The Momentum Amplituhedron Boundaries

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&

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Based on: arXiv:2003.13704

with T. Lukowski, R. Moerman

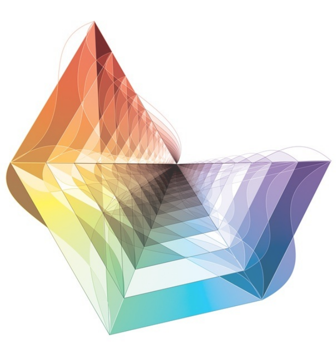
Outline

* Introduction and Motivation
* Reminder: Momentum Amplituhedron
* Singularities of Amplitudes from its Boundaries
* Conclusions

Introduction

Scattering amplitudes in (planar) N=4 sYM:
encoded in "Amplituhedra"

More general paradigm:
positive geometries
-> regions with boundaries equipped with rational differential forms
-> the forms are logarithmic on any boundary



(picture by A. Gilmore)

Introduction

Scattering amplitudes in (planar) N=4 sYM:
encoded in "Amplituhedra"

More general paradigm:
positive geometries

* amplituhedron (N. Arkani-Hamed, J. Trnka)
* kinematic associahedron (N. Arkani-Hamed, Y. Bai, S. He, G. Yan)
* cosmological polytope (N. Arkani-Hamed, P. Benincasa, A. Postnikov)
* CFTs (B. Eden, P. Heslop, L.Mason; N. Arkani-Hamed, Y.-T. Huang, S.-H. Shao)
* ...
* momentum amplituhedron (D. Damgaard, LF, T. Lukowski, M. Parisi)

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Introduction

Scattering amplitudes in (planar) N=4 sYM:
encoded in "Amplituhedra"

Conjecture:
amplituhedra encode physical singularities of amplitudes in the structure of their boundaries
-> Facets (= codim 1): straightforward
-> What happens deeper in geometry?

In this talk:
Boundaries of Mom Ampl<->Singularities of Tree Ampls
Boundary Stratification + Full Classification of Sings

Singularities for Scattering Ampls

Color-ordered amplitudes: pure Yang-Mills

* Multi-particle poles when sum of adjacent momenta goes on-shell:

$$P_{i,j}^2 = (p_i + p_{i+1} + \ldots + p_j)^2 \to 0$$

$$A_n^{\text{tree}}(1,\ldots,n) \xrightarrow{P_{i,j}^2 \to 0} \sum_{h=\pm 1} A_{j-i+2}^{\text{tree}}(i,\ldots,j,P_{i,j}^h)\frac{1}{P_{i,j}^2} A_{n-j+i}^{\text{tree}}(P_{i,j}^{-h}, j+1,\ldots,i-1)$$

* Collinear and soft singularities: universal factorization properties

$$A_n^{\text{tree}}(\ldots,i^{h_i},(i+1)^{h_{i+1}},\ldots) \xrightarrow{p_i || p_{i+1}} \sum_{h=\pm 1} A_{n-1}^{\text{tree}}(\ldots,P_{i,i+1}^h,\ldots)\ \text{Split}_{-h}^{\text{tree}}(i^{h_i},(i+1)^{h_{i+1}})$$

$$A_n^{\text{tree}}(\ldots,i,s^h,j,\ldots) \xrightarrow{p_s \to 0} A_{n-1}^{\text{tree}}(\ldots,i,j,\ldots)\ \text{Soft}^{\text{tree}}(i,s^h,j)$$

Singularities for Scattering Ampls

Color-ordered amplitudes: N=4 super Yang-Mills

* Multi-particle poles when sum of adjacent momenta goes on-shell:

$$A_{n,k}(1,\ldots,n) \xrightarrow{P_{i,j}^2\to 0} \sum_{k'=1}^{k} \int d^4\eta_{P_{i,j}} A_{j-i+2,k'}(i,\ldots,j,P_{i,j}) \frac{1}{P_{i,j}^2} A_{n-j+i,k-k'+1}(P_{i,j},j+1,\ldots,i-1)$$

* Collinear and soft singularities:

$p_1||p_2$ $$A_{n,k}(1,2,3,\ldots,n) \xrightarrow{P_{12}^2\to 0} \int d^4\eta_{P_{12}} \left[\mathrm{Split}_0^{\mathrm{tree}} A_{n-1,k}(P_{12},3,\ldots,n) + \mathrm{Split}_{-1}^{\mathrm{tree}} A_{n-1,k-1}(P_{12},3,\ldots,n)\right]$$

helicity-preserving super-splitting function — parametrised by $\langle 12\rangle \to 0$

helicity-decreasing super-splitting function — parametrised by $[12] \to 0$

$$\mathrm{Split}_0^{\mathrm{tree}}(z;\eta_1,\eta_2,\eta_{P_{12}}) \equiv \frac{1}{\sqrt{z(1-z)}}\frac{1}{\langle 12\rangle}\prod_{A=1}^{4}(\eta_{P_{12}A} - \sqrt{z}\eta_{1A} - \sqrt{1-z}\eta_{2A})$$

z momentum fraction

$$\mathrm{Split}_{-1}^{\mathrm{tree}}(z;\eta_1,\eta_2,\eta_{P_{12}}) \equiv \frac{1}{\sqrt{z(1-z)}}\frac{1}{[12]}\prod_{A=1}^{4}(\eta_{1A}\eta_{2A} + \sqrt{1-z}\eta_{1A}\eta_{P12A} - \sqrt{z}\eta_{2A}\eta_{P_{12}A})$$

Singularities for Scattering Ampls

Color-ordered amplitudes: N=4 super Yang-Mills

* Multi-particle poles when sum of adjacent momenta goes on-shell:

$$A_{n,k}(1,\ldots,n) \xrightarrow{P_{i,j}^2\to 0} \sum_{k'=1}^{k} \int d^4\eta_{P_{i,j}} A_{j-i+2,k'}(i,\ldots,j,P_{i,j}) \frac{1}{P_{i,j}^2} A_{n-j+i,k-k'+1}(P_{i,j},j+1,\ldots,i-1)$$

* Collinear and soft singularities:

$p_1 || p_2$: $$A_{n,k}(1,2,3,\ldots,n) \xrightarrow{P_{12}^2\to 0} \int d^4\eta_{P_{12}} \left[\text{Split}_0^{\text{tree}} A_{n-1,k}(P_{12},3,\ldots,n) + \text{Split}_{-1}^{\text{tree}} A_{n-1,k-1}(P_{12},3,\ldots,n)\right]$$

parametrised by $\langle 12\rangle \to 0 \quad \langle 23\rangle \to 0$

parametrised by $[12] \to 0 \quad [23] \to 0$

g^+ $\qquad$ g^-

$p_2 \to 0$: $$A_{n,k}(1,2,3,\ldots,n) \xrightarrow{p_2\to 0} \text{Soft}_0^{\text{tree}} A_{n-1,k}(1,3,\ldots,n) \qquad A_{n,k}(1,2,3,\ldots,n) \xrightarrow{p_2\to 0} \text{Soft}_{-1}^{\text{tree}} A_{n-1,k-1}(1,3,\ldots,n)$$

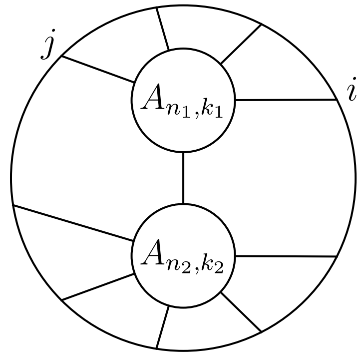
helicity-preserving super-soft function $\quad \text{Soft}_0^{\text{tree}} = \dfrac{\langle 13\rangle}{\langle 12\rangle\langle 23\rangle}$

helicity-decreasing super-soft function $\quad \text{Soft}_{-1}^{\text{tree}} = \eta_2^4 \dfrac{[13]}{[12][23]}$

Singularities for Scattering Ampls

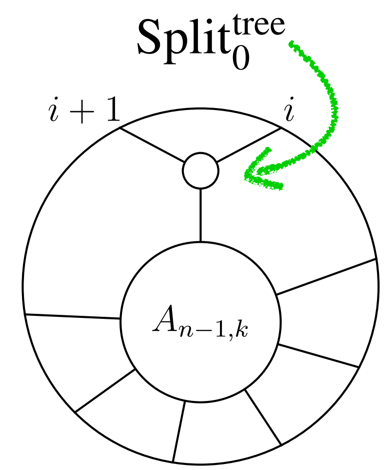
Color-ordered amplitudes: N=4 super Yang-Mills

multi-particle factorizations

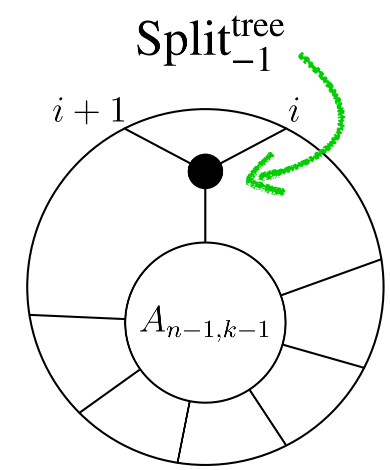


$$s_{i,i+1\ldots,j} = 0$$

collinear limits

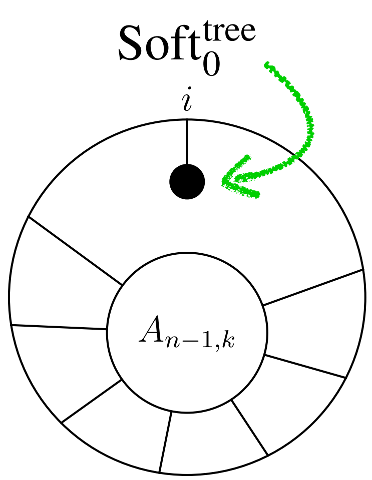


$$\langle i\, i+1\rangle = 0$$

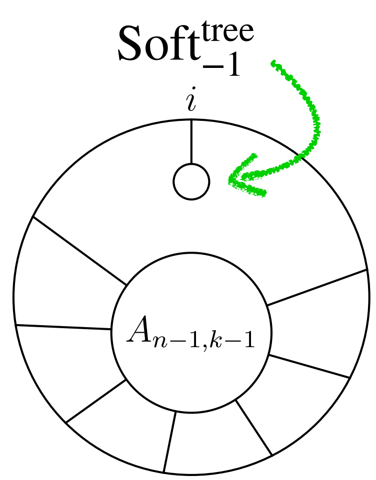


$$[i\, i+1] = 0$$

soft limits

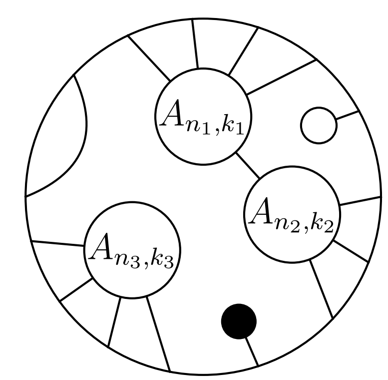


$$\langle i\, i+1\rangle = 0 = \langle i-1\, i\rangle$$



$$[i\, i+1] = 0 = [i-1\, i]$$

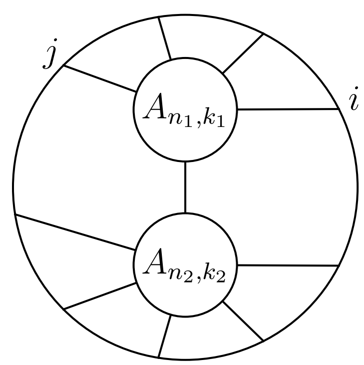
and further



Singularities for Scattering Ampls

Color-ordered amplitudes: N=4 super Yang-Mills

multi-particle factorizations



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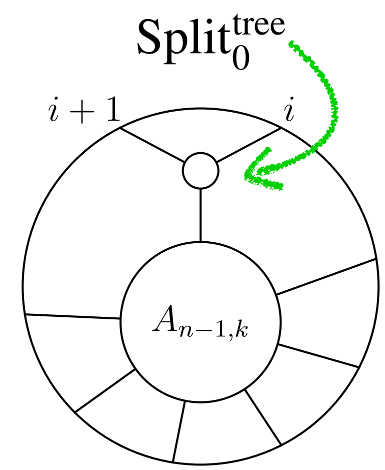
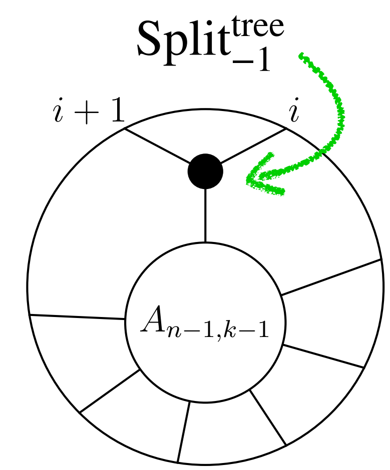


$\langle i\, i+1 \rangle = 0$



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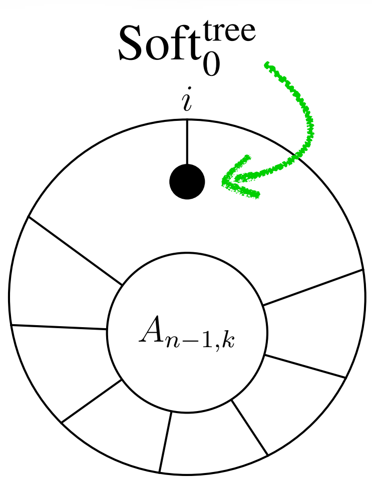
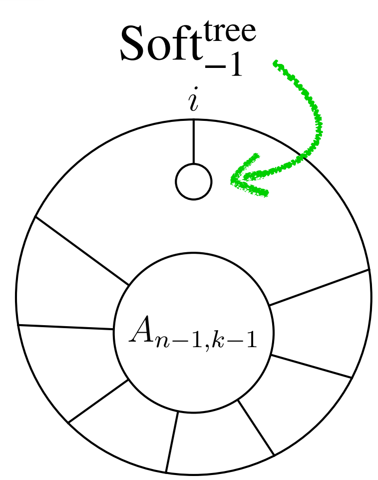


$\langle i\, i+1 \rangle = 0 = \langle i-1\, i \rangle$



$[i\, i+1] = 0 = [i-1\, i]$

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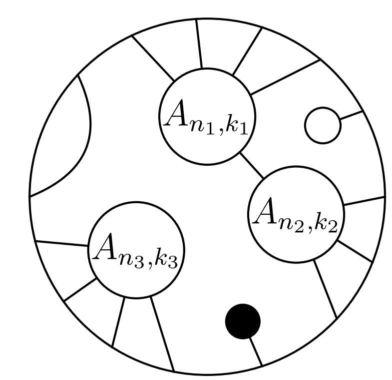
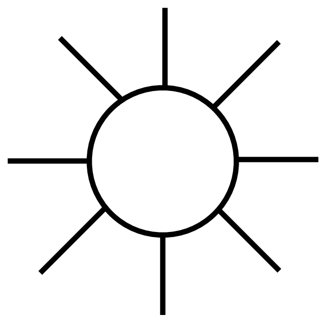


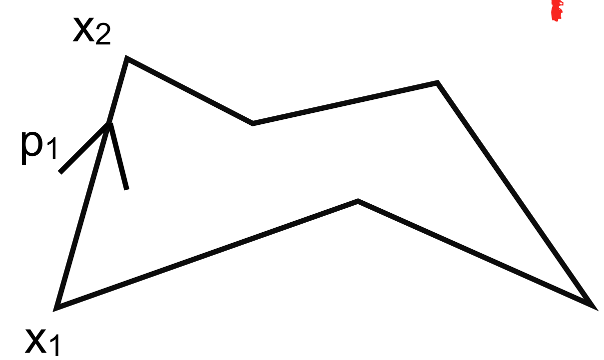
we want to classify these using the momentum amplituhedron

Scattering amplitudes in N=4 sYM

duality
⟷

Amplitude | **Wilson loop**





$$p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$q_i^{\alpha A} = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$

on-shell superspace

$(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$

+ bosonization

⟷

dual superspace

$(\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$

momentum amplituhedron

Fourier transform on λ_i^α

Incidence relations

$$\tilde\mu_i^{\dot\alpha} := x_i^{\dot\alpha\alpha}\lambda_{i\alpha}$$
$$\chi_i^A := \theta_i^{\alpha A}\lambda_{i\alpha}$$

twistor superspace

$\mathcal{W}_i^A = (\mu_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$

momentum-twistor superspace

$\mathcal{Z}_i^A = (\lambda_i^\alpha, \tilde\mu_i^{\dot\alpha}, \chi_i^A)$

+ bosonization

amplituhedron

Scattering amplitudes in N=4 sYM

on-shell superspace

$$(\lambda_i^{\alpha}, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$$

Non-chiral superspace

$$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1{,}2$$

$$\begin{cases} \tilde{q}^{\dot{a}r} = \sum_{i=1}^{n} \tilde{\lambda}_i^{\dot{a}} \eta_i^r \\ q^{ar} = \sum_{i=1}^{n} \lambda_i^a \tilde{\eta}_i^{\dot{r}} \end{cases}$$

Associate a,ȧ with SU(2)xSU(2) R-symmetry indices:

$$\eta^a \to d\lambda^a\,, \quad \tilde{\eta}^{\dot{a}} \to d\tilde{\lambda}^{\dot{a}}$$

n-point super-amplitude in non-chiral space <-> 2n form

(S. He, C. Zhang)

Amplitudes as forms

$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1{,}2$

n-point super-amplitude in non-chiral space <-> 2n form

$$\mathcal{A}_{n,k} := (dq)^4 \wedge \mathbf{\Omega_{n,k}}$$

2n-4 form

$$\begin{cases} d\tilde{q}^{\dot{a}r} = \sum_{i=1}^{n} \tilde{\lambda}_i^{\dot{a}} d\lambda^r \\ dq^{ar} = \sum_{i=1}^{n} \lambda_i^a d\tilde{\lambda}_i^{\dot{r}} \end{cases}$$

Example: 4-point MHV

$$\mathbf{\Omega}_{4,2} = \frac{(d\tilde{q})^4}{st} = \mathrm{dlog}\frac{\langle 12\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 23\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 34\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 41\rangle}{\langle 13\rangle}$$

Geometry whose canonical form gives the amplitude form?

Momentum amplituhedron M!

Momentum Amplituhedron

Bosonized spinor helicity variables:

$$\tilde{\Lambda}_i^{\dot{A}} = \begin{pmatrix} \tilde{\lambda}_i^{\dot{a}} \\ \tilde{\phi}_{\dot{a}}^{\dot{\alpha}} \cdot \tilde{\eta}_i^{\dot{a}} \end{pmatrix}, \qquad \dot{A} = (\dot{a}, \dot{\alpha}) = 1, \ldots, k+2 \qquad \Lambda_i^{A} = \begin{pmatrix} \lambda_i^{a} \\ \phi_{a}^{\alpha} \cdot \eta_i^{a} \end{pmatrix}, \qquad A = (a, \alpha) = 1, \ldots, n-k+2$$

$$\left\{ \text{matrix } \tilde{\Lambda} \text{ positive} \quad \text{and} \quad \text{matrix } \Lambda^{\perp} \text{ positive} \right\}$$

Positive region

Momentum amplituhedron: Image of the positive Grassmannian $G_+(k,n)$ through the map

$$\Phi_{(\Lambda,\tilde{\Lambda})} : G_+(k,n) \to G(k,k+2) \times G(n-k,n-k+2)$$

defined as:

$$\tilde{Y}_{\dot{\alpha}}^{\dot{A}} = \sum_{i=1}^{n} c_{\dot{\alpha}i} \tilde{\Lambda}_i^{\dot{A}} \qquad Y_{\alpha}^{A} = \sum_{i=1}^{n} c_{\alpha i}^{\perp} \Lambda_i^{A}$$

Momentum Amplituhedron

$$\tilde{Y}_{\dot\alpha}^{\dot A} = \sum_{i=1}^{n} c_{\dot\alpha i}\,\tilde\Lambda_i^{\dot A} \qquad Y_\alpha^A = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda_i^A$$

Facets: codimension-1 boundaries

$$\langle Y\, i\, i+1\rangle = 0\,, \qquad [\tilde Y\, i\, i+1] = 0 \qquad \text{Collinear limits}$$

$$S_{i,i+1\ldots,i+p} = 0\,, \qquad p = 2,\ldots,n-4 \qquad \text{Factorizations}$$

Uplift of planar Mandelstam variables

$$S_{i,i+1\ldots,i+p} = \sum_{i\le j_1<j_2\le i+p} \langle Y j_1 j_2\rangle [\tilde Y j_1 j_2]$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Facets: codimension-1 boundaries

$\langle Y\, i\, i+1 \rangle = 0\,,$	$[\tilde{Y}\, i\, i+1] = 0$	Collinear limits
$S_{i,i+1\ldots,i+p} = 0\,,$	$p = 2,\ldots,n-4$	Factorizations

Volume form
=
Differential form with log singularities on all boundaries
=
Sum over cells of push-forwards of canonical diff-form

Momentum Amplituhedron

$$\tilde{Y}_{\dot\alpha}^{\dot A} = \sum_{i=1}^{n} c_{\dot\alpha i}\,\tilde\Lambda_i^{\dot A} \qquad\qquad Y_\alpha^A = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda_i^A$$

Facets: codimension-1 boundaries

$\langle Y\, i\, i+1\rangle = 0\,,\qquad [\tilde Y\, i\, i+1] = 0$ Collinear limits

$S_{i,i+1\ldots,i+p} = 0\,,\qquad p = 2,\ldots,n-4$ Factorizations

$\mathbf{\Omega}_{n,k}$: volume form with logarithmic singularities on all boundaries of the space

$$\mathbf{\Omega}_{n,k} = \sum_{\sigma} \mathrm{dlog}\,\alpha_1^\sigma \wedge \mathrm{dlog}\,\alpha_2^\sigma \wedge \ldots \wedge \mathrm{dlog}\,\alpha_{2n-4}^\sigma$$

Logarithmic differential form

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_{i} \qquad\qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_{i}$$

➡ Volume form

$$\mathbf{\Omega}_{n,k} = \sum_{\sigma} \mathrm{dlog}\, \alpha_1^{\sigma} \wedge \mathrm{dlog}\, \alpha_2^{\sigma} \wedge \ldots \wedge \mathrm{dlog}\, \alpha_{2n-4}^{\sigma}$$

➡ Volume function

$$\mathbf{\Omega}_{n,k} \wedge d^4P\, \delta^4(P) = \prod_{\alpha=1}^{n-k} \langle Y_1 \ldots Y_{n-k} d^2 Y_{\alpha} \rangle \prod_{\dot{\alpha}=1}^{k} [\tilde{Y}_1 \ldots \tilde{Y}_k d^2 \tilde{Y}_{\dot{\alpha}}]\, \delta^4(P)\, \Omega_{n,k}$$

➡ Amplitude

Reference subspaces

$$\mathscr{A}^{\mathtt{tree}}_{n,k} = \delta^4(p) \int d\phi_a^1 \ldots d\phi_a^{n-k} \int d\tilde{\phi}^1_{\dot{a}} \ldots d\tilde{\phi}^k_{\dot{a}}\; \Omega_{n,k}(Y^*, \tilde{Y}^*, \Lambda, \tilde{\Lambda})$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Example

* MHV$_4$ amplitude:

$$\alpha_1 = \frac{\langle Y12 \rangle}{\langle Y13 \rangle}, \alpha_2 = \frac{\langle Y23 \rangle}{\langle Y13 \rangle}, \alpha_3 = \frac{\langle Y34 \rangle}{\langle Y13 \rangle}, \alpha_4 = \frac{\langle Y14 \rangle}{\langle Y13 \rangle}$$

$$\mathbf{\Omega}_{4,2} = \bigwedge_{j=1}^{4} \mathrm{dlog}\alpha_j = \mathrm{dlog}\frac{\langle Y12 \rangle}{\langle Y13 \rangle} \wedge \mathrm{dlog}\frac{\langle Y23 \rangle}{\langle Y13 \rangle} \wedge \mathrm{dlog}\frac{\langle Y34 \rangle}{\langle Y13 \rangle} \wedge \mathrm{dlog}\frac{\langle Y14 \rangle}{\langle Y13 \rangle}$$

$$= \frac{\langle 1234 \rangle^2}{\langle Y12 \rangle \langle Y23 \rangle \langle Y34 \rangle \langle Y41 \rangle} \langle Y d^2 Y_1 \rangle \langle Y d^2 Y_2 \rangle$$

Divergences on the 4 facets of the momentum amplituhedron:

$$\langle Yii+1 \rangle = 0 \,, i = 1,\ldots,4$$

Momentum Amplituhedron

Examples

* NMHV$_6$ amplitudes:

$$\tilde{Y}^{\dot A}_{\dot\alpha} = \sum_{i=1}^{n} c_{\dot\alpha i}\,\tilde\Lambda^{\dot A}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_i$$

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{\left(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\right)^2\left([\tilde Y45]\langle 12345\rangle + [\tilde Y46]\langle 12346\rangle + [\tilde Y56]\langle 12356\rangle\right)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde Y45][\tilde Y56]\langle Y1|5+6|4\tilde Y]\langle Y3|4+5|6\tilde Y]}$$

Spurious singularities

Divergences on the 15 facets of the momentum amplituhedron:

$$\langle Yii+1\rangle = 0\,,\ i = 1,\ldots,6\,, \quad [\tilde Y ii+1] = 0\,,\ i = 1,\ldots,6\,, \quad S_{i,i+1,i+2} = 0\,,\ i = 1,2,3$$

Singularities from Boundaries

Boundary stratification from Mathematica package amplituhedronBoundaries.m (T. Lukowski, R. Moerman)
up to n=9 and all k (extended to n=11 via dual graphs) based on positroids.m (J. Bourjaily)

Singularities from Boundaries

Boundary stratification from Mathematica package amplituhedronBoundaries.m
up to n=9 and all k (extended to n=11 via dual graphs)

(T. Lukowski, R. Moerman)
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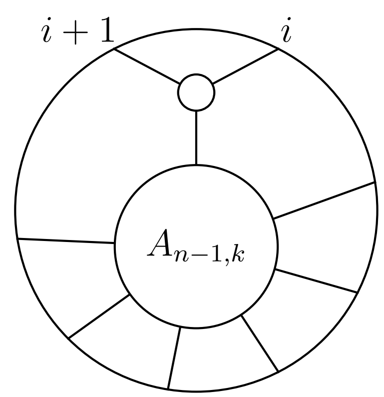
(J. Bourjaily)

-> labeled by plabic diagrams of cells in $G_+(k,n)$

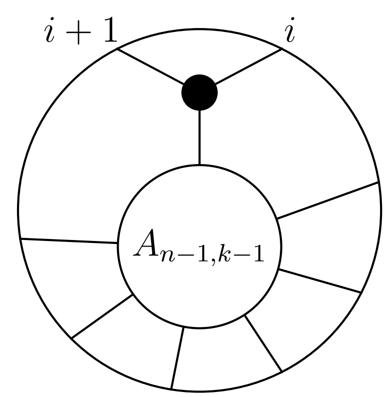
Codim 1 boundaries

collinear



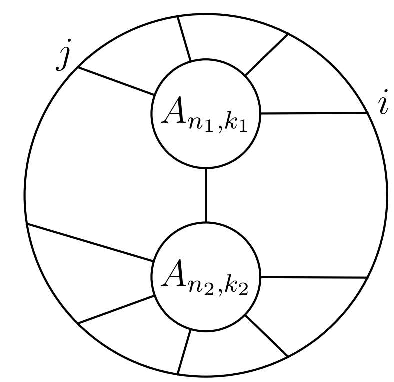
$\langle Y\, i\, i+1 \rangle = 0$

collinear



$[\tilde{Y}\, i\, i+1] = 0$

factorization



$S_{i,i+1\ldots,j} = 0$

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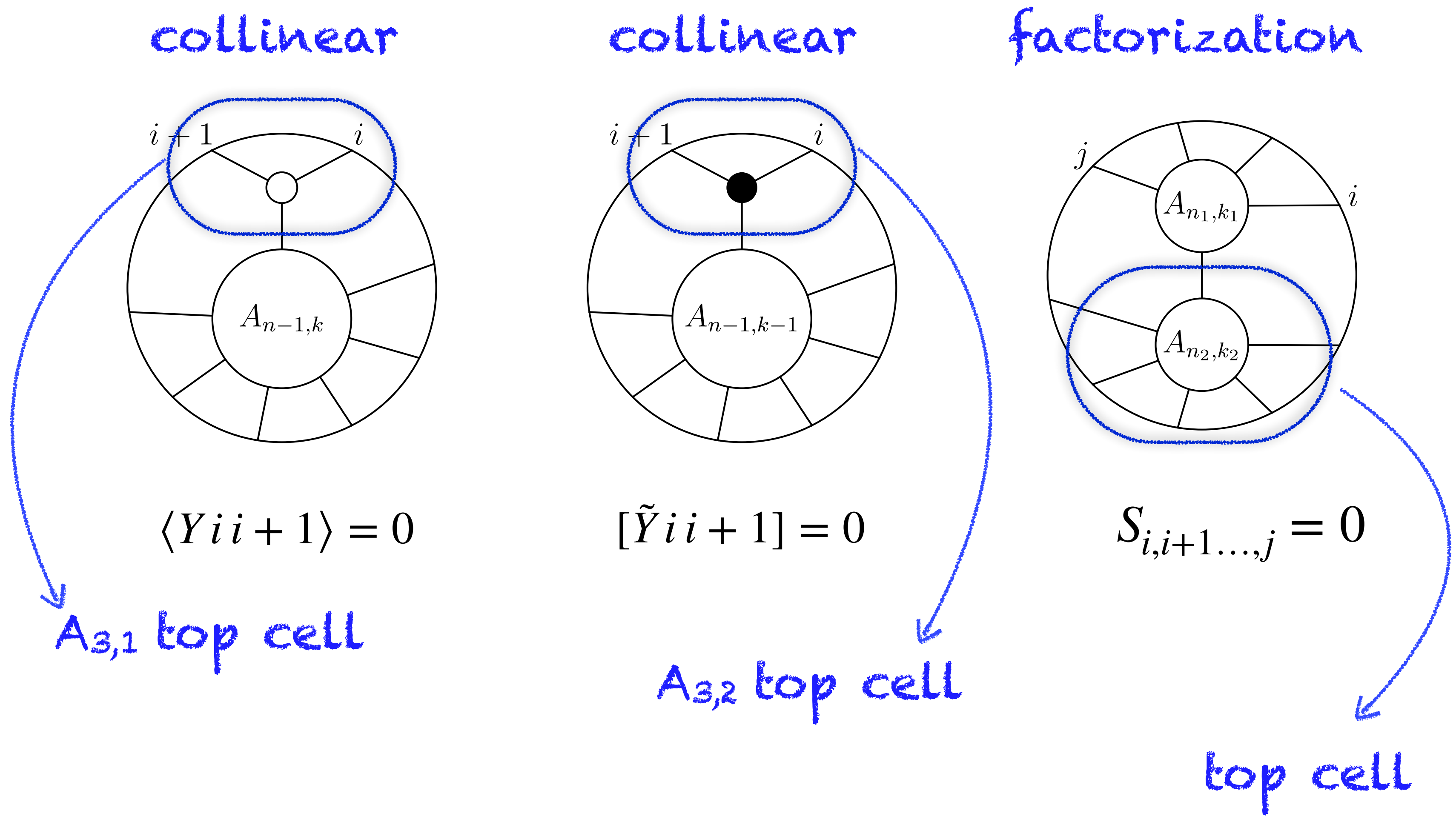

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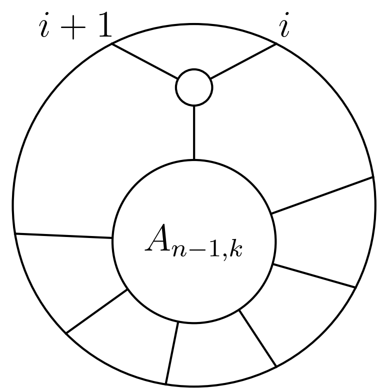
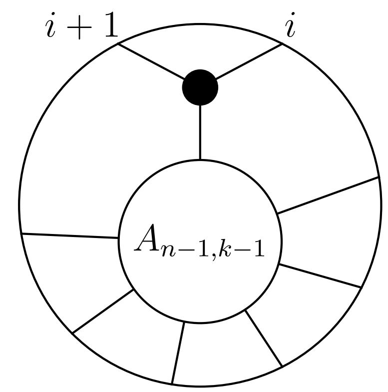
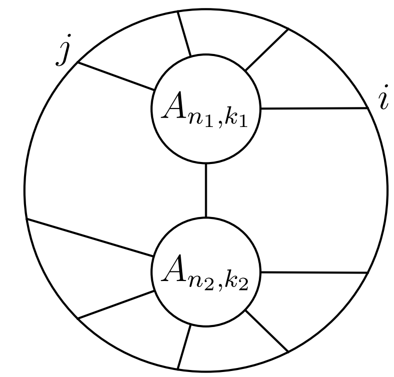
Codim 1 boundaries



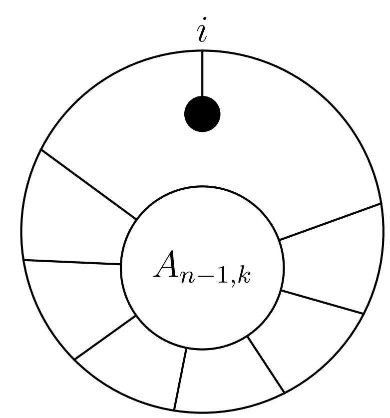




Codim 2 boundaries

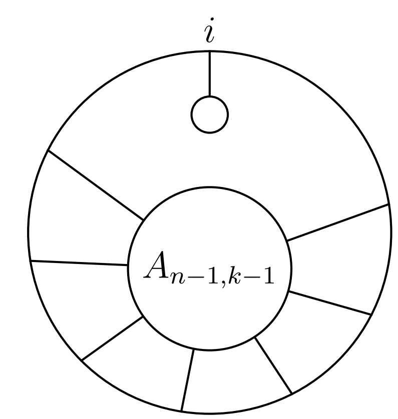
further factorizations and collinear limits

soft



$\langle Y\, i\, i+1\rangle = 0 = \langle Y\, i-1\, i\rangle$

soft



$[\tilde{Y}\, i\, i+1] = 0 = [\tilde{Y}\, i-1\, i]$

Intersection of two consecutive codim 1 collinear boundaries

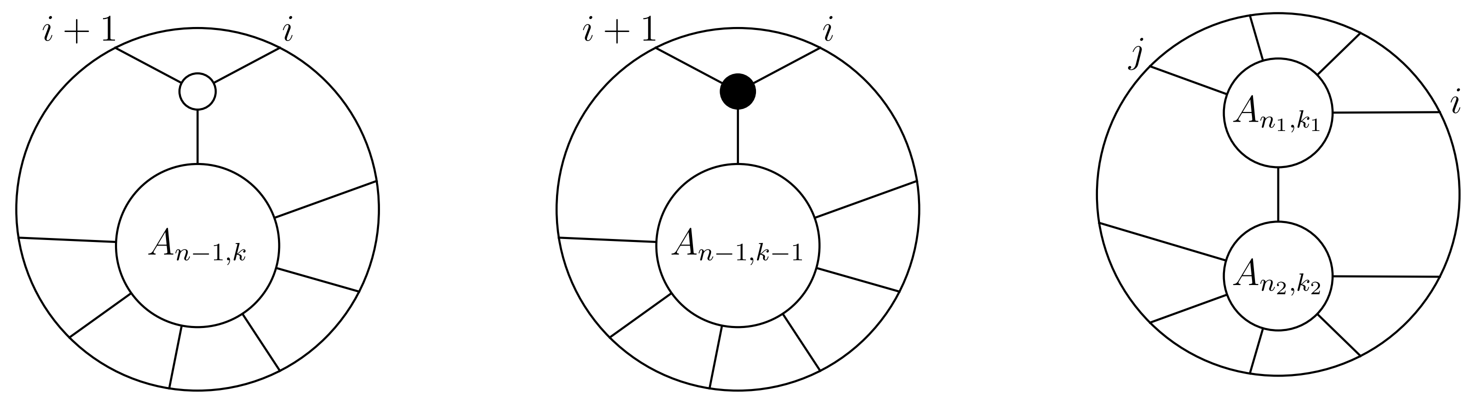
Singularities from Boundaries

Boundary stratification from Mathematica package amplituhedronBoundaries.m up to n=9 and all k (extended to n=11 via dual graphs)

(T. Lukowski, R. Moerman) based on positroids.m (J. Bourjaily)

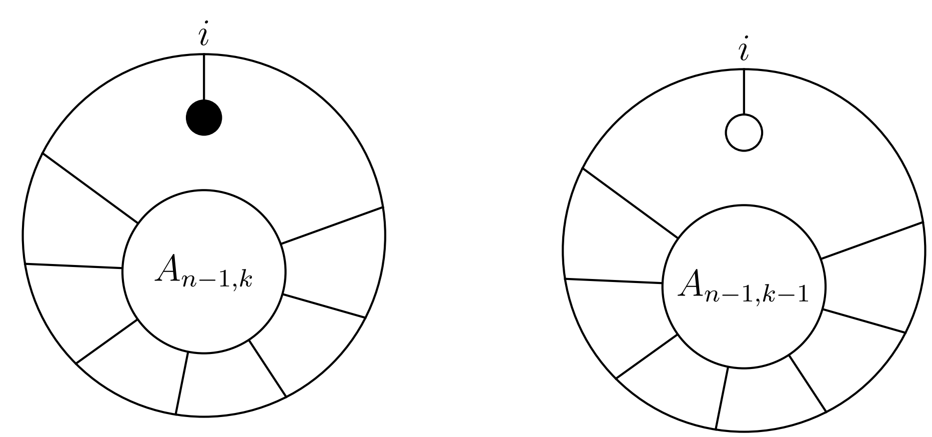
-> labeled by plabic diagrams of cells in $G_+(k,n)$

Codim 1 boundaries



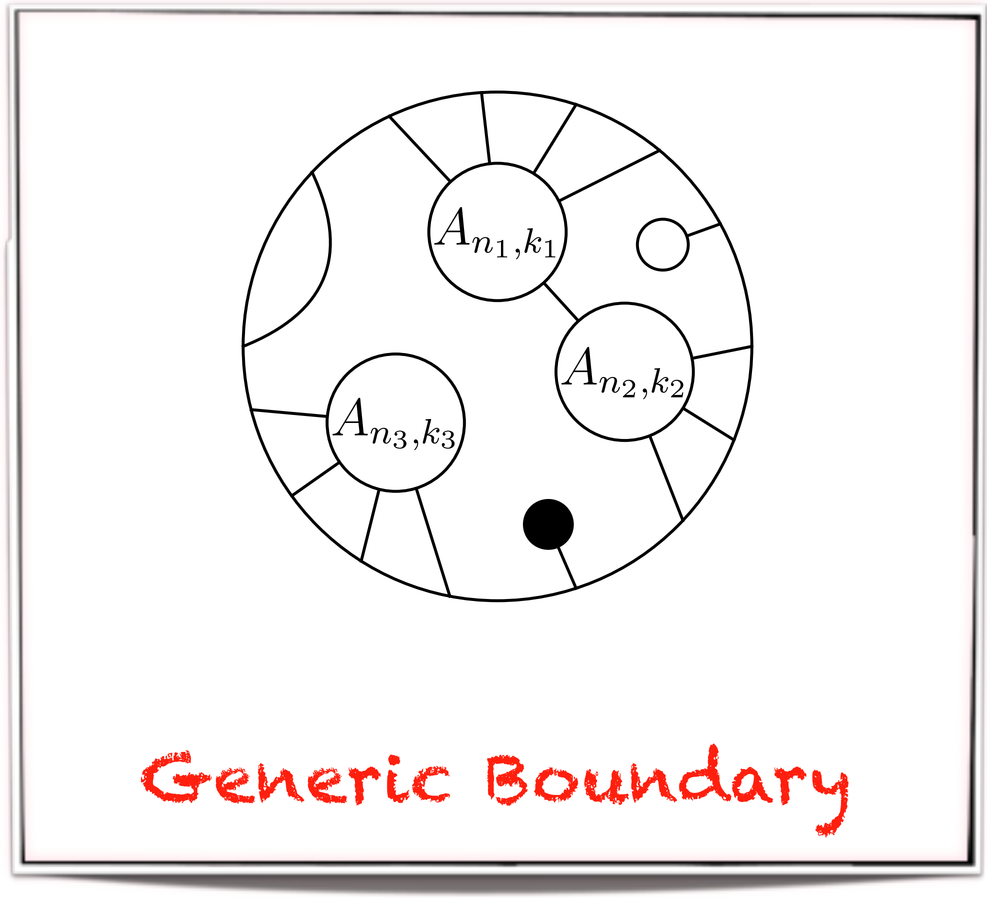
Codim 2 boundaries

further factorizations and collinear limits



+ generalization deeper in the geometry

Singularities from Boundaries



* black lollipop - helicity-preserving soft limit
* white lollipop - helicity-reducing soft limit
* single line - forward-limit
* top cell, collinear limit or factorization channel for an amplitude $A_{n'k'}$ with $n'<n$ and $k'\leq k$. Can be any boundary of M as long as it is a connected diagram

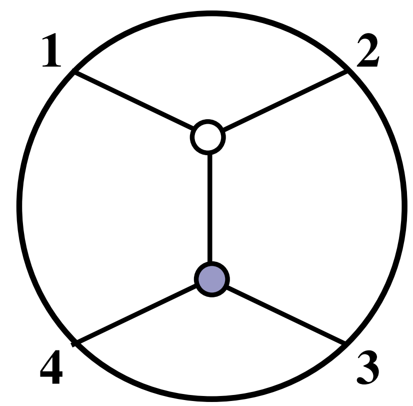
Singularities from Boundaries

Examples

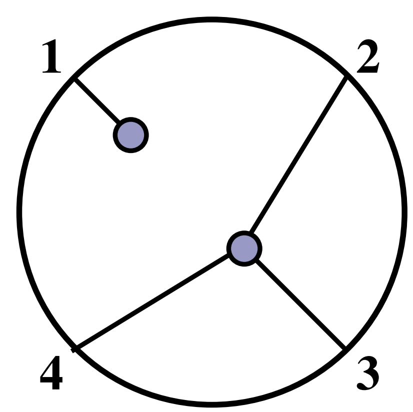
⭐ MHV_n <-> for k=2 it agrees with boundary stratification of $Gr_+(2,n)$

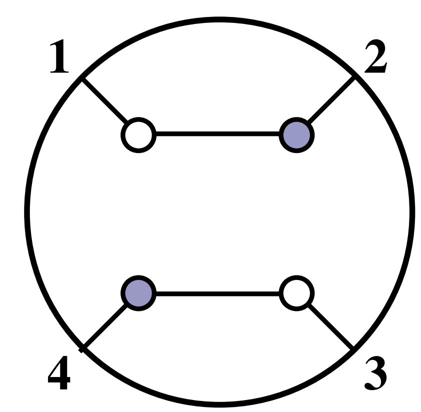
Representatives:

Codim 1 boundaries

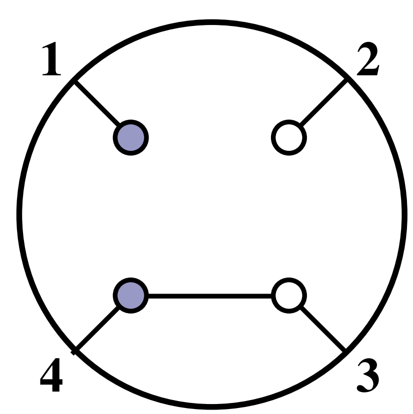


Codim 2 boundaries

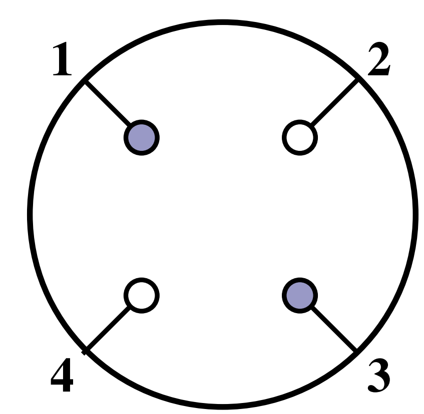




Codim 3 boundaries



Codim 4 boundaries



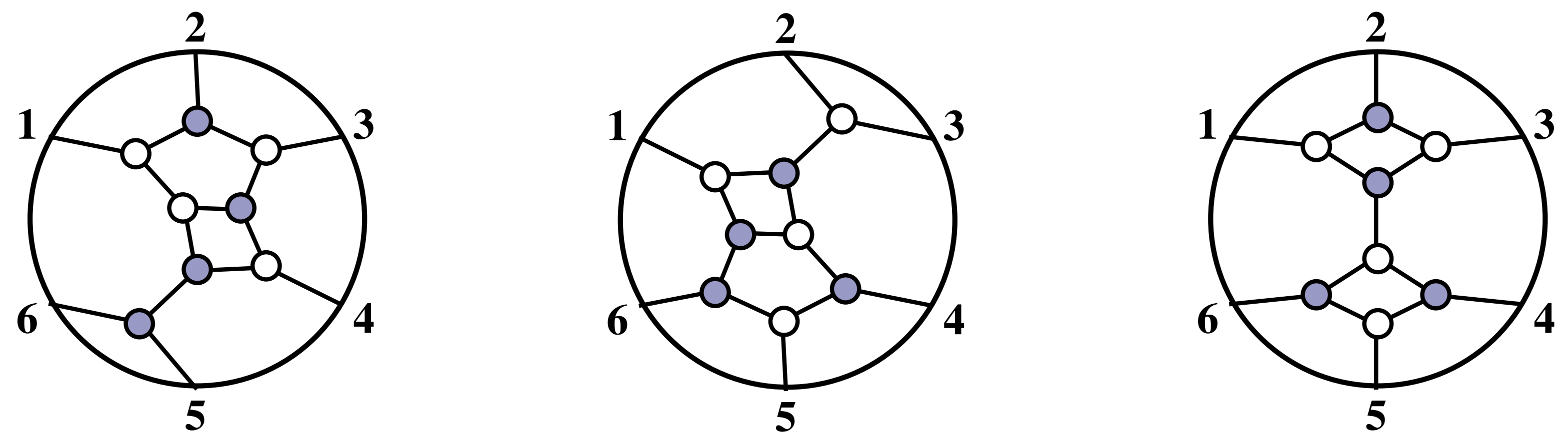
Singularities from Boundaries

Examples

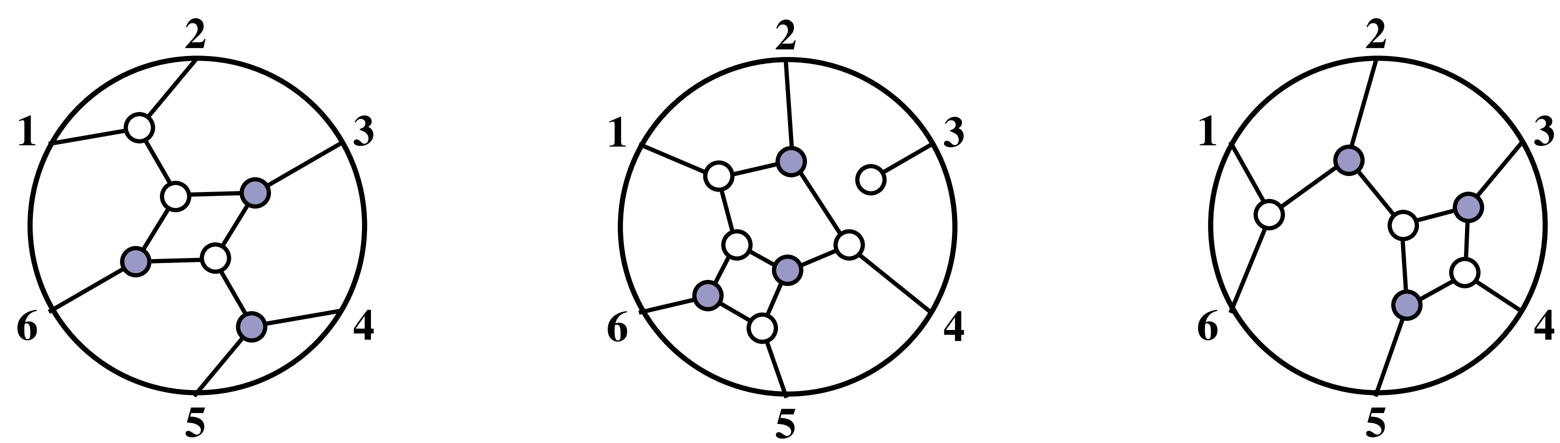
☆ $NMHV_6$

Representatives:

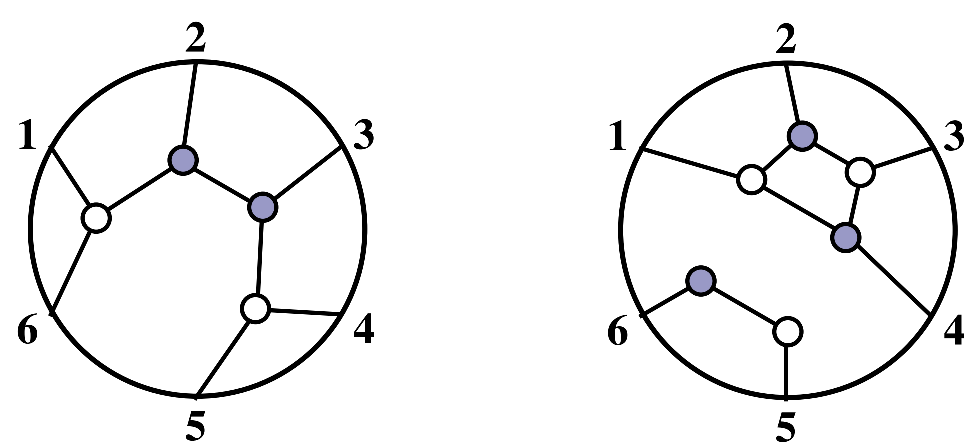
Codim 1 boundaries



Codim 2 boundaries

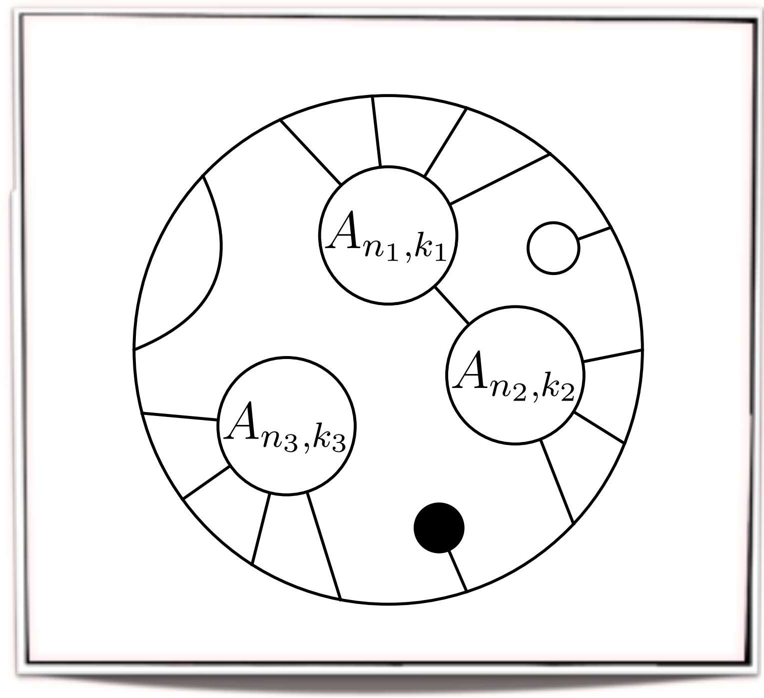


Codim 3 boundaries



...

Singularities from Boundaries



- ☆ Number of boundaries for a given dimension:

$(n,k)\backslash\ d$	0	1	2	3	4	5	6	7	8	9	10	11	12
$(4,2)$	6	12	10	4	1								
$(5,2)$	10	30	40	30	15	5	1						
$(6,2)$	15	60	110	120	90	50	21	6	1				
$(6,3)$	20	90	180	215	180	114	54	15	1				
$(7,2)$	21	105	245	350	350	266	161	77	28	7	1		
$(7,3)$	35	210	560	910	1050	938	665	350	119	21	1		
$(8,2)$	28	168	476	840	1050	1008	784	504	266	112	36	8	1
$(8,3)$	56	420	1400	2870	4200	4788	4424	3262	1820	720	188	28	1
$(8,4)$	70	560	1960	4200	6426	7672	7420	5696	3264	1280	300	32	1

+ parity conjugates (n,k) -> (n,n-k)

Generating fc

$$F_{n,k}(x) = \sum_{\sigma_{n,k}} (-x)^{\dim\sigma}$$

$$F_{n,k}(1) = 1$$

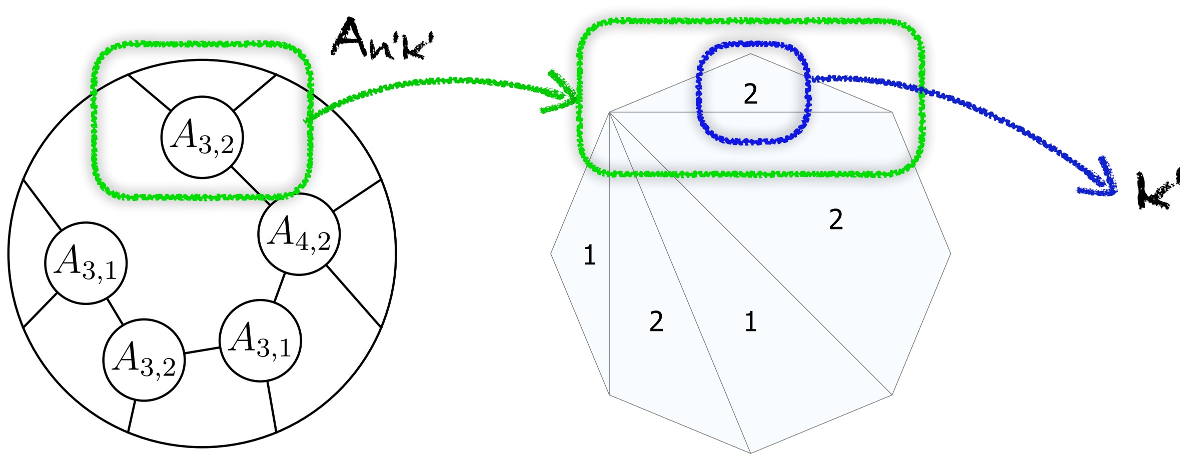
- ☆ Euler characteristic = 1 -> indicates that M is (2n-4)-diml ball

Singularities from Boundaries

Dual graphs: enumeration enabling to go up to n=11 any k

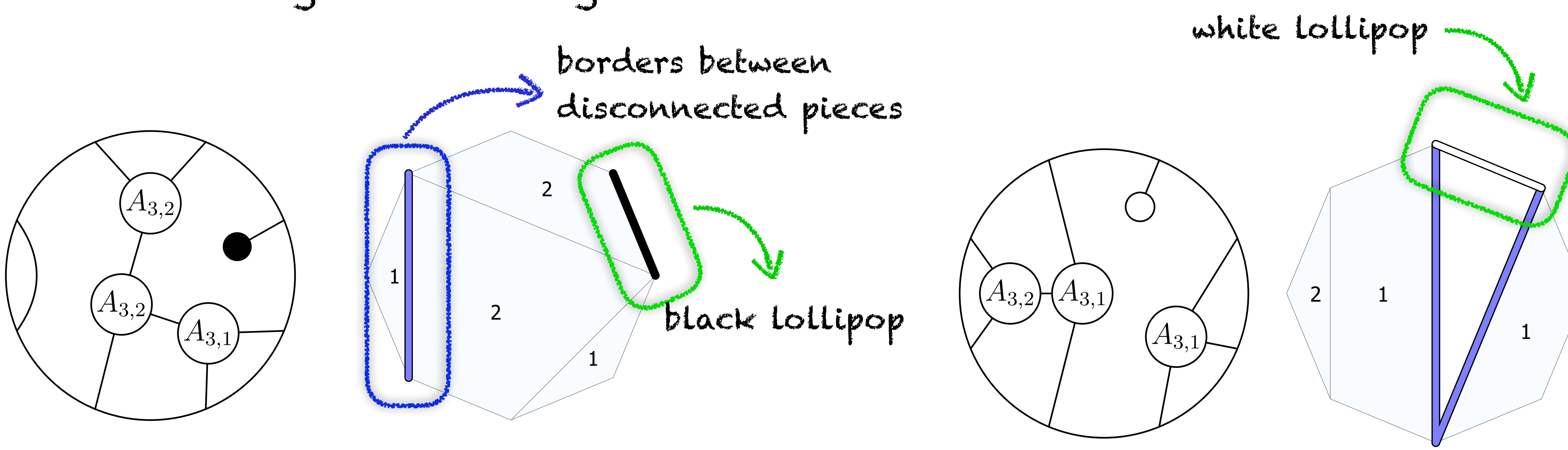
-> Each boundary labelled by a partial triangulation of a regular n-gon + additional decorations

Connected



n external legs   n-gon

Disconnected



Conclusions

Momentum amplituhedron:
positive geometry for tree-level amplitudes in N=4 sYM in spinor helicity space

- ☑ Classified all physical singularities of tree-level ampls in N=4 sYM by studying its boundaries
- ☑ Each singularity comes from a subsequent multi-particle factorization and collinear limit
- ☑ Singularities translated to geometry as appropriate intersection of facets

- ☐ Proof that momentum amplituhedron is a ball?
- ☐ Loop amplitudes?

Thank you!