{9} c_{(\psi^4)_1}\,.$$

Can also redefine operators.

$$\boxed{\gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (D^2\phi^4)_1} = 0 \qquad \gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (D^2\phi^4)_2} = 0}$$

We can choose scheme to generate even more two loop zeros

Color Selection Rules

As simple example:

$$\mathcal{O}_{(\psi^4)_1} \to \{\mathcal{O}_{(\phi^2F^2)_1}, \mathcal{O}_{(\phi^2F^2)_2}\}$$

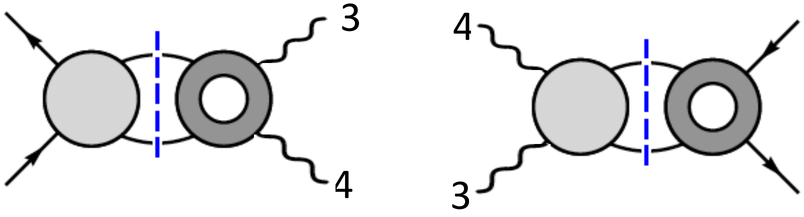
Simplified model with no Yukawa's

$$\mathcal{O}_{(\psi^4)_1} = (\bar{\psi}^m \gamma^\mu \psi^n)(\bar{\psi}^p \gamma_\mu \psi^r)$$

$$\mathcal{O}_{(\phi^2F^2)_1} = (\phi^\dagger \phi) F^a_{\mu\nu} F^a_{\mu\nu}$$

$$\mathcal{O}_{(\phi^2F^2)_2} = d^{abc} (\phi^\dagger T^a \phi) F^b_{\mu\nu} F^c_{\mu\nu}$$

$$\gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (\phi^2F^2)_1} F^{(0)}_{(\phi^2F^2)_1} + \gamma^{\mathrm{UV}(2)}_{(\psi^4)_1 \to (\phi^2F^2)_2} F^{(0)}_{(\phi^2F^2)_2} + \gamma^{\mathrm{UV}(1)}_{(\psi^4)_1 \to (D\phi^2\psi^2)_2} F^{(1)}_{(D\phi^2\psi^2)_2}$$
$$= -\frac{1}{\pi}\Big((\mathcal{M}^{12}_{2\to2})^{(0)} \otimes \mathrm{Re}F^{(1)}_{(\psi^4)_1}(\psi\overline{\psi}FF) + (\mathcal{M}^{34}_{2\to2})^{(0)} \otimes \mathrm{Re}F^{(1)}_{(\psi^4)_1}(\phi\overline{\phi}\psi\overline{\psi})\Big)$$



$$A^{(1)}_{(\psi^4)_1}(1^+_\psi\, 2^-_{\overline{\psi}}\, 3^+ 4^+) = -\frac{\widetilde{g}^2 N_f s[14]\langle 24\rangle\, [T^{a_3}, T^{a_4}]_{i_2 i_1}}{3\langle 34\rangle^2}$$

$$A^{(1)}_{(\psi^4)_1}(1^-_\psi\, 2^+_{\overline{\psi}}\, 3^+ 4^+) = -\frac{\widetilde{g}^2 N_f \langle 12\rangle[23][24]\, [T^{a_3}, T^{a_4}]_{i_2 i_1}}{3\langle 34\rangle}$$

$$A^{(1)}_{(D\phi^2\psi^2)_2}(1_\phi 2_{\overline{\phi}}\, 3^+ 4^+) = \frac{\widetilde{g}^2 N_f s(s+2t)\, [T^{a_3}, T^{a_4}]_{i_2 i_1}}{3\langle 34\rangle^2}$$

Contributions have antisymmetric color but target operators have symmetric color.

Can get additional zeros simply because color factors don't match.

Summary of Two-Loop Anomalous Dimension Zeros

ZB, Parra-Martinez, Sawyer

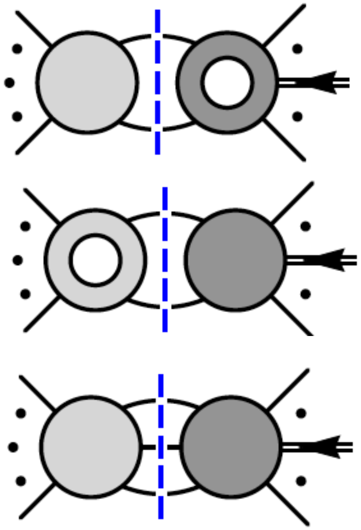
operator insertion

renormalized operator

	$\mathcal{O}_{F^3}$	$\mathcal{O}_{(\phi^2F^2)_1}$	$\mathcal{O}_{(\phi^2F^2)_2}$	$\mathcal{O}_{(D^2\phi^4)_1}$	$\mathcal{O}_{(D^2\phi^4)_2}$	$\mathcal{O}_{(D\phi^2\psi^2)_1}$	$\mathcal{O}_{(D\phi^2\psi^2)_2}$	$\mathcal{O}_{(\psi^4)_1}$	$\mathcal{O}_{(\psi^4)_2}$	$\mathcal{O}_{\phi^6}$
$\mathcal{O}_{F^3}$				0	0	0	0	0	0	$\emptyset$
$\mathcal{O}_{(\phi^2F^2)_1}$								0	0	0
$\mathcal{O}_{(\phi^2F^2)_2}$								0	0	0
$\mathcal{O}_{(D^2\phi^4)_1}$								0^*	0^*	0
$\mathcal{O}_{(D^2\phi^4)_2}$								0^*	0^*	0
$\mathcal{O}_{(D\phi^2\psi^2)_1}$										$\emptyset$
$\mathcal{O}_{(D\phi^2\psi^2)_2}$										$\emptyset$
$\mathcal{O}_{(\psi^4)_1}$				0	0					$\emptyset$
$\mathcal{O}_{(\psi^4)_2}$				0	0					$\emptyset$
$\mathcal{O}_{\phi^6}$										

Two loop zeros in anomalous dim matrix

In simplified SMEFT,
with no Yukawas.



- **Plenty of 2-loop zeros!**
- **Possibly even more zeros can be found.**

$\emptyset$: Trivial zeros. No diagrams.

0 : Nonrenormalization theorems, color, helicity and "accidental"

0^*: Good scheme choice.

Summary of Anomalous Dimension Zeros

With traditional Feynman diagrams: If you can write a diagram you expect nonzero anomalous dimension.

Now we have many different (overlapping) reasons for zeros.

1. **Susy embeddings.** Alonso, Jenkins, Manohar and Trott; Elias-Miro, Espinosa, Pomarol
2. **Helicity selection rules.** Cheung and Shen
3. **Angular momentum selection rules.** Jaing, Shu, Xiao, Zheng
4. **Scaleless integrals and nonrenormalization theorems.**
5. **Color selection rules.** ZB, Parra-Martinez, Sawyer
6. **Good choices of renormalization schemes (finite shifts).**
7. **“Accidental zeros”, need to inspect the unitarity cuts.**

Surely not the end of the story.

Summary

1. For good reason SMEFT is becoming very topical in collider physics. Systematic approach to BSM physics.
2. Amplitude methods streamline calculations of anomalous dimensions. Confirmed many one-loop SMEFT anomalous dimensions.
3. Nontrivial zeros in anomalous dimensions explained and predicted. Theorems valid beyond one loop.
4. Calculated one-loop amplitudes showing new zeros that feed into vanishing two-loop anomalous dims.
5. Even more two-loop zeros: adjust renormalization scheme, vanishings of 1 loop rational terms.

Good opportunities for amplitudes people to help out with SMEFT or HEFT, both on conceptual and calculational sides.

See talks by Shadmi and Craig