Conformal Detectors
in + beyond perturbation thy

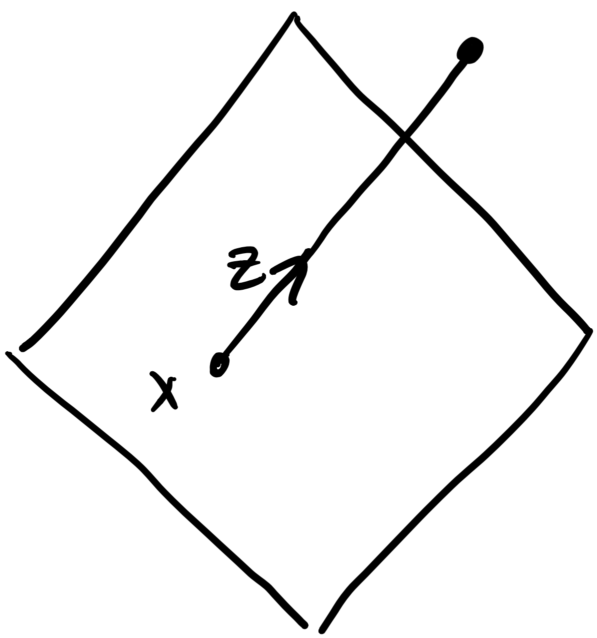
- w/ Chang, Kologlu, Kravchuk, Zhiboedov
- w/ Caron-Huot, Kologlu, Komatsu, Kravchuk, Meltzer

(both to appear)

Detectors

example: $\displaystyle\int_{-\infty}^{\infty} dx^{+}\, \mathcal{O}_{++\cdots+}$

$$\longrightarrow \quad L[\mathcal{O}](x,z) = \int_{-\infty}^{\infty} d\alpha\, (-\alpha)^{-\Delta-J}\, \mathcal{O}\left(x - \frac{z}{\alpha}, z\right)$$

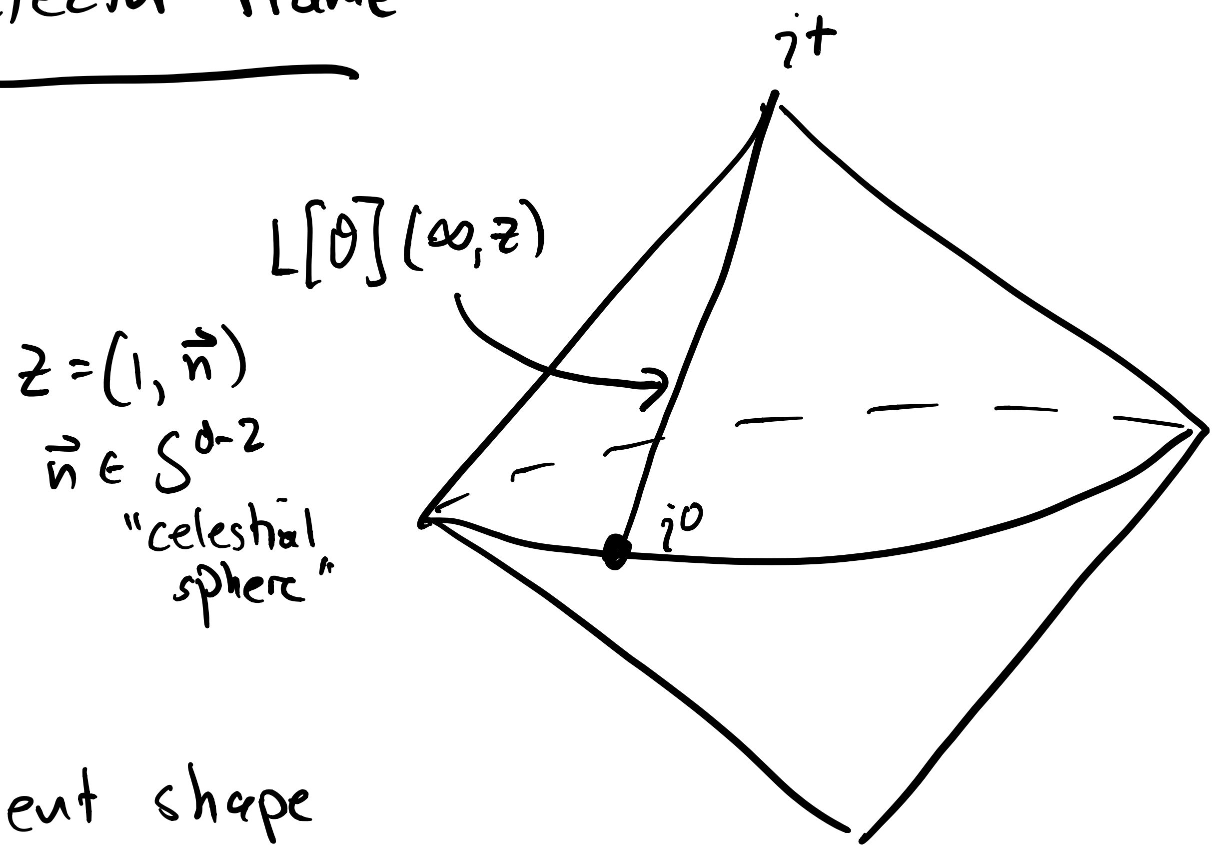


$$\mathcal{O}(x,z) = \mathcal{O}_{\mu_1\cdots\mu_J}\, z^{\mu_1}\cdots z^{\mu_J}$$

null vector

- conformally invariant
- "light transform"

Detector frame



event shape

$$\langle \Psi | L[\mathcal{O}_1](\infty, z_1) \cdots L[\mathcal{O}_k](\infty, z_k) | \Psi \rangle$$

e.g. energy correlators $\mathcal{E}(\vec{n}) = 2L[T](\infty, z)$

Example

$$L[\phi\, \partial^J \phi] \equiv L[X_J] \qquad \text{(free theory)}$$

$$\begin{aligned} \nu(z; p) &\equiv \langle \phi(p) |\, L[X_J](\infty, z)\, | \phi(p) \rangle \\ &= \int d\beta\, \beta^{J+d-4}\, \delta(p - \beta z) \end{aligned}$$

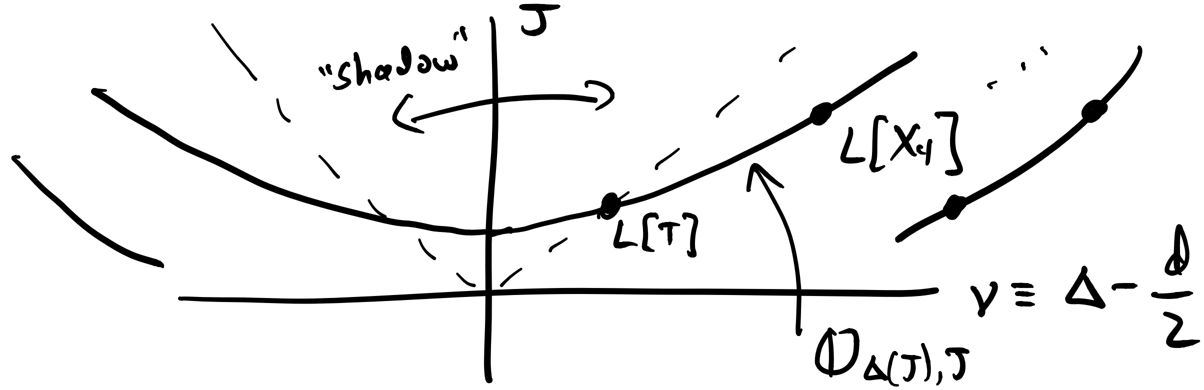
- counts particles in z-dir., weighted by E^{J-1}
- turn on interactions
  $\Longrightarrow$ not IR safe if $J \neq 2$

Let the theory tell you its detectors!

local op	detector
"anything you can measure at a pt."	"anything you can weight a cross-section with"
UV-div.	IR-div.
renormalize!	renormalize!
thy-dep.	thy-dep.
OPE	light-ray OPE / ??
Hilbert space/ radial quant.	??

Analyticity in spin

- detectors lie on Regge traj. (Caron-Huot '17)
- in pert. thy, can renormalize, e.g. $L[X_J]$ for $J \in \mathbb{C}$
- general nonpert. construction of detectors w/ $J \in \mathbb{C}$ (Kravchuk, DSD '18)

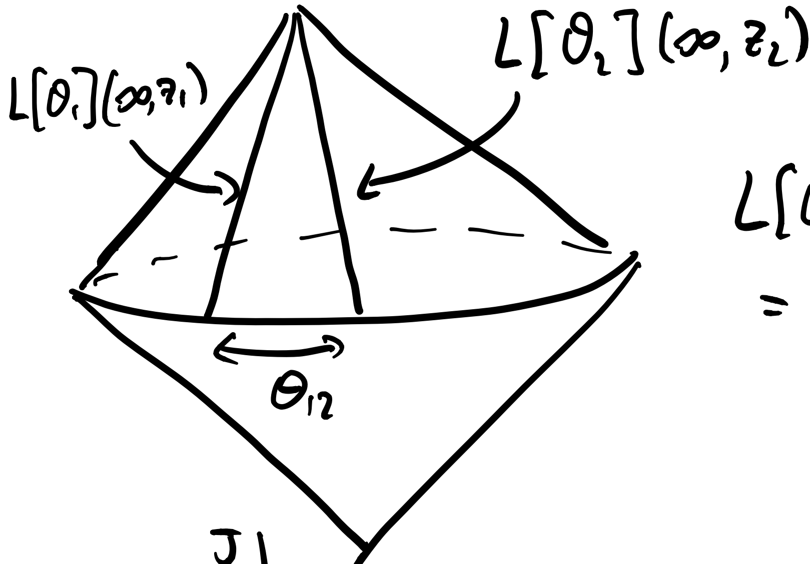


$$\mathbb{O}_{\Delta(J),J}\Big|_{J=\text{even integer}} = L[\mathcal{O}_{\Delta,J}]$$

This talk

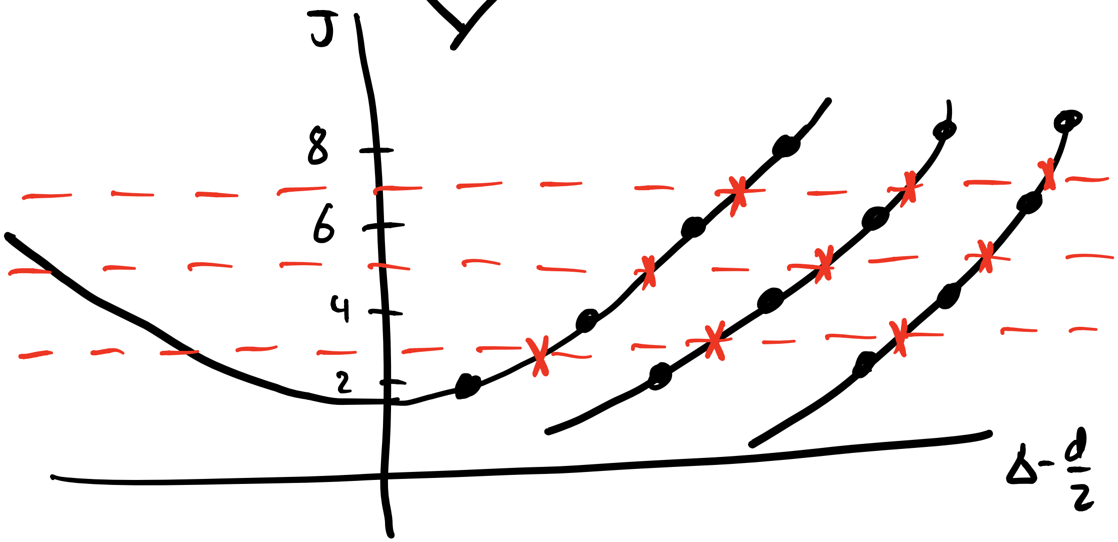
- Light-ray OPE
- Pomeron in Wilson-Fisher

Light-Ray OPE



$$L[\mathcal{O}_1](\infty, z_1)\, L[\mathcal{O}_2](\infty, z_2) = \sum_i \theta_{12}^{\Delta_i - 1}\, \mathbb{O}_i(\infty, z_2)$$

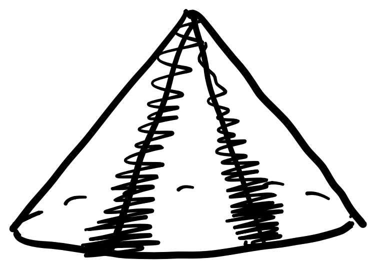
e.g. for $\mathcal{E} \times \mathcal{E}$, $J = 3, 5, 7, \ldots$ $\mathbb{O}$'s appear



Light-ray OPE (sketch)

$$\mathbb{O}_{\Delta_i(J),J} = \int dx_1\, dx_2\ K_{\Delta_i(J),J}(x_1,x_2)\, \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)$$

$\lim_{J \to 3,5,\dots} K_{\Delta,J}$ ~ localizes on null cone for both x_1, x_2



- undo smearing w/ harmonic analysis

$J=3$: leading term (Hofman-Maldacena)

$J=5,7,\dots$: higher spin on celestial sphere

Light-ray OPE : celestial blocks

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle_{\mathcal{O}_{20'}} = F_{\mathcal{E}}\left(\frac{1-\vec{n}_1\cdot\vec{n}_2}{2}\right)$$

"celestial block"

$$F_{\mathcal{E}}(\zeta) = \sum_i P_{\Delta_i} \frac{4\pi^4\,\Gamma(\Delta_i-2)}{\Gamma\left(\frac{\Delta_i-1}{2}\right)^3\Gamma\left(\frac{3-\Delta_i}{2}\right)} f_{\Delta_i}(\zeta)$$

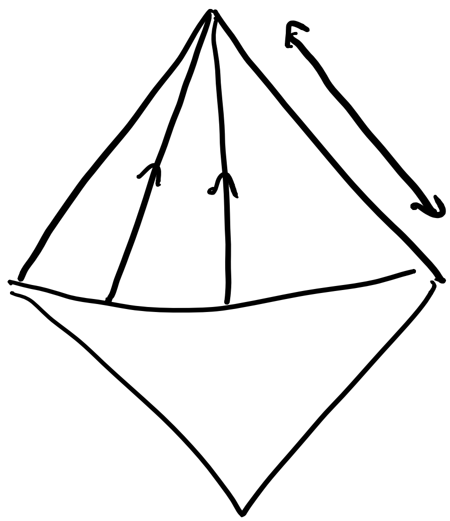
$\Delta_i(J=3)$

$$f_\Delta(\zeta) = \zeta^{\frac{\Delta-7}{2}}\,{}_2F_1\left(\frac{\Delta-1}{2},\frac{\Delta-1}{2},\Delta-1,\zeta\right)$$

- checked in $\mathcal{N}=4$ SYM up to 4 loops, at strong coupling

[also: Dixon, Moult, Zhu; Korchemsky]

Light-ray OPE: transverse spin



dilatation symmetry:

$$L[\mathcal{O}_1]\,L[\mathcal{O}_2] \sim L[\text{spin } J_1+J_2-1]$$

how does $J_1+J_2-1 \underline{+n}$ appear?

special conf.-inv't differential operators

$$\mathcal{D}_n = \Big(\partial_x \cdot \big[(w\cdot\partial_w - z\cdot\partial_z)\,w + z\; w\cdot\partial_z\big]\Big)^n$$

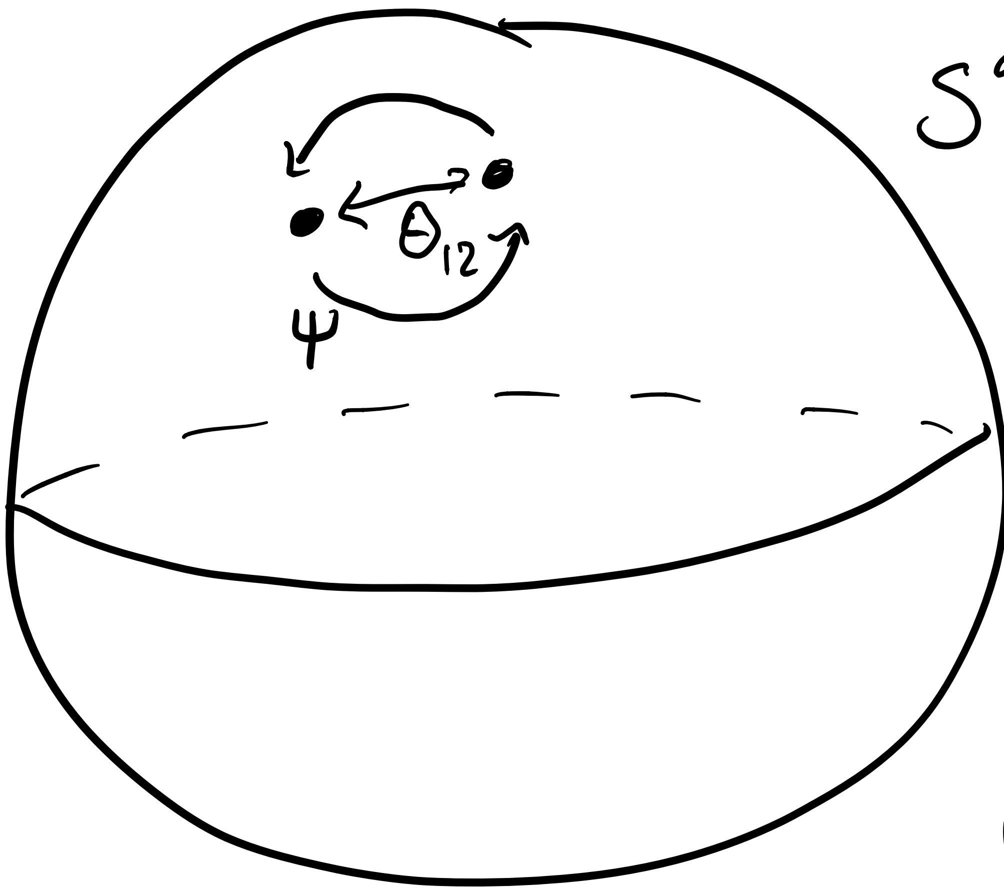
$$: \quad \begin{array}{|c|c|c|}\hline n+j-1 & & \\ \hline j & & \\ \cline{1-2}\end{array} \longrightarrow \begin{array}{|c|c|c|}\hline j-1 & & \\ \hline j+n & \multicolumn{2}{c}{} \\ \cline{1-1}\end{array}$$

spin

transverse spin

$$\mathcal{D}_n\, \mathbb{O}_{\Delta_i,\, J=n+J_1+J_2-1} \in L[\mathcal{O}_1]\,L[\mathcal{O}_2]$$

Light-ray OPE: transverse spin

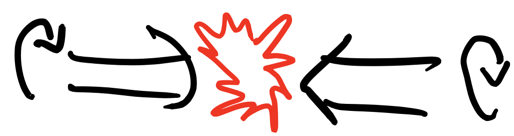


celestial S^{d-2}

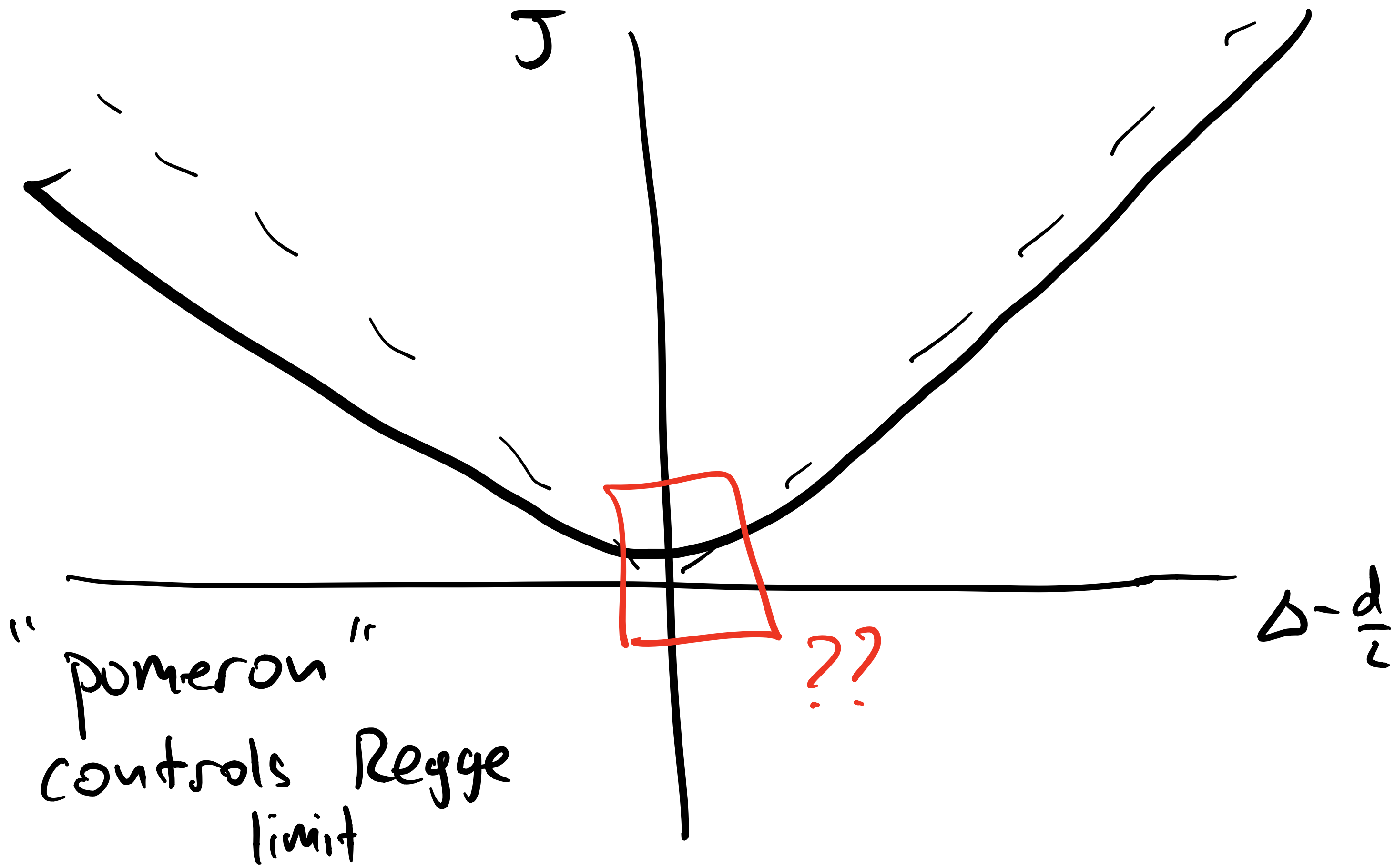
$$\theta_{12}^{\Delta_i - 1} e^{ij\psi}$$

- non-rot-sym states

e.g. polarized beams

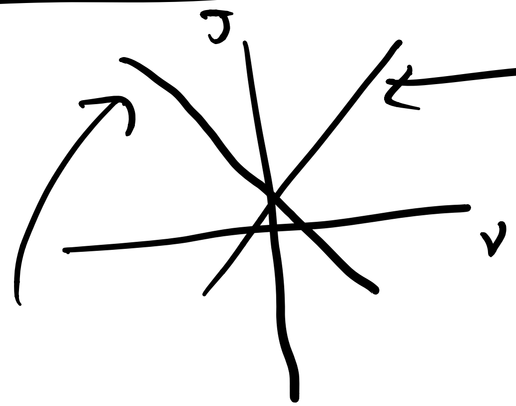


Pomeron in Wilson-Fisher



Pomeron in Wilson-Fisher

free



$$V_\nu = \int d\beta\, \beta^{\nu+\frac{d-4}{2}}\, \delta(p-\beta z)$$

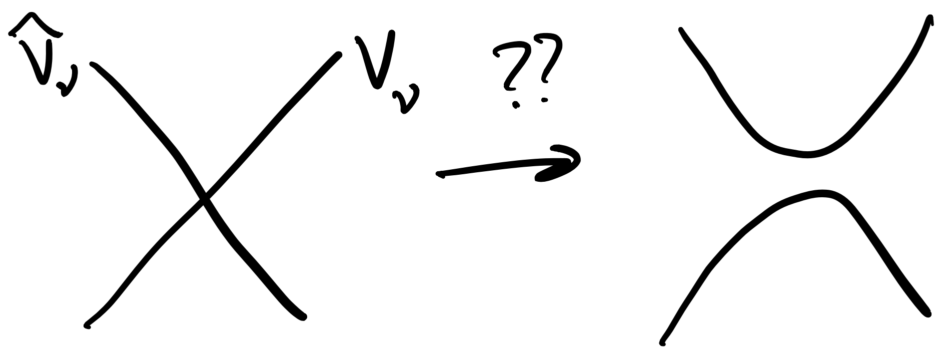
(local on S^{d-2})

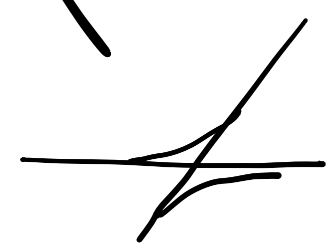
$$\tilde{V}_\nu = S[V_\nu] = 2(-2z\cdot p)^{\nu-\frac{d-2}{2}}\, \delta(p^2)$$

"shadow"

(non-local on S^{d-2})

interactions :



(cf. BFKL + DGLAP recombination )

Pomeron in Wilson-Fisher

$$V_{\gamma=0} = \int d\beta\, \beta^{\frac{d-4}{2}}\, \delta(p-\beta z)$$

$$\tilde{V}_{\gamma=0} \stackrel{?}{=} \underbrace{2(-2z\cdot p)^{-\frac{d-2}{2}}\,\delta(p^2)}_{\text{bad distribution!}}$$

(like $\frac{1}{x}$)

$$\frac{1}{x} \to \left[\frac{1}{x}\right]_+ \quad \text{(subtract log divergence)}$$

$$\left[\frac{1}{\mu x}\right]_+ = \frac{1}{\mu}\left[\frac{1}{x}\right]_+ + \frac{\log\mu}{\mu}\,\delta(x)$$

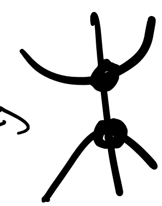
Pomeron in Wilson-Fisher

$$\tilde{V}_0' = \mu^{\frac{2-d}{2}}\, 2\left[(-2z\cdot p/\mu)\right]_+^{-\frac{d-2}{2}} + \int d\beta \log\frac{\beta}{\mu}\, \beta^{\frac{d-4}{2}}\, \delta(p-\beta z)$$

- must introduce scale
- log mixing w/ $V_{\gamma=0}$

$$D\begin{pmatrix} V_0 \\ \tilde{V}_0' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} V_0 \\ \tilde{V}_0' \end{pmatrix}$$

- at loop level, log rep splits $\longrightarrow$



- 1-loop Pomeron is lin. comb. of
  $V_0 + \tilde{V}_0'$

Future

- space of detectors / light-ray ops?
- explore Chew-Frautschi plot
- multi-point event shapes?
- non-conformal theories?
- $\mathcal{L}HC = \prod_k L[\mathcal{O}_k](\infty, z_k) = \sum_i f_i(\theta_{12}, \ldots)\, \mathcal{O}_i$