Stefano Frixione

The partonic structure of the electron

Based on: 1909.03886 (SF), 1911.12040 (Bertone, Cacciari, SF, Stagnitto)

Cambridge, 4/6/2020

Assumption:

Somewhere, someone will build an e^+e^- collider

(linear or circular)

Goal: increase the accuracy in the computations of e^+e^- cross sections

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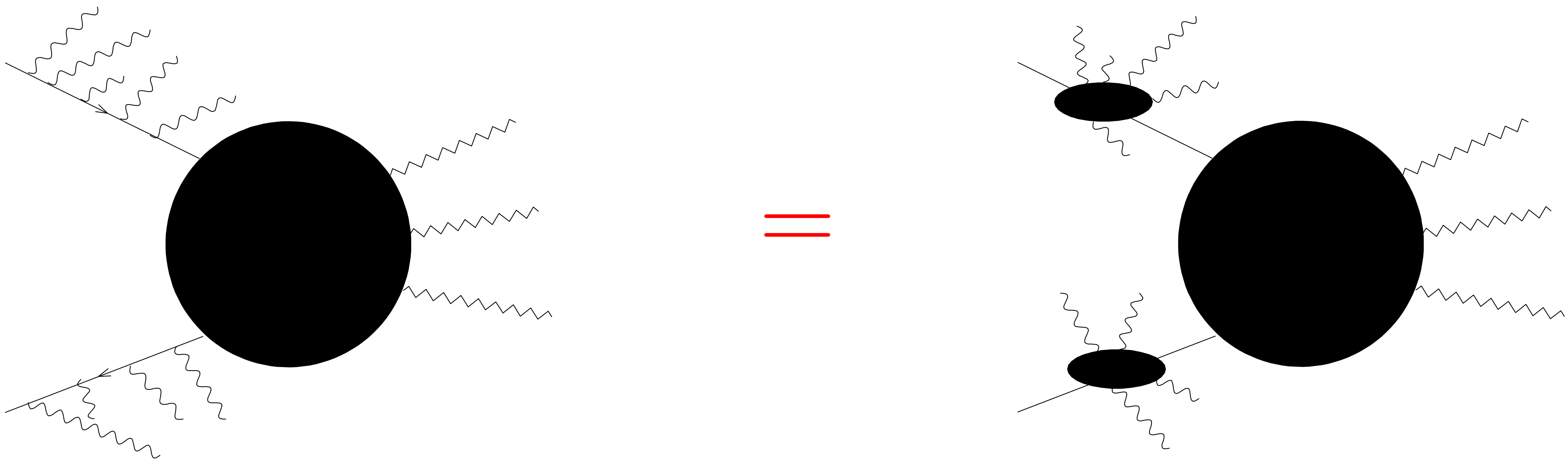
Framework: a factorisation formula

- aka structure-function approach: best to *not* use this terminology

Factorisation



$$\sigma = \text{PDF} \star \text{PDF} \star \hat{\sigma}$$

PDFs collect (universal) small-angle dynamics

Goal: increase the accuracy in the computations of e^+e^- cross sections

Framework: a factorisation formula

- aka structure-function approach: best to *not* use this terminology

By means of: more accurate PDFs

- PDFs aka structure functions: best to *not* use this terminology
- improve the LL+LO accuracy, $(\alpha \log(E/m))^k$, by including NLL+NLO terms, $(\alpha \log(E/m))^k + \alpha\,(\alpha \log(E/m))^{k-1}$, in the PDFs
- the corresponding increased accuracy of short-distance cross sections is widely available, and is understood here

Current z-space LO+LL PDFs $(\alpha \log(E/m))^k$:

- $0 \le k \le \infty$ for $z \simeq 1$ (Gribov, Lipatov)
- $0 \le k \le 3$ for $z < 1$ (Skrzypek, Jadach; Cacciari, Deandrea, Montagna, Nicrosini; Skrzypek)
- matching between these two regimes

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- matching between these two regimes

Sought z-space NLO+NLL PDFs $(\alpha\log(E/m))^k + \alpha\,(\alpha\log(E/m))^{k-1}$:

- $0 \le k \le \infty$ for $z \simeq 1$
- $0 \le k \le 3$ for $z < 1 \iff \mathcal{O}(\alpha^3)$
- matching between these two regimes
- for e^+, e^-, and γ
- both numerical and analytical

Main tool: the solution of PDFs evolution equations

Consider the production of a system X at an e^+e^- collider:

$$e^+(P_{e^+}) + e^-(P_{e^-}) \longrightarrow X$$

Its cross section is written as follows:

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

To be definite, let's stipulate that:

$$k \in \{e^+, \gamma\}, \qquad l \in \{e^-, \gamma\}$$

which is immediate to generalise, if need be. Then:

- $d\Sigma_{e^+e^-}$: the collider-level cross section
- $d\sigma_{kl}$: the particle-level cross section
- $\mathcal{B}_{kl}(y_+, y_-)$: describes beam dynamics
- e^+, e^- on the lhs: the beams
- e^+, e^-, γ on the rhs: the particles

I'll only talk about particles and particle-level cross sections

The parametrisation of beam dynamics is supposed to be given

I sum over polarisations

Write any particle cross section by means of a factorisation formula, quite similar to its QCD counterpart $\longrightarrow$

$$d\bar{\sigma}_{kl}(p_k,p_l) \;=\; \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+,\mu^2,m^2)\, \Gamma_{j/l}(z_-,\mu^2,m^2)$$
$$\times\, d\hat{\sigma}_{ij}(z_+p_k, z_-p_l, \mu^2) + \Delta$$

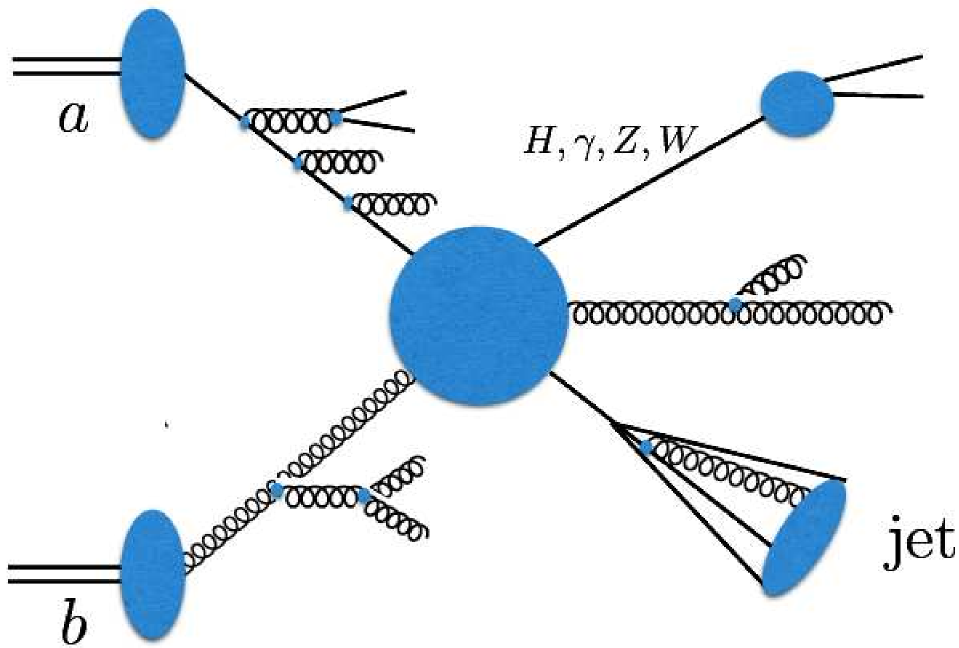
with:

$$d\bar{\sigma}_{kl} = d\sigma_{kl} + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right), \qquad s=(p_k+p_l)^2\,, \qquad p\geq 1$$

- $d\bar{\sigma}_{kl}$: the particle-level cross section, with power-suppressed terms discarded
- $d\hat{\sigma}_{ij}$: the subtracted parton-level cross section. Independent of m
- $e^+\,,e^-\,,\gamma$ on the lhs: the particles
- $e^+\,,e^-\,,\gamma$ on the rhs: the partons
- $\Gamma_{i/k}$: the PDF of parton i inside particle k. It can be computed perturbatively
- μ: the hard scale, $m^2 \ll \mu^2 \sim s$

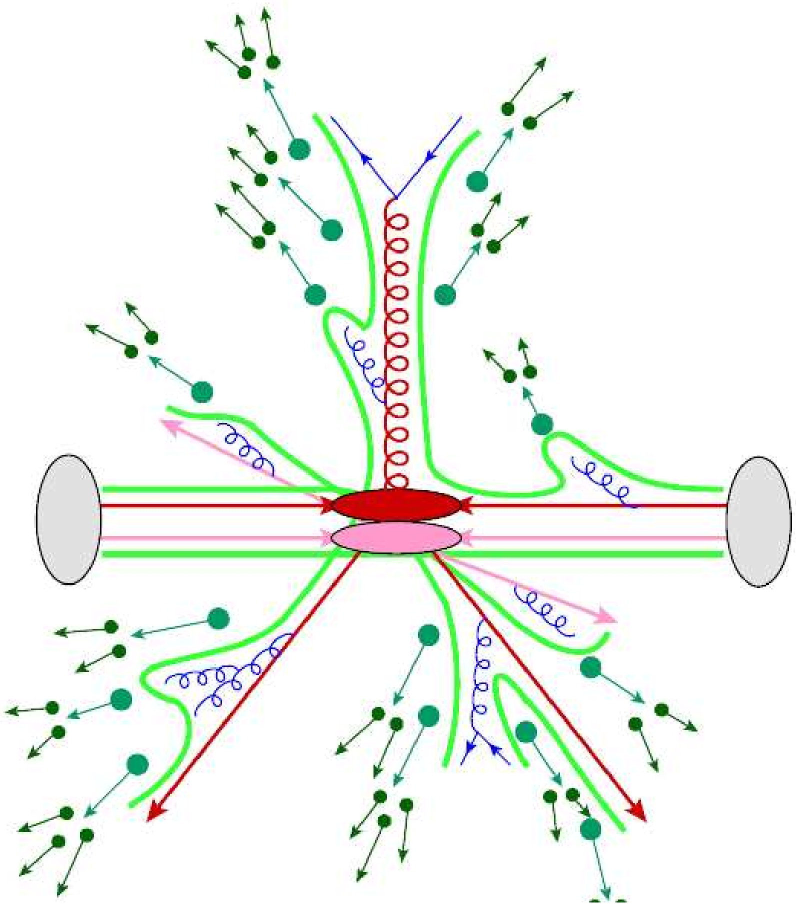
In QCD:

$$d\sigma_{H_1 H_2 \to X}(S) = \sum_{ij} \int dx_1 dx_2 f_i^{(H_1)}(x_1, \mu^2) f_j^{(H_2)}(x_2, \mu^2) \times d\hat{\sigma}_{ij \to X}(\hat{s} = x_1 x_2 S, \mu^2) + \mathcal{O}\left(\left(\frac{1\ \text{GeV}}{\mu}\right)^k\right)$$



Plot: D. de Florian

In QCD, power corrections may become the dominant source of uncertainty



Plot: S. Prestel

An unmitigated horror for the purist

- Hadronisation, UE, ..., all based on models, fitted to data
- Sometimes poorly understood
- Universality assumed
- Extrapolations (e.g. LHC→FCC-hh) dubious. Always better in situ

Unpleasant but unavoidable

Differences of QED wrt QCD:

- PDFs and power-suppressed terms can be computed perturbatively
- An object (e.g. e^-) may play the role of both particle and parton

As in QCD, a particle is a physical object, a parton is not

An aside

The fact that, unlike in QCD, PDFs and PSTs can be computed perturbatively in QED does *not* mean that cross sections computed at a given order in α are physically sensible

ISR effects spoil "convergence", and must be resummed

$\longrightarrow$ PDFs (or YFS [e.g. KKMC], or showers [e.g. Babayaga])

While there has been a significant activity in cross section computations

(ie a necessary but *not* sufficient operation),

we've been living with KK MCs and LL PDFs since the early 90s

$$
\begin{aligned}
d\bar{\sigma}_{kl}(p_k, p_l) \;\; &= \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2) \, \Gamma_{j/l}(z_-, \mu^2, m^2) \\
&\qquad \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \Delta
\end{aligned}
$$

This formula can be used in several ways:

A: to solve for the PDFs, given the particle and parton cross sections

B: for the computation of the particle cross section, given the parton cross section and the PDFs

C: for cross checks, given both cross sections and the PDFs

An exact expression: Δ accounts for possibly different treatments of mass effects on the lhs and rhs

BTW: my cross sections are either fully inclusive on final state objects, or such that final-state objects are defined through fragmentation functions

Note that e.g. bare-electron or bare-photon cross sections are:

a) a very bad idea;

b) unphysical quantities as soon as one considers EW corrections

$$
\begin{aligned}
d\bar{\sigma}_{kl}(p_k, p_l) \quad &= \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_-\, \Gamma_{i/k}(z_+, \mu^2, m^2)\, \Gamma_{j/l}(z_-, \mu^2, m^2) \\
&\qquad \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \Delta
\end{aligned}
$$

This formula can be used in several ways:

A: to solve for the PDFs, given the particle and parton cross sections

- Compute *both* cross sections to the desired perturbative order
- Formally expand the PDFs to the same order
- Set $\Delta = 0$ and solve for the PDFs

This is one of the possible procedures to compute the initial conditions for PDF evolution – see 1909.03886 for NLO results

$$
\begin{aligned}
d\bar{\sigma}_{kl}(p_k,p_l) \quad &= \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+,\mu^2,m^2)\,\Gamma_{j/l}(z_-,\mu^2,m^2) \\
&\quad \times d\hat{\sigma}_{ij}(z_+p_k,z_-p_l,\mu^2) + \Delta
\end{aligned}
$$

This formula can be used in several ways:

B: for the computation of the particle cross section, given the parton cross section and the PDFs

- This is the standard usage: set $\Delta = 0$ and use the available parton cross section and PDFs
- PDFs are understood to be evolved – if so, the lhs does not contain large logs of the mass
- If PDFs are not evolved, but expanded perturbatively, the lhs does contain large logs of the mass: no phenomenological interest

Being done, using the NLL-evolved PDFs obtained in 1911.12040

$$
\begin{aligned}
d\bar{\sigma}_{kl}(p_k, p_l) &= \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2) \, \Gamma_{j/l}(z_-, \mu^2, m^2) \\
&\quad \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \Delta
\end{aligned}
$$

This formula can be used in several ways:

C: for cross checks, given both cross sections and the PDFs

- If all quantities are computed at the same perturbative order: must obtain $\Delta = 0$
- If the PDFs are evolved: Δ features large logs of the mass

Of no interest phenomenologically

Henceforth, I consider the dominant production mechanism at an e^+e^- collider, namely that associated with partons inside an electron*

Simplified notation:

$$\Gamma_i(z, \mu^2) \;\equiv\; \Gamma_{i/e^-}(z, \mu^2)$$

*The case of the positron is identical, at least in QED, and will be understood

NLO initial conditions (1909.03886)

Conventions for the perturbative coefficients:

$$\Gamma_i = \Gamma_i^{[0]} + \frac{\alpha}{2\pi}\,\Gamma_i^{[1]} + \mathcal{O}(\alpha^2)$$

Results:

$$\begin{aligned}
\Gamma_i^{[0]}(z,\mu_0^2) &= \delta_{ie^-}\,\delta(1-z) \\
\Gamma_{e^-}^{[1]}(z,\mu_0^2) &= \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ + K_{ee}(z) \\
\Gamma_{\gamma}^{[1]}(z,\mu_0^2) &= \frac{1+(1-z)^2}{z}\left(\log\frac{\mu_0^2}{m^2} - 2\log z - 1\right) + K_{\gamma e}(z) \\
\Gamma_{e^+}^{[1]}(z,\mu_0^2) &= 0
\end{aligned}$$

Note:

- Meaningful only if $\mu_0 \sim m$
- In $\overline{\mathrm{MS}}$, $K_{ij}(z) = 0$; in general, these functions *define* an IR scheme

NLL evolution (1911.12040)

General idea: solve the evolution equations starting from the initial conditions computed previously

$$\frac{\partial \Gamma_i(z,\mu^2)}{\partial \log \mu^2} = \frac{\alpha(\mu)}{2\pi} \left[P_{ij} \otimes \Gamma_j \right] (z,\mu^2) \quad \Longleftrightarrow \quad \frac{\partial \Gamma(z,\mu^2)}{\partial \log \mu^2} = \frac{\alpha(\mu)}{2\pi} \left[\mathbb{P} \otimes \Gamma \right] (z,\mu^2) \,,$$

Done conveniently in terms of non-singlet, singlet, and photon

Two ways:

- Mellin space: suited to both numerical solution and all-order, large-z analytical solution (called *asymptotic solution*)
- Directly in z space in an integrated form: suited to fixed-order, all-z analytical solution (called *recursive solution*)

A technicality: owing to the running of α, it is best to evolve in t rather than in μ, with: (~ Furmanski, Petronzio)

$$\begin{aligned} t &= \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)} \\ &= \frac{\alpha(\mu)}{2\pi} L - \frac{\alpha^2(\mu)}{4\pi} \left(b_0 L^2 - \frac{2b_1}{b_0} L \right) + \mathcal{O}(\alpha^3)\,, \qquad L = \log \frac{\mu^2}{\mu_0^2}\,. \end{aligned}$$

Note:

- $t \longleftrightarrow \mu$; notation-wise, the dependence on t is equivalent to the dependence on μ
- $t = 0 \iff \mu = \mu_0$
- L is my "large log"
- Tricky: fixed-α expressions are obtained with $t = \alpha L/(2\pi)$ (and not $t = 0$)

Mellin space

Introduce the evolution operator $\mathbb{E}_N$

$$\Gamma_N(\mu^2) = \mathbb{E}_N(t)\,\Gamma_{0,N}\,, \qquad \mathbb{E}_N(0) = I\,, \qquad \Gamma_{0,N} \equiv \Gamma_N(\mu_0^2)$$

The PDFs evolution equations are then re-expressed by means of an evolution equation for the evolution operator:

$$\begin{aligned} \frac{\partial \mathbb{E}_N(t)}{\partial t} &= \frac{b_0 \alpha^2(\mu)}{\beta(\alpha(\mu))} \sum_{k=0}^{\infty} \left(\frac{\alpha(\mu)}{2\pi}\right)^k \mathbb{P}_N^{[k]}\,\mathbb{E}_N(t) \\ &= \left[\mathbb{P}_N^{[0]} + \frac{\alpha(\mu)}{2\pi}\left(\mathbb{P}_N^{[1]} - \frac{2\pi b_1}{b_0}\,\mathbb{P}_N^{[0]}\right)\right]\mathbb{E}_N(t) + \mathcal{O}(\alpha^2) \end{aligned}$$

- Can be solved numerically
- Can be solved analytically in a closed form under simplifying assumptions. Chiefly: large-z is equivalent to large-N
- I'll mainly discuss the (technically simpler) case of the the non-singlet

Show first that this formalism allows one to quickly re-obtain the known LL result:

$$\Gamma_{0,N}^{[0]} = 1 \quad \Longrightarrow \quad \Gamma_{\rm LL}(z,\mu^2) = M^{-1}\big[\exp\big(\log E_N\big)\big]$$

From the explicit expression of the AP ff kernel:

$$\log E_N = \frac{\alpha}{2\pi}\, P_N^{[0]}\, L \;\xrightarrow{N\to\infty}\; -\eta_0\left(\log \bar{N} - \lambda_0\right)$$

$$\eta_0 = \frac{\alpha}{\pi} L\,, \qquad \bar{N} = N\, e^{\gamma_{\rm E}}\,, \qquad \lambda_0 = \frac{3}{4}$$

The computation of the inverse Mellin transform is trivial:

$$\Gamma_{\rm LL}(z,\mu^2) = \frac{e^{-\gamma_{\rm E}\eta_0}\, e^{\lambda_0\eta_0}}{\Gamma(1+\eta_0)}\, \eta_0 (1-z)^{-1+\eta_0}$$

The usual form, bar for the "-1" of soft origin (we're resumming collinear logs here)

The NLL case is only slightly more complicated; we use:

$$\Gamma_{\mathrm{NLL}}(z,\mu^2) = M^{-1}\left[\exp\left(\log E_N\right)\right] \otimes \Gamma_{\mathrm{NLO}}(z,\mu_0^2)$$

which is convenient because the form of the evolution operator is functionally the same as at the LL:

$$\log E_N \xrightarrow{N\to\infty} -\xi_1 \log \bar{N} + \hat{\xi}_1$$

with:

$$\begin{aligned}
\xi_1 &= 2t - \frac{\alpha(\mu)}{4\pi^2 b_0}\left(1 - e^{-2\pi b_0 t}\right)\left(\frac{20}{9} n_F + \frac{4\pi b_1}{b_0}\right) \\
&= 2t + \mathcal{O}(\alpha t) = \eta_0 + \dots \\
\hat{\xi}_1 &= \frac{3}{2} t + \frac{\alpha(\mu)}{4\pi^2 b_0}\left(1 - e^{-2\pi b_0 t}\right)\left(\lambda_1 - \frac{3\pi b_1}{b_0}\right) \\
&= \frac{3}{2} t + \mathcal{O}(\alpha t) = \lambda_0 \eta_0 + \dots \\
\lambda_1 &= \frac{3}{8} - \frac{\pi^2}{2} + 6\zeta_3 - \frac{n_F}{18}(3 + 4\pi^2)
\end{aligned}$$

Thence:

$$\begin{aligned}\Gamma_{\mathrm{NLL}}(z,\mu^2) = \frac{e^{-\gamma_{\mathrm{E}}\xi_1}\, e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)}\, \xi_1 (1-z)^{-1+\xi_1} \\ \times \left\{ 1 + \frac{\alpha(\mu_0)}{\pi} \left[\left(\log\frac{\mu_0^2}{m^2} - 1 \right) \left(A(\xi_1) + \frac{3}{4} \right) - 2B(\xi_1) + \frac{7}{4} \right.\right. \\ \left.\left. + \left(\log\frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1) \right) \log(1-z) - \log^2(1-z) \right] \right\}\end{aligned}$$

where:

$$\begin{aligned} A(\kappa) &= -\gamma_{\mathrm{E}} - \psi_0(\kappa) \\ B(\kappa) &= \frac{1}{2}\,\gamma_{\mathrm{E}}^2 + \frac{\pi^2}{12} + \gamma_{\mathrm{E}}\,\psi_0(\kappa) + \frac{1}{2}\,\psi_0(\kappa)^2 - \frac{1}{2}\,\psi_1(\kappa) \end{aligned}$$

z space

Use integrated PDFs (so as to simplify the treatment of endpoints)

$$\mathcal{F}(z,t) = \int_0^1 dy\,\Theta(y-z)\,\Gamma(y,\mu^2) \quad \Longrightarrow \quad \Gamma(z,\mu^2) = -\frac{\partial}{\partial z}\mathcal{F}(z,t)$$

in terms of which the formal solution of the evolution equation is:

$$\mathcal{F}(z,t) = \mathcal{F}(z,0) + \int_0^t du\,\frac{b_0\alpha^2(u)}{\beta(\alpha(u))}\left[\mathbb{P}\,\overline{\otimes}\,\mathcal{F}\right](z,u)$$

By inserting the representation:

$$\mathcal{F}(z,t) = \sum_{k=0}^{\infty}\frac{t^k}{k!}\left(\mathcal{J}_k^{\mathrm{LL}}(z) + \frac{\alpha(t)}{2\pi}\,\mathcal{J}_k^{\mathrm{NLL}}(z)\right)$$

on both sides of the solution, one obtains recursive equations, whereby a $\mathcal{J}_k$ is determined by all $\mathcal{J}_p$ with $p < k$. The recursion starts from $\mathcal{J}_0$, which are the integrated initial conditions

For the record, the recursive equations are:

$$
\begin{aligned}
\mathcal{J}_k^{\mathrm{LL}} &= \mathbb{P}^{[0]} \,\overline{\otimes}\, \mathcal{J}_{k-1}^{\mathrm{LL}} \\
\mathcal{J}_k^{\mathrm{NLL}} &= (-)^k (2\pi b_0)^k \mathcal{F}^{[1]}(\mu_0^2) \\
&\quad + \sum_{p=0}^{k-1} (-)^p (2\pi b_0)^p \Bigg(\mathbb{P}^{[0]} \,\overline{\otimes}\, \mathcal{J}_{k-1-p}^{\mathrm{NLL}} + \mathbb{P}^{[1]} \,\overline{\otimes}\, \mathcal{J}_{k-1-p}^{\mathrm{LL}} \\
&\qquad\qquad - \frac{2\pi b_1}{b_0} \mathbb{P}^{[0]} \,\overline{\otimes}\, \mathcal{J}_{k-1-p}^{\mathrm{LL}} \Bigg)
\end{aligned}
$$

We have computed these for $k \leq 3$ ($\mathcal{J}^{\mathrm{LL}}$) and $k \leq 2$ ($\mathcal{J}^{\mathrm{NLL}}$), ie to $\mathcal{O}(\alpha^3)$

Results in 1911.12040 and its ancillary files

Large-z singlet and photon

As for the non-singlet, start from the asymptotic AP kernel expressions:

$$\mathbb{P}_{\mathrm{S},N} \quad \xrightarrow{N\to\infty} \quad \begin{pmatrix} -2\log\bar{N} + 2\lambda_0 & 0 \\ 0 & -\frac{2}{3}\,n_F \end{pmatrix}$$

$$+ \frac{\alpha}{2\pi}\begin{pmatrix} \frac{20}{9} n_F \log\bar{N} + \lambda_1 & 0 \\ 0 & -n_F \end{pmatrix} + \mathcal{O}(1/N) + \mathcal{O}(\alpha^2)$$

This implies

$$\begin{aligned} (\mathbb{E}_N)_{SS} &= E_N \\ M^{-1}\left[\,(\mathbb{E}_N)_{\gamma\gamma}\,\right] &= \frac{\alpha(\mu_0)}{\alpha(\mu)}\,\delta(1-z) \end{aligned}$$

$\Rightarrow$ Singlet $\equiv$ non-singlet

Photon $\equiv$ initial condition + $\alpha(0)$ scheme

Photon $\equiv$ initial condition + $\alpha(0)$ scheme $\Longrightarrow$

$$\Gamma_\gamma(z,\mu^2) = \frac{1}{2\pi}\,\frac{\alpha(\mu_0)^2}{\alpha(\mu)}\,\frac{1+(1-z)^2}{z}\left(\log\frac{\mu_0^2}{m^2} - 2\log z - 1\right)\,.$$

Or: $\sim$ Weizsaecker-Williams function, plus the natural emergence of a small scale in the argument of α

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Or: $\sim$ Weizsaecker-Williams function, plus the natural emergence of a small scale in the argument of α

But: vastly different from the numerical (exact) solution

$\longrightarrow$ $1/N$ suppression of off-diagonal terms in the evolution operator is over-compensated by the δ-like peak of the electron initial-condition

Photon $\equiv$ initial condition $+\ \alpha(0)$ scheme $\Longrightarrow$

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Or: $\sim$ Weizsaecker-Williams function, plus the natural emergence of a small scale in the argument of α

But: vastly different from the numerical (exact) solution

$\longrightarrow$ $1/N$ suppression of off-diagonal terms in the evolution operator is over-compensated by the δ-like peak of the electron initial-condition

By solving the 2×2 system $(\xi_{1,0} = 2 + \mathcal{O}(\alpha))$:

$$\Gamma_\gamma(z,\mu^2) = \frac{\alpha(\mu_0)^2}{\alpha(\mu)}\,\frac{3}{2\pi\xi_{1,0}}\,\log(1-z) - \frac{\alpha(\mu_0)^3}{\alpha(\mu)}\,\frac{1}{2\pi^2\xi_{1,0}}\,\log^3(1-z)$$

A remarkable fact

Our asymptotic solutions, expanded in α, feature *all* of the terms:

$$\frac{\log^q(1-z)}{1-z} \qquad \text{singlet, non-singlet}$$
$$\log^q(1-z) \qquad \text{photon}$$

of our recursive solutions

Non-trivial; stems from keeping subleading terms (at $z \to 1$) in the AP kernels

However:

It turns out that, at the NLL, all of the $\log^k(1-z)$ terms not proportional to $\log \mu_0^2/m^2$ are artifacts of the $\overline{\mathrm{MS}}$ scheme

They are artifacts because they are non-physical: at the NLO, they cancel against similar terms in the cross sections

They do contribute at higher (ie out of control) orders. It might be convenient to get rid of them from the beginning

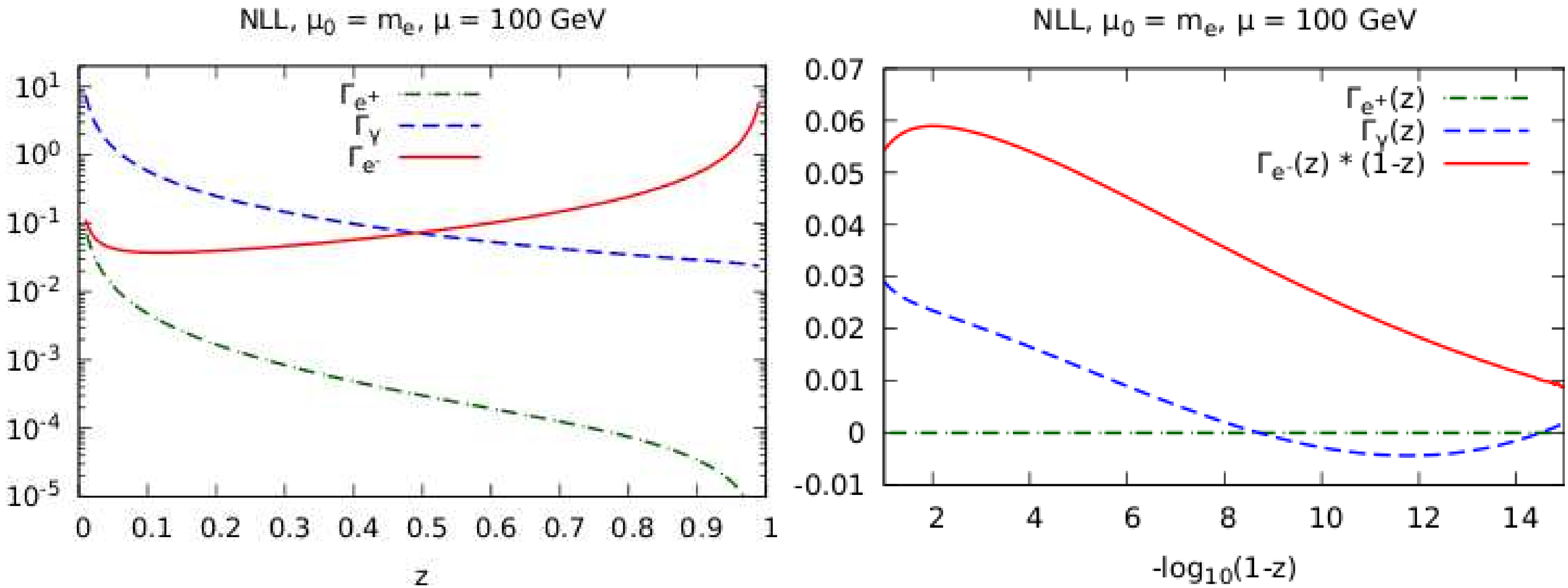
$\longrightarrow$ Different IR scheme: work in progress

Illustrative results for PDFs

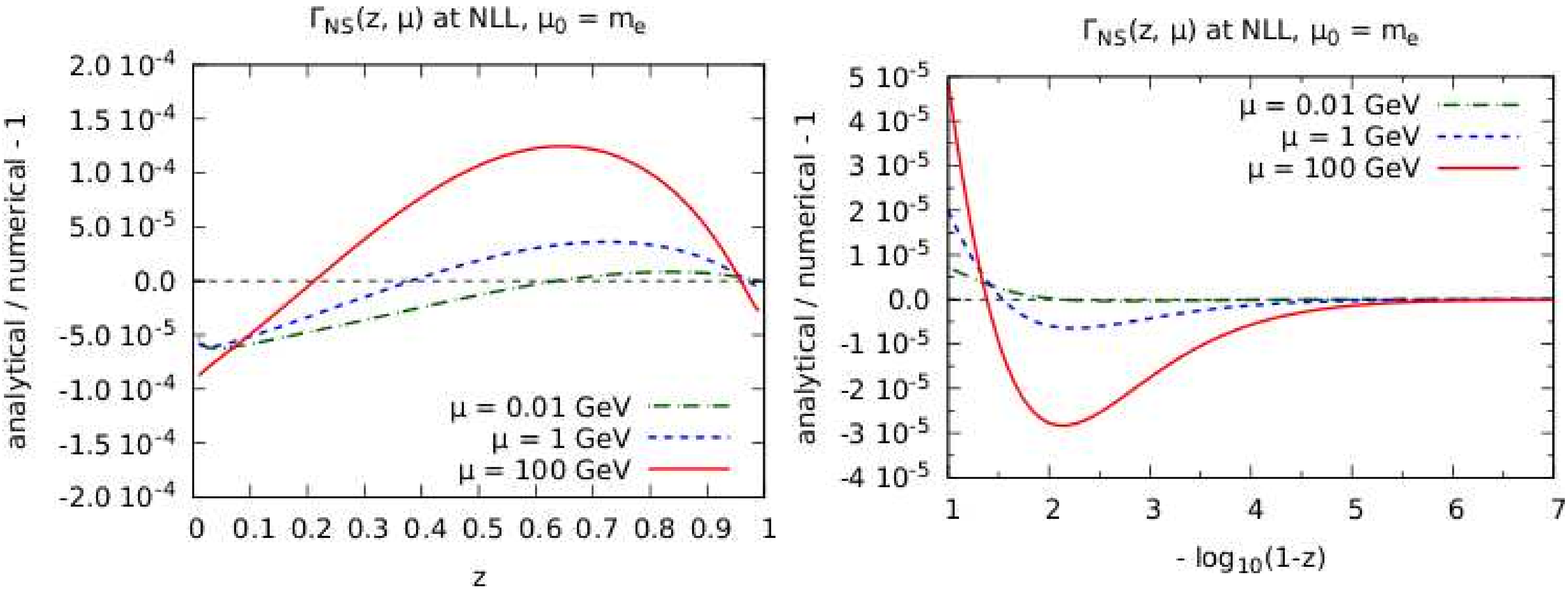
- Analytical results obtained by means of an additive matching between the recursive and the asymptotic solutions

- All are in $\overline{\mathrm{MS}}$

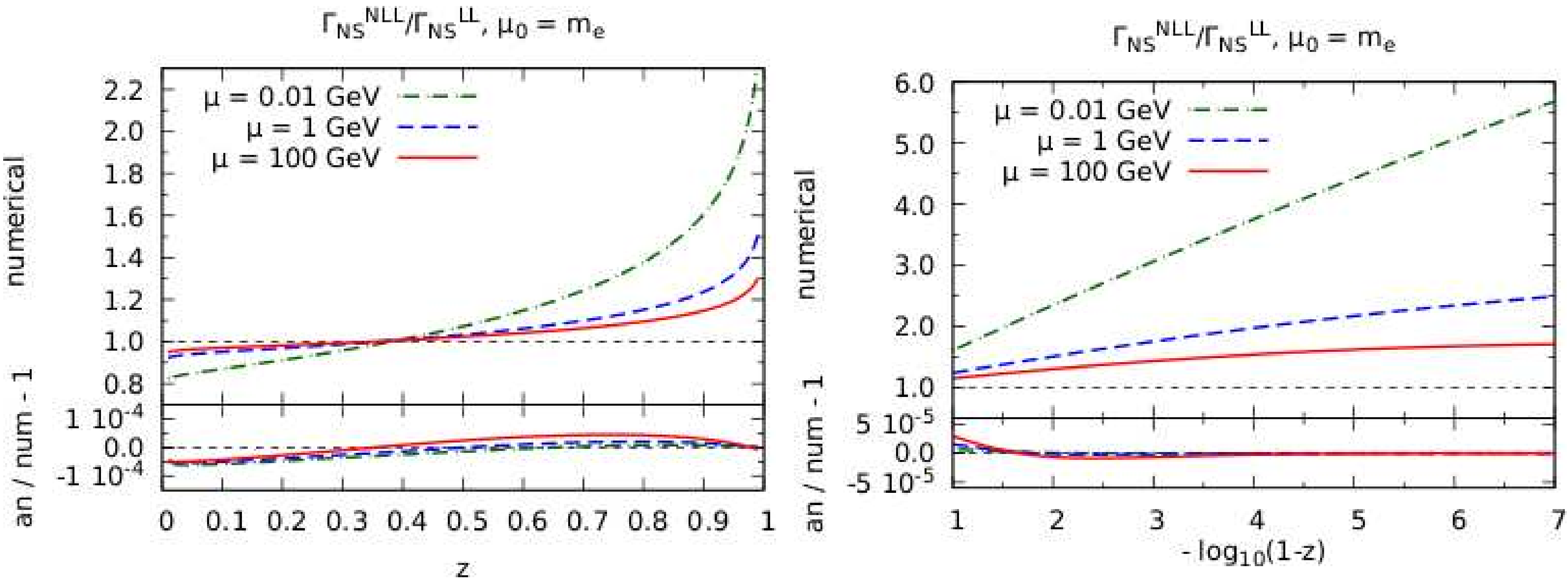
- Bear in mind that PDFs are unphysical quantities



e^- vs γ vs e^+. Note that e^- in the right-hand panel is strongly damped



Numerical vs analytical, non-singlet



NLL vs LL, non-singlet. The insets show the double ratio, ie numerical vs analytical

In order to understand the large-z bit of the previous plots:

$$\Gamma_{\rm LL}(z,\mu^2) = \frac{e^{-\gamma_{\rm E}\eta_0} e^{\lambda_0\eta_0}}{\Gamma(1+\eta_0)}\, \eta_0 (1-z)^{-1+\eta_0}$$

$$\Gamma_{\rm NLL}(z,\mu^2) = \frac{e^{-\gamma_{\rm E}\xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)}\, \xi_1 (1-z)^{-1+\xi_1}$$
$$\times \left\{1 + \frac{\alpha(\mu_0)}{\pi}\left[\left(\log\frac{\mu_0^2}{m^2} - 1\right)\left(A(\xi_1) + \frac{3}{4}\right) - 2B(\xi_1) + \frac{7}{4}\right.\right.$$
$$\left.\left. + \left(\log\frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1)\right)\log(1-z) - \log^2(1-z)\right]\right\}$$

with:

$$\xi_1 \simeq \eta_0\,, \qquad \hat{\xi}_1 \simeq \lambda_0\eta_0$$

$$A(\kappa) = \frac{1}{\kappa} + \mathcal{O}(\kappa) \quad \Longrightarrow \quad \log(1-z) \text{ dominates}$$

$$B(\kappa) = -\frac{\pi^2}{6} + 2\zeta_3\kappa + \mathcal{O}(\kappa^2)$$

Conclusions

- We have computed all NLO initial conditions for PDFs and FFs (1909.03886), unpolarised

- We have NLL-evolved those relevant to the electron PDFs (1911.12040), both analytically and numerically

- These can be obtained at:

  `https://github.com/gstagnit/ePDF`

Many results are based on establishing a "dictionary" QCD $\longrightarrow$ QED, which works at any order in α_S and α

Being done/to be done

- Assess the impact of PDFs NLL effects on physical cross sections

- The inclusion of these results in MG5_aMC@NLO v3.X is the only missing ingredient in the latter for the computation of NLO QED corrections in e^+e^- collisions

  NLO QCD+EW in hh collisions and NLO QCD in e^+e^- collisions already OK

- γ PDFs; soft effects; alternative IR schemes; FFs

- Polarisations?