UCLA

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Graviton dominance from extremal black hole scattering

Julio Parra-Martinez

w/ Ruf, Zeng [2005.04236] & Bern, Ita, Ruf [2002.02459]

building on [Caron-Huot, Zahraee 1810.04694]

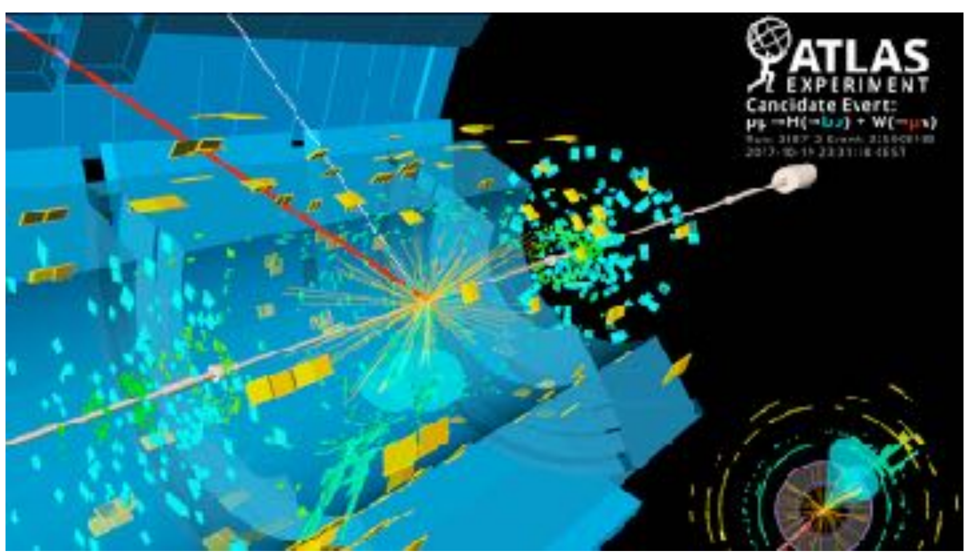
@ Zoomplitudes 2020

Looking to the future

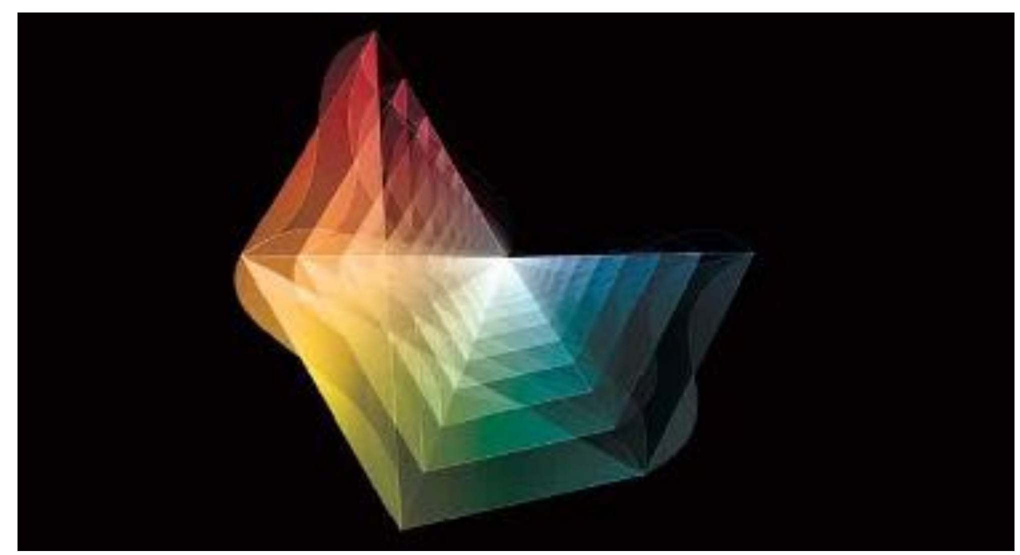
- Many interesting talks today - Amplitudes has caught up to the state of the art in calculations relevant for GW See previous talks today!
- We need to look ahead. What's next? Where to start?
- Historical lesson:

To go here!

See e.g. Johannes', Lance's talk!



QCD



$\mathcal{N} = 4$ SYM

Start here!

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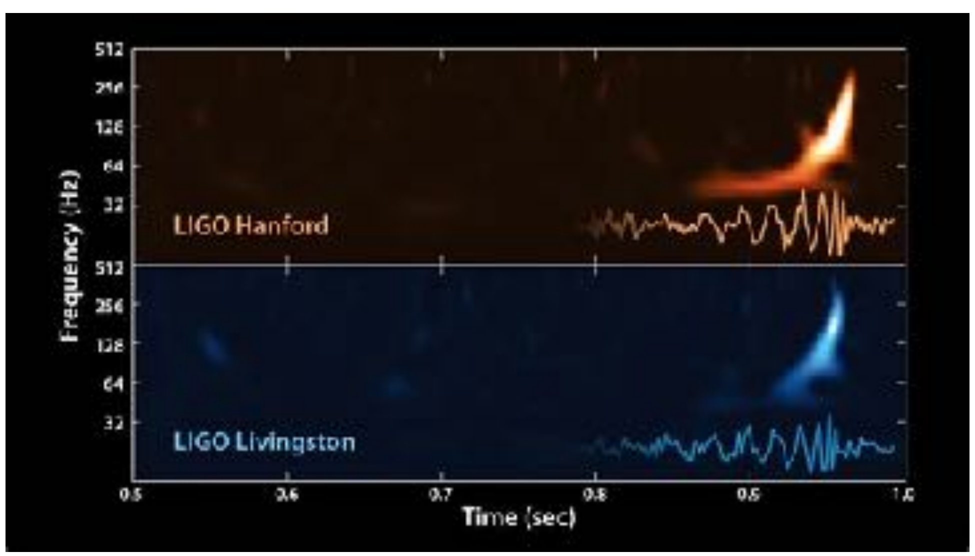
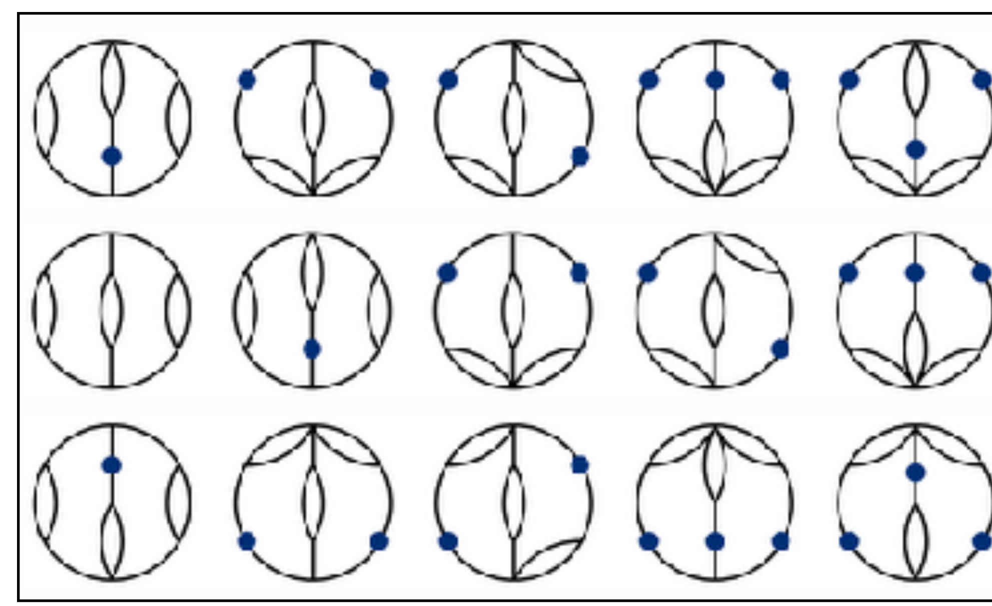


Einstein Gravity



$\mathcal{N} = 8$ SUGRA

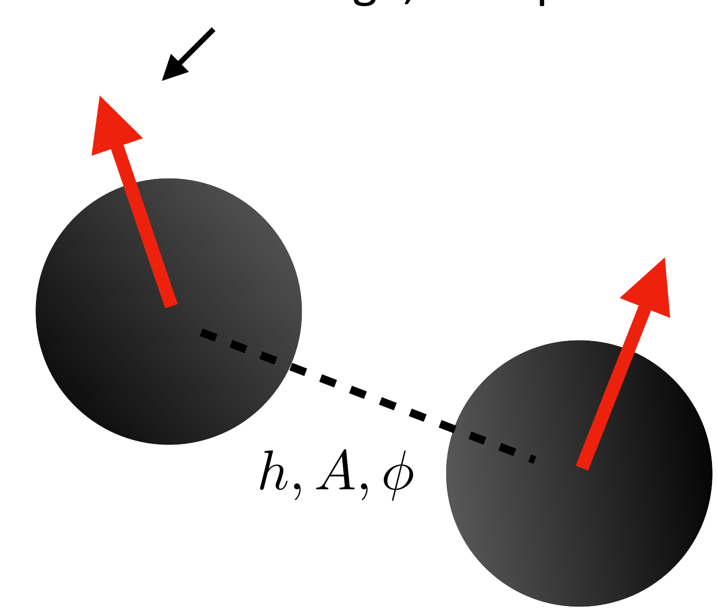
Start here!

Extremal BH in $\mathcal{N} = 8$ supergravity

[Andrianopoli, D'Auria, Ferrara, Fre, Trigiante]

- Graviton multiplet (massless)

$$h_{\mu\nu}\,, \quad A^{\mu}_{IJ}\,, \quad \phi_{IJKL} \;+\; \text{fermions}$$



- Extremal BH ~ half-BPS multiplet (focus on scalar component)

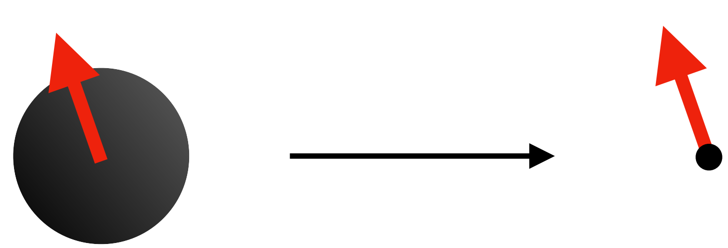
Central charges C_{IJ} [Caron-Huot, Zahraee]

$$C_1 = m_1 \begin{pmatrix} 0 & 1_{4\times4} \\ -1_{4\times4} & 0 \end{pmatrix}, \qquad C_2 = m_2 \begin{pmatrix} 0 & \Phi \\ -\Phi & 0 \end{pmatrix}$$

$$\Phi = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}, e^{i\phi_4}) \qquad \sum_i \phi_i = 0$$

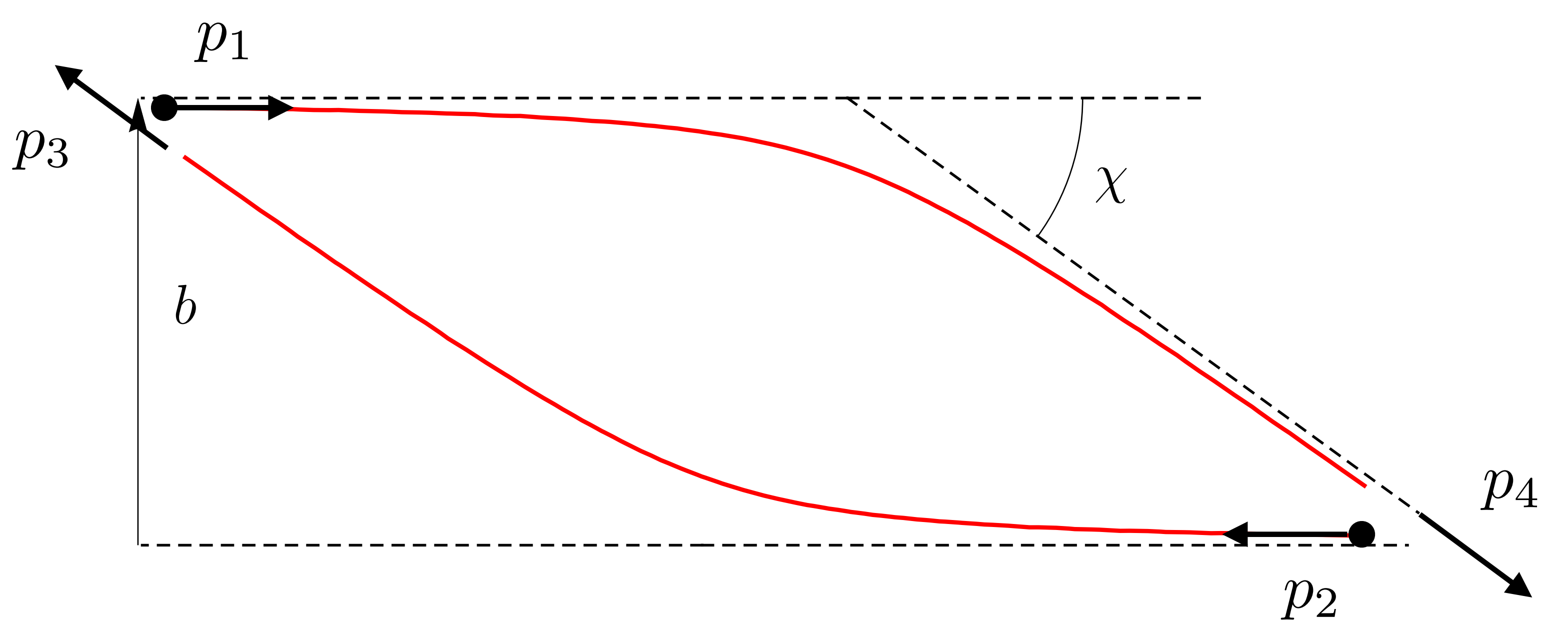
- Force between BH depends on alignment of charges (vanishes for aligned charges + static limit)

- We will ignore finite-size effects and model BH as a point-like BPS superparticle



$+$ finite-size corrections

The calculation



Two-loop ($\mathcal{O}(G^3)$) results

Einstein Gravity

[Bern, Cheung, Roiban, Shen, Solon, Zeng]

- Integrand: Unitarity construction
- Integration: EFT subtraction, non-relativistic integration (4D) & velocity resummation
- Scattering angle from Hamiltonian, classical dynamics

↑

Methods questioned in [Damour]

$\mathcal{N} = 8$ Supergravity

[JPM, Ruf, Zeng]

- Integrand: KK reduction
- Integration: dimensional regularization, relativistic velocity differential equations
- Scattering angle from eikonal phase

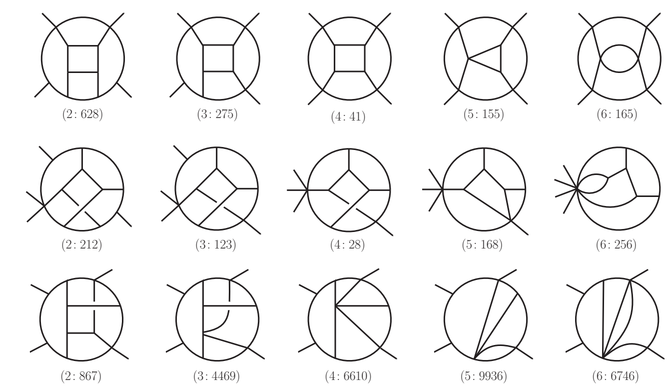
↑

Independent methods

Integrands from KK reduction

- Loop integrand known up to five loops

  [Bern, Brink, Carrasco, Chen, Dixon, Edison, Green, Johansson, JPM, Kosower, Perelstein, Roiban, Rozowski, Schwarz, Zeng]

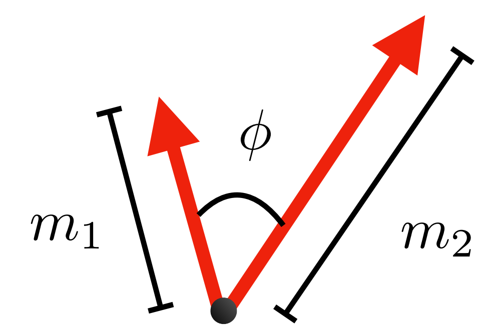


- Massless particle in D+n-dim = BPS state in D-dim

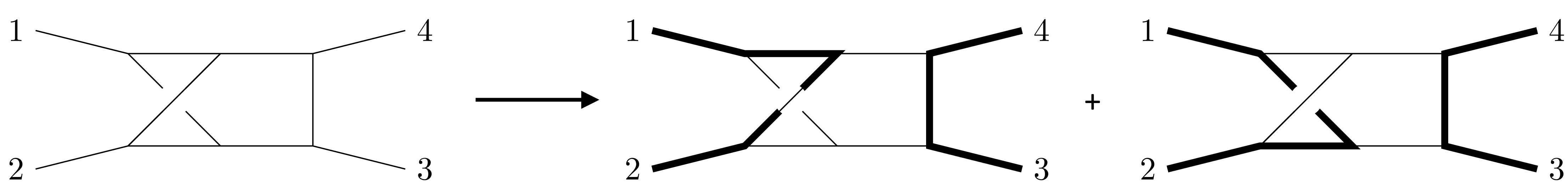
$$P^M = p^\mu + \tilde{p}^m \qquad \Gamma^M_{AB} \sim \Gamma^\mu_{\alpha\beta} \oplus \Sigma^m_{IJ} \qquad \tilde{p}_1 \cdot \tilde{p}_2 = m_1 m_2 \cos\phi$$

10D 4D 6D

$$\{Q_A, Q_B\} = \Gamma_{AB} \cdot P \quad \rightarrow \quad \{Q_{I\alpha}, Q_{J\beta}\} = \delta_{IJ}\gamma_{\alpha\beta} \cdot p + \delta_{\alpha\beta}\overbrace{\Sigma_{IJ} \cdot \tilde{p}}^{C_{IJ}}$$



- Kaluza-Klein reduction - massless exchange



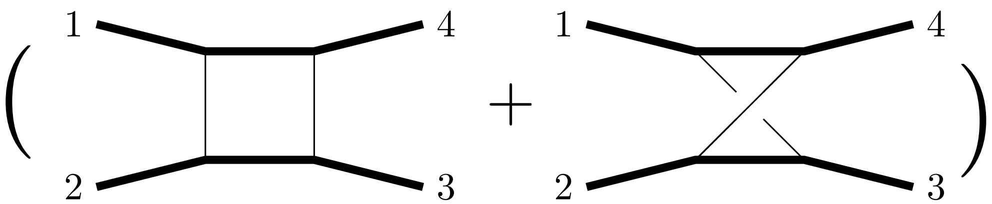
Integrands from KK reduction

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$t = -q^2$$

- One-loop integrand [Brink, Green, Schwarz; Caron-Huot, Zahraee]

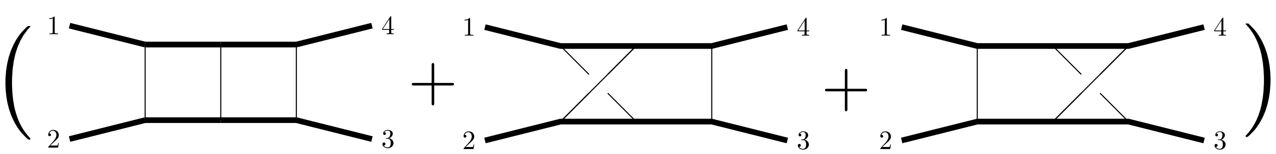
$$\mathcal{M}_4^{(1)} = -i(8\pi G)^2 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$



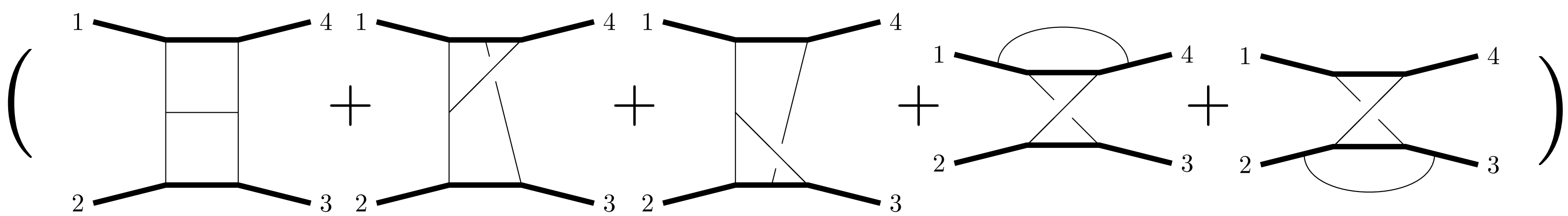
- Two-loop integrand [Bern, Dixon, Perelstein, Rozowski; JPM, Ruf, Zeng]

$$\mathcal{M}_4^{(2)} = -(8\pi G)^3 16 m_1^4 m_2^4 (\sigma - \cos\phi)^4$$

$$\Big[4 m_1^2 m_2^2 (\sigma - \cos\phi)^2$$



$$+(q^2)^2$$



$$+ (2 \leftrightarrow 3) \Big]$$

Only scalar integrals

(Un)surprisingly simple!

Integration: Regions

- Classical limit = Large angular momentum $\frac{m_i^2}{-q^2} \sim \frac{s}{-q^2} \sim J \gg 1$
- Method of regions [Beneke, Smirnov] $|\boldsymbol{v}| = q^0/|\boldsymbol{q}|$

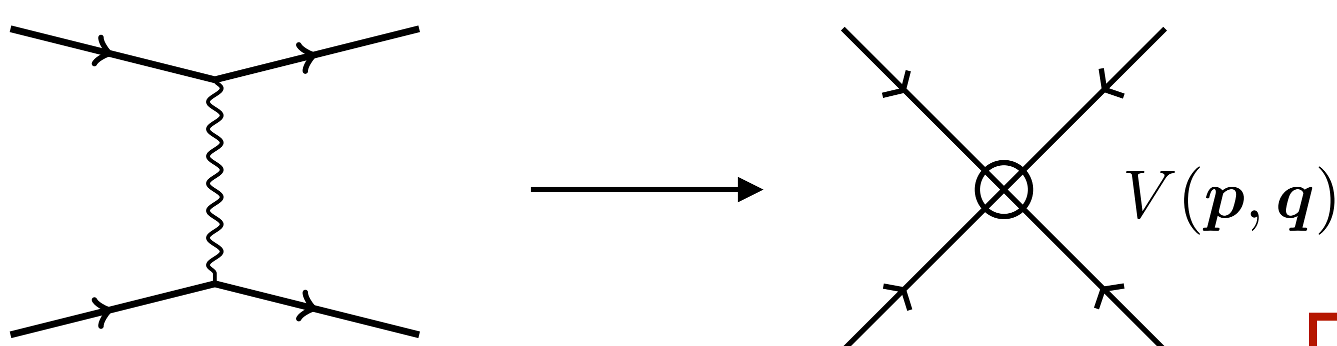
hard: $(\omega, \boldsymbol{\ell}) \sim (m, m)$

soft: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}|, |\boldsymbol{q}|) \sim J^{-1}\,(m|\boldsymbol{v}|, m|\boldsymbol{v}|)$

→ potential: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}|) \sim J^{-1}\,(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|)$ ←

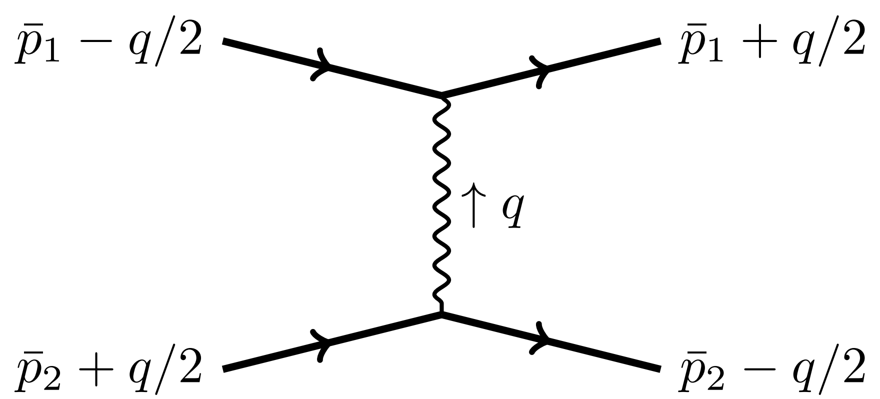
radiation: $(\omega, \boldsymbol{\ell}) \sim (|\boldsymbol{q}||\boldsymbol{v}|, |\boldsymbol{q}||\boldsymbol{v}|) \sim J^{-1}\,(m|\boldsymbol{v}|^2, m|\boldsymbol{v}|^2)$

- Potential gravitons off-shell, mediate instantaneous interactions



Conservative dynamics

Velocity differential equations



- Special variables [Sudakov] $\bar{p}_i \cdot q = 0$
- Differential equations for soft integrals

$$\ell^2 \quad \to \quad \ell^2$$
$$(\ell + p_i)^2 - m_i^2 \quad \to \quad 2\ell \cdot u_i \,, \qquad u_i = \bar{p}_i / \bar{m}_i$$

- Single variable! canonical form [Henn]

$$I(q, \bar{p}_i, \bar{m}_i) = (-q^2)^a \, I(y) \qquad y = \frac{\bar{p}_1 \cdot \bar{p}_2}{\bar{m}_1 \bar{m}_2} = \sigma + \mathcal{O}(q^2)$$

$$\mathrm{d}\vec{I}(y) = \epsilon \sum_i A_i \mathrm{dlog}\, \alpha_i(y) \, \vec{I}(y)$$

- Symbol alphabet: $\{x, x+1, x-1\} \to$ new functions e.g. $\mathrm{Li}_2(1 - x^2)$ ← Relevant at $\mathcal{O}(G^4)$?

$$y = \frac{1+x^2}{2x} \qquad \log x \sim \mathrm{arcsinh} \sqrt{\frac{\sigma - 1}{2}}$$

More generally, Harmonic polylogs (HPL) [Remiddi, Vermaseren]

Static boundary conditions

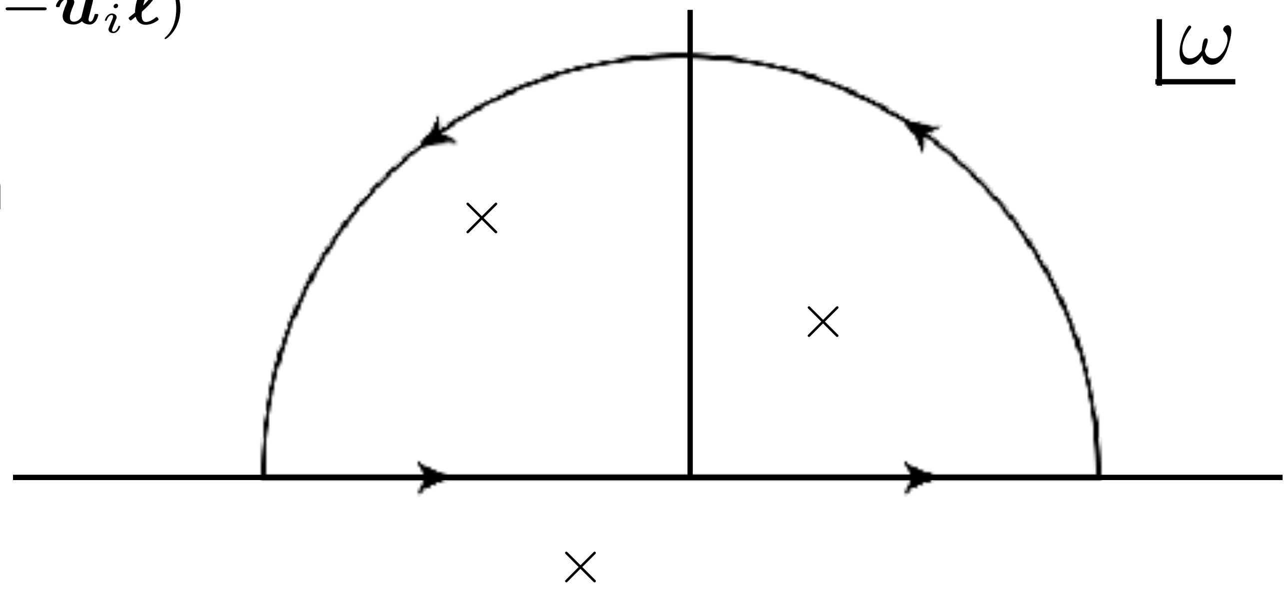
- Radiation and potential regions split in near-static limit $v \ll 1$
- Integrals in potential region satisfy same differential equations as soft integrals! Only need to calculate appropriate boundary conditions.
- Potential region integral with cut matter propagators

Graviton: $\frac{1}{\ell^2} = \frac{1}{\omega^2 - \boldsymbol{\ell}^2} = -\frac{1}{\boldsymbol{\ell}^2} - \frac{\omega^2}{(\boldsymbol{\ell}^2)^2} - \frac{\omega^4}{(\boldsymbol{\ell}^2)^3} + \cdots$

Matter: $\frac{1}{2u_i \cdot \ell} = \frac{1}{2(u_i^0\,\omega - \boldsymbol{u}_i \boldsymbol{\ell})}$

- Evaluated by residue prescription

(Similar to NRQCD/NRGR)



The amplitudes

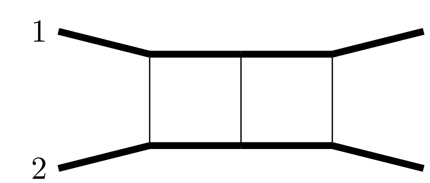
- One-loop

$$\mathcal{M}_{4(p)}^{(1)} = 64G^2\, m_1^3 m_2^3 (\sigma - \cos\phi)^4 \left(\frac{-q^2}{\bar{\mu}^2}\right)^{-\epsilon} \left\{ \frac{1}{(-q^2)} \frac{i\pi}{2\sqrt{\sigma^2-1}} \frac{e^{\epsilon}\Gamma(-\epsilon)^2\Gamma(1+\epsilon)}{\Gamma(-2\epsilon)} - \epsilon \frac{1}{} \frac{\sqrt{\pi}(m_1+m_2)}{m_1 m_2(\sigma^2-1)} \frac{e^{\epsilon}\Gamma\left(\frac{1}{2}-\epsilon\right)^2\Gamma\left(\epsilon+\frac{1}{2}\right)}{\Gamma(1-2\epsilon)} - \epsilon \frac{i\pi\left(m_1^2+m_2^2+2m_1m_2\sigma\right)}{8m_1^2m_2^2\left(\sigma^2-1\right)^{3/2}} \frac{e^{\epsilon}\Gamma(-\epsilon)^2\Gamma(1+\epsilon)}{\Gamma(-2\epsilon)} \right\} + \cdots$$

[Cristofoli, Damgaard, di Vecchia, Heissenberg]

- Two-loop

$$I_{\mathrm{III}}^{(p)} = -\frac{1}{(4\pi)^4}\left(\frac{-q^2}{\bar{\mu}^2}\right)^{-2\epsilon} \left\{ \frac{1}{(-q^2)} \frac{\pi^2}{2m_1^2m_2^2(\sigma^2-1)} \left[\frac{1}{\epsilon^2} - \frac{\pi^2}{6} + \frac{2}{3}\log^2(x)\right] + \frac{1}{\sqrt{-q^2}} \frac{i\pi^3(m_1+m_2)}{2m_1^3m_2^3(\sigma-1)\sqrt{\sigma^2-1}} - \frac{\pi^2\left(2m_2m_1\sigma+m_1^2+m_2^2\right)}{8m_1^4m_2^4(\sigma^2-1)^2}\frac{1}{\epsilon} \right\} + \cdots$$



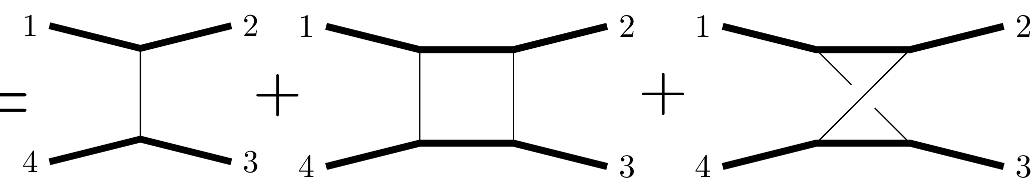
$$\mathcal{M}_{4,(p)}^{(2)} = -32\pi G^3\, m_1^4 m_2^4 (\sigma-\cos\phi)^4 \left(\frac{-q^2}{\bar{\mu}^2}\right)^{-2\epsilon} \left\{ \frac{1}{(-q^2)} \frac{2(\sigma-\cos\phi)^2}{(\sigma^2-1)} \left[\frac{1}{\epsilon^2} - \frac{\pi^2}{6}\right] + \frac{1}{\sqrt{-q^2}} \frac{4i\pi(m_1+m_2)(\sigma-\cos\phi)^2}{m_1m_2(\sigma^2-1)^{3/2}} - \frac{1}{\epsilon}\left[\frac{(m_1^2+m_2^2+2\sigma m_1m_2)(\sigma-\cos\phi)^2}{2m_1^2m_2^2(\sigma^2-1)^2} - 2\frac{\operatorname{arcsinh}\left(\sqrt{\frac{\sigma-1}{2}}\right)}{m_1m_2\sqrt{\sigma^2-1}}\right] \right\} + \cdots$$

Eikonal exponentiation

- Exponentiation in momentum space

$$i\mathcal{M}(\sigma, q_\perp) = \mathrm{cexp}\left(i\delta(\sigma, q_\perp)\right) - 1 = i\delta(\sigma, q_\perp) - \tfrac{1}{2!}\delta(\sigma, q_\perp) \otimes \delta(\sigma, q_\perp) + \cdots$$

e.g. leading phase $i\delta^{(0)} =$



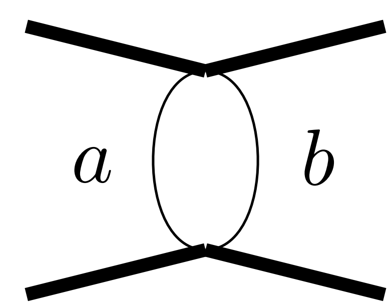
$+ \cdots$ "Resums" ladders at leading power

Inverse relation: $\delta = \mathcal{M} - \frac{i}{2}\mathcal{M} \otimes \mathcal{M} - \frac{1}{3}\mathcal{M} \otimes \mathcal{M} \otimes \mathcal{M} + \cdots$

- Convolutions are simple (propagator integrals)

$$f_1(q_\perp) \otimes f_2(q_\perp) = \frac{1}{N}\int \frac{\mathrm{d}^{D-2}\ell_\perp}{(2\pi)^{D-2}}\, f_1(\ell_\perp)\, f_2(q_\perp - \ell_\perp)$$

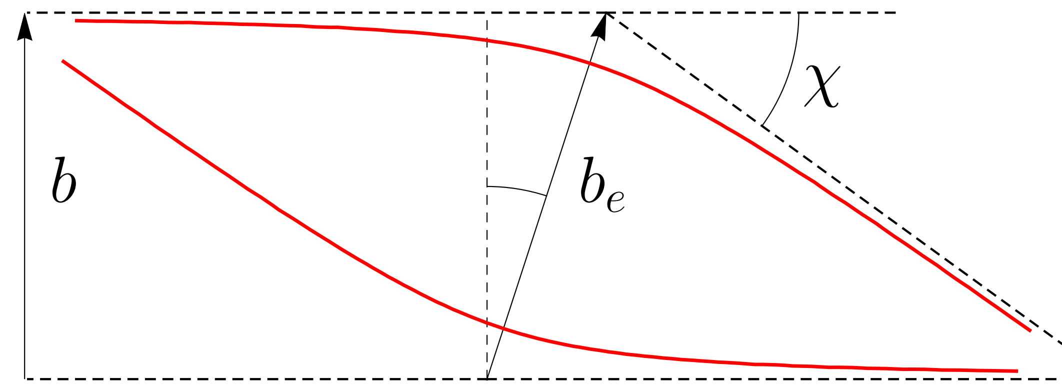
$$\frac{1}{q_\perp^a} \otimes \frac{1}{q_\perp^b} = \frac{1}{q_\perp^{a+b-D+2}} F(\epsilon)$$



- Ladder contributions cancel exactly! Only H matters.

$$\delta^{(2)}(\sigma, q_\perp) = -64\pi(Gm_1m_2)^3 \frac{(\sigma - \cos\phi)^4}{\sqrt{\sigma^2-1}} \mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}\, \frac{1}{\epsilon}\left(\frac{q_\perp^2}{\bar{\mu}^2}\right)^{-2\epsilon}$$

Scattering angle



- Scattering angle from eikonal phase **See Emil's talk!**

$$i\mathcal{M}_4(\sigma, q_\perp) \propto \int d^2 b_e e^{i b_e \cdot q_\perp} \left(e^{i\delta(\sigma, b_e)} - 1\right)$$

Stationary phase approx. $\vec{q} = -\frac{\partial}{\partial \vec{b}_e}\delta(\sigma, \vec{b}_e) \longrightarrow \sin\frac{\chi}{2} = -\frac{1}{2p}\frac{\partial}{\partial b_e}\delta(\sigma, b_e)$

- In terms of angular momentum $J = p\, b_e \cos\chi/2$

$$\chi = \frac{G m_1 m_2}{J}\frac{4(\sigma - \cos\phi)^2}{\sqrt{\sigma^2-1}} + \frac{G^2 m_1^2 m_2^2}{J^2}\times 0$$
$$- \frac{G^3 m_1^3 m_2^3}{J^3} 16\left[\frac{(\sigma-\cos\phi)^6}{3(\sigma^2-1)^{3/2}} + \frac{4 m_1 m_2 (\sigma - \cos\phi)^4}{m_1^2 + m_2^2 + 2 m_1 m_2 \sigma}\text{arcsinh}\sqrt{\frac{\sigma-1}{2}}\right]$$

- Checks: probe limit, static limit (BPS)

$$\frac{1}{2}\chi_{\rm p} = \arctan\left[\frac{G M^2 \nu_{\rm p}}{J}\frac{2(\sigma_{\rm p} - \cos\phi_{\rm p})^2}{(\sigma_{\rm p}^2 - 1)^{1/2}}\right] \qquad \nu_{\rm p} = \frac{m_{\rm p}}{M}$$

⟵ Newtonian result!

- Identical result when using EFT matching & classical EOM ✓ **See Chia-Hsien's talk!**

EFT vs. Eikonal

EFT for PM scattering

[Cheung, Rothstein, Solon]

- Non-relativistic
- EFT integrand subtraction avoids calculating IR divergent integrals
- Non-relativistic loop-by loop integration avoids ϵ/ϵ & q/q.
  Special scheme. Very efficient to target classical contributions

Eikonal exponentiation

[Old subject, e.g. Amati, Ciafaloni, Veneziano]

- Fully relativistic, but need to establish exponentiation
- Evaluate infrared divergent integrals
- Requires higher orders in ϵ and q to check exponentiation. More standard scheme

Applications

Integrability of BH orbits

[Caron-Huot, Zahraee]

- No precession at order G^2 (no triangle) and all orders in probe limit! Conjectured to hold to al orders.
- No precession possible at $\mathcal{O}(G^3)$ $\Delta\Phi = \chi(J) + \chi(-J)$ [Kaelin, Porto]
- But angle deviates from (integrable) Newtonian result

$$\chi = \frac{G m_1 m_2}{J} \frac{4(\sigma - \cos\phi)^2}{\sqrt{\sigma^2 - 1}} + \frac{G^2 m_1^2 m_2^2}{J^2} \times 0$$
$$- \frac{G^3 m_1^3 m_2^3}{J^3} 16 \left[\frac{(\sigma - \cos\phi)^6}{3(\sigma^2 - 1)^{3/2}} + \boxed{\frac{4 m_1 m_2 (\sigma - \cos\phi)^4}{m_1^2 + m_2^2 + 2 m_1 m_2 \sigma} \operatorname{arcsinh} \sqrt{\frac{\sigma - 1}{2}}} \right]$$

- EOB [Buonanno, Damour] related to Dual Conformal Invariance (Extension of Laplace-Runge-Lenz symmetry)

[Caron-Huot, Henn; Caron-Huot QCDMG18]

Is it there in physical angle?

Graviton dominance

GRAVITON DOMINANCE IN ULTRA-HIGH-ENERGY SCATTERING

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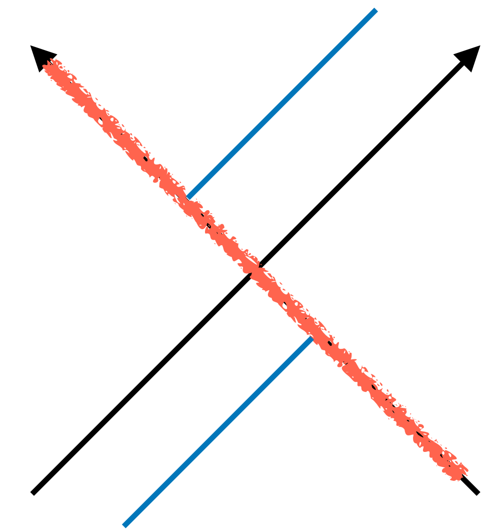
Received 7 July 1987

The scattering process of two pointlike particles at CM energies in the order of Planck units or beyond, is very well calculable using known laws of physics, because graviton exchange dominates over all other interaction processes. At energies much higher than the Planck mass black hole production sets in, accompanied by coherent emission of real gravitons.

- High-energy limit = probe particle moving in Aichelburg-Sexl shockwave

$$\mathcal{M}_4 \sim \frac{\Gamma(1-iGs)}{\Gamma(1+iGs)} \frac{Gs}{t} (-t)^{iGs}$$



- Leading eikonal phase and angle dominated by graviton exchange

$$\chi^{1\mathrm{PM}} = \frac{4Gm_1m_2\sigma}{J} \sim \frac{2Gs}{J}$$

- How about subleading G?

Graviton dominance @ $\mathcal{O}(G^3)$ (massive)

- Classical high-energy limit $J \gg \sigma \gg 1$
- High-energy limit of $\mathcal{N}=8$ angle [JPM, Ruf, Zeng]

$$\chi^{3\text{PM}}_{=8} \overset{\sigma\to\infty}{=} -\frac{16 G^3 m_1^3 m_2^3 \sigma^3 \log(\sigma)}{J^3} + \cdots$$

- High-energy limit of Einstein gravity angle [Bern, Cheung, Roiban, Shen, Solon, Zeng]

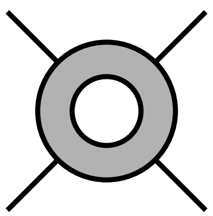
$$\chi^{3\text{PM}}_{\text{EG}} \overset{\sigma\to\infty}{=} -\frac{16 G^3 m_1^3 m_2^3 \sigma^3 \log(\sigma)}{J^3} + \cdots$$

- Graviton seems to dominate also at $\mathcal{O}(G^3)$. Requires explanation.

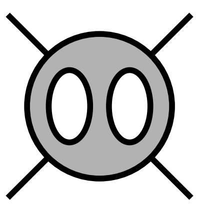
Graviton dominance @ $\mathcal{O}(G^3)$ (massless)

- Recent calculation of full 4-pt two-loop graviton amplitude in GR
  [Abreu, Jaquier, Febres Cordero, Ita, Page, Ruf, Sotnikov] See Samuel's talk!
- Previous results for $\mathcal{N} \geq 4$ [Dixon, Boucher-Veronneau]

Caravel



$$\frac{\mathcal{M}^{(1)}}{\mathcal{M}^{(0)}} = K\left[-\frac{2}{\epsilon}\left(i\pi + \frac{-t}{s}\log\left(\frac{-t}{s}\right) + \frac{-u}{s}\log\left(\frac{-u}{s}\right)\right) + F^{(1)}\right]$$



$$\frac{\mathcal{M}^{(2)}}{\mathcal{M}^{(0)}} = \frac{1}{2}\left[\frac{\mathcal{M}^{(1)}}{\mathcal{M}^{(0)}}\right]^2 + K^2 F^{(2)} \qquad K = \frac{r_\Gamma G s}{2\pi}\left(\frac{\bar{\mu}^2}{-t}\right)^{\epsilon}$$

- Relevant contributions at two loops

$$\mathcal{M}^{(2)} \underset{s/q^2\to\infty}{\sim} -\frac{2\pi i}{\epsilon}\frac{s}{q^2}F^{(1)} + \frac{s}{q^2}F^{(2)}$$

Massless HE limit

$\sigma \gg J \gg 1$

- All universal!

$$\mathrm{Im}\ \frac{F^{(1)}}{s} = \frac{q^2}{s}\,2\pi i\ \log\frac{s}{q^2} \qquad \mathrm{Re}\ \frac{F^{(2)}}{s} = \frac{q^2}{s}\left(-2\pi^2\log^2\frac{s}{q^2} + 4\pi^2\log\frac{s}{q^2} + \mathrm{const.}\right)$$

Massless angle is universal through $\mathcal{O}(G^3)$

[Bern, JPM, Ita, Ruf]

One loop angle, quantum, non-universal

Two loop angle universal

$$\sin\frac{\chi^{\mathcal{N}\geq 4}}{2} = \frac{Gs}{J} + \frac{1}{2}\left(\frac{Gs}{J}\right)^3 = \sin\frac{\chi^{\mathrm{GR}}}{2}$$

It also agrees with [Amati, Ciafaloni, Veneziano] and disagrees with [Damour]

Requires an explanation!

Summary

- Developed new methods for calculation of classical scattering amplitudes
- Combined with old techniques (eikonal) to calculate scattering angle of extremal BH in $\mathcal{N}=8$ supergravity
- Eikonal and EFT methods agree - confirmation of BCRSSZ
- No precession at $\mathcal{O}(G^3)$, but angle deviates from Newton
- 't Hooft's graviton dominance seems to extend to subleading order for both massive and massless (why? proof?)
- $\mathcal{N}=8$ supergravity is an excellent theoretical laboratory
- Looking to the future: What's next? Spin? $\mathcal{O}(G^4)$?

Thank you!

KLT BCJ EFT Cuts SUSY DCI
Dyons Universality Integrable
Extremal Conservative Potential
Glauber Exponentiation EOB
BPS Effective field theory Soft
PM Large Angular Momentum
Elastic Froissart–Gribov Eikonal
Classical Partial Waves Inversion
Unitarity Double Copy Radiation
Soft Limits Finite Size Dispersion
Spin