High Loop Higher-Spin Frontier

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The Niels Bohr International Academy

Why do Precision Gravity?

- Sure, there is real-world data:
  High demand on accurate theoretical gravitational waveform templates, and all the compelling motives you heard today...
- In all honesty, driven by pure science:
  - How well can perturbative gravity theory capture strong gravity?
  - How can classical theory inform us about the quantum one, when we thought we know all there is about the former?
  - How can we draw parallels between gauge theories and gravity, to better our fundamental understanding of them?
- With a field theory mindset all of these Qs can be & are addressed, while doing post-Newtonian (PN) Gravity, $v \ll 1 \rightarrow$

$nPN \equiv v^{2n}$ correction in classical gravity to Newtonian gravity

- It so happens that PN gravity is the basis for theoretical waveforms...

State of the Art in PN Gravity

State of the art of PN theory for compact binary dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO
S^0	1	0	3	0	25
S^1	2	7	32	**174**	
S^2	2	2	18	**52**	
S^3	4	**24**			
S^4	3	**5**			

- Each (n,l) entry at PN order: $n + l + \text{Parity}(l)/2$
- l=0 is the non-spinning sector:
  - n=0 1687 Newton (he had no friends)
  - n=1 1938 Einstein et al.
  - n=2 1973-4 Ohta et al.
  - n=3 1998-2001 Jaranowski, Schafer, Blanchet, Damour...
  - n=4 2012-2019 Foffa et al., Jaranowski, Schafer, Damour, Blanchet et al...
  - n=5 2019- Foffa et al., Blumlein et al., Bini et al...
  - n=6 2020- Bini et al...

State of the Art in PN Gravity

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- Each (n,l) entry at PN order: $n + l + \text{Parity}(l)/2$
- l=2: $L_{S^2} \simeq L_{S_1^2} + L_{S_1 S_2}$; l=4 $L_{S^4} \simeq L_{S_1^4} + L_{S_1^3 S_2} + L_{S_1^2 S_2^2}$
- n=0 is tree level: l=1 1959 Tulczyjew, l=2 1975 Barker & O'Connell, l=3,4,... 2014-5 **ML** & Steinhoff...
- Measure for loop computational scale: number of n-loop graphs at $N^n LO$
- To push precision frontier consistently - push across diagonals!

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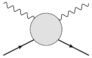
- Gray area corresponds to where we can no longer take $p^\mu \simeq \frac{m}{u} u^\mu$, but have to take into account corrections from non-minimal coupling part of spinning particle action. What happens then?



- Gray area also corresponds to gravitational Compton scattering with $s \geq 3/2$ as classical S^l corresponds to quantum $s = l/2$
- Can we get insight on the graviton Compton amplitude with $s \geq 5/2$?

Why is Spin so Challenging?

- Relativistic spin has a minimal finite measure S/M,
  and this clashes with the EFT/point-particle viewpoint.
- There is no unique 'center' for the spin in relativistic physics.
  This puzzling issue appears already at the LO PN spin correction.
- Accelerations already show up in the LO spin-orbit potential.
  Dealing with higher-order time derivative terms adds complexity.
- Spin is derivatively-coupled:
  - All related computations/integrands carry heavier tensorial load.
  - More time derivatives giving rise to higher-order time derivative terms.
- Requires to deal with Levi-Civita in d dimensions.

Some Amplitudes groups joined efforts on classical spin:

Guevara et al. 2017-9, Chung et al. 2018–20, Damgaard et al. 2019,
Aoude et al. 2020, Bern et al. 2020,...

State of the Art in PN Gravity

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Flash across some new results:

- N^3LO spin-orbit sector, **ML**, Mcleod, von Hippel, arXiv:2003.02827
- N^3LO quadratic-in-spin sector, **ML**, Mcleod, von Hippel, arXiv:2003.07890
- NLO cubic-in-spin sector, **ML**, Mougiakakos, Vieira, arXiv:1912.06276
- NLO quartic-in-spin sector, **ML**, Teng, arXiv:2005.xxxxx

EFT of Gravitating Spinning Particles

[Goldberger & Rothstein 2004, Porto 2005, **ML** & Steinhoff 2015]

$$S_{\rm eff} = S_{\rm g}[g_{\mu\nu}] + \sum_{a=1}^{2} S_{\rm pp}(\lambda_a),$$

$$S_{\rm g}[g_{\mu\nu}] = -\frac{1}{16\pi G_d}\int d^{d+1}x\sqrt{g}\,R + \frac{1}{32\pi G_d}\int d^{d+1}x\sqrt{g}\,g_{\mu\nu}\Gamma^{\mu}\Gamma^{\nu},$$

$$G_d \equiv G_N\left(\sqrt{4\pi e^{\gamma}}\,R_0\right)^{d-3},$$

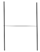
To facilitate computations in PN context: [Kol & Smolkin 2008]

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \equiv e^{2\phi}\left(dt - A_i dx^i\right)^2 - e^{-2c_d\phi}\gamma_{ij}dx^i dx^j, \quad c_d \equiv \frac{2(d-1)}{(d-2)}$$

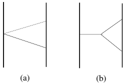
$$\langle\ \phi(x_1)\ \ \phi(x_2)\ \rangle = \quad = \frac{16\pi\ G_d}{c_d}\cdot\delta(t_1-t_2)\int_{\vec{k}}\frac{e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2},$$

$$\langle A_i(x_1)\ A_j(x_2)\rangle = \quad = -16\pi\ G_d\cdot\delta(t_1-t_2)\int_{\vec{k}}\frac{e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2}\ \delta_{ij}.$$

Graph Topologies in Conservative Sector



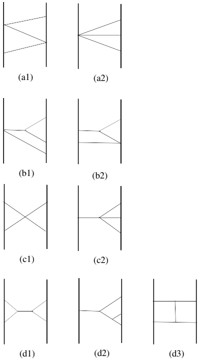
Single topology at $O(G)$:
One-graviton exchange.



Topologies at $O(G^2)$:
(a) Two-graviton exchange;
(b) Cubic self-interaction
$\equiv$ One-loop topology.

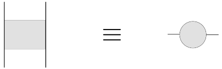
$$\int_{\vec{p}_1} \frac{e^{i\vec{p}_1\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_1^{\,2}} \int_{\vec{p}_2} \frac{e^{i\vec{p}_2\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_2^{\,2}},$$

$$p_1 + p_2 \to p, \quad p_2 \to k_1,$$



Topologies at $O(G^3)$

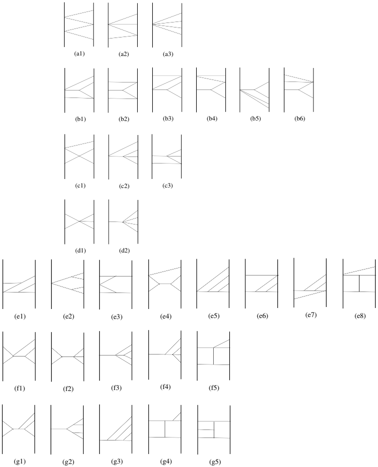
$$\to \int_{\vec{p}} e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)} \int_{\vec{k}_1} \frac{1}{\vec{k}_1^{\,2}(\vec{p}-\vec{k}_1)^2}$$



[Kol & Shir 2013]
Standard multi-loop:
n-loop master integrals and IBPs

Graph Topologies in Conservative Sector

[**ML** Mcleod von Hippel 2020]



Topologies at $O(G^4)$

At G^n the loop order n_L

$$n_L \equiv 2n - \sum_{i=1}^{n+1} m_i$$

with m_i gravitons on insertion i

We define a topology at order G^{n+1} to be of rank r, when r of the basic n-loop integral types are required in order to express its n-loop integral form.

Integration and Scalability

[**ML** Mcleod von Hippel 2020]

- Building on the publicly-available `EFTofPNG` code, **ML** & Steinhoff 2017, `https://github.com/miche-levi/pncbc-eftofpng`
- Generation of rules and contractions/graphs in `FeynRul`, `FeynGen` modules
- E.g., in spin-orbit sector 388 graphs; 93 are higher-rank=require reduction
- Higher-rank graphs are reduced using IBP method
- Significant upgrade to `NLoop` and `Main` modules of `EFTofPNG`
- Main techniques: projection method for integrand numerators as high as rank-8, and variations on Laporta's algorithm
- Altogether the code runs over ~ 2 days
- The upgrade of `EFTofPNG` will be made publicly available too

Special Features

[**ML** Mcleod von Hippel 2020]

Zeros

Apart from sporadic vanishing graphs, there are 2 topologies which generically yield zeros. These result from nested factorizable topologies,

$$\text{Fig. (c1),(e4)} \propto \zeta(2)[\Gamma(-\epsilon)]^{-1} \sim \zeta(2)\epsilon + \mathcal{O}(\epsilon^2), \quad \epsilon \equiv d-3,\, \zeta(2) = \pi^2/6$$

The leading factorizable topologies stand for purely short-distance contributions, which are contact interaction terms of the form $\delta(\vec{r})$

Riemann Zeta Values

These uniquely arise from 3-rank topologies which contain the previous - nested factorizable - integral type, and since the IBP relations entail $\sim \epsilon^{-1}$ coefficients, the $\zeta(2)$ factors are uncovered

Simple poles and Logarithms

Most 3-loop graphs yield simple poles in conjunction with logarithms, from the factor $\Gamma(\epsilon)(r/R_0)^{-4\epsilon}$. These drop from observable quantities.

Extending Non-Minimal Couplings with Spin

Leading non-minimal couplings to all orders in spin

[**ML** & Steinhoff 2015]

$$L_{\rm NMC(R)} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

Extending beyond linear in curvature

[**ML** Mcleod von Hippel 2020]

$$L_{\rm NMC(R^2)} = C_{E^2} \frac{E_{\alpha\beta} E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{B^2} \frac{B_{\alpha\beta} B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + \ldots$$
$$+ C_{E^2S^2} S^\mu S^\nu \frac{E_{\mu\alpha} E_\nu^{\alpha}}{\sqrt{u^2}^{\,3}} + C_{B^2S^2} S^\mu S^\nu \frac{B_{\mu\alpha} B_\nu^{\alpha}}{\sqrt{u^2}^{\,3}}$$
$$+ C_{\nabla EBS} S^\mu \frac{D_\mu E_{\alpha\beta} B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{E\nabla BS} S^\mu \frac{E_{\alpha\beta} D_\mu B^{\alpha\beta}}{\sqrt{u^2}^{\,3}}$$
$$+ C_{(\nabla E)^2S^2} S^\mu S^\nu \frac{D_\mu E_{\alpha\beta} D_\nu E^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + C_{(\nabla B)^2S^2} S^\mu S^\nu \frac{D_\mu B_{\alpha\beta} D_\nu B^{\alpha\beta}}{\sqrt{u^2}^{\,3}} + \ldots$$

N^{3}LO Gravitational Quadratic-in-Spin Action at G^4

[**ML** Mcleod von Hippel 2020]

$$L^{\text{N}^3\text{LO}}_{\text{S}^2} = L^{\text{N}^3\text{LO}}_{\text{S}_1\text{S}_2} + L^{\text{N}^3\text{LO}}_{\text{S}_1^2} + L^{\text{N}^3\text{LO}}_{\text{C}_{1ES^2}\text{S}_1^2} + (1 \leftrightarrow 2)\,,$$

$$L^{\text{N}^3\text{LO}}_{\text{S}_1\text{S}_2} = -\frac{G^4}{r^6}\frac{1}{m_1 m_2}\Bigg[\vec{S}_1\cdot\vec{S}_2\left(5\,m_1^4 m_2 + 63\,m_1^3 m_2^2\right) - \vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\left(25\,m_1^4 m_2 + 300\,m_1^3 m_2^2\right)\Bigg],$$

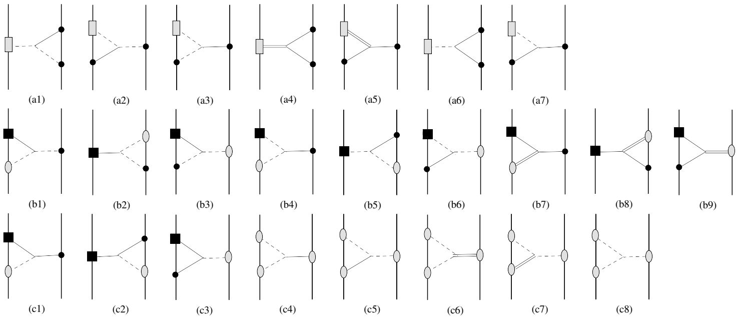
$$L^{\text{N}^3\text{LO}}_{\text{S}_1^2} = \frac{G^4}{r^6}\frac{1}{m_1^2}\Bigg[S_1^2\left(\frac{1}{14}\,m_1^4 m_2 - \frac{73}{70}\,m_1^3 m_2^2 - \,m_1^2 m_2^3\right) + (\vec{S}_1\cdot\vec{n})^2\left(\frac{23}{7}\,m_1^4 m_2 + \frac{2851}{70}\,m_1^3 m_2^2 + 31\,m_1^2 m_2^3\right)\Bigg],$$

$$L^{\text{N}^3\text{LO}}_{\text{C}_{1ES^2}\text{S}_1^2} = -\frac{G^4}{r^6}\frac{C_{1ES^2}}{m_1^2}\left(S_1^2 - 3(\vec{S}_1\cdot\vec{n})^2\right)\left(\frac{23}{28}\,m_1^4 m_2 + \frac{341}{14}\,m_1^3 m_2^2 + 57\,m_1^2 m_2^3 + 9\,m_1 m_2^4\right).$$

NLO Cubic-in-Spin Sector

ML Mougiakakos Vieira 2019

One-loop Feynman graphs



- Graphs include all relevant interactions among the spin-induced quadrupole, octupole, and the mass and spin
- There are nonlinearities originating strictly from minimal coupling
- Cubic vertices with time derivatives, similar to NLO (odd P) spin-orbit sector

NLO Cubic-in-Spin Sector

ML Mougiakakos Vieira 2019

New Feature: Extra one- and two-graviton exchange

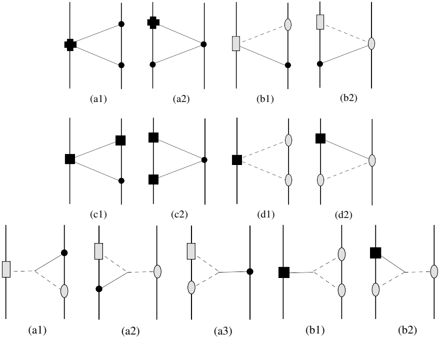
(a1) (a2) (b1) (b2)

- $p_\mu = \frac{m}{u} u_\mu + \Delta p_\mu (RS^2) \Rightarrow L_{S^3}$
- New type of worldline-graviton couplings to "composite" octupole expressed in terms of "elementary" spin multipoles

NLO Quartic-in-Spin Sector

ML & Teng, arXiv:2005.xxxxx

Non-linear "elementary" Feynman graphs



- Though the "elementary" graphs in this sector are fewer and simpler, the situation gets quite complicated in terms of new contributions
- More types of worldline-graviton couplings to "composite" hexadecapole in terms of "elementary" spin multipoles, incl. product of Wilson coefficients
- Relevant operators quadratic in the curvature

Field Theory for Gravity at All Scales

ML, Rept. Prog. Phys. 2019
ML, Mougiakakos, Vieira, arXiv:1912.06276
ML, Mcleod, von Hippel, arXiv:2003.02827
ML, Mcleod, von Hippel, arXiv:2003.07890
ML & Teng, arXiv:2005.xxxxx

- Real-world scalability:
  - Precision frontier with spins pushed to 4.5 & 5PN orders
  - EFT of gravitating spinning objects - self-contained framework
  - Continuous development of public computational tools
- Higher-spin extension:
  - New features in NLO higher-spin sectors resonate with picture of composite (rather than elementary) particles at higher quantum spins
  - Going into the "gray area" becomes extremely intricate also classically
  - Possible insight for graviton Compton amplitude with higher spins
  - Can amplitudes approaches capture such classical effects?

Thank You & See You All in Real-Life in Amplitudes 2021!

– On behalf of the organizers @CPH