Higher genus monodromy relations and Color-Kinematics

Zoomplitudes 2020

Edoardo Casali

QMAP, UC DAVIS

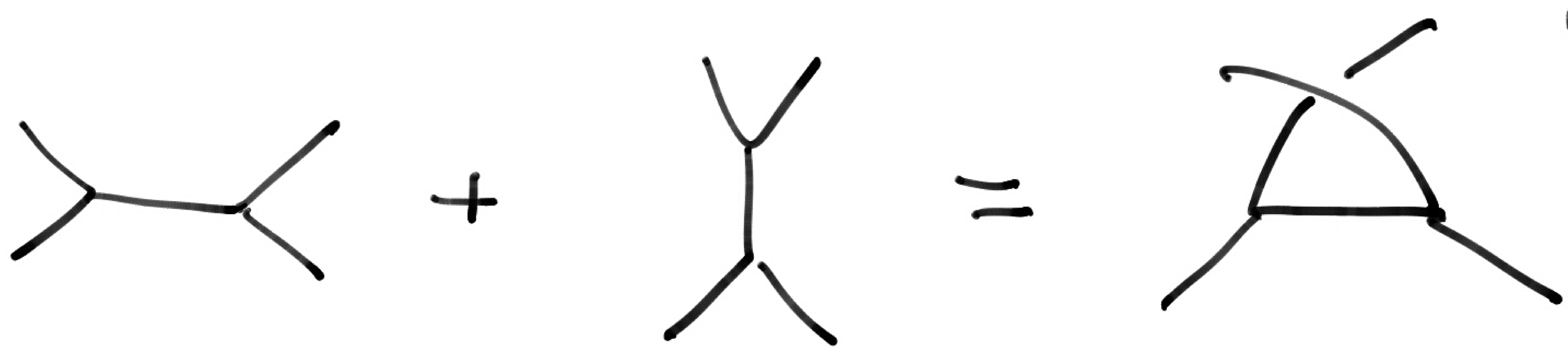
E.C., S. Mizera, P. Tourkine (1910.08514, 2005.05329)

Color-Kinematics and double-copy

$$A^{YM} = \int \prod_{i=1}^{L} \frac{d^d l_i}{(2\pi)^d} \sum_{\gamma \in \Gamma} \frac{1}{S_\gamma} \frac{n_\gamma c_\gamma}{D_\gamma} ;$$

Γ set of trivalent graphs

c_γ are color factors (Traces)



$c_s \quad + \quad c_t \quad = \quad c_u$

} Consequence of Jacobi id.

Color-Kinematics and double copy

→ If $\underbrace{n_s + n_t = n_u}_{\text{CK numerators}}$ for all triples of graphs γ then

↳ CK numerators (Bern, Carrasco, Johansson - 08/10)

$$A^{G} = \int \prod_{i=1}^{L} d^d \ell_i \sum_{\gamma \in \Gamma} \frac{1}{S_\Gamma} \frac{n_\gamma n_\gamma}{D_\Gamma}$$

is a gravitational amplitude

→ Double copy

- Tree-level proof
- Numerous loop-level checks
- Interesting non-perturbative gen.

Color-Kinematics and Double Copy

- @ loop-level n_γ's are ambiguous since ℓ's are defined up to shifts for each γ

- Finding CK satisfying n's difficult at loop level

More fundamental questions

- Where does color-kinematics come from, and does it still holds unmodified to all loops?

- Origin/proof of double copy at higher loops and relation to C-K

String theory can help!

- @ tree-level monodromy relations in the $\alpha' \to 0$ limit are related to color-kinematics (Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove -10)

- @ tree-level BCJ numerator $\longleftrightarrow$ Residue theorem in $\mathcal{M}_{0,n}$

(Mizera 19, Mason, Frost 19)
↳ See talk earlier

String theory can help ⁈

- @ tree-level KLT relations is a sort of double-copy for amplitudes

$$\mathcal{M} = \sum A_i H_{ij} A_j$$

Full YM amplitudes

Gravity amplitude

This talk:

Field theory ($\alpha' \to 0$) limit of loop-level monodromy relations

(Tourkine, Vanhove 16
" , " , Ochirov 17
Hohenegger, Stieberger 17
Casali, Mizera, Tourkine 19)

Monodromy Relations (g=0)

Open String amplitudes (g=0) are not all independent (Plahte 70/ Bjerrum-Bohr; Damgaard Sondergaard, Vanhove 10/11)

YM Vertex op.

$$\sum_{i=1}^{n-1} e^{\pm \pi i k_1 \cdot \sum_{j=2}^{i} k_j} A^{g=0}(2,\ldots,i,1,i+1,\ldots,n) = 0$$

Monodromy Relations $(g=0)$

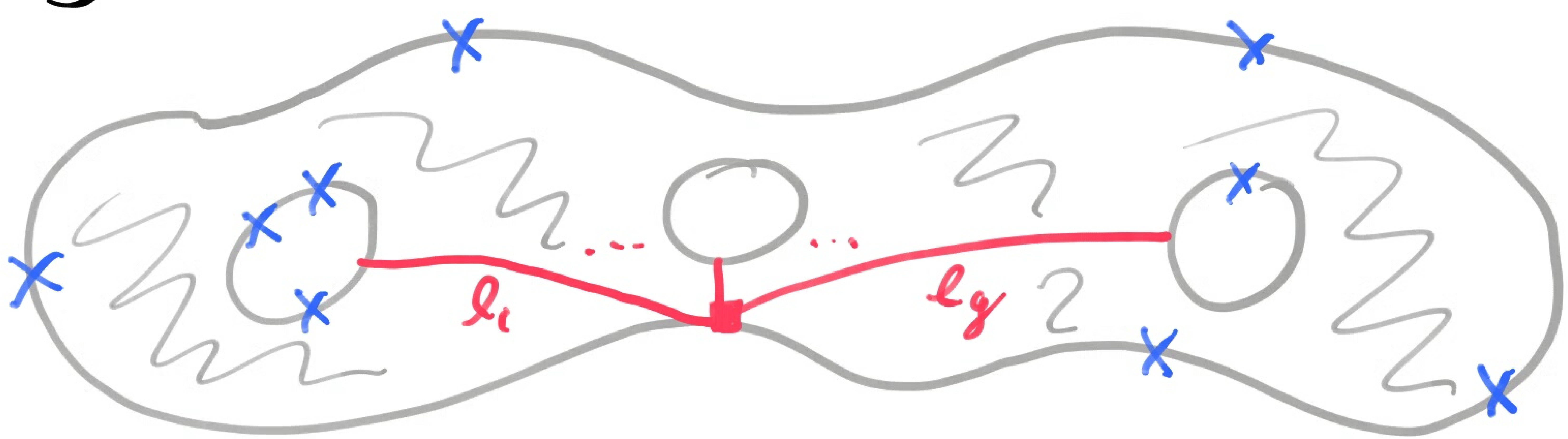
Originate from multy-valuedness of String integrand

$$A^{g=0} = \int d\mu_{0,n} \, \underbrace{\varphi(k,\epsilon,z)}_{\text{Theory dependent}} \underbrace{\prod_{i<j} (z_i - z_j)^{\alpha' k_i \cdot k_j}}_{\text{Universal Koba-Nielsen (KB) factor}}$$

$g>0$ Monodromy Relations

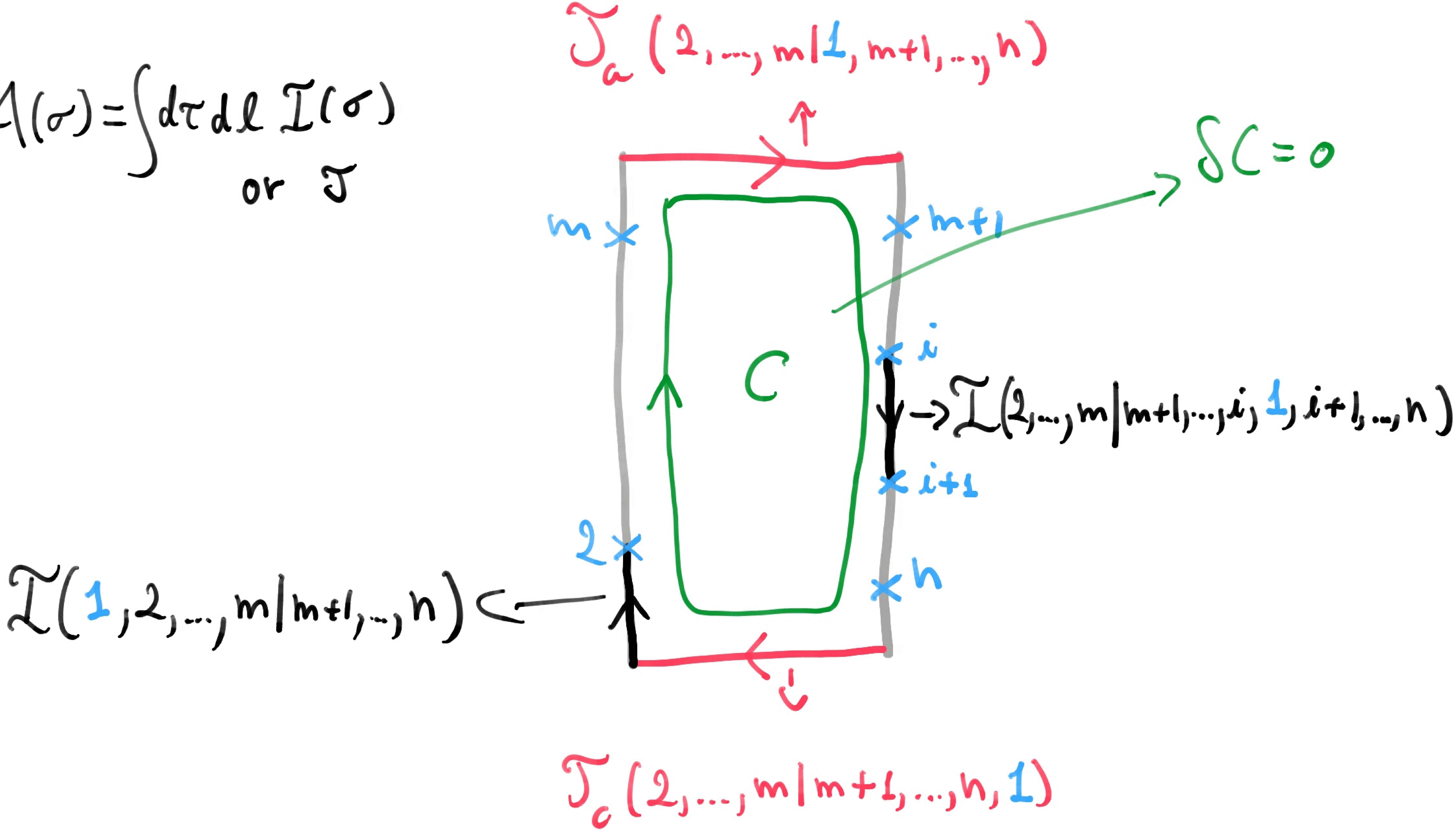
$g>0$ open string amplitude:

$$A^g = \int d\mu_{g,n} \prod_{i=1}^{g} d\ell_i^d \, \varphi(k,\epsilon,z,\ell) \, e^{2\pi i \alpha' k_I \cdot \sum \ell_i z_i} \prod_{i<j} E(z_i,z_j)^{\alpha' k_i \cdot k_j}$$



$\underline{g=1}$ Monodromy Relations

$$A(\sigma)=\int d\tau\, d\ell\; \mathcal{I}(\sigma) \quad \text{or} \quad \mathcal{J}$$



$g=1$ Monodromy Relations

$$\sum_i e^{\pm \pi i \alpha' k_1 \cdot \sum_j k_j} \mathcal{I}(2,3,\ldots,i,1,i+1,\ldots,m|m+1,\ldots,n)$$

$$+ \sum_i e^{\pm \pi i \alpha' k_1 \cdot (\ell + \sum_j k_j)} \mathcal{I}(2,\ldots,m|m+1,\ldots,i,1,i+1,\ldots,n)$$

$$= \mp e^{\pm \pi i \alpha' k_1 \cdot \ell} \Big(e^{\pm \pi \alpha' k_1 \cdot \sum_j k_j} \mathcal{J}_{a\pm}(2,\ldots,m|1,m+1,\ldots,n)$$

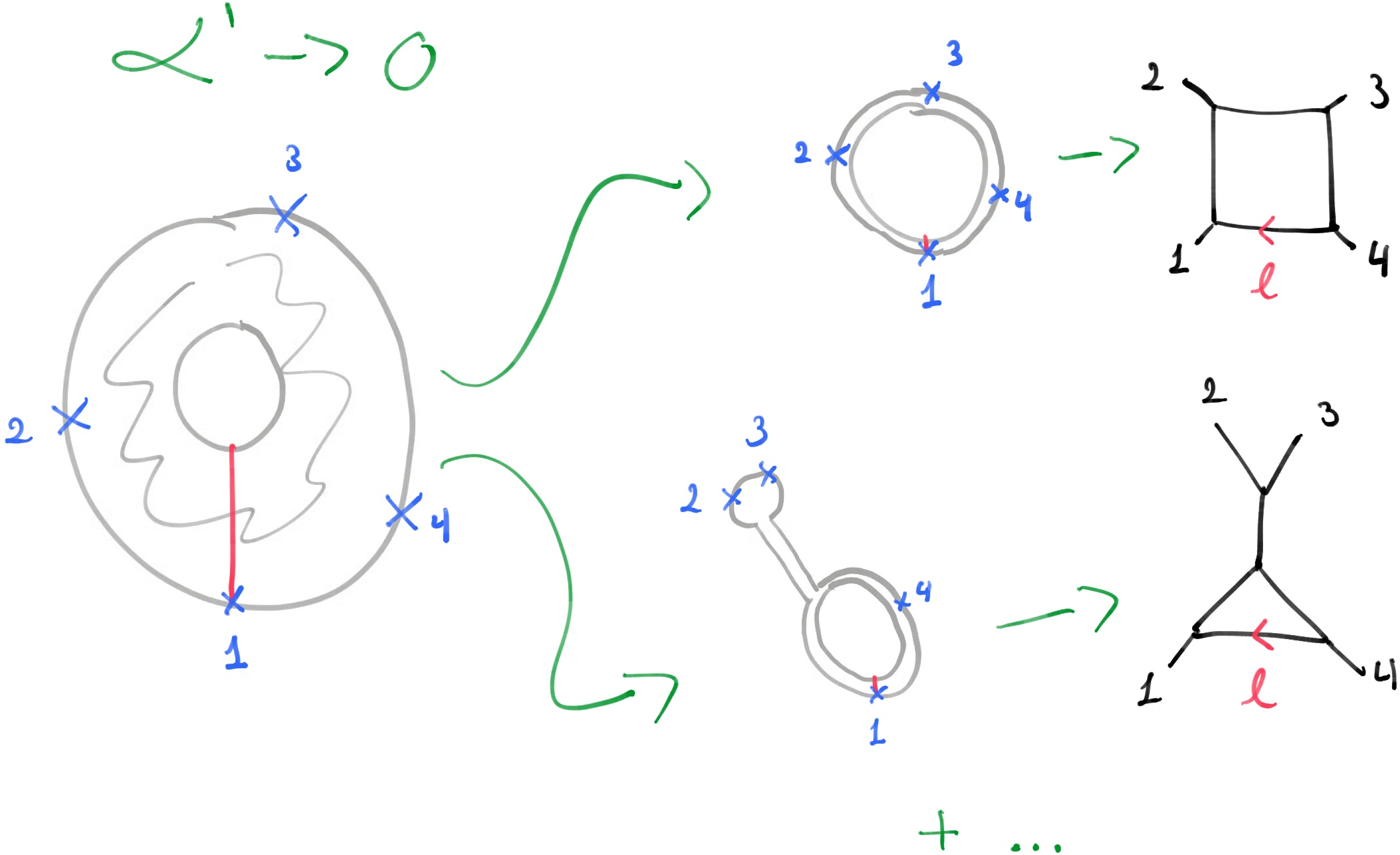
$$- \mathcal{J}_{c\pm}(2,\ldots,m|m+1,\ldots,1) \Big)$$

$\alpha' \to 0$

$\to$ Field theory limit of $\mathcal{I}$ terms is well known,
Bern-Kosower rules (Bern, Kosower 91)

$\to$ $\mathcal{J}$ terms are new, must examine their
limit carefully!

(Conjectured structure in
Ochirov, Tourkine, Vanhove 17)



α' -> 0
3
2
4
1
3
2
4
1
2
3
1
4
ℓ
3
2
4
1
2
3
1
4
ℓ
+ ...

Bern-Kosower Rules for $\mathcal{I}$

$$A' = \int d\mu_{1,n}\, \varphi\, KB \; ; \; \varphi = \dot{G}\, \phi_1 + \phi_2$$

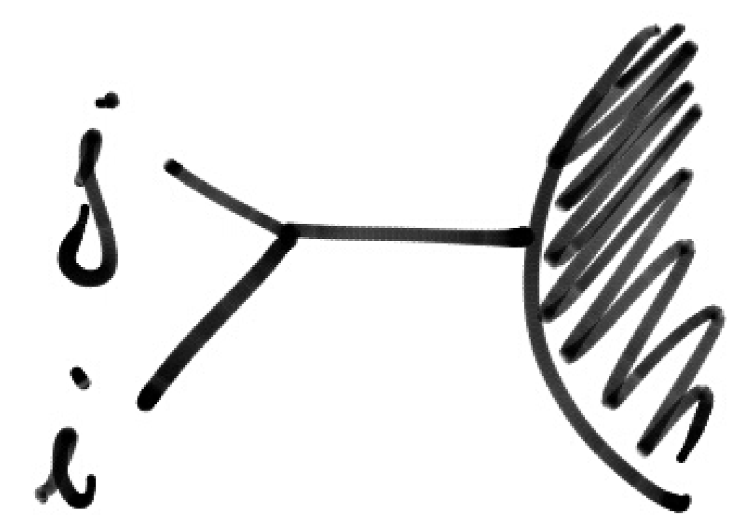
$\phi_1 + \phi_2 \to$ No simple pole as $z_i \to z_j$

$\dot{G} \to$ Simple pole as $z_i \to z_j$

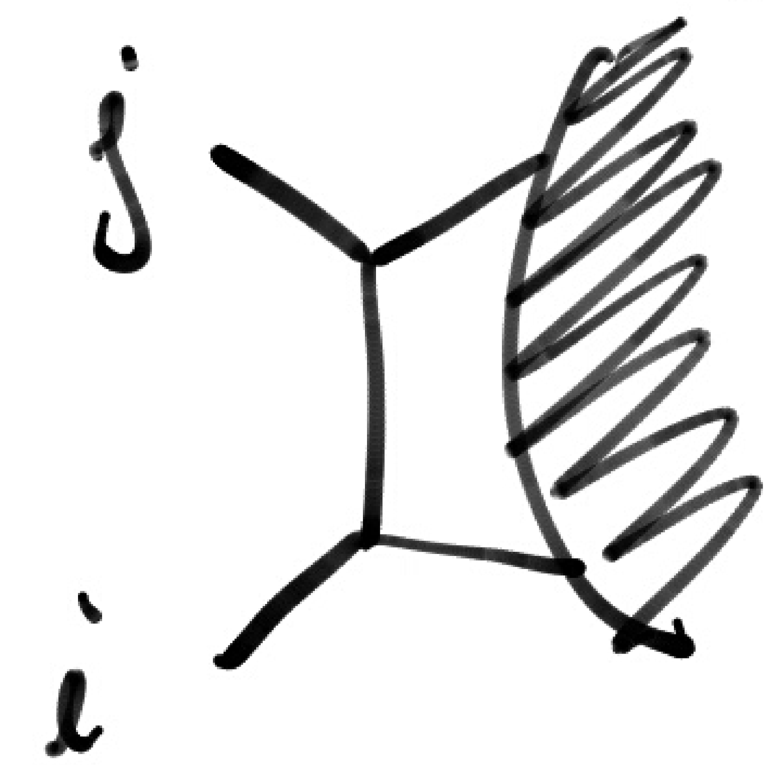
$\alpha' \to 0$

$$\sum \left(\text{Trivalent graphs}\right)$$

ϕ_1



$(\phi_1 + \phi_2)$



Distinct boundaries of $\mu_{1,n}$

$\alpha' \to 0$ of $\mathcal{J}$'s

$$\mathcal{J}_{a\pm} = \int_0^{\pm 1/2} T_1(x+it)\,\varphi(x+it)$$

$$T_1(z_1) = e^{2\pi i \alpha' k_1 \cdot l} \prod_{j=2}^{m} \left(-i\,\Theta_1(iy_1 - z_1)\right)^{\alpha' k_1 \cdot k_j} \prod_{j=m+1}^{n} \Theta_2(iy_2 - z_1)^{\alpha' k_1 \cdot k_j}$$

$$\Theta_1(z_1 - z_i) \xrightarrow{z_1 \to z_i} (z_1 - z_i) + \dots$$

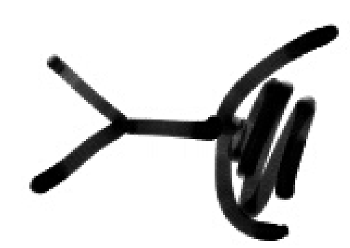
Tropical scaling $\quad \mathrm{Im}\, z_j = \dfrac{y_j}{\pi\alpha'} \; ; \; \mathrm{Im}\,\tau = \dfrac{T}{\pi\alpha'}$

$$|\Theta(z_i - z_j)| \sim -\log\left(|\sinh(y_i - y_j)|/\alpha'\right) \sim \underbrace{-|y_j - y_i|/\alpha'}_{\text{Worldline propagator}} + \mathcal{O}\left(e^{y/\alpha'}\right)$$

Taking this limit on J reveals two new things:

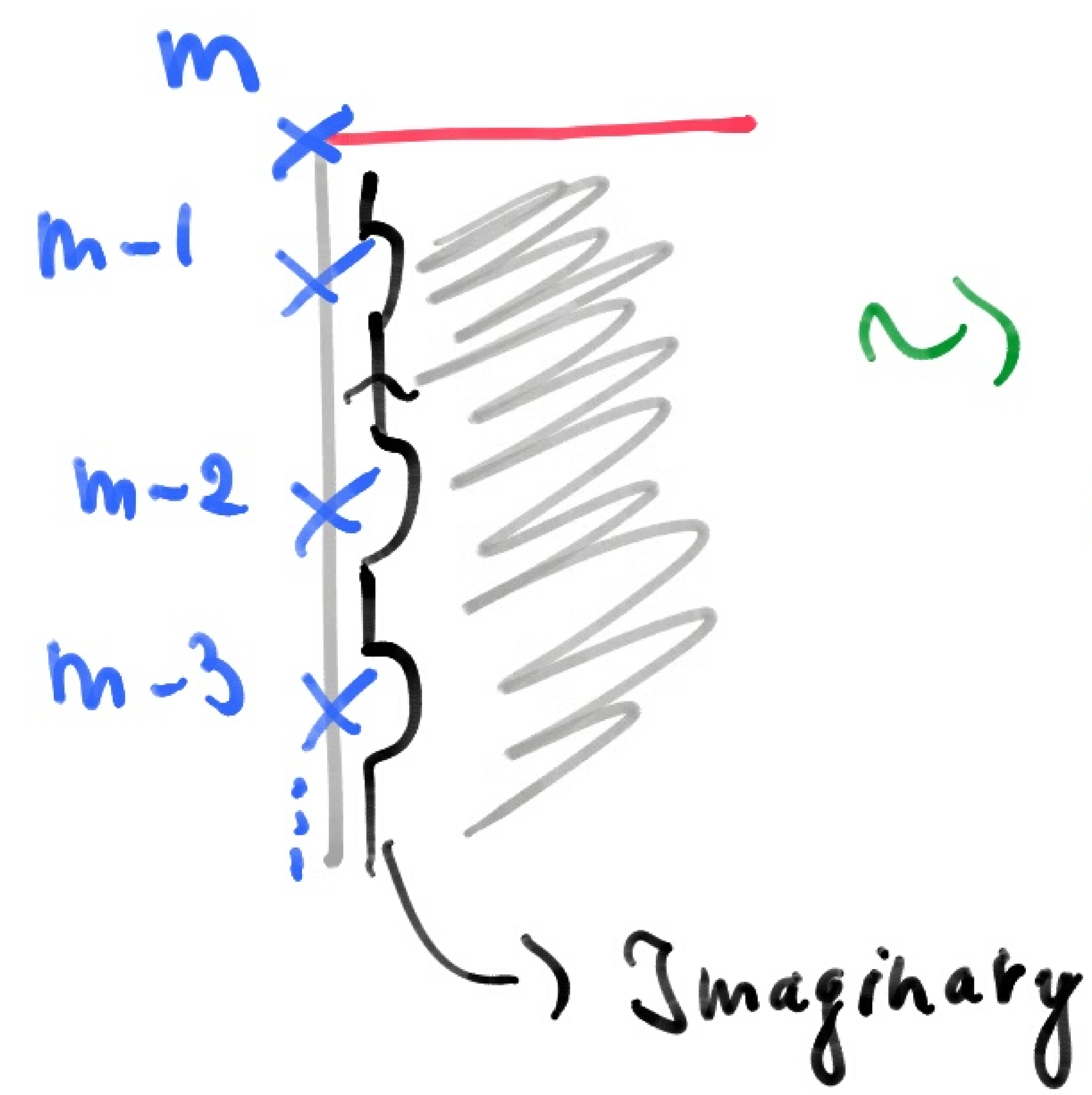
- A phase $J \overset{\alpha' \to 0}{\rightsquigarrow} \left(\exp i\pi\alpha' \sum_{j=2}^{m-1} k_1 \cdot k_j + \frac{k_1 \cdot k_m}{2} \right) \times \text{graphs}$

- Contact terms; 

(and new  terms)

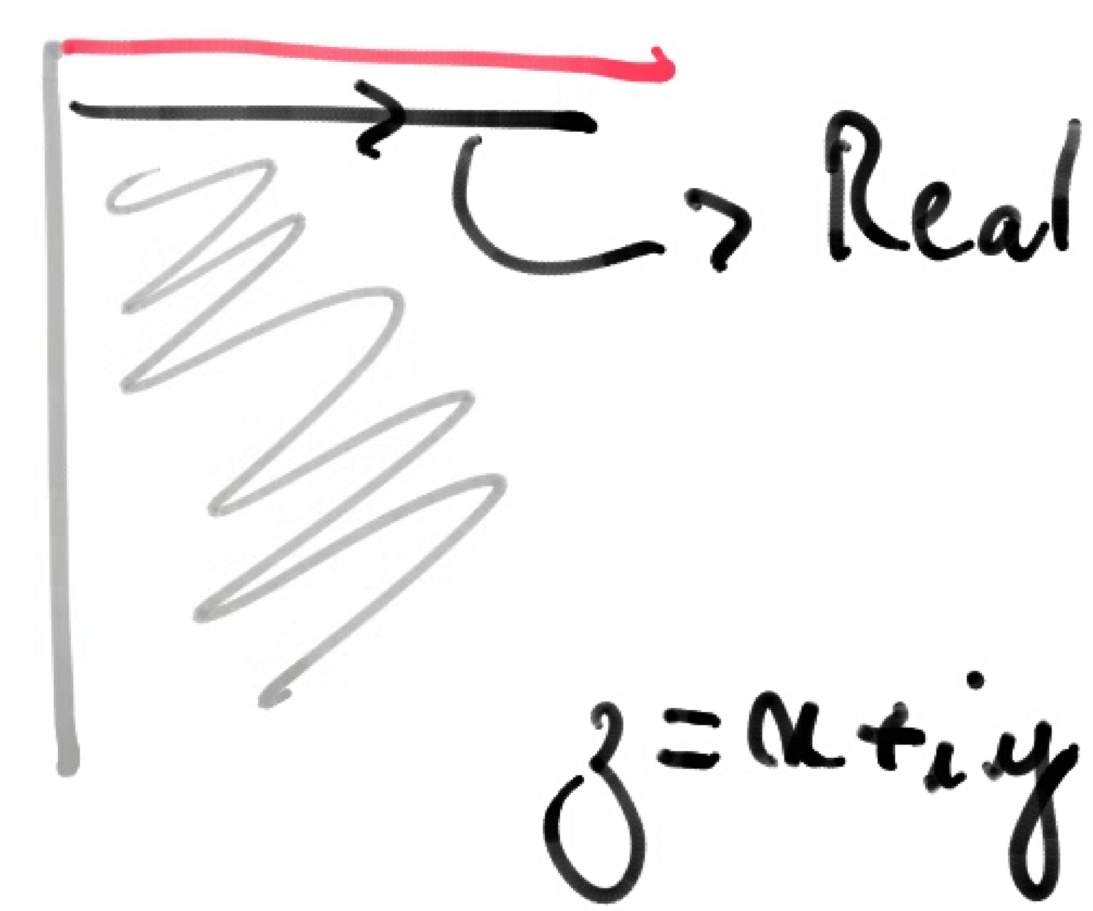
Phases:

$$\sum_{i=2}^{m-1} K_1 \cdot K_i \longleftrightarrow$$



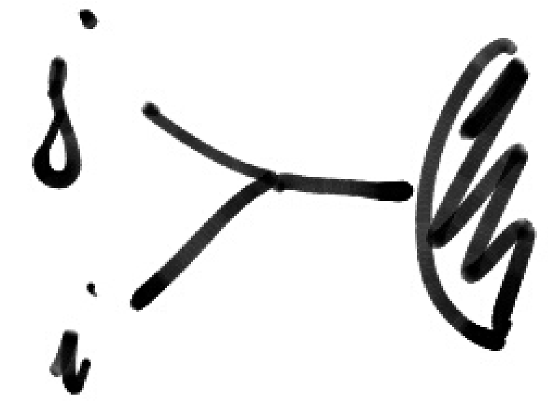
⇝ Conventional, expected from form of KB

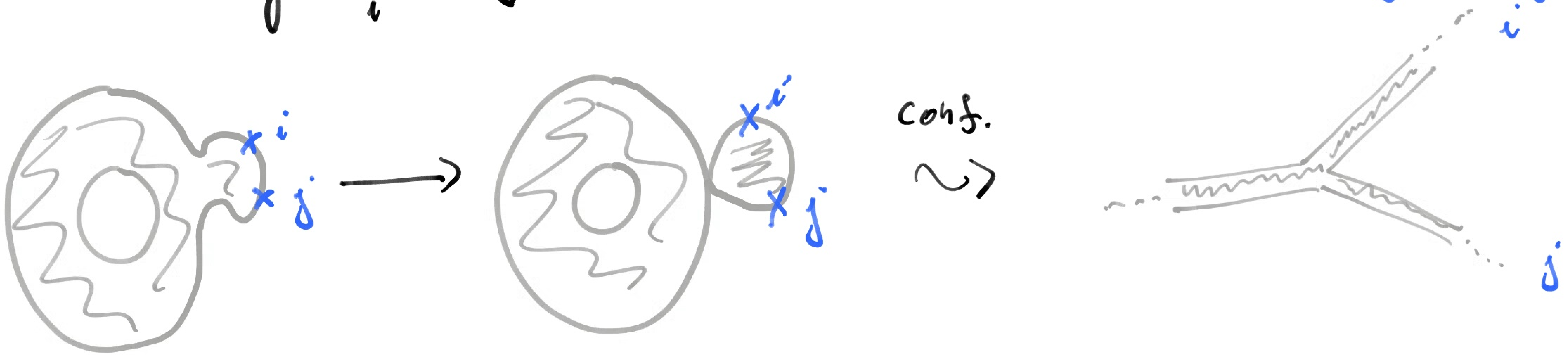
$$\frac{K_1 \cdot K_m}{2} \longleftrightarrow$$



⇝ Unusual, also appears at $g>1$

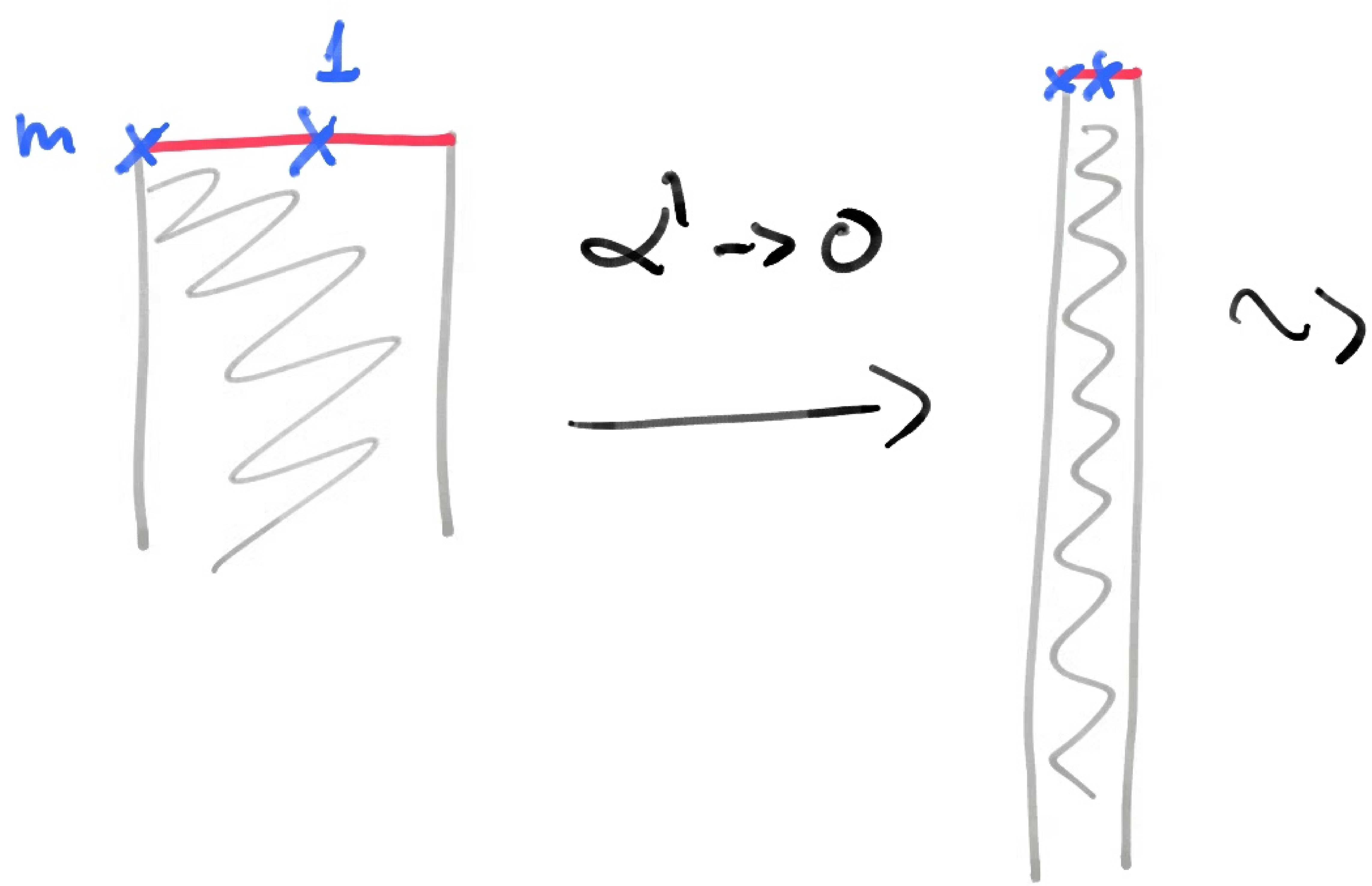
Graphs:

Usually  created from $\dot{G}(z_i - z_j)$ as $z_j \to z_i$

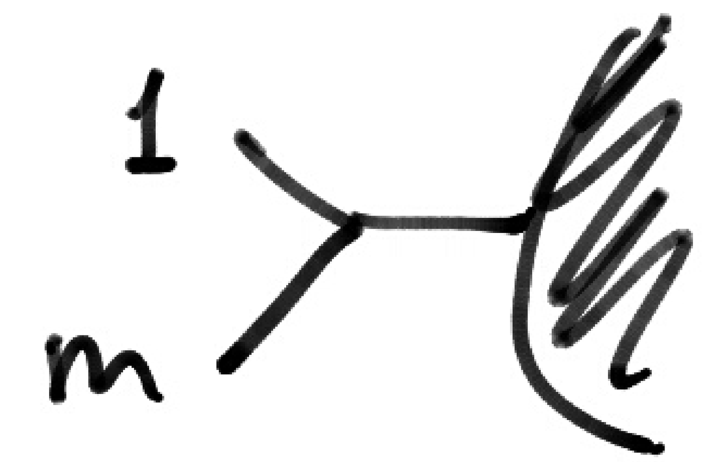


$\rightsquigarrow$ locally $\displaystyle\int dy \, \frac{1}{y} \, e^{-k_i \cdot k_j \ln y} \propto \frac{1}{k_i \cdot k_j}$

Graphs: For J



$\rightsquigarrow$

ϕ_1 

$\rightsquigarrow$ gauge choice

like massive contribution
(but is not)

$\phi_2 \frac{\alpha'}{2}$ 

$\rightsquigarrow$ here for $g>1$ too

$\rightsquigarrow$ size of strip

$$A = \int (\dot{G}\,\phi_1 + \phi_2)\, KB$$

$\alpha' \to 0$ of monodromy relations

$$\sum_i e^{i\alpha'\phi_i} \mathcal{I}_i + \sum_j e^{i\alpha'\theta_j} \mathcal{J}_j = 0$$

$$\mathcal{I}_i = \mathcal{I}_i^{FT} + \mathcal{O}(\alpha'^2)$$

$$\mathcal{J}_j = - \mathcal{J}_j^{FT} - i\alpha' \mathcal{J}_j^{CT} + \mathcal{O}(\alpha'^2)$$

$\alpha' \to 0$ of monodromy relations

$$\underbrace{\left(\sum_i \tilde{I}_i^{FT} + \sum_j \tilde{J}_j^{FT} \right)}_{} + i\alpha' \left(\sum_i \phi_i \tilde{I}_i^{FT} + \underbrace{\sum_j \theta_j \tilde{J}_j^{FT} + \tilde{J}_j^{CT}}_{} \right) + \mathcal{O}(\alpha'^2) = 0$$

Bern-Dixon-Dunbar-Kosower relations

Boels-Isermann (planar)

Feng-Jia-Huang

Analogous to Bern-Carrasco-Johansson

$\rightsquigarrow$ Schematic form valid at higher loops

For $\mathcal{O}(1)$ rel. $\left(\sum_i \mathcal{I}_i^{FT} + \sum_j \mathcal{J}_j^{FT} \right) = 0$

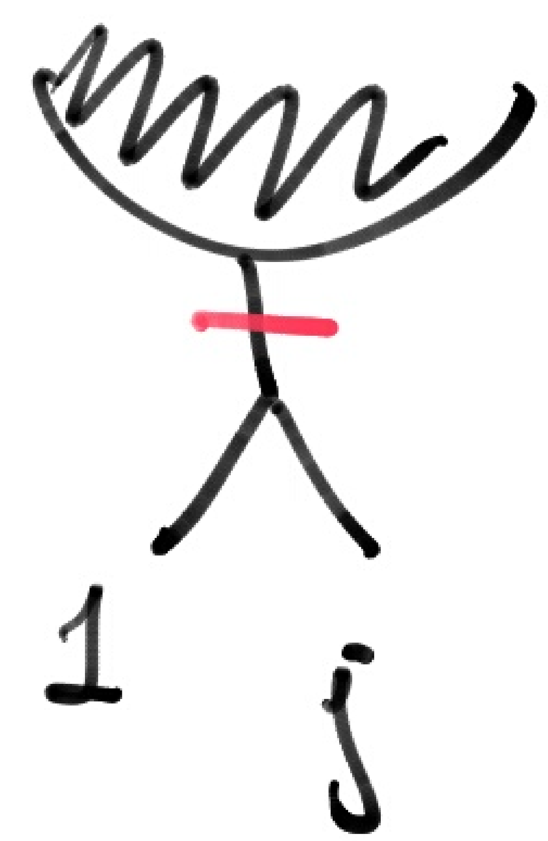
tree graphs cancel term by term

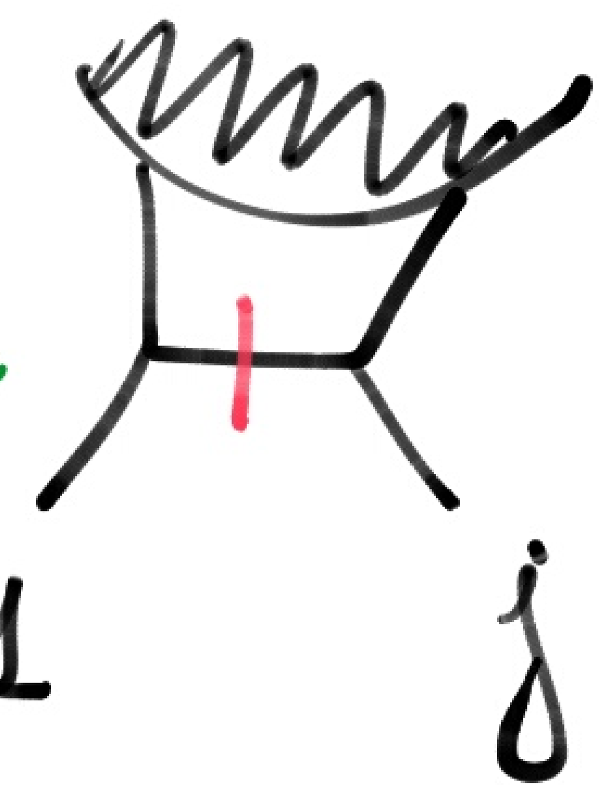
$\rightsquigarrow$ Can be upgraded to relations among amplitudes ($\int d^D\ell \sum \mathcal{J}_i = 0$)

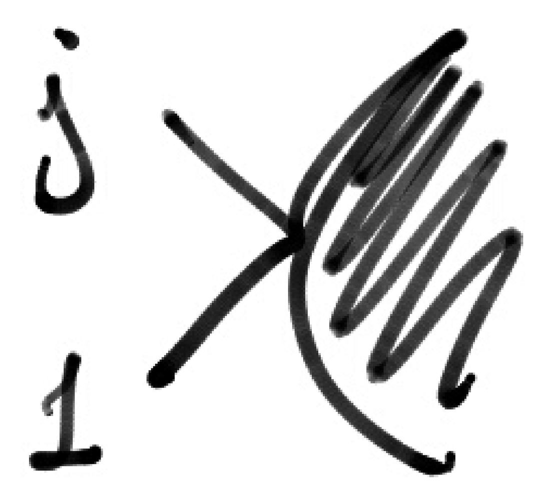
For $\left(\sum_i \phi_i I_i^{FT} + \sum_j \theta_j J_j^{FT} + J_j^{CT}\right)$, phases cancel propagators

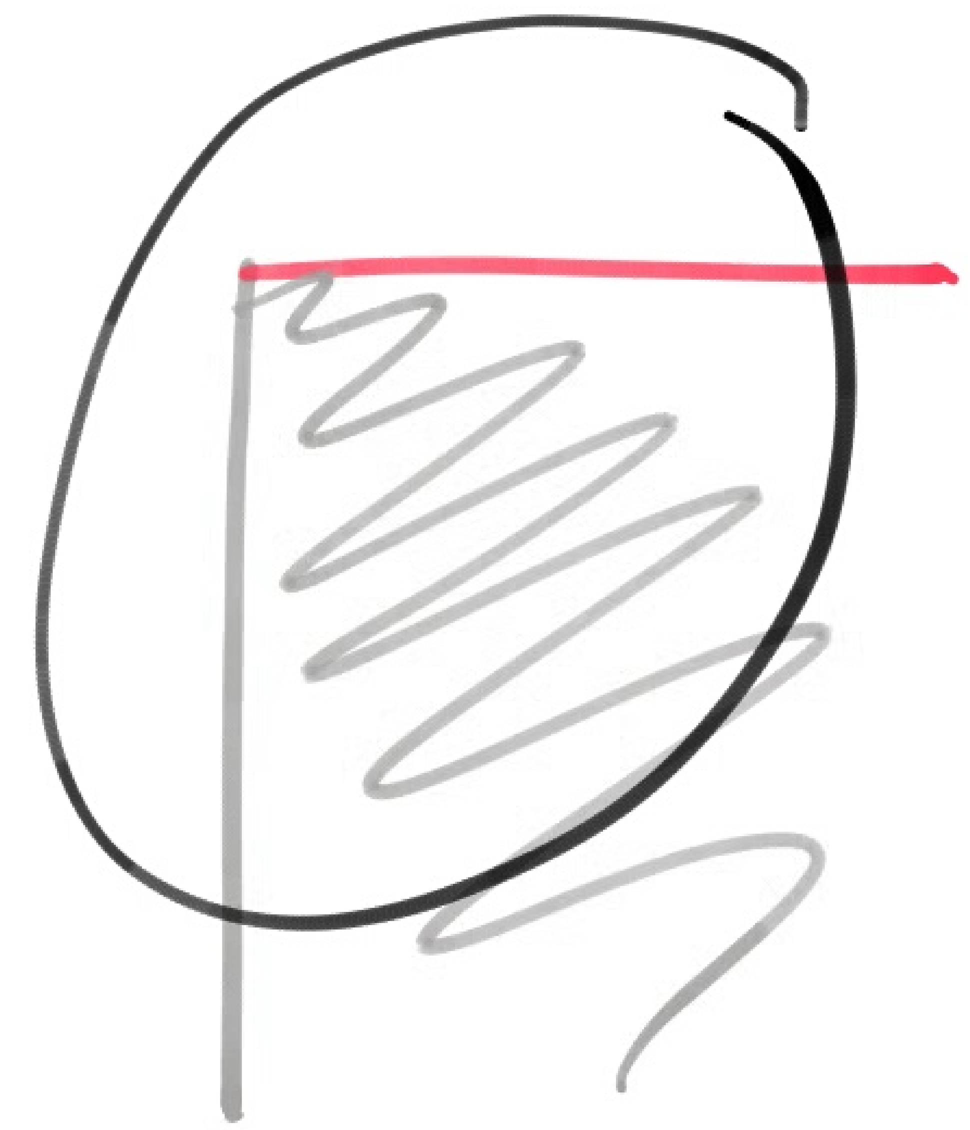
s.t.

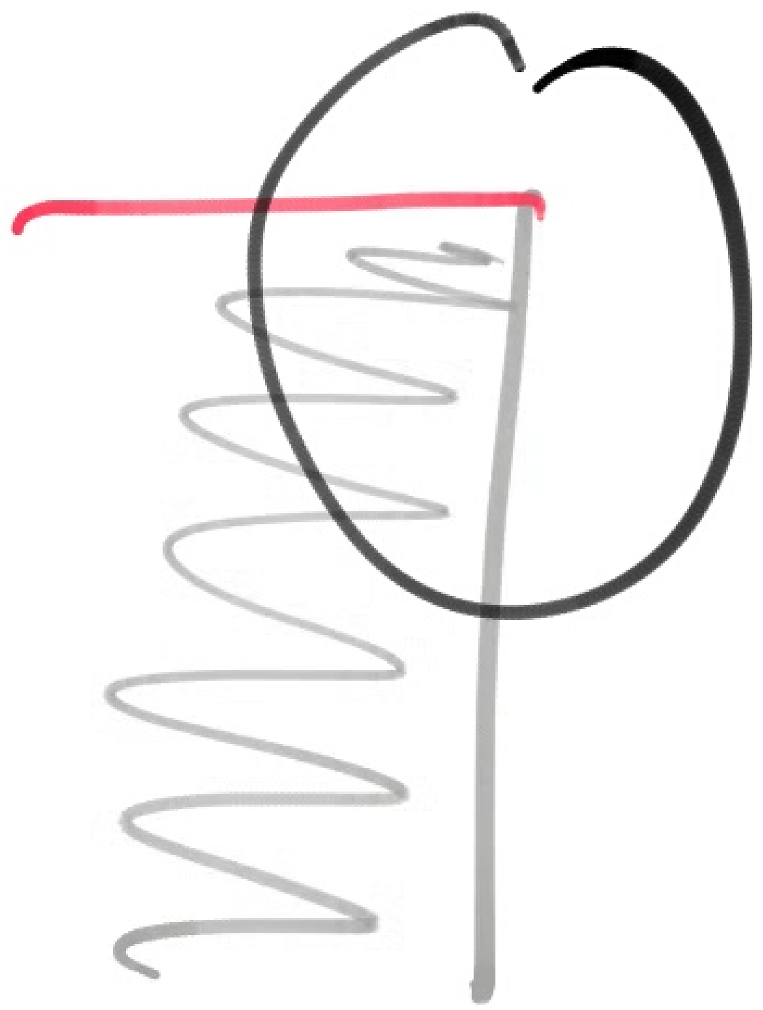
n_s 

n_t 

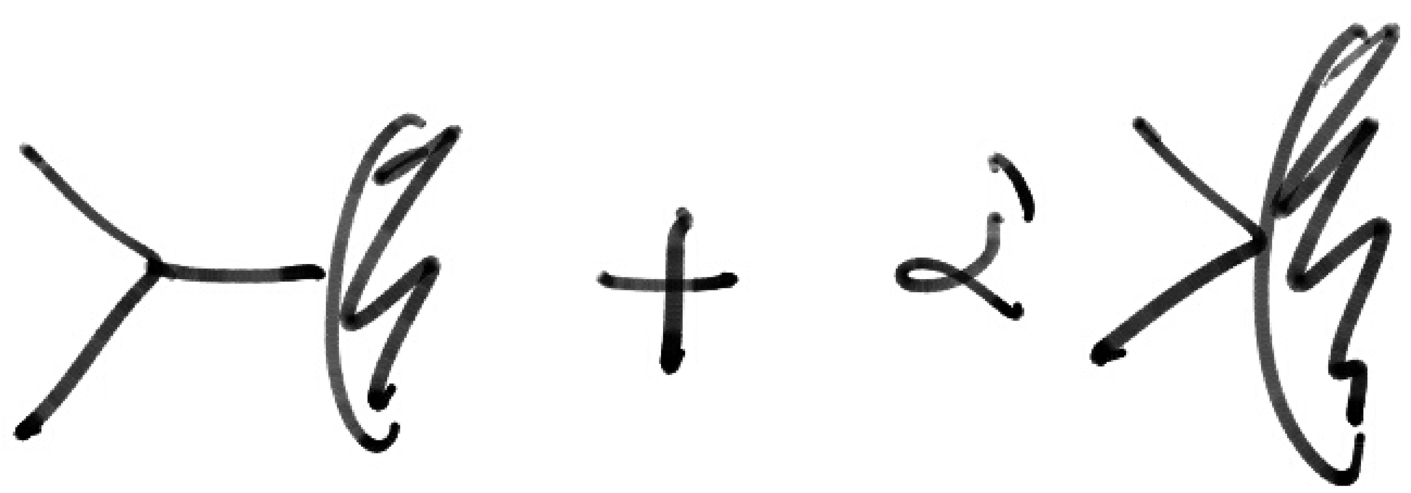
n_u 

$$= \underbrace{\left(n_s - n_t - n_u\right)}_{=0 \text{ by C-K}}$$ 

, except near 

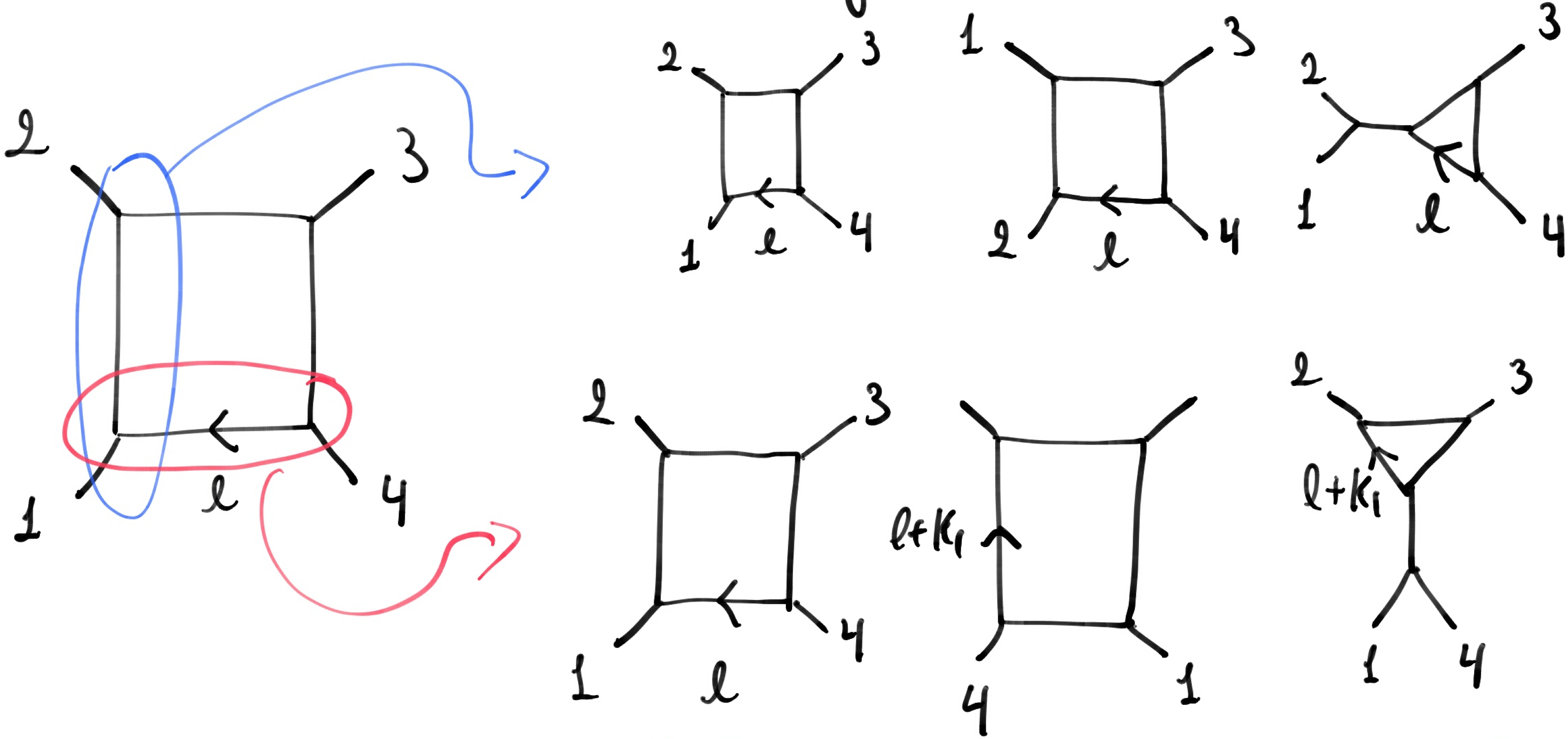


Near edges there are not enough n's to form BCJ triples.

 from J cancel directly

the leftover n's

Due to J, the labelling problem is avoided



<=> BCJ move changes loop momentum

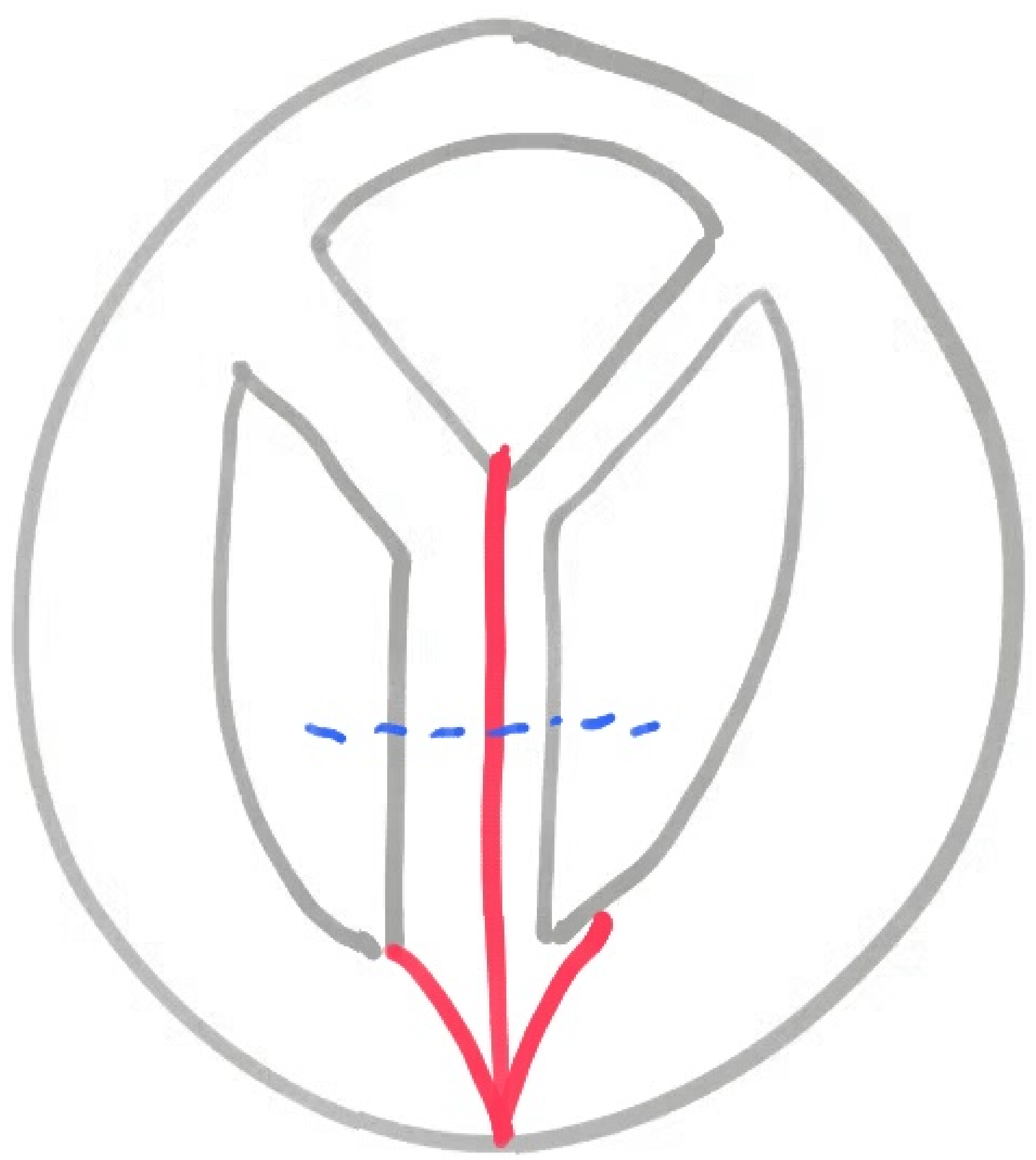
In Short:

- $\alpha' \to 0$ of monodromy rel's produce BCJ satisfying numerators away from edges (in BK rep.)

- New contact terms and phase in $\mathcal{J}$ avoid labelling ambiguity by removing unwanted terms

Higher loops

- Almost everything is the same but new kind of limit at $g \geq 3$ must be investigated

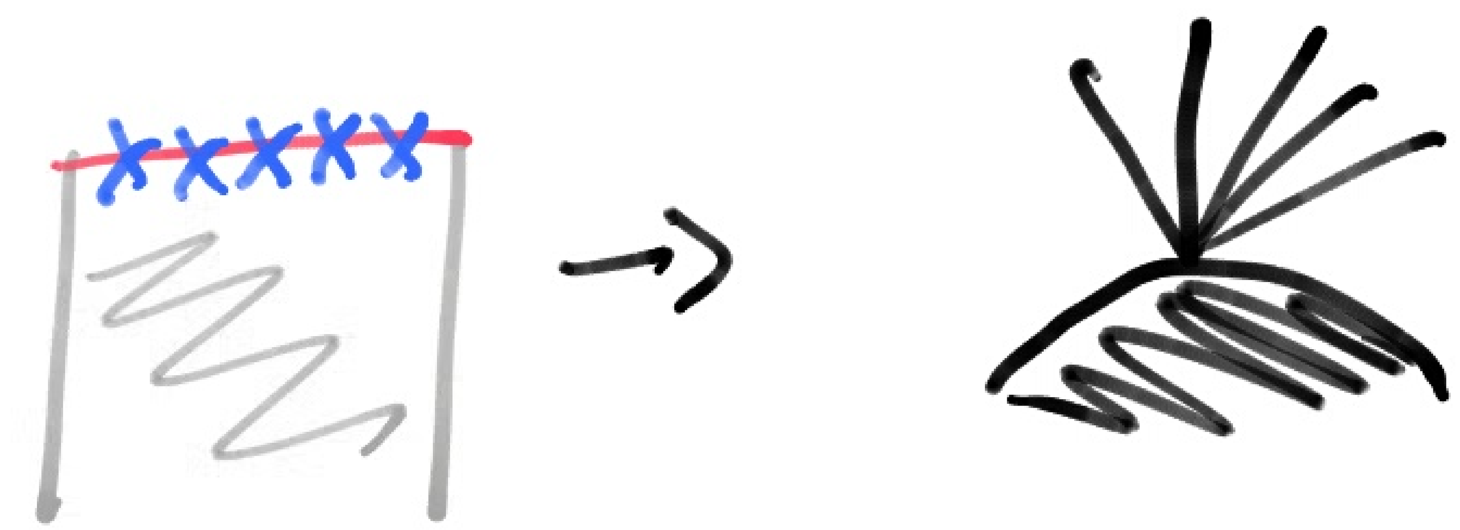


- Away from edges h's obey BCJ (in BK rep.)

- Relations only involve BCJ triples with external legs

Lessons for KLT

- A generic basis of cycles will have punctures along the A-cycles



Higher genus KLT involves contact terms in the limit $\alpha' \to 0$ $\rightsquigarrow$ Generalized BCJ?

Residues in $\mathcal{M}_{n,g}$

- Generalize $\displaystyle\sum_{\Gamma} \frac{\mathrm{Res}(\varphi_-)\,\mathrm{Res}(\varphi_+)}{\prod_{e\in\Gamma} p_e^2}$ to higher loops

- Connection to intersection theory on $\mathcal{M}_{n,g}$

Thank you !