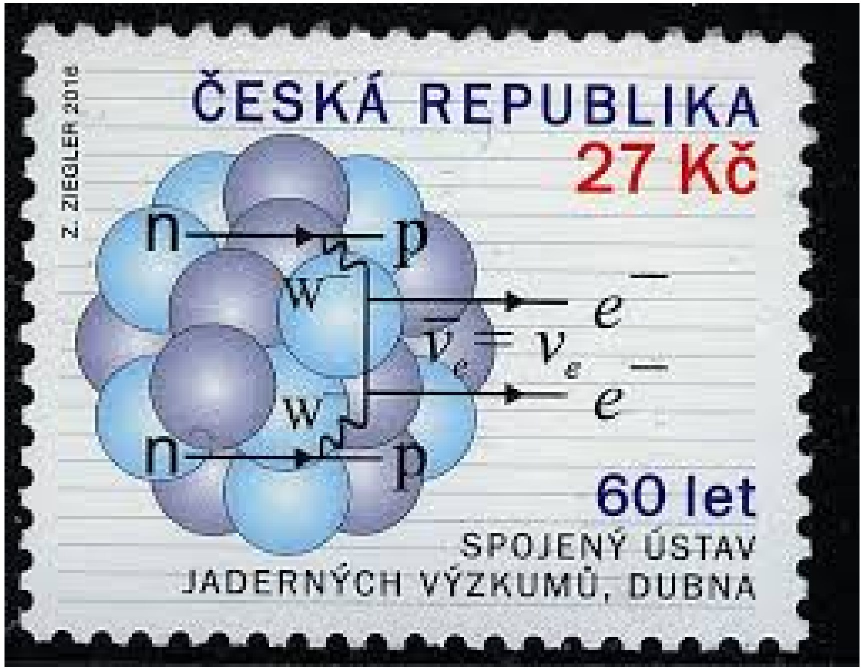
Looking for Majorana in All the Wrong Places

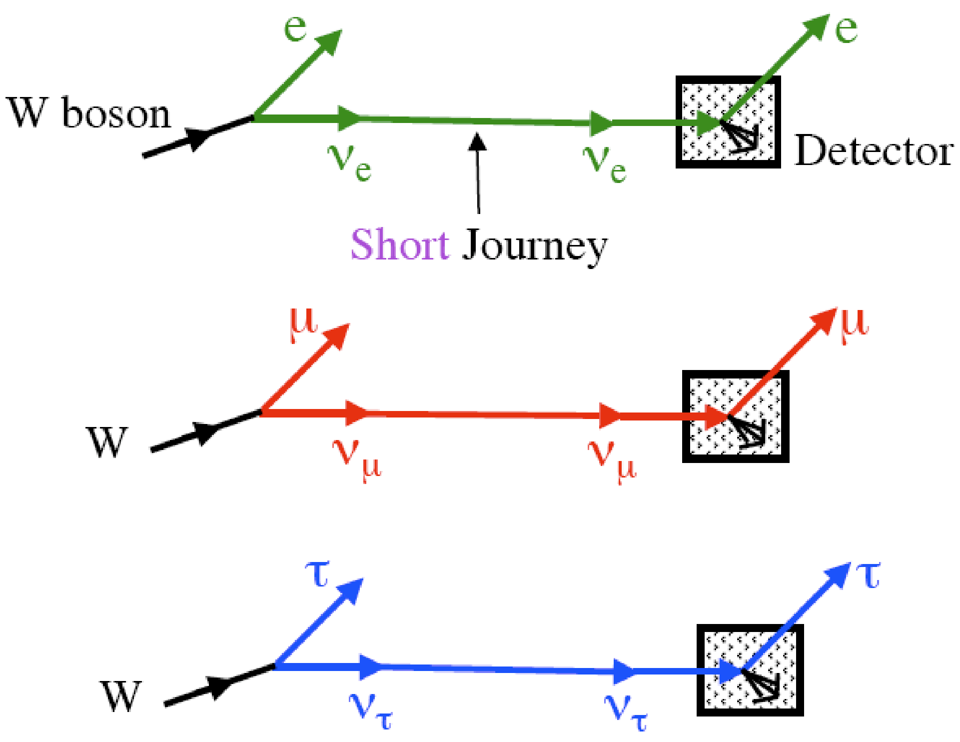


André de Gouvêa – Northwestern University

COFI Seminar

April 22, 2020

In the 20th Century, this is how we pictured neutrinos:



- come in three flavors (see figure);
- interact only via weak interactions ($W^{\pm}, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number (conserved):
  - $L(\nu) = L(\ell) + 1$,
  - $L(\bar{\nu}) = L(\bar{\ell}) = -1$.

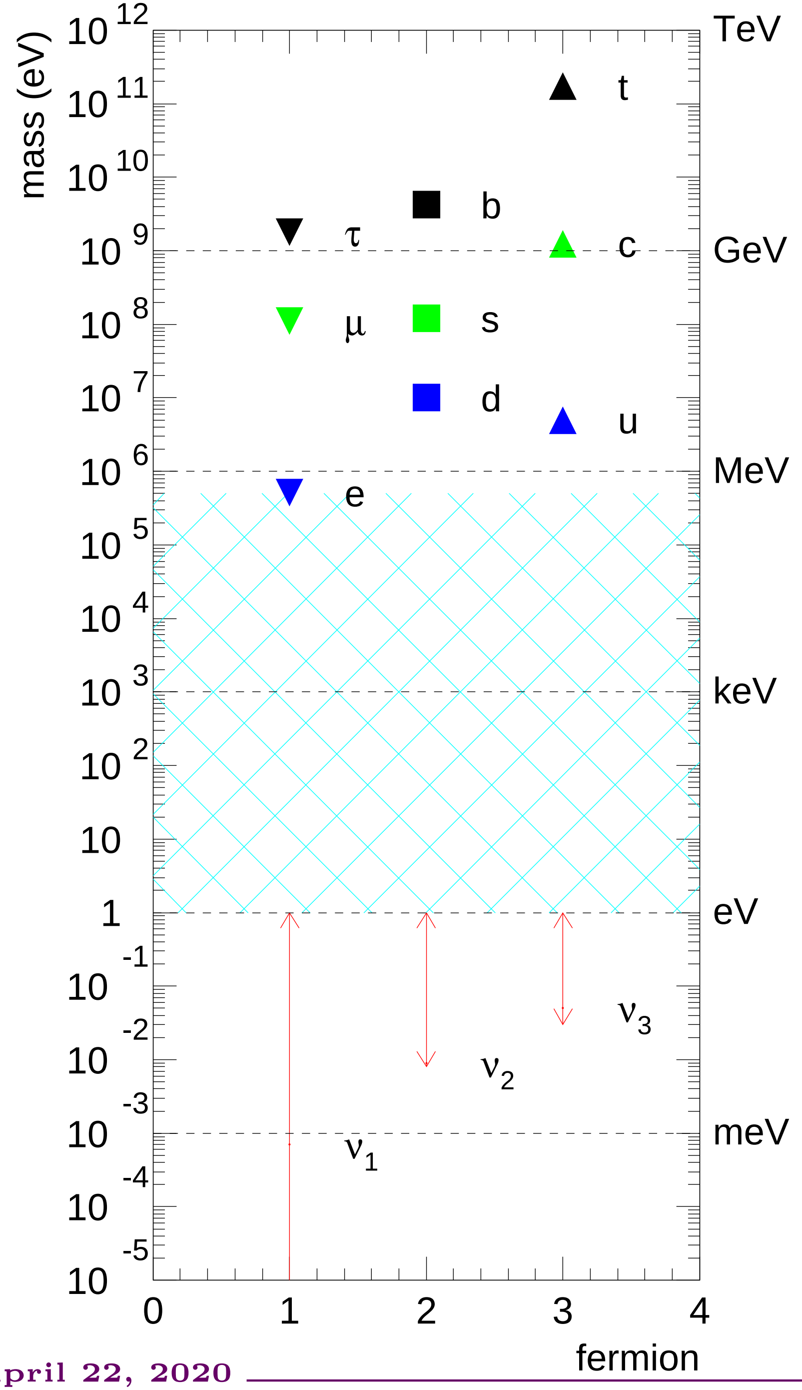
Something Funny Happened on the Way to the 21st Century

ν Flavor Oscillations

Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L. The evidence is overwhelming.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric and accelerator experiments;
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments;
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor experiments;
- $\nu_\mu \to \nu_{\rm other}$ and $\bar{\nu}_\mu \to \bar{\nu}_{\rm other}$— atmospheric and accelerator expts;
- $\nu_\mu \to \nu_e$ — accelerator experiments.

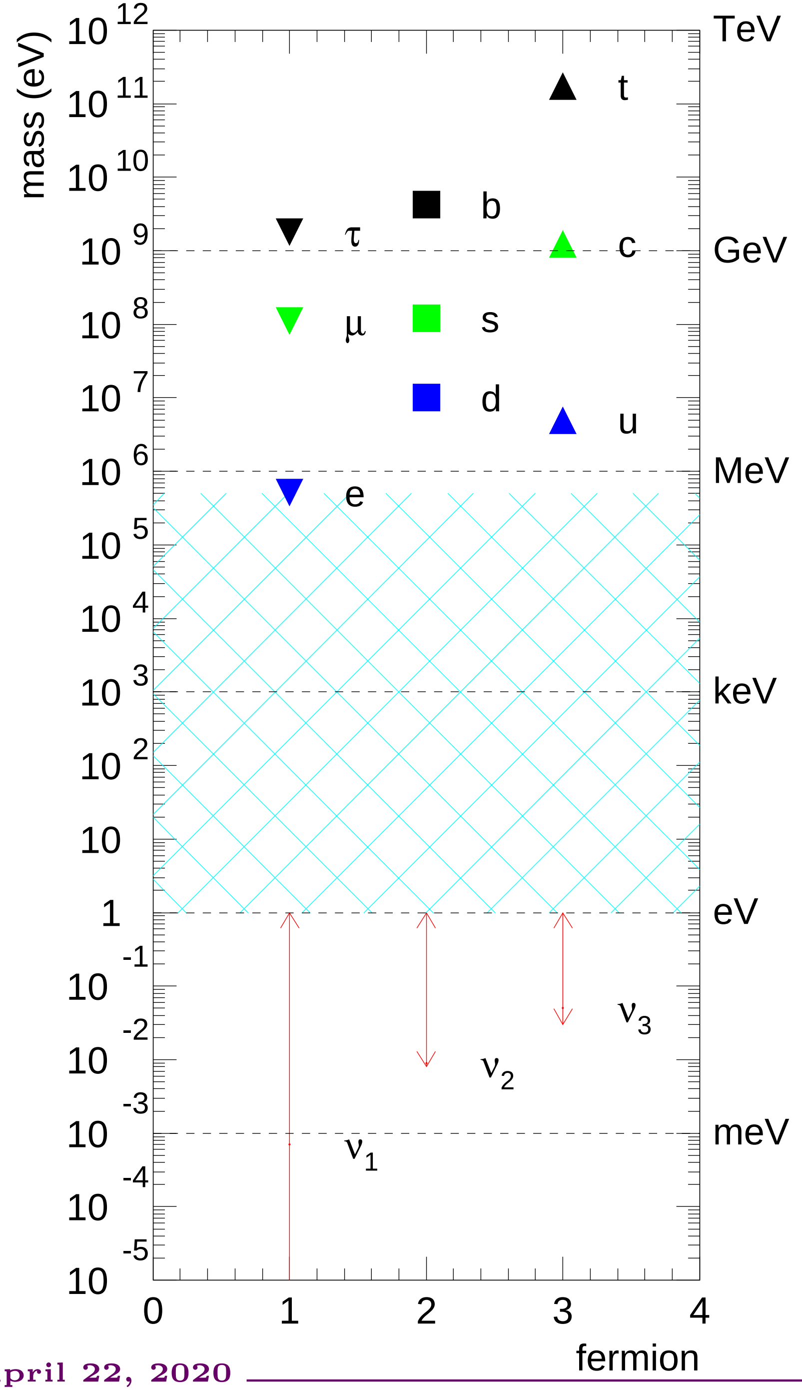
The simplest and **only satisfactory** explanation of **all** this data is that neutrinos have distinct masses, and mix.



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?



NEUTRINOS HAVE MASS

[albeit very tiny ones...]

So What?

⇓

NEW PHYSICS

In Summary: Neutrino Masses are the Only* "Palpable" Evidence of Physics Beyond the Standard Model

---

* There is only a handful of questions our model for fundamental physics cannot explain (my personal list. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- How come there is so much matter relative to radiation in the Universe? [Baryogenesis] (not in SM).
- Why is the expansion of the Universe accelerating? Why does it appear that the expansion of the Universe underwent rapid acceleration in the past [Dark Energy & Inflation]? (not in SM).

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$\Updownarrow$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input.

Neutrino Masses, EWSB, and a New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

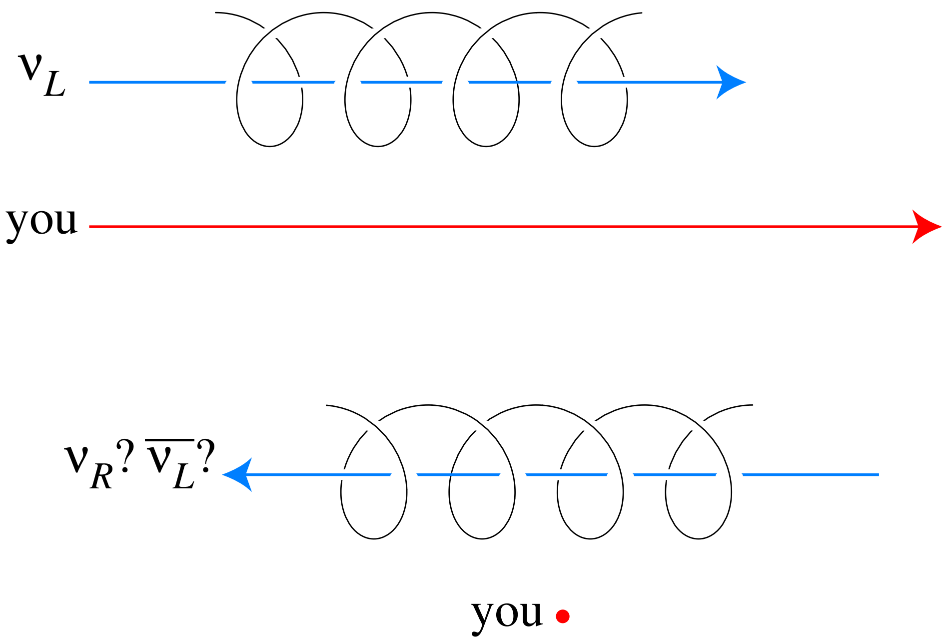
1. Neutrinos talk to the Higgs boson very, very **weakly** (Dirac neutrinos);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking! (Majorana neutrinos);
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism (Majorana neutrinos).

Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including ...

- **understanding the fate of lepton-number**. Neutrinoless double beta decay.
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- precision studies of charged-lepton properties ($g-2$, edm), and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

Are Neutrinos Majorana or Dirac Fermions?



A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$

$$\updownarrow \text{“Lorentz”}$$

$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$

$$\updownarrow \text{“Lorentz”} \qquad \text{‘DIRAC’}$$

$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$

‘MAJORANA’ $\qquad \updownarrow$ “Lorentz”

$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry **any** quantum number — including lepton number.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m_\nu \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m_\nu}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \text{ followed by } \nu_e + X \to e^+ + X, \quad P \simeq \left(\frac{m_\nu}{E}\right)^2$$

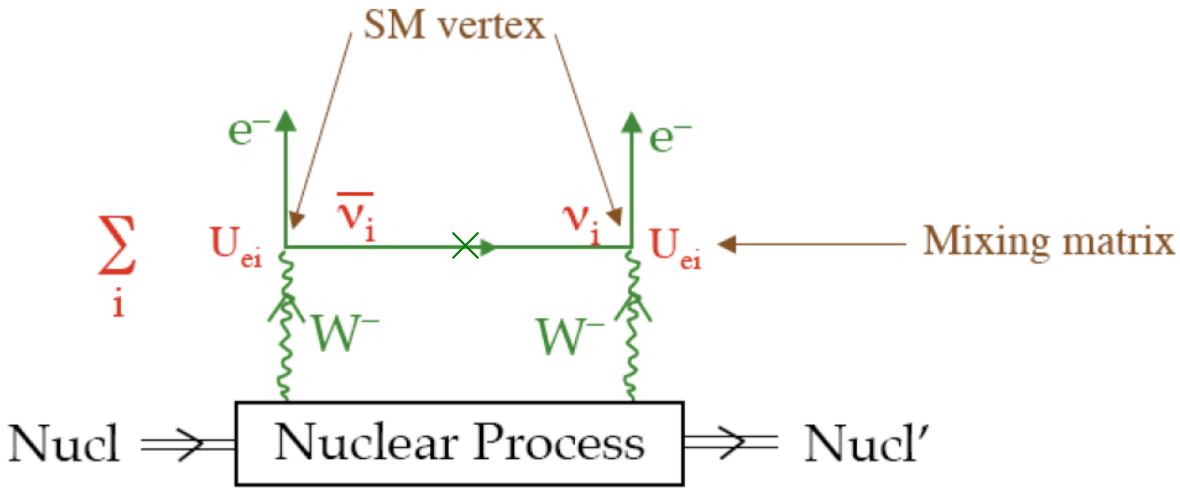
Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1\text{ eV}/100\text{ MeV})^2 = 10^{-18}$. VERY Challenging!

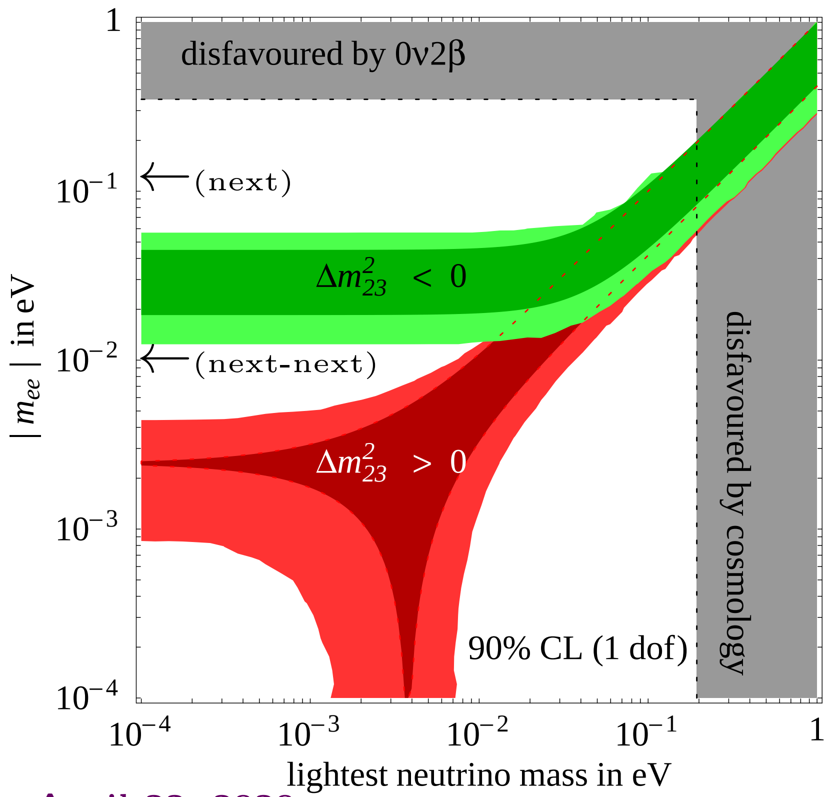
Search for the Violation of Lepton Number (or $B-L$)

Best Bet: search for

Neutrinoless Double-Beta

Decay: $Z \to (Z+2)e^-e^-$





Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

$\Leftarrow$ **no longer lamp-post physics!**

What Else is There?

1. How about other searches for lepton number violation? Can they ever be competitive? How?

2. Are there other ways to tell whether the neutrinos are Majorana or Dirac fermions?

How about other searches for lepton number violation? Can they ever be competitive? How?

There are two major challenges one must face before embracing other searches for lepton number violation (LNV).

1. **Constraints from searches for $0\nu\beta\beta$ are too strong.** There is an "easy" way out: play with the flavor structure of the LNV physics.

2. **Neutrino masses are very small. Neutrino masses are a consequence of LNV physics.** The relation between the LNV physics and the neutrino masses, however, is indirect so the real question is whether there are scenarios where LNV is accessible to laboratory experiments while, at the same time, the neutrino masses are tiny.

I will discuss what this means with concrete examples.

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- \, {}^{32}S \to e^+ \, {}^{32}Si^*)$ / $\sigma(\mu^- \, {}^{32}S \to \nu_\mu \, {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \, {}^{127}I \to e^+ \, {}^{127}Sb^*)$ / $\sigma(\mu^- \, {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- \pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- \pi^-)/\Gamma_{total}$		$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- K^-)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{total}$		$<3.3 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- K^-)/\Gamma_{total}$		$<4.8 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ K^- K^-)/\Gamma_{total}$		$<4.7 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to p\mu^- \mu^-)/\Gamma_{total}$		$<4.4 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\mu^+ \mu^-)/\Gamma_{total}$		$<3.3 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\gamma)/\Gamma_{total}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0)/\Gamma_{total}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}2\pi^0)/\Gamma_{total}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\eta)/\Gamma_{total}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p}\pi^0\eta)/\Gamma_{total}$		$<2.7 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		$<7.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \Lambda\pi^-)/\Gamma_{total}$		$<1.4 \times 10^{-7}$, CL = 90%
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2e^-)$ $\Leftarrow$ $0\nu\beta\beta$		$>1.9 \times 10^{25}$ yr, CL = 90%
$\Gamma(\pi^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$	[q]	$<1.5 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ e^+)/\Gamma_{total}$		$<5.0 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- e^+ e^+)/\Gamma_{total}$		$<6.4 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0 e^+ \overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^- 2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^- 2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^- 2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to K^*(892)^- 2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2e^+ + c.c.)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2e^+ + c.c.)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2e^+ + c.c.)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^- \pi^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to pe^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p}e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- 2\mu^+)/\Gamma_{total}$		$<1.2 \times 10^{-7}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<8.4 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^- 2e^+)/\Gamma_{total}$		$<5.2 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^- 2\mu^+)/\Gamma_{total}$		$<1.3 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<6.1 \times 10^{-6}$, CL = 90%
$\Gamma(D_s^+ \to K^*(892)^- 2\mu^+)/\Gamma_{total}$		$<1.4 \times 10^{-3}$, CL = 90%
$\Gamma(B^+ \to \pi^- e^+ e^+)/\Gamma_{total}$		$<2.3 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.0 \times 10^{-9}$, CL = 95%
$\Gamma(B^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- e^+ e^+)/\Gamma_{total}$		$<1.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.2 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to \rho^- e^+ \mu^+)/\Gamma_{total}$		$<4.7 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^- e^+ e^+)/\Gamma_{total}$		$<3.0 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^- \mu^+ \mu^+)/\Gamma_{total}$		$<4.1 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<1.6 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- e^+ e^+)/\Gamma_{total}$		$<4.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- \mu^+ \mu^+)/\Gamma_{total}$		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- e^+ \mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ \mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- \mu^+ \mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-} \mu^+ \mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^- \mu^+ \mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0 \pi^- \mu^+ \mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0 e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to p e)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p \mu)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- {}^{32}S \to e^+ {}^{32}Si^*)$ / $\sigma(\mu^- {}^{32}S \to \nu_\mu {}^{32}P^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}I \to e^+ {}^{127}Sb^*)$ / $\sigma(\mu^- {}^{127}I \to$ anything)		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- Ti \to e^+ Ca)$ / $\sigma(\mu^- Ti \to$ capture)		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- \pi^-)/\Gamma_{total}$		$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- \pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+ \pi^- K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+ \pi^- K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \mu^+ K^- K^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to p \mu^- \mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} \mu^+ \mu^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} \gamma)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} \pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} 2\pi^0)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} \eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{p} \pi^0 \eta)/\Gamma_{total}$		
$\Gamma(\tau^- \to \Lambda \pi^-)/\Gamma_{total}$		
$\Gamma(\tau^- \to \overline{\Lambda} \pi^-)/\Gamma_{total}$		
$t_{1/2}({}^{76}Ge \to {}^{76}Se + 2e^-)$		
$\Gamma(\pi^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^- \mu^+ e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^- e^+ e^+)/\Gamma_{total}$		
$\Gamma(K^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{total}$	[q]	$<1.1 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{total}$	[q]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0 e^+ \overline{\nu}_e)/\Gamma_{total}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^- 2\mu^+)/\Gamma_{total}$		$<2.2 \times 10^{-8}$, CL = 90%
$\Gamma(D^+ \to \pi^- e^+ \mu^+)/\Gamma_{total}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \rho^- 2\mu^+)/\Gamma_{total}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^- 2e^+)/\Gamma_{total}$		$<9 \times 10^{-7}$, CL = 90%
$\Gamma(D^+ \to K^- 2\mu^+)/\Gamma_{total}$		$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^- e^+ \mu^+)/\Gamma_{total}$		$<1.9 \times 10^{-6}$, CL = 90%

$\Gamma(B^0 \to \Lambda_c^+ \mu^-)/\Gamma_{total}$	$<1.4 \times 10^{-6}$, CL = 90%
$\Gamma(B^0 \to \Lambda_c^+ e^-)/\Gamma_{total}$	$<4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to \pi^+ e^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^+ \mu^-)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^- e^+)/\Gamma_{total}$	$<4 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to \pi^- \mu^+)/\Gamma_{total}$	$<6 \times 10^{-7}$, CL = 90%
$\Gamma(\Lambda \to K^+ e^-)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^+ \mu^-)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^- e^+)/\Gamma_{total}$	$<2 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^- \mu^+)/\Gamma_{total}$	$<3 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda \to K^0_S \nu)/\Gamma_{total}$	$<2 \times 10^{-5}$, CL = 90%
$\Gamma(\Xi^- \to p \mu^- \mu^-)/\Gamma_{total}$	$<4 \times 10^{-8}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p} 2e^+)/\Gamma_{total}$	$<2.7 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p} 2\mu^+)/\Gamma_{total}$	$<9.4 \times 10^{-6}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \overline{p} e^+ \mu^+)/\Gamma_{total}$	$<1.6 \times 10^{-5}$, CL = 90%
$\Gamma(\Lambda_c^+ \to \Sigma^- \mu^+ \mu^+)/\Gamma_{total}$	$<7.0 \times 10^{-4}$, CL = 90%

$\Gamma(D^+ \to K^*(892)^- 2\mu^+)/\Gamma_{total}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2e^+ + c.c.)/\Gamma_{total}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2\pi^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2e^+ + c.c.)/\Gamma_{total}$		$<2.06 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<3.9 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2e^+ + c.c.)/\Gamma_{total}$		$<1.52 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- 2\mu^+ + c.c.)/\Gamma_{total}$		$<9.4 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \pi^- \pi^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<7.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<2.18 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to 2K^- e^+ \mu^+ + c.c.)/\Gamma_{total}$		$<5.7 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to p e^-)/\Gamma_{total}$	[r]	$<1.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to \overline{p} e^+)/\Gamma_{total}$	[s]	$<1.1 \times 10^{-5}$, CL = 90%
$\Gamma(D_s^+ \to \pi^- 2e^+)/\Gamma_{total}$		$<4.1 \times 10^{-6}$, CL = 90%
		$<1.2 \times 10^{-7}$, CL = 90%
		$<8.4 \times 10^{-6}$, CL = 90%
		$<5.2 \times 10^{-6}$, CL = 90%
		$<1.3 \times 10^{-5}$, CL = 90%
		$<6.1 \times 10^{-6}$, CL = 90%
		$<1.4 \times 10^{-3}$, CL = 90%
		$<2.3 \times 10^{-8}$, CL = 90%
		$<4.0 \times 10^{-9}$, CL = 95%
		$<1.5 \times 10^{-7}$, CL = 90%
		$<1.7 \times 10^{-7}$, CL = 90%
		$<4.2 \times 10^{-7}$, CL = 90%
		$<4.7 \times 10^{-7}$, CL = 90%
		$<3.0 \times 10^{-8}$, CL = 90%
		$<4.1 \times 10^{-8}$, CL = 90%
		$<1.6 \times 10^{-7}$, CL = 90%
		$<4.0 \times 10^{-7}$, CL = 90%
		$<5.9 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to K^*(892)^- e^+ \mu^+)/\Gamma_{total}$		$<3.0 \times 10^{-7}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ e^+)/\Gamma_{total}$		$<2.6 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- e^+ \mu^+)/\Gamma_{total}$		$<1.8 \times 10^{-6}$, CL = 90%
$\Gamma(B^+ \to D^- \mu^+ \mu^+)/\Gamma_{total}$		$<6.9 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to D^{*-} \mu^+ \mu^+)/\Gamma_{total}$		$<2.4 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to D_s^- \mu^+ \mu^+)/\Gamma_{total}$		$<5.8 \times 10^{-7}$, CL = 95%
$\Gamma(B^+ \to \overline{D}^0 \pi^- \mu^+ \mu^+)/\Gamma_{total}$		$<1.5 \times 10^{-6}$, CL = 95%
$\Gamma(B^+ \to \Lambda^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \Lambda^0 e^+)/\Gamma_{total}$		$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 \mu^+)/\Gamma_{total}$		$<6 \times 10^{-8}$, CL = 90%
$\Gamma(B^+ \to \overline{\Lambda}^0 e^+)/\Gamma_{total}$		$<8 \times 10^{-8}$, CL = 90%

Some of the results presented here are from Berryman *et al*, arXiv:1611.00032.

Other studies include Geib *et al*, arXiv:1609.09088 and
Geib and Merle *et al*, arXiv:1612.00452

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to pe)/\Gamma_{\rm total}$	$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p\mu)/\Gamma_{\rm total}$	$<1.8 \times 10^{-6}$, CL = 95%
⇒ limit on $\mu^- \to e^+$ conversion ⇐ (Concentrate Here)	
$\sigma(\mu^- {}^{32}\mathrm{S} \to e^+ {}^{32}\mathrm{Si}^*)$ / $\sigma(\mu^- {}^{32}\mathrm{S} \to \nu_\mu {}^{32}\mathrm{P}^*)$	$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- {}^{127}\mathrm{I} \to e^+ {}^{127}\mathrm{Sb}^*)$ / $\sigma(\mu^- {}^{127}\mathrm{I} \to \text{anything})$	$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \mathrm{Ti} \to e^+ \mathrm{Ca})$ / $\sigma(\mu^- \mathrm{Ti} \to \text{capture})$	$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-\pi^-)/\Gamma_{\rm total}$	$<2.0 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to \mu^+\pi^-\pi^-)/\Gamma_{\rm total}$	$<3.9 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+\pi^-K^-)/\Gamma_{\rm total}$	$<3.2 \times 10^{-8}$, CL = 90%
$\Gamma(\tau^- \to e^+K^-K^-)/\Gamma_{\rm total}$	$<3.3 \times 10^{-8}$, CL = 90%

Experimental Sensitivities

KamLAND-Zen: $T_{0\nu\beta\beta} > 1.07 \times 10^{26}$ yr (90% CL; ^{136}Xe)

arXiv:1605.02889; KamLAND-ZEN Collaboration

SINDRUM II:

- $\mu^- \to e^-$ conversion: $$R^{\rm Au}_{\mu^- e^-} \equiv \frac{\Gamma(\mu^- + {\rm Au} \to e^- + {\rm Au})}{\Gamma(\mu^- + {\rm Au} \to \nu_\mu + {\rm Pt})} < 7 \times 10^{-13}\ (90\%\ {\rm CL})$$
- $\mu^- \to e^+$ conversion: $$R^{\rm Ti}_{\mu^- e^+} \equiv \frac{\Gamma(\mu^- + {\rm Ti} \to e^+ + {\rm Ca})}{\Gamma(\mu^- + {\rm Ti} \to \nu_\mu + {\rm Sc})} < \begin{cases} 1.7 \times 10^{-12}\ ({\rm GS},\ 90\%\ {\rm CL}) \\ 3.6 \times 10^{-11}\ ({\rm GDR},\ 90\%\ {\rm CL}) \end{cases}$$

*Eur. Phys. J. **C47**, 337 (2006); SINDRUM II Collaboration*
*Phys. Lett. **B422**, 334 (1998); SINDRUM II Collaboration*

Apples–to–apples comparison of $\mu^- \to e^-$ and $\mu^- \to e^+$?

- 1993 – simultaneous analysis!
- Apply this to future experiments

$$R^{\rm Ti}_{\mu^- e^-} < 4.3 \times 10^{-12}\ (90\%\ {\rm CL})$$
$$R^{\rm Ti}_{\mu^- e^+} < 4.3 \times 10^{-12}\ (90\%\ {\rm CL})$$

*Phys. Lett. **B317**, 631 (1993); SINDRUM II Collaboration*

Experimental Sensitivities

Upcoming experiments:

$$\begin{aligned}
\text{DeeMe:} \quad & R^{\rm SiC}_{\mu^- e^-} > 5 \times 10^{-14} \ (90\%\ \text{CL}), \\
\text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^-} > 6.6 \times 10^{-17} \ (90\%\ \text{CL}), \\
\text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^-} > 7.2 \times 10^{-15} \ (90\%\ \text{CL}), \\
\text{COMET Phase-II:} \quad & R^{\rm Al}_{\mu^- e^-} > 6 \times 10^{-17} \ (90\%\ \text{CL}), \\
\text{PRISM:} \quad & R^{\rm Al}_{\mu^- e^-} > 5 \times 10^{-19} \ (90\%\ \text{CL}).
\end{aligned}$$

Who could do this measurement?

- *Possibly* Mu2e and COMET Phase-I – similar to SINDRUM II
- *Probably* not DeeMe, COMET Phase-II or PRISM

$$\begin{aligned}
\text{Mu2e:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-16} \\
\text{COMET Phase-I:} \quad & R^{\rm Al}_{\mu^- e^+} \gtrsim 10^{-14}
\end{aligned}$$

νSM – EFT Path

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\text{SM}} \supset -y_{ij}\frac{L^iHL^jH}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB } \mathcal{L}_{\nu\text{SM}} \supset \frac{m_{ij}}{2}\nu^i\nu^j; \quad m_{ij} = y_{ij}\frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \to m_\nu \ll m_f$ $(f = e, \mu, u, d, \text{etc})$
- Neutrinos are Majorana fermions – Lepton number is violated!
- νSM effective theory – not valid for energies above at most Λ.
- What is Λ? First naive guess is that Λ is the Planck scale – does not work. Data require $\Lambda \sim 10^{14}$ GeV (related to GUT scale?) [note $y^{\max} \equiv 1$]

What else is this "good for"? Depends on the ultraviolet completion!

Tree-Level Realization of the Weinberg Operator

If $\mu = \lambda v \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

"Higher Order" Neutrino Masses from $\Delta L = 2$ Physics

Imagine that there is new physics that breaks lepton number by 2 units at some energy scale Λ, but that it does not, in general, lead to neutrino masses at the tree level.

We know that neutrinos will get a mass at some order in perturbation theory – which order is model dependent!

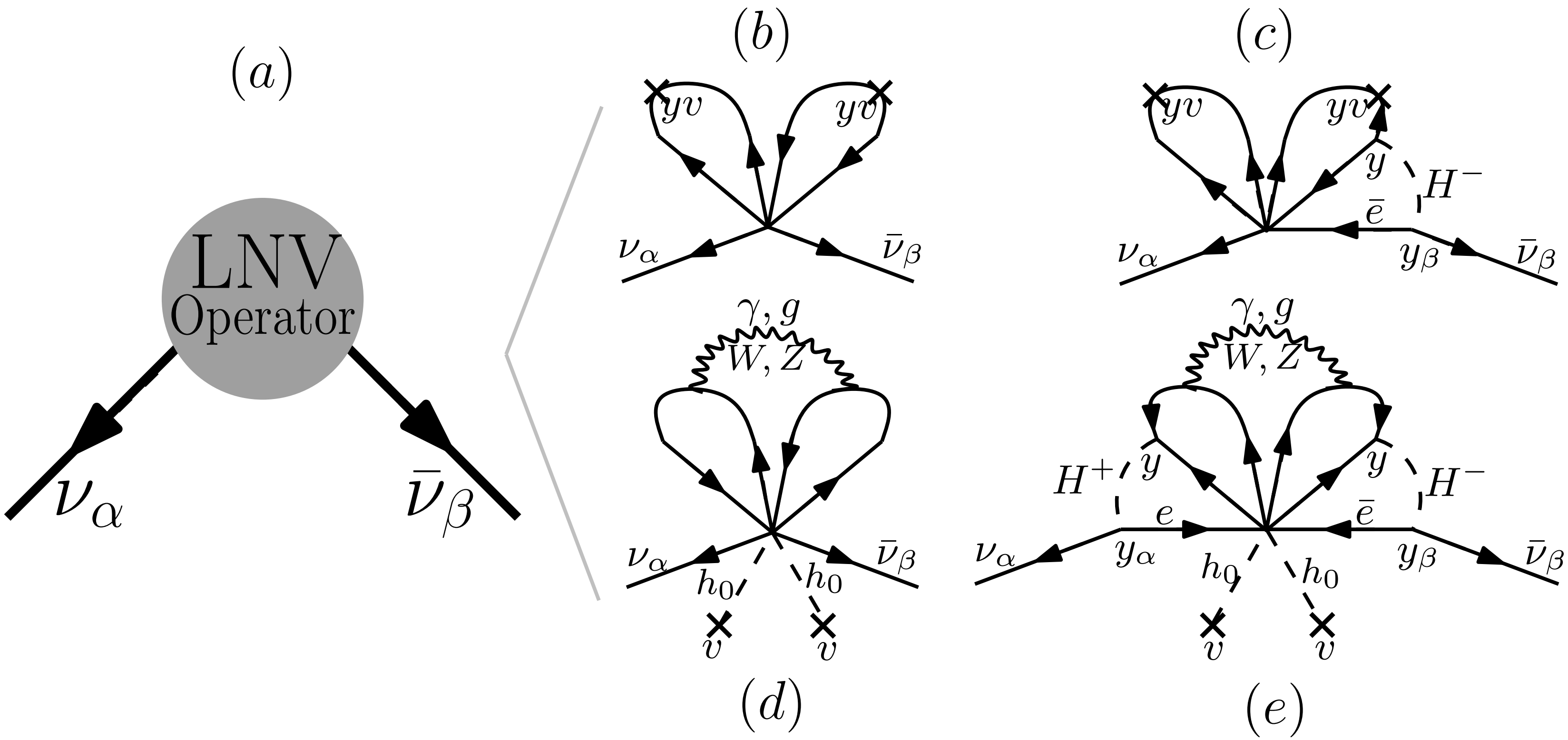
For example:

- SUSY with trilinear R-parity violation – neutrino masses at one-loop;
- Zee models – neutrino masses at one-loop;
- Babu and Ma – neutrino masses at two loops;
- Chen et al, 0706.1964 – neutrino masses at two loops;
- Angel et al, 1308.0463 – neutrino masses at two loops;
- etc.

One Approach Aimed at Phenomenology

- Only consider $\Delta L = 2$ operators;
- Operators made up of only standard model fermions and the Higgs doublet (no gauge bosons);
- Electroweak symmetry breaking characterized as prescribed in SM;
- Effective operator couplings assumed to be "flavor indifferent", unless otherwise noted;
- Operators "turned on" one at a time, assumed to be leading order (tree-level) contribution of new lepton number violating physics.
- We can use the effective operator to estimate the coefficient of all other lepton-number violating lower-dimensional effective operators (loop effects, computed with a hard cutoff).

Results presented are order of magnitude *estimates*, not precise quantitative results. Q: Does this really make sense? A: Sometimes...


(a)
LNV
Operator
ν_α
$\bar{\nu}_\beta$
(b)
yv
yv
ν_α
$\bar{\nu}_\beta$
(c)
yv
yv
y
H^-
$\bar{e}$
ν_α
y_β
$\bar{\nu}_\beta$
γ, g
W, Z
ν_α
h_0
h_0
$\bar{\nu}_\beta$
v
v
(d)
γ, g
W, Z
H^+
y
y
H^-
e
$\bar{e}$
ν_α
y_α
h_0
h_0
y_β
$\bar{\nu}_\beta$
v
v
(e)

$\mathcal{O}$	Operator	Λ [TeV]
$\mathcal{O}_1$	$(LH)(LH)$	$6\times10^{10-11}$
$\mathcal{O}_2$	$(LL)(LH)e^c$	$4\times10^{6-7}$
$\mathcal{O}_{3_a}$	$(LL)(QH)d^c$	$2\times10^{4-5}$
$\mathcal{O}_{3_b}$	$(LQ)(LH)d^c$	$1\times10^{7-8}$
$\mathcal{O}_{4_a}$	$(L\overline{Q})(LH)\overline{u^c}$	$4\times10^{8-9}$
$\mathcal{O}_{4_b}$	$(LL)(\overline{Q}H)\overline{u^c}$	$2-7$
$\mathcal{O}_8$	$(LH)\overline{e^c}\overline{u^c}d^c$	$6\times10^{2-3}$

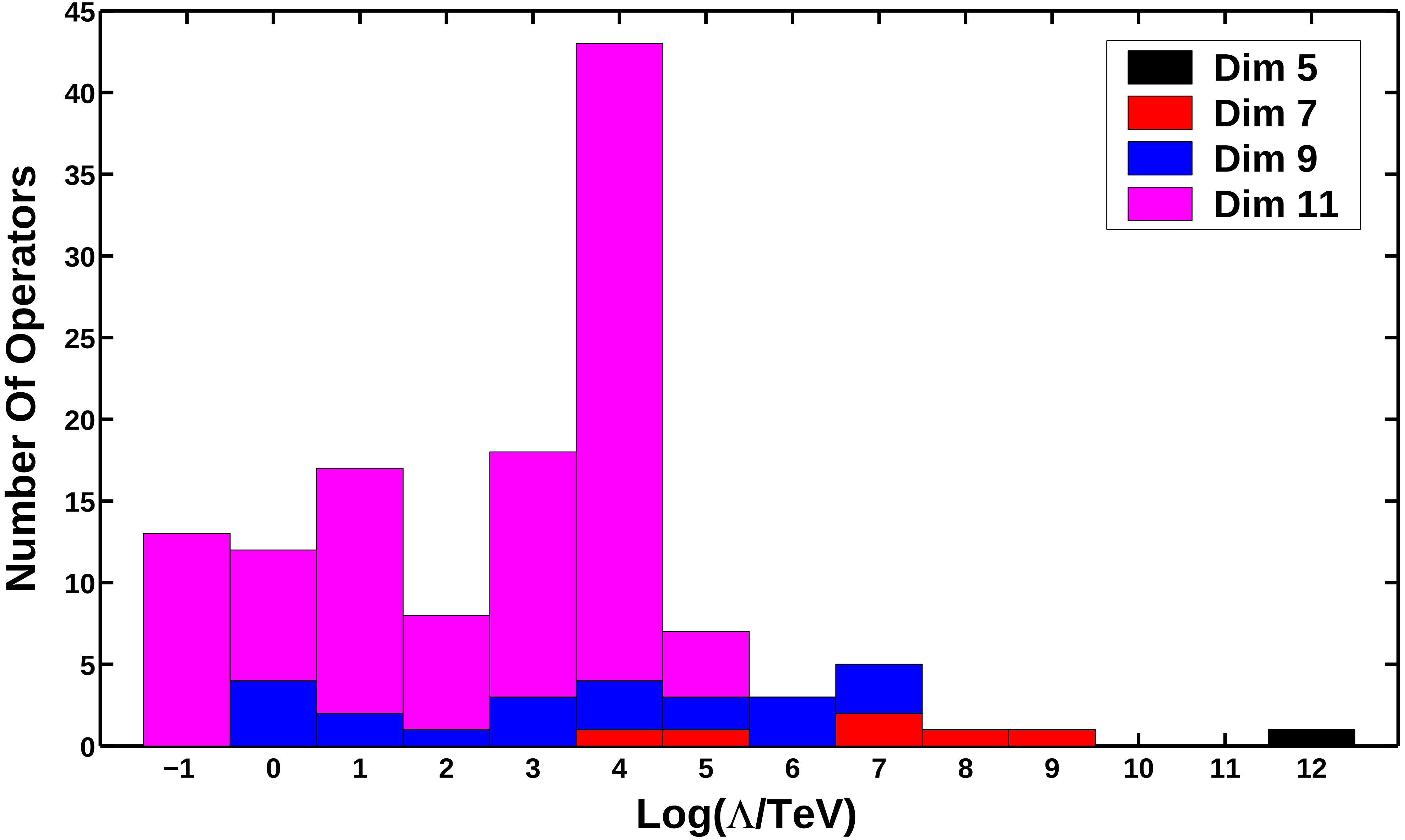
$\mathcal{O}$	Operator	Λ [TeV]
$\mathcal{O}_5$	$(L\overline{H})(LH)(QH)d^c$	$6\times10^{4-5}$
$\mathcal{O}_6$	$(LH)(L\overline{H})(\overline{Q}H)\overline{u^c}$	$2\times10^{6-7}$
$\mathcal{O}_7$	$(LH)(QH)(\overline{Q}H)\overline{e^c}$	$4\times10^{1-2}$
$\mathcal{O}_9$	$(LL)(LL)e^ce^c$	$3\times10^{2-3}$
$\mathcal{O}_{10}$	$(LL)(LQ)e^cd^c$	$6\times10^{2-3}$
$\mathcal{O}_{11_a}$	$(LL)(QQ)d^cd^c$	$3-30$
$\mathcal{O}_{11_b}$	$(LQ)(LQ)d^cd^c$	$2\times10^{3-4}$

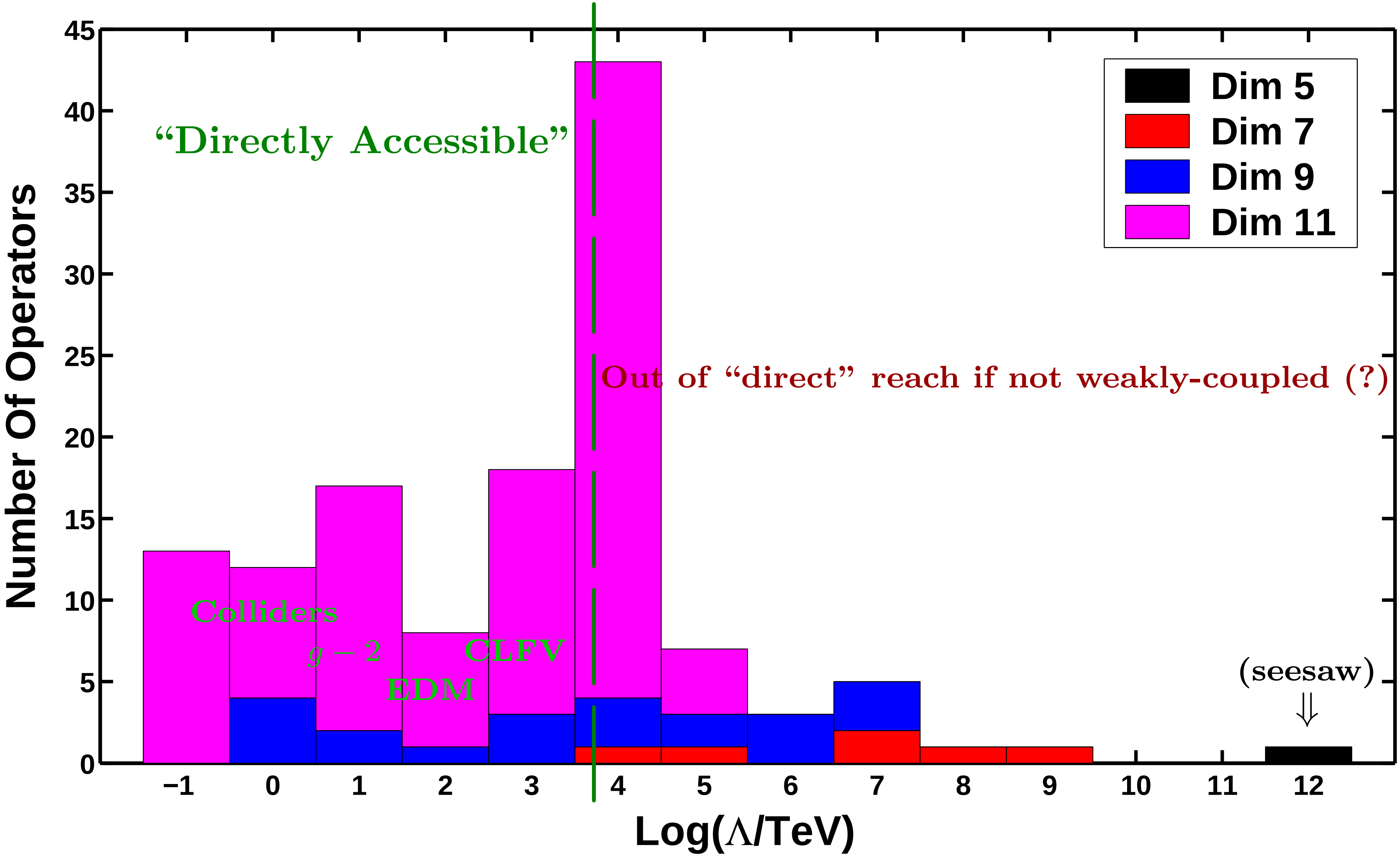
$\mathcal{O}_{12_a}$	$(L\overline{Q})(L\overline{Q})\overline{u^c}\overline{u^c}$	$2\times10^{6-7}$
$\mathcal{O}_{12_b}$	$(LL)(\overline{Q}\overline{Q})\overline{u^c}\overline{u^c}$	$0.3-0.6$
$\mathcal{O}_{13}$	$(L\overline{Q})(LL)\overline{u^c}e^c$	$2\times10^{4-5}$
$\mathcal{O}_{14_a}$	$(LL)(Q\overline{Q})\overline{u^c}d^c$	10^{2-3}
$\mathcal{O}_{14_b}$	$(L\overline{Q})(LQ)\overline{u^c}d^c$	$6\times10^{4-5}$
$\mathcal{O}_{15}$	$(LL)(L\overline{L})d^c\overline{u^c}$	10^{2-3}
$\mathcal{O}_{16}$	$(LL)e^cd^c\overline{e^c}\overline{u^c}$	$0.2-2$
$\mathcal{O}_{17}$	$(LL)d^cd^c\overline{d^c}\overline{u^c}$	$0.2-2$

$\mathcal{O}_{18}$	$(LL)d^cu^c\overline{u^c}\overline{u^c}$	$0.2-2$
$\mathcal{O}_{19}$	$(LQ)d^cd^c\overline{e^c}\overline{u^c}$	$0.1-1$
$\mathcal{O}_{20}$	$(L\overline{Q})d^c\overline{u^c}\overline{e^c}\overline{u^c}$	$4-40$
$\mathcal{O}_s$	$e^ce^cu^cu^c\overline{d^c}\overline{d^c}$	10^{-3}

- Ignore Lorentz, $SU(3)_C$ structure
- $SU(2)_L$ contractions denoted with parentheses
- Λ indicates range in which $m_\nu \in [0.05\text{ eV}, 0.5\text{ eV}]$

hep-ph/0106054; K.S. Babu & C.N. Leung
arXiv:0708.1344; A. de Gouvêa & J. Jenkins
arXiv:1212.6111; P.W. Angel, et al.
arXiv:1404.4057; A. de Gouvêa, at al.

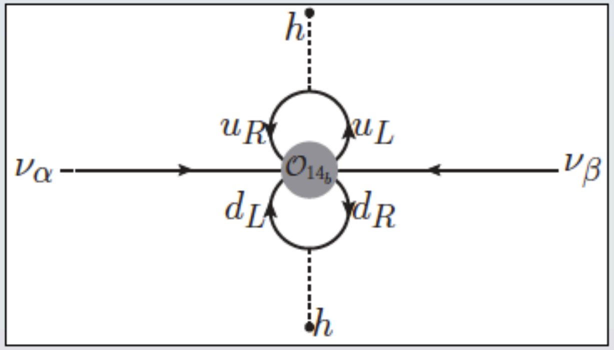

Number Of Operators
Log(Λ/TeV)
Dim 5
Dim 7
Dim 9
Dim 11
0
5
10
15
20
25
30
35
40
45
−1
0
1
2
3
4
5
6
7
8
9
10
11
12


Number Of Operators
45
40
35
30
25
20
15
10
5
0
−1
0
1
2
3
4
5
6
7
8
9
10
11
12
Log(Λ/TeV)
Dim 5
Dim 7
Dim 9
Dim 11
"Directly Accessible"
Out of "direct" reach if not weakly-coupled (?)
Colliders
g − 2
EDM
CLFV
(seesaw)
⇓

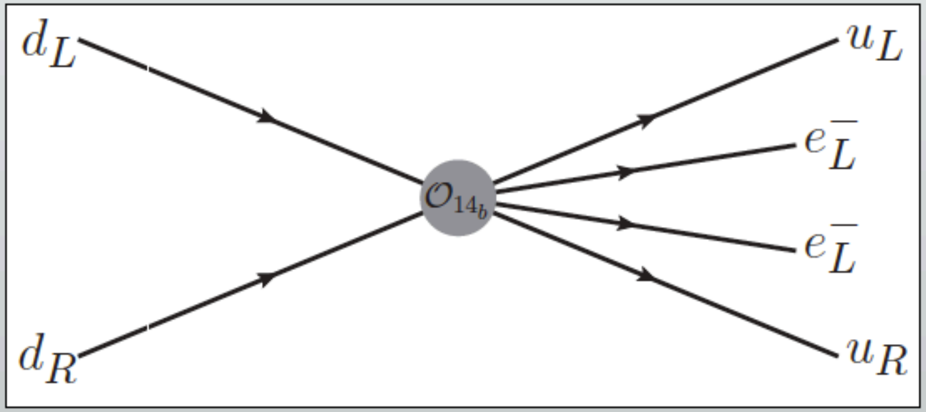
LNV from Effective Operators

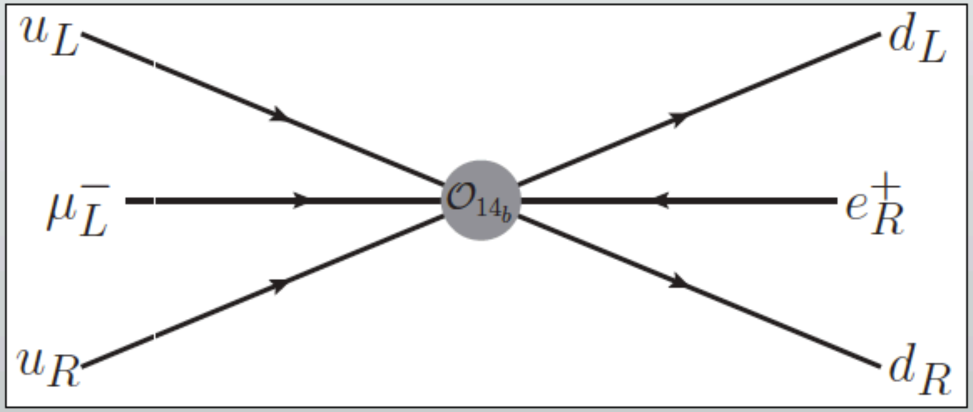
What do these operators do? Consider $\mathcal{O}_{14b}=(LQ)(LQ)u^c d^c$.

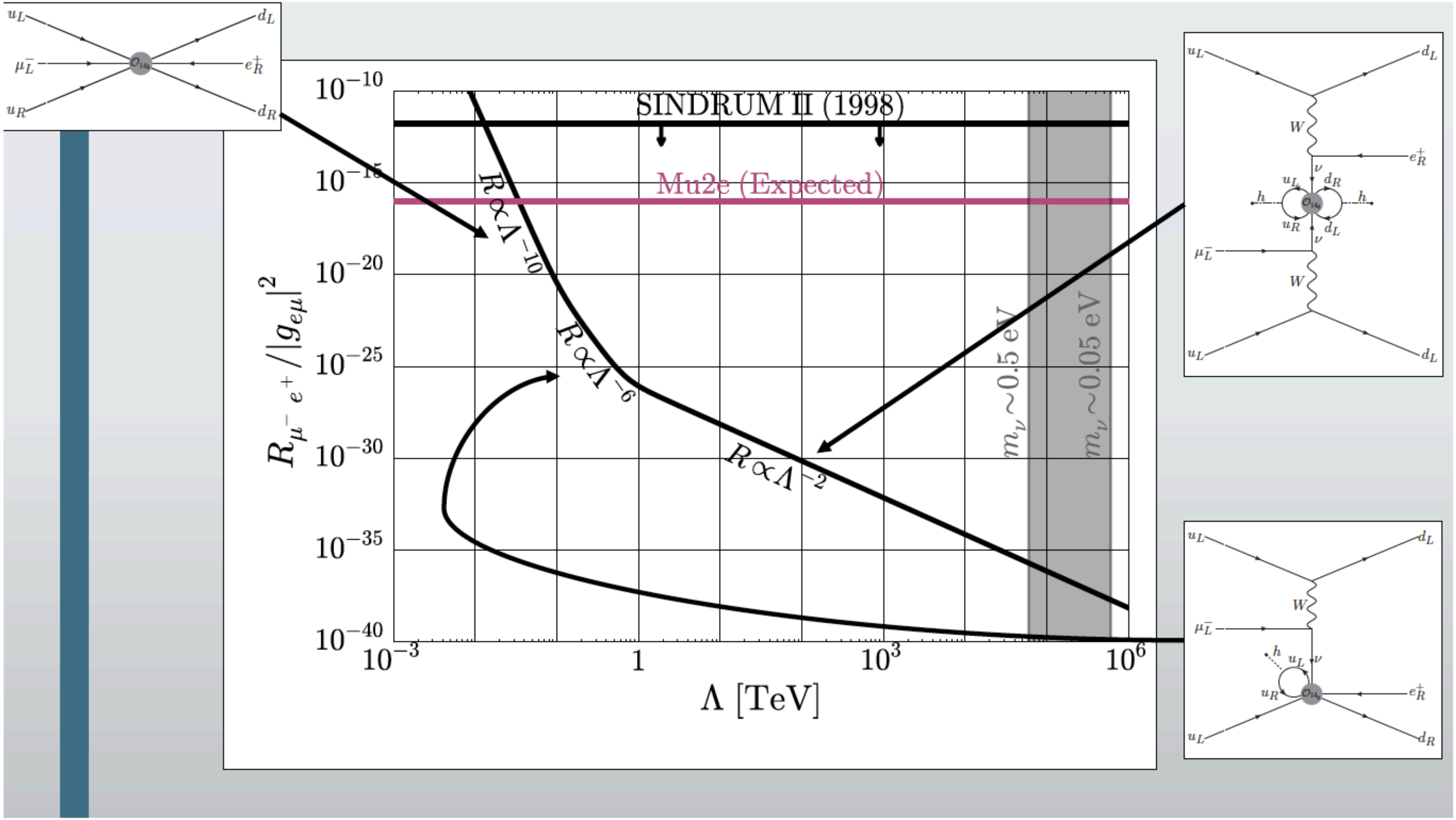
- They generate neutrino masses:

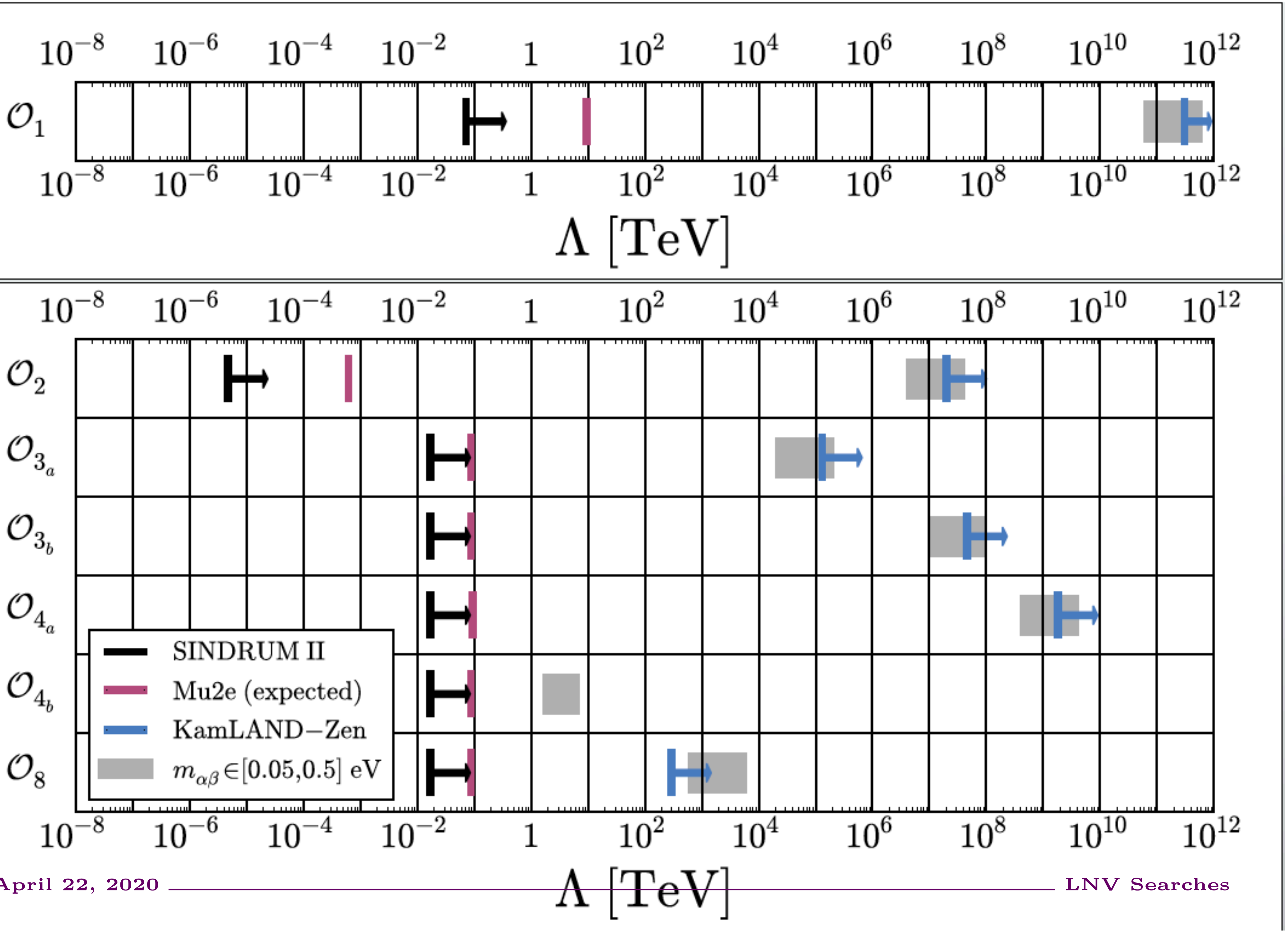


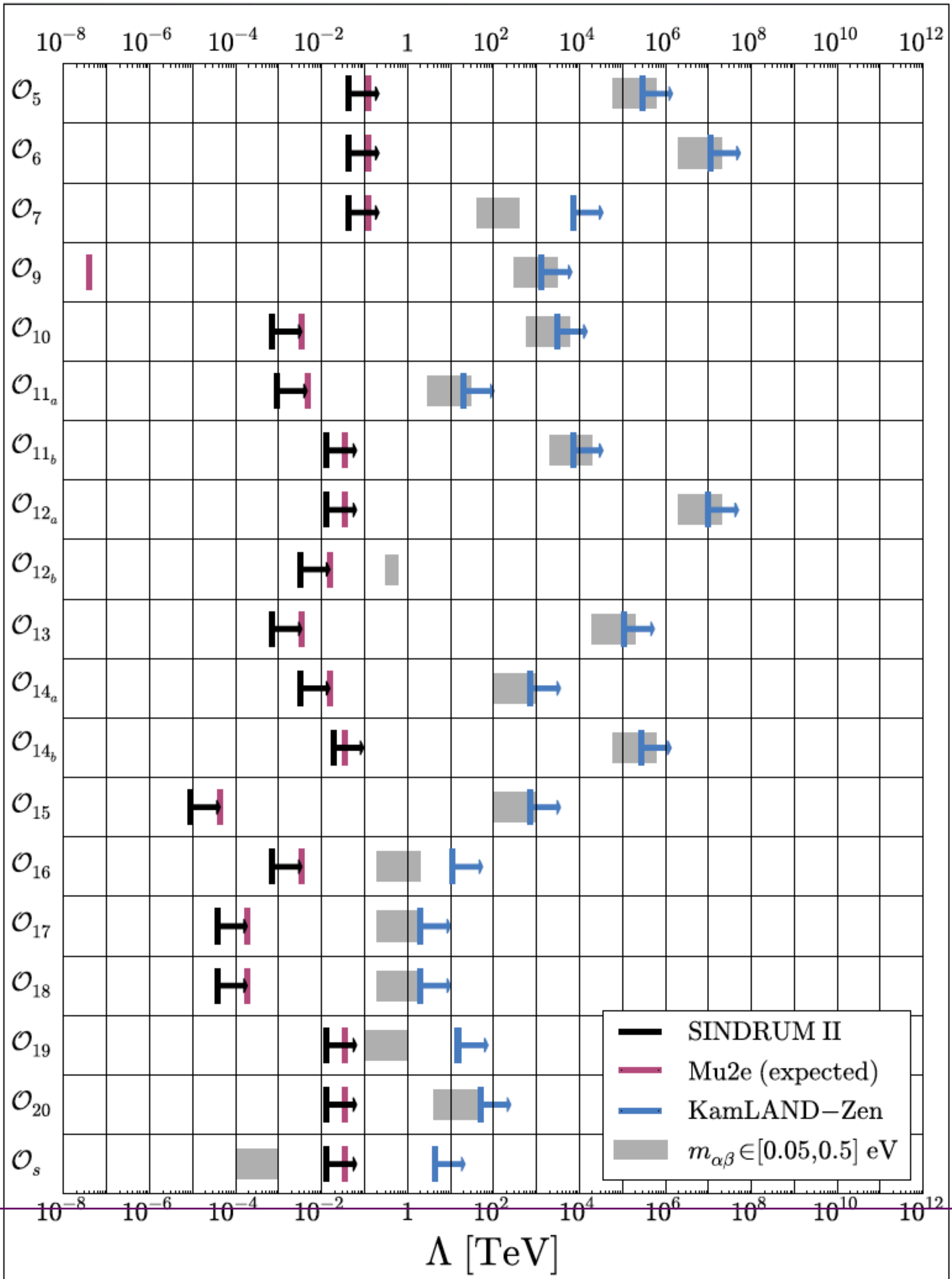
- They generate various LNV phenomena:






SINDRUM II (1998)
Mu2e (Expected)
$R \propto \Lambda^{-10}$
$R \propto \Lambda^{-6}$
$R \propto \Lambda^{-2}$
$m_\nu \sim 0.5$ eV
$m_\nu \sim 0.05$ eV
$R_{\mu^- e^+}/|g_{e\mu}|^2$
Λ [TeV]
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^{-35}
10^{-40}
10^{-3}
1
10^{3}
10^{6}
W


$\mathcal{O}_1$
$\mathcal{O}_2$
$\mathcal{O}_{3_a}$
$\mathcal{O}_{3_b}$
$\mathcal{O}_{4_a}$
$\mathcal{O}_{4_b}$
$\mathcal{O}_8$
10^{-8} 10^{-6} 10^{-4} 10^{-2} 1 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12}
Λ [TeV]
SINDRUM II
Mu2e (expected)
KamLAND−Zen
$m_{\alpha\beta}\in[0.05,0.5]$ eV


$\mathcal{O}_5$
$\mathcal{O}_6$
$\mathcal{O}_7$
$\mathcal{O}_9$
$\mathcal{O}_{10}$
$\mathcal{O}_{11_a}$
$\mathcal{O}_{11_b}$
$\mathcal{O}_{12_a}$
$\mathcal{O}_{12_b}$
$\mathcal{O}_{13}$
$\mathcal{O}_{14_a}$
$\mathcal{O}_{14_b}$
$\mathcal{O}_{15}$
$\mathcal{O}_{16}$
$\mathcal{O}_{17}$
$\mathcal{O}_{18}$
$\mathcal{O}_{19}$
$\mathcal{O}_{20}$
$\mathcal{O}_s$
10^{-8} 10^{-6} 10^{-4} 10^{-2} 1 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12}
Λ [TeV]
SINDRUM II
Mu2e (expected)
KamLAND−Zen
$m_{\alpha\beta}\in$[0.05,0.5] eV

How Do We Do More (or At Least Better)?

Questions:

- Are these results reliable? Which ones? How reliable?

  We assume, for example, that we can "turn on" one effective operator at a time. We also assume that the LNV physics, when integrated at tree-level, leads to effective operators of a certain mass dimension but not lower dimensional ones.

- How about constraints from lepton-number-conserving processes?

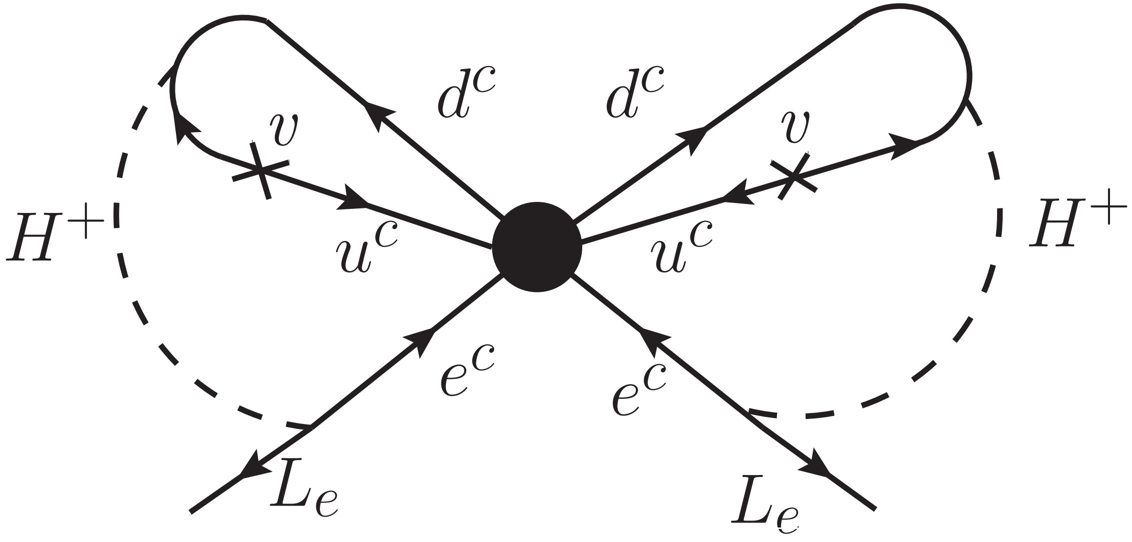
  The idea is that we can do a good job when it comes to low-energy, LNV observables (neutrino masses, $0\nu\beta\beta$). This EFT approach as "nothing to say" about lepton-number conserving phenomena.

Approach: try out some UV completions. Concentrate on $\mathcal{O}_s$.

[AdG et al, arXiv:1907.02541]

[AdG et al, arXiv:1907.02541]

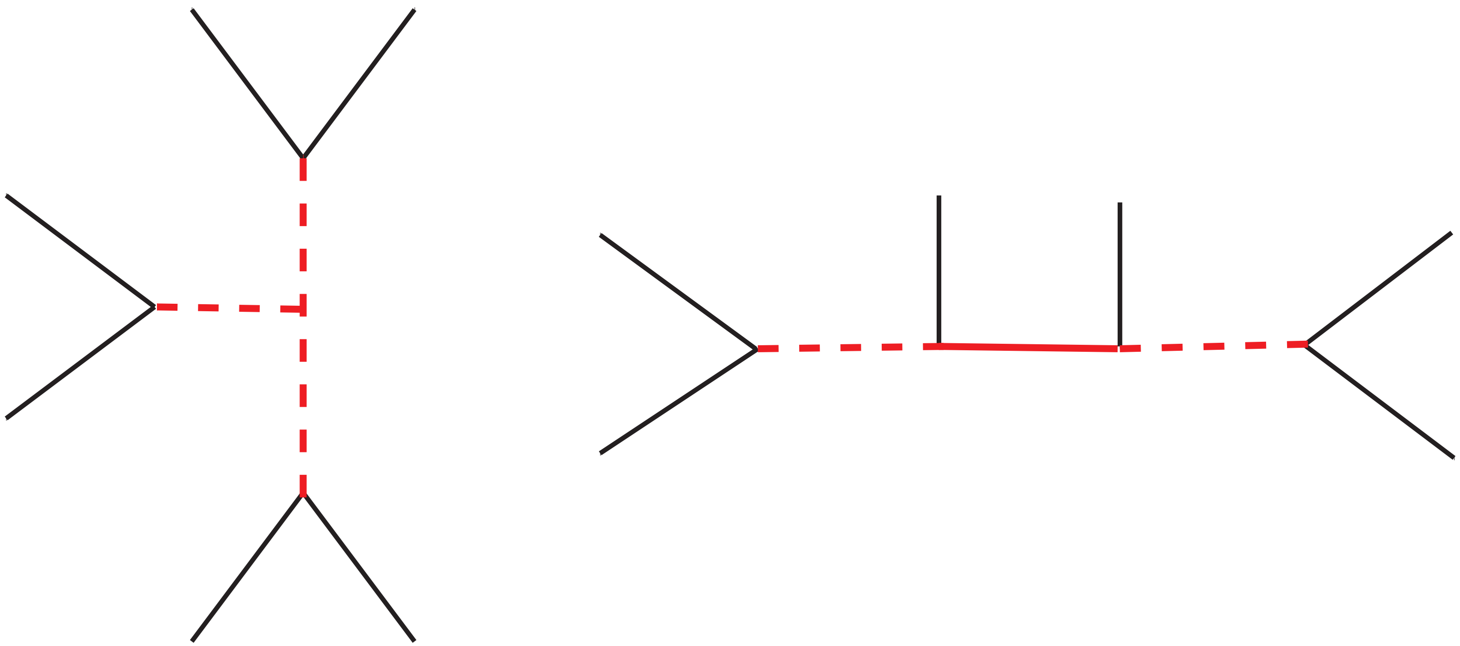
$$\mathcal{O}_s^{\alpha\beta} = \ell^c_\alpha \ell^c_\beta u^c u^c \overline{d^c}\,\overline{d^c}$$



$$m_{\alpha\beta} = \frac{g_{\alpha\beta}}{\Lambda}\frac{y_\alpha y_\beta (y_t y_b v)^2}{(16\pi^2)^4}$$

[AdG et al, arXiv:1907.02541]

$$\mathcal{O}_s^{\alpha\beta} = \ell^c_\alpha \ell^c_\beta u^c u^c \overline{d^c}\,\overline{d^c}$$



New particles in red. Easy to figure out their quantum numbers given what we know about e^c, d^c, u^c. Given what we know about L, Q, we can also figure out what quantum numbers we don't want in order to prevent other dimension-nine operators at the tree-level.

Table 1: All new particles required for all different tree-level realizations of the all-singlets dimension-nine operator $\mathcal{O}_s^{\alpha\beta}$.The fermions ψ, ζ, and χ come with a partner (ψ^c, ζ^c, and χ^c respectively), not listed. We don't consider fields that couple only to the antisymmetric combination of same-flavor quarks.

New particles	$\left(\mathrm{SU}(3)_\mathrm{C}, \mathrm{SU}(2)_\mathrm{L}\right)_{\mathrm{U}(1)_\mathrm{Y}}$	Spin
$\Phi \equiv (\overline{l^c}\,\overline{l^c})$	$(1,1)_{-2}$	scalar
$\Sigma \equiv (\overline{u^c}\,\overline{u^c})$	$(6,1)_{4/3}$	scalar
$\Delta \equiv (\overline{d^c}\,\overline{d^c})$	$(6,1)_{-2/3}$	scalar
$C \equiv (\overline{u^c}\,d^c)$	$(1,1)_1, \ (8,1)_1$	vector
$\psi \equiv (u^c\,l^c\,l^c)$	$(\overline{3},1)_{4/3}$	fermion
$\zeta \equiv (d^c\,\overline{l^c}\,\overline{l^c})$	$(\overline{3},1)_{-5/3}$	fermion
$\chi \equiv (l^c\,u^c\,u^c)$	$(\overline{6},1)_{-1/3}$	fermion
$N \equiv (l^c\,\overline{d^c}\,u^c)$	$(1,1)_0, \ (8,1)_0$	fermion

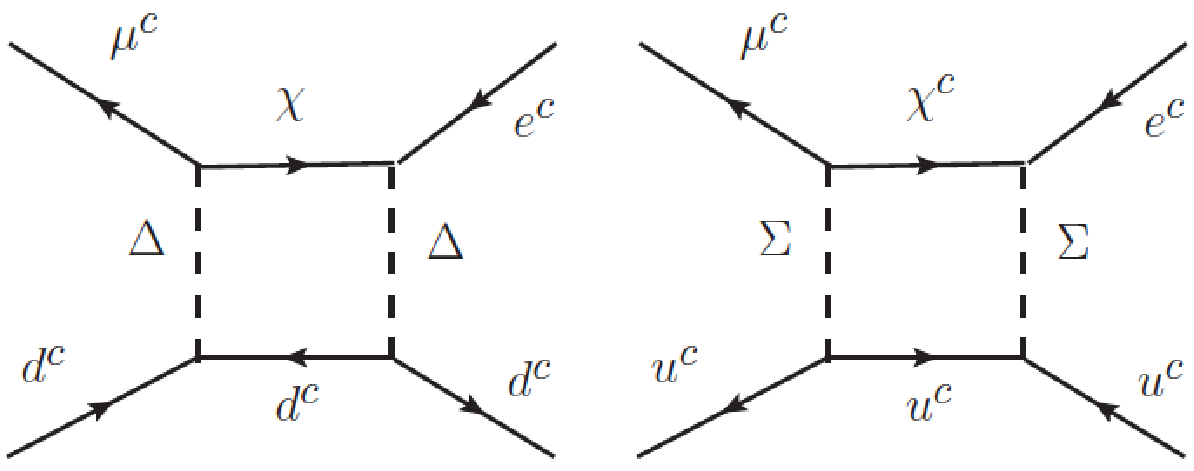


FIG. 8: Feynman diagrams (box-diagrams) contributing to the CLFV process $\mu^- \to e^-$-conversion, in Model $\chi\Delta\Sigma$.

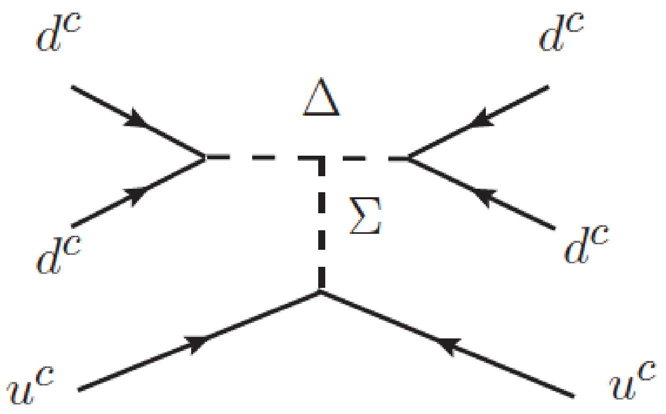
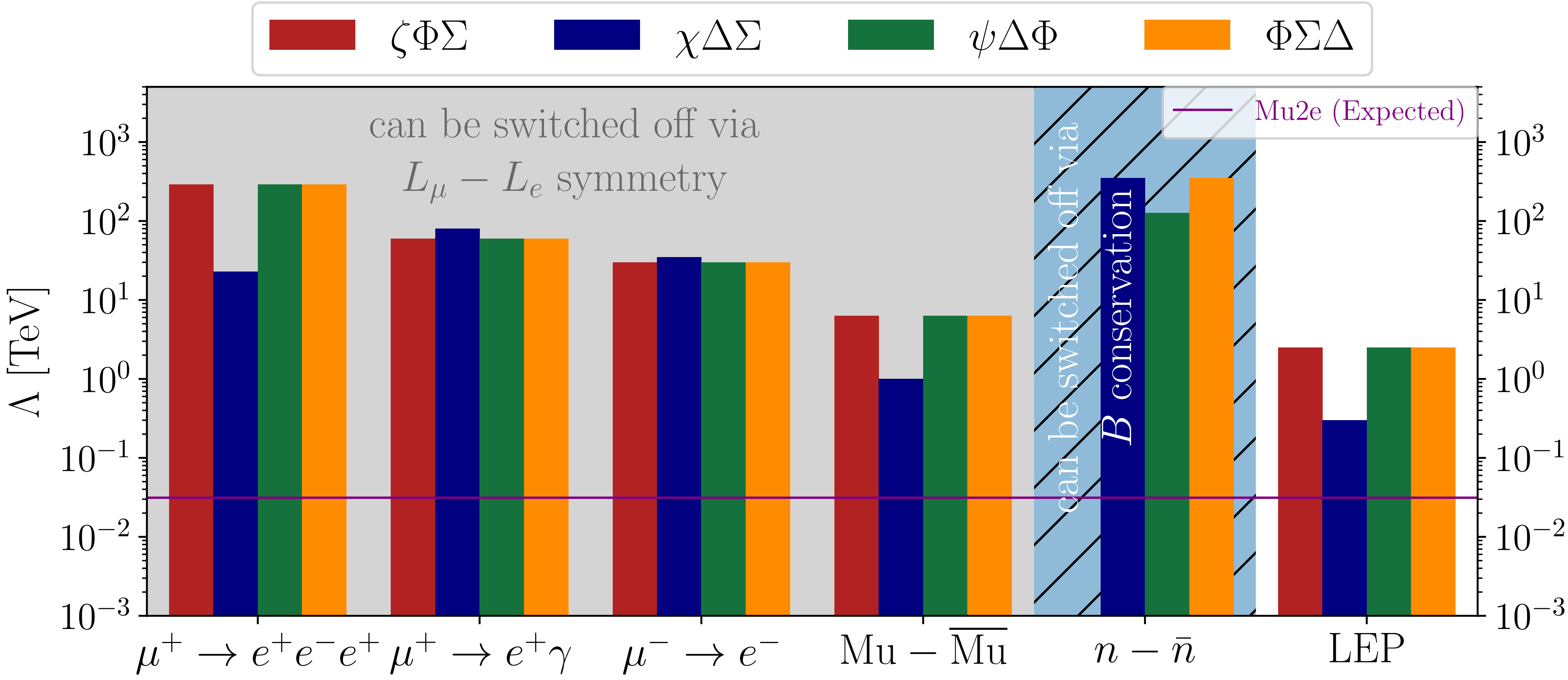


FIG. 9: Tree-level Feynman diagram that mediates $n - \overline{n}$ oscillations in Model $\chi\Delta\Sigma$.

[AdG et al, arXiv:1907.02541]



(models with new vector bosons not included)

[AdG et al, arXiv:1907.02541]

Are there other ways to tell whether the neutrinos are Majorana or Dirac fermions?

The answer is a qualified 'yes.' However, it requires **non-relativistic neutrinos**.

The qualification is that we have to know the relevant physics – new physics may spoil everything! One also has to "get lucky" sometimes. There are no "theorems" as far as I know…

Again: Why Don't We Know the Answer?

Neutrino Masses are Very Small*! [e.g. $|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle \simeq\rangle \sim |L\rangle$]

In fact, except for neutrino oscillation experiments, no consequence of a nonzero neutrino mass has ever been observed in any experiment. As far as all non-oscillation neutrino experiments are concerned, neutrinos are massless fermions.

*Very small compared to what? Compared to the typical energies and momentum transfers in your experiment. Another way to think about this: neutrinos are always **ultrarelativistic** in the lab frame.

There are two ways around it:

1. Find something that only Majorana fermions know how to do [e.g. violate lepton number] or
2. **find some non-ultrarelativistic neutrinos to work with!**

Examples, or

Where Can I Get Some Non-Relativistic Neutrinos?

- Reactions with (Not-To-Be-Detected) Neutrinos in the Final State;
- Decaying Neutrinos;
- The Cosmic Neutrino Background (brief comment).

The Burden of Working with Non-Ultrarelativistic Neutrinos

In a nutshell: the weak interactions are weak. Remember, at low energies

$$\sigma \propto E \quad \text{(or worse)}$$

On the other hand, telling Majorana From Dirac neutrinos is “trivial.” Indeed, it is an order one effect.

Final-State Neutrinos Near Threshold

We looked at

$$e\gamma \to e\nu\bar{\nu}$$

at sub-eV energies, because it can be done, in principle (electron at rest, infrared photon). Best to do it in the mass basis! Using the Fermi theory…

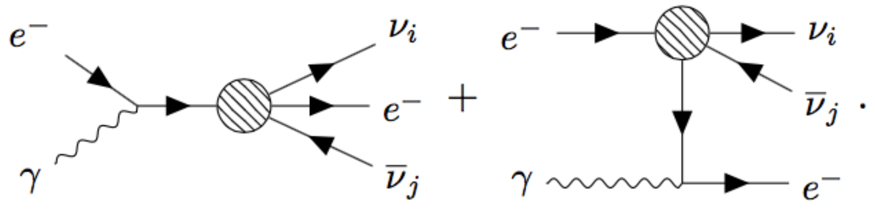
$$\mathcal{L}_{\rm CC} + \mathcal{L}_{\rm NC} = -\sqrt{2}G_F\left(\overline{\nu}_j\gamma^\mu P_L\nu_i\right)\left[\overline{\ell}_\alpha\gamma_\mu\left(g_V^{\alpha\beta ij}\mathbb{1} - g_A^{\alpha\beta ij}\gamma_5\right)\ell_\beta\right], \quad \text{(II.4)}$$

where we introduce the vector and axial couplings

$$g_V^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\left(1-4\sin^2\theta_W\right)\delta_{ij}\delta_{\alpha\beta}, \qquad g_A^{\alpha\beta ij} = U_{\alpha i}U^*_{\beta j} - \frac{1}{2}\delta_{ij}\delta_{\alpha\beta}. \quad \text{(II.5)}$$

Since the only charged leptons considered in this work are electrons, we will make the simplification $g^{ij}_{V,A} \equiv g^{eeij}_{V,A}$.

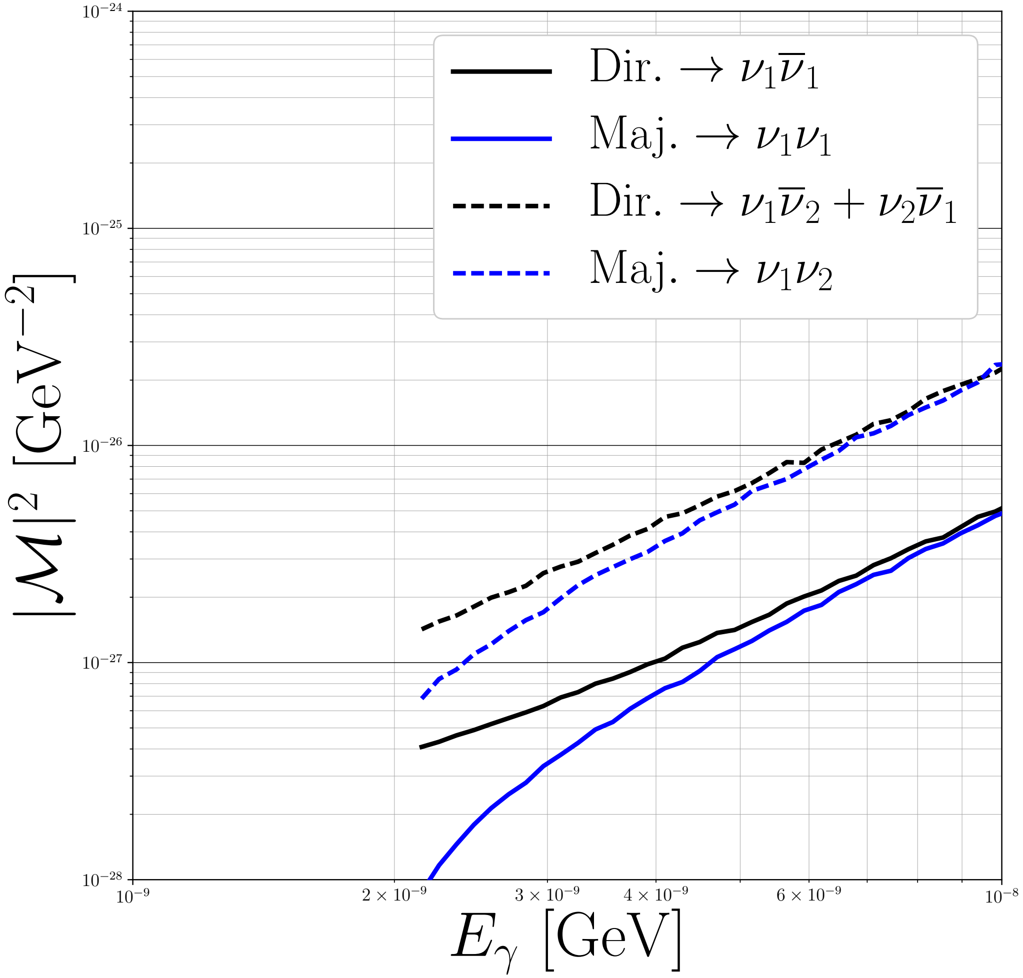
The following diagrams are relevant to the evaluation of the amplitude:

$i\mathcal{M} \approx$  (II.6)

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]

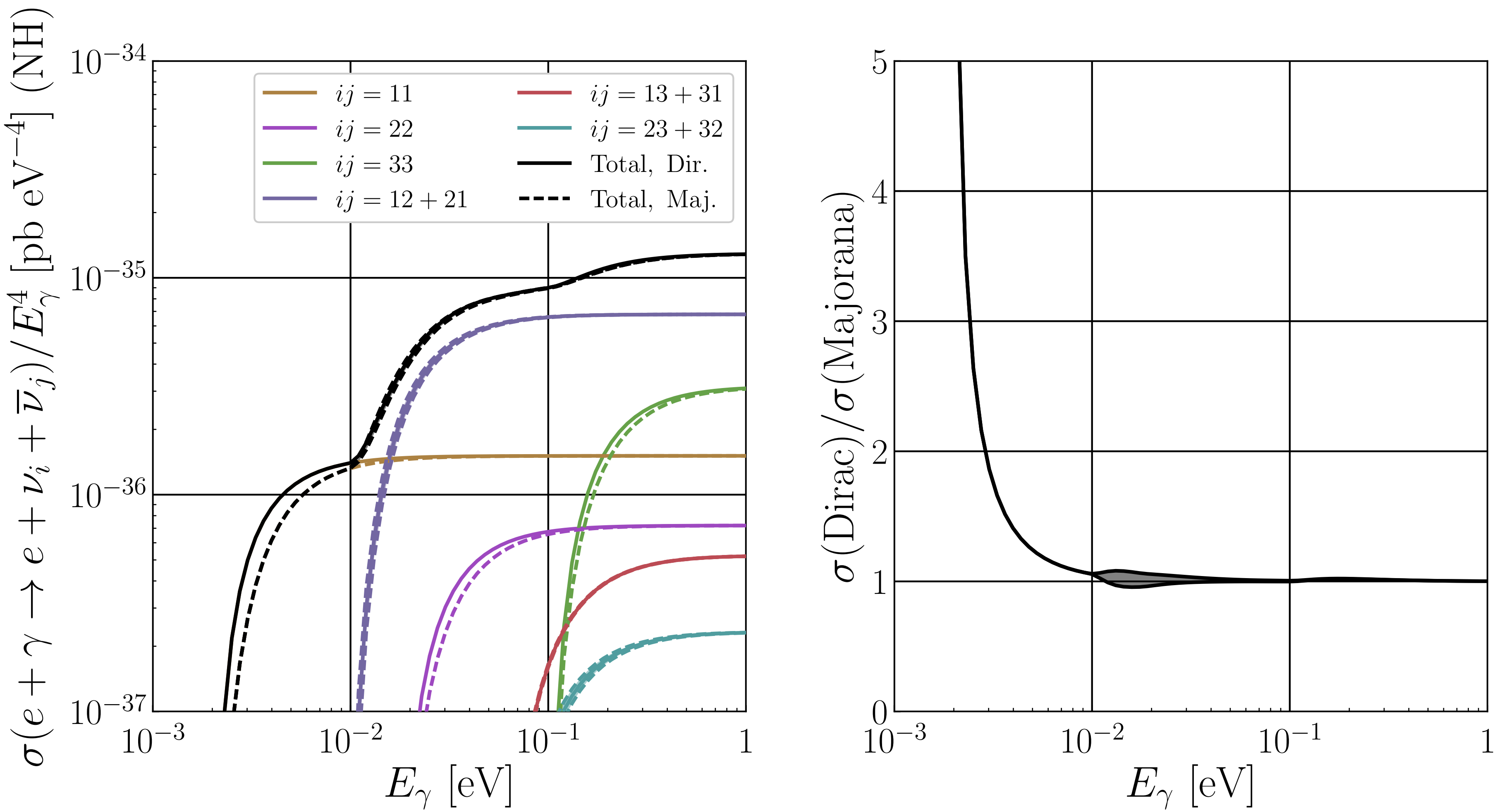
Final-State Neutrinos Near Threshold

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Final-State Neutrinos Near Threshold

[Berryman, AdG, Kelly, Schmitt, arXiv:1805.10294]



Another Example of Neutrinos Near Threshold (Brief)

Atomic process: $A^* \to A\gamma$, where A (A^*) is a neutral atom (in some excited state). Now replace the γ with an off-shell Z, which manifests itself as two neutrinos:

$$A^* \to A\nu\bar{\nu}.$$

It is easy to imagine sub-eV energies and hence the neutrinos are not ultra-relativistic.

For all the details including rates – tiny – and difference between Majorana and Dirac neutrinos – large – see, for example, Yoshimura, hep-ph/0611362, Dinh *et al.*, arXiv:1209.4808, and Song *et al.* arXiv:1510.00421, and references therein.

Neutrino Decay (Hint – Only Massive Particles Decay)

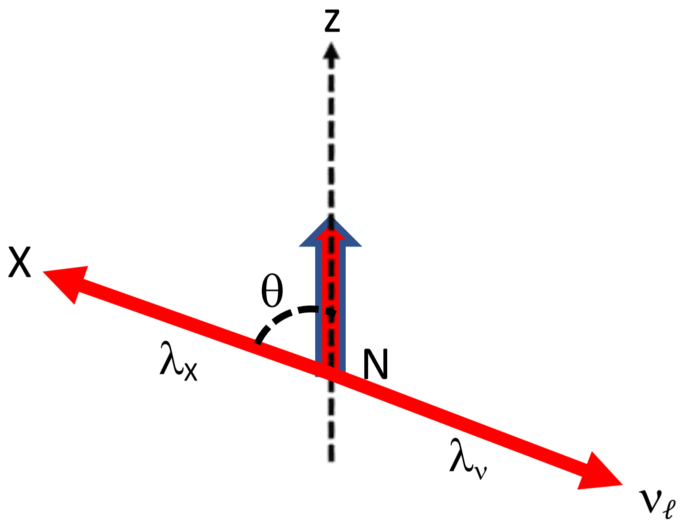
[Balantekin, AdG, Kayser, arXiv:1808.10518]

The two heavy neutrinos are expected to decay. E.g., if the neutrino mass ordering is normal, the decay modes $\nu_3 \to \nu_1\gamma$ and $\nu_3 \to \nu_1\nu_2\bar{\nu}_1$ are not only kinematically allowed, they are mediated by the weak interactions once mixing is taken into account.

Dirac and Majorana neutrinos "decay differently." In particular, the number of accessible final states, and the way in which they can potentially interfere, is such that the partial widths, and the lifetimes are different – assuming the same mixing and mass parameters – if the neutrinos are Majorana or Dirac.

Obvious challenges. $\Gamma \propto (m_\nu)^n$ [n is some positive power] so the neutrino lifetimes are expected to be cosmological. Insult to injury, the $\nu \to \nu$'s decay mode is significant, which renders studying the final products of the decay a rather daunting task. Nonetheless, we proceed ...

CPT invariance [at leading order]



We showed

$$\frac{d\Gamma(N \to \nu_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 + \alpha\cos\theta)$$

$$\frac{d\Gamma(\bar{N} \to \bar{\nu}_\ell + X)}{d(\cos\theta)} = \frac{\Gamma}{2}(1 - \alpha\cos\theta)$$

Since $\alpha = -\bar{\alpha}$, for Majorana neutrinos we get $\alpha = 0$. This result holds for any self-conjugate boson X.

The two-body decay of a Majorana fermion into a self-conjugate final state is isotropic

A.B. Balantekin, B. Kayser, Ann. Rev. Nucl. Part. Sci. **68** (2018) 313-338 (arXiv:1805.00922)
A.B. Balantekin, A. de Gouvêa, B. Kayser, arXiv:1808.10518

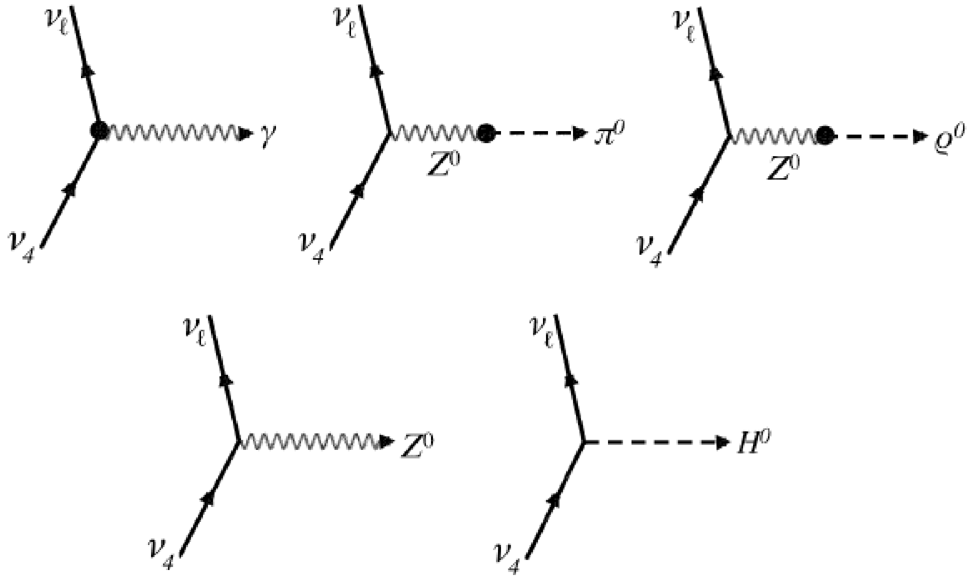
A More Realistic (?) Application – Neutral Heavy Leptons

If a neutral heavy lepton ν_4 is discovered somewhere – LHC, MicroBooNE, ICARUS, DUNE, SuperB Factory, SHiP, etc – in the future, after much rejoicing, we will want to establish whether this fermion is *Majorana* or *Dirac*.

How do we do it?

- Check for lepton-number violation. What does it take?
  - A lepton-number asymmetric initial state (easy). Or an even-by-event lepton number "tag" of the neutral heavy lepton (e.g. LHC environment).
  - Charge identification capability in the detector (sometimes absent or partially absent).
- **Kinematics**. Not only are the decay widths different – not useful, since it requires we know unknown parameters – the kinematics are qualitatively different, as I showed in the last slide.

Heavy Neutral Leptons – More Realistic (?) Application



All of these decays are isotropic for a Majorana parent. Otherwise (weak interactions)...

Boson	γ	π^0	ρ^0	Z^0	H^0
α	$\frac{2\Im(\mu d^*)}{\lvert\mu\rvert^2+\lvert d\rvert^2}$	1	$\frac{m_4^2-2m_\rho^2}{m_4^2+2m_\rho^2}$	$\frac{m_4^2-2m_Z^2}{m_4^2+2m_Z^2}$	1

The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

Assuming the Standard Model Cosmology, at least two of the three neutrinos are mostly non-relativistic today:

$$T_\nu \sim 2K \sim 2 \times 10^{-4}\ \text{eV}.$$

Furthermore, it turns out that hitting a Majorana neutrino at rest is easier than hitting a Dirac neutrino at rest, assuming the weak interactions.

When you interact with a polarized neutrino at rest, it will either choose to behave like the left-chiral component or the right-chiral component, with the same probability.

In the Dirac case, the right-chiral component is sterile, i.e., it does not participate in the weak interactions and you can't interact with it. Furthermore, the antineutrinos have the opposite lepton number and can't be detected via $\nu(Z,A) \to e^-(Z+1,A)$.

In the Majorana case, the right-chiral component is the object we usually refer to as the antineutrino. In this case, both can interact via the weak interactions. When it comes to the cosmic neutrino background being detected via $\nu(Z,A) \to e^-(Z+1,A)$, we get a hit from the neutrinos – just like in the Dirac case – but we also get a hit from the "antineutrino," with the same rate.

The Cosmic Neutrino Background

[see, e.g., Long, Lunardini, Sabancilar, arXiv:1405.7654]

This means that if we ever observe the cosmic neutrino background, we can determine the nature of the neutrino. If all neutrinos were at rest, for the same neutrino (+ antineutrino, in the Dirac case) flux, we expect twice as many events in the experiment if the neutrinos are Majorana fermions. One can easily include finite temperature effects, effects related to the neutrino mass ordering, a potential primordial lepton asymmetry, etc.

Some challenges:

- We have never detected the cosmic neutrino background! (see, however, PTOLEMY [arXiv:1808.01892] for a great idea that may work one day);
- We measure flux times cross-section. While we know the average neutrino number density of the universe very well from the Standard Model of Cosmology, we don't know the number density of neutrinos *here* very well [Uncertainty around 100%?].

Quick Summary

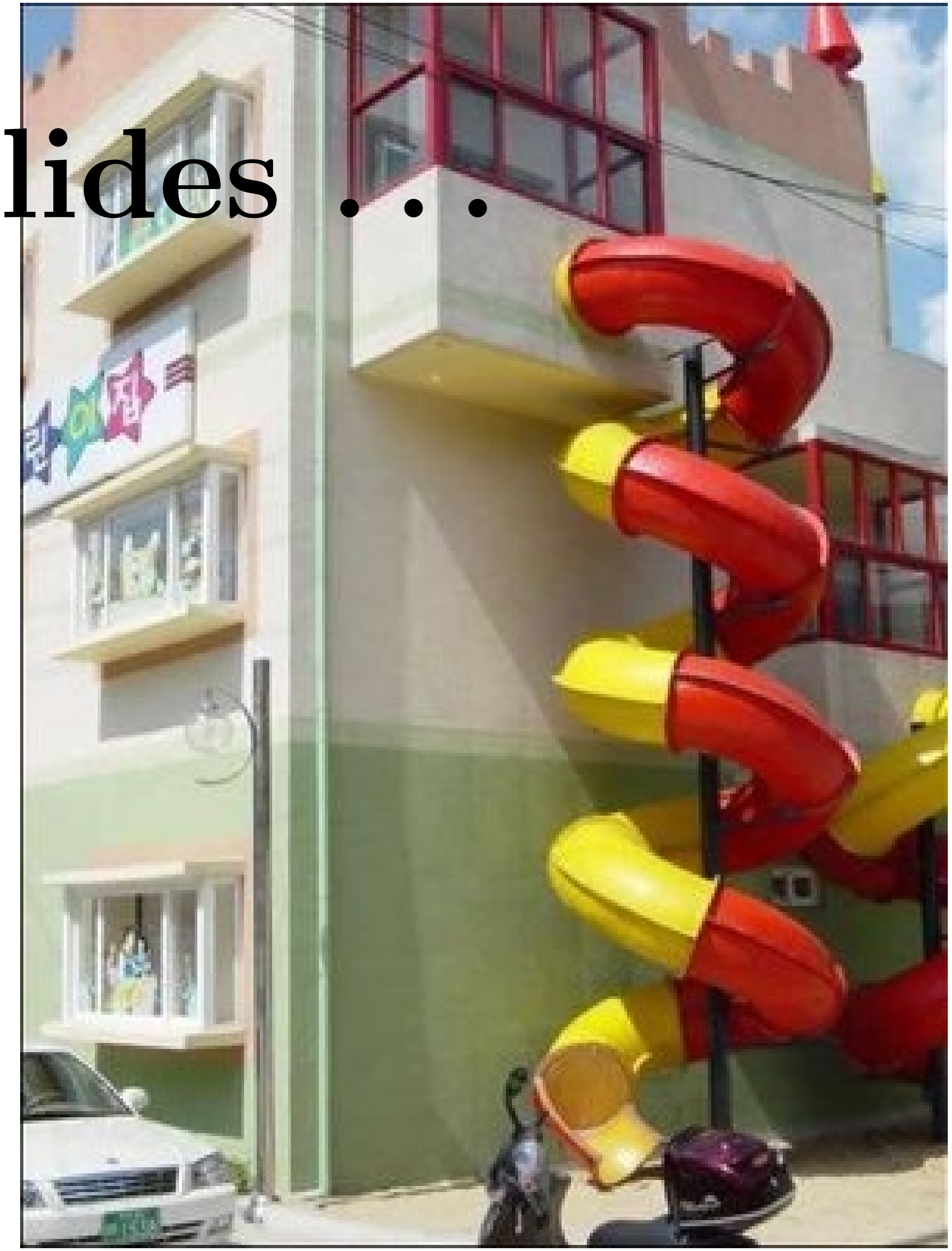
- Majorana and Dirac Fermions are Qualitatively Different. However, massless Majorana and Dirac fermions are "the same" – Majorana-versus-Dirac is a nonquestion! Since neutrinos are always ultra-relativistic, it is very difficult to address whether they are Majorana or Dirac. Neutrinos are massless as far as most experiments are concerned.

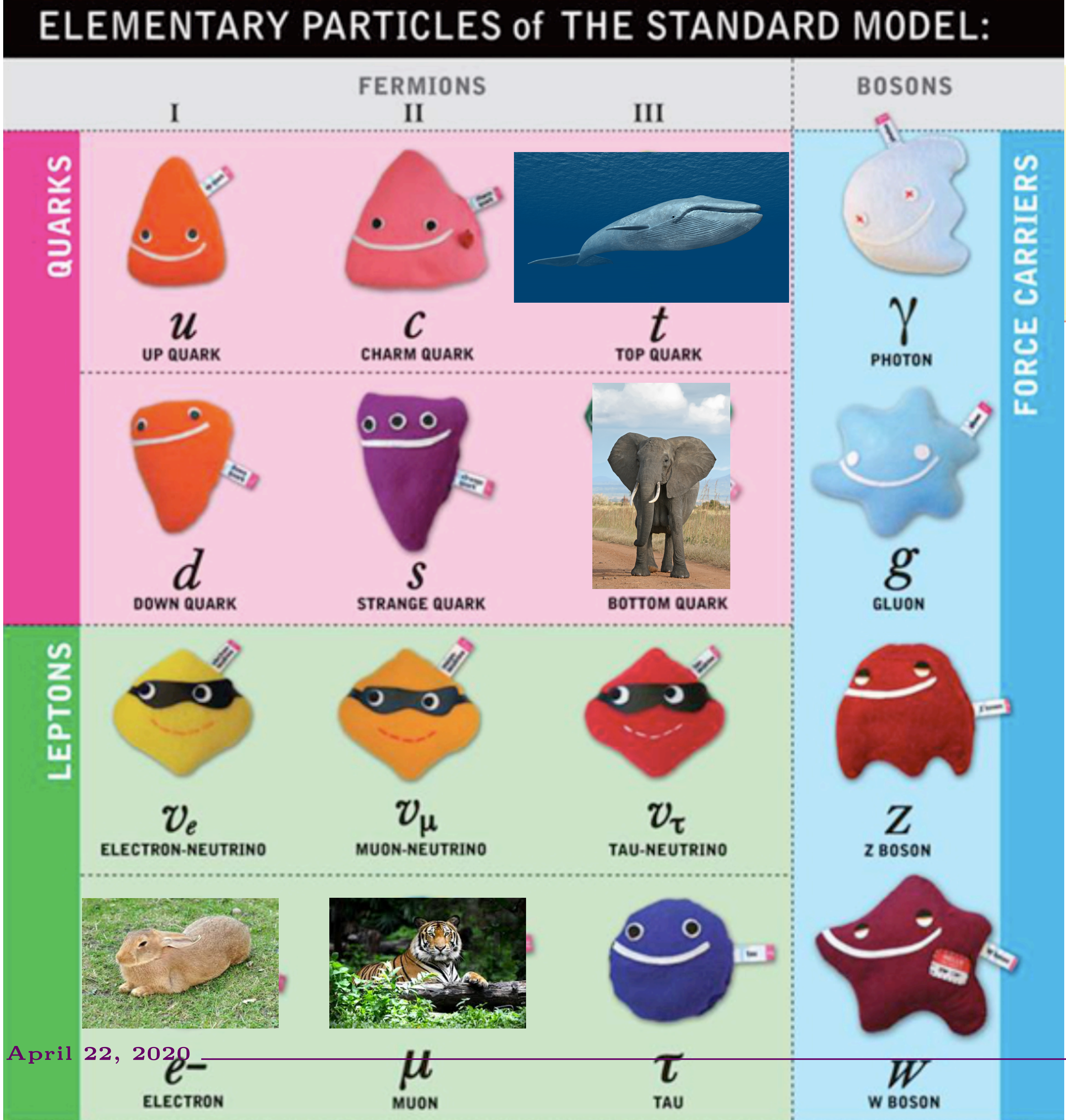
- One solution is to look for phenomena that can only occur if the neutrino is a Majorana fermion (e.g., LNV). Even for very rare phenomena, any positive result establishes that neutrinos are Majorana fermions.

- It is hard to compete with $0\nu\beta\beta$, but possible. It is hard to explain tiny neutrino masses and hope for positive results from other LNV searches, but possible. Trick is to "distance" the neutrino masses from the source of LNV.

- The other way is to find circumstances where the neutrinos are not ultra-relativistic. In this case, the Majorana versus Dirac differences are large. The rates, on the other hand…

Backup Slides . . .





(Now with Higgs boson!)

21st Century Periodic Table

http://www.particlezoo.net

On Higher Dimensional Operators [Kobach, arXiv:1604.05726 + refs therein]

Very generically, there is relationship between ΔL, the lepton number of a given operator, ΔB, the baryon number of a given operator, and D, the mass-dimension of the operator, assuming only Lorentz and hypercharge invariance.

$$\left|\frac{1}{2}\Delta B + \frac{3}{2}\Delta L\right| \in \mathbb{N} \begin{cases} \text{odd} & \leftrightarrow \quad D \text{ is odd,} \\ \text{even} & \leftrightarrow \quad D \text{ is even.} \end{cases}$$

- Operators with $|\Delta L| = 2$, $\Delta B = 0$ have odd mass dimension. The lowest such operator is dimension five.
- Operators with odd mass-dimension must have non-zero ΔB or ΔL. In more detail, it is easy to show that, for operators with odd mass-dimension, $|\Delta(B-L)|$ is an even number not divisible by four $(2, 6, 10, \ldots)$. All odd-dimensional operators violate $B-L$ by at least two units. For operators with even mass-dimension, $|\Delta(B-L)|$ is a multiple of four, including zero $(0, 4, 8, 12, \ldots)$.

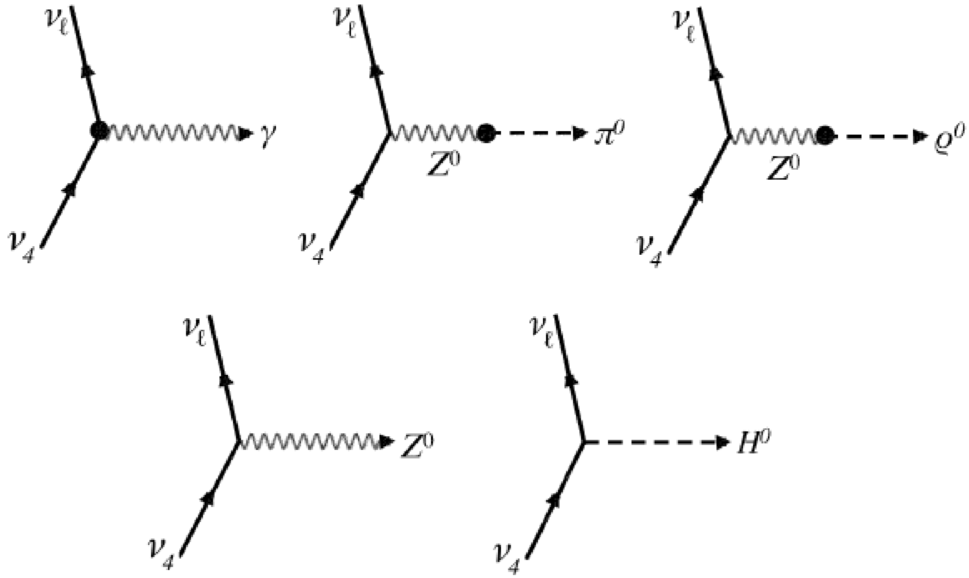
A More Realistic (?) Application – Neutral Heavy Leptons

If a neutral heavy lepton ν_4 is discovered somewhere – LHC, MicroBooNE, ICARUS, DUNE, SuperB Factory, SHiP, etc – in the future, after much rejoicing, we will want to establish whether this fermion is *Majorana* or *Dirac*.

How do we do it?

- Check for lepton-number violation. What does it take?
  - A lepton-number asymmetric initial state (easy). Or an even-by-event lepton number "tag" of the neutral heavy lepton (e.g. LHC environment).
  - Charge identification capability in the detector (sometimes absent or partially absent).
- **Kinematics**. Not only are the decay widths different – not useful, since it requires we know unknown parameters – the kinematics are qualitatively different, as I showed in the last slide.

Heavy Neutral Leptons – More Realistic (?) Application



All of these decays are isotropic for a Majorana parent. Otherwise (weak interactions)...

Boson	γ	π^0	ρ^0	Z^0	H^0
α	$\frac{2\Im(\mu d^*)}{\lvert\mu\rvert^2+\lvert d\rvert^2}$	1	$\frac{m_4^2-2m_\rho^2}{m_4^2+2m_\rho^2}$	$\frac{m_4^2-2m_Z^2}{m_4^2+2m_Z^2}$	1

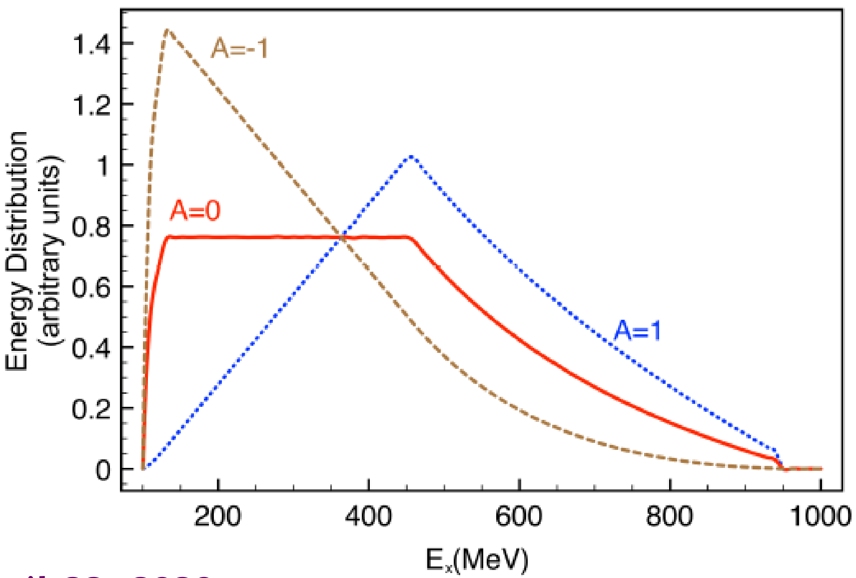
Energy distribution in the Laboratory ("same" as angular distribution)

Parent's rest frame for $N \to \nu_\ell + X$

$$\frac{dn_X}{d\cos\theta_X} \propto (1 + A\cos\theta_X), \quad A = \alpha \times \text{polarization}$$

Lab frame with $r = m_X^2/m_N^2 < 1$

$$\frac{dn_X(E_N, E_X)}{dE_X} \propto \frac{2}{p_N(1-r)}\left[1 + A\left(\frac{2}{(1-r)}\frac{E_X}{p_N} - \left(\frac{1+r}{1-r}\right)\frac{E_N}{p_N}\right)\right]$$



$m_X = 100\,\text{MeV}$

$m_N = 300\,\text{MeV}$

$500\,\text{MeV} < E_N < 100\,\text{MeV}$

Final comment: We Can Use Charged Final States Too!

The two-body final states here all involve a neutrino and a neutral boson. Impossible to reconstruct the parent rest-frame and it requires measuring the properties of a neutral boson, which is sometimes challenging. Can we use the charged final states? E.g.

$$\nu_4 \to \mu^+ \pi^-$$

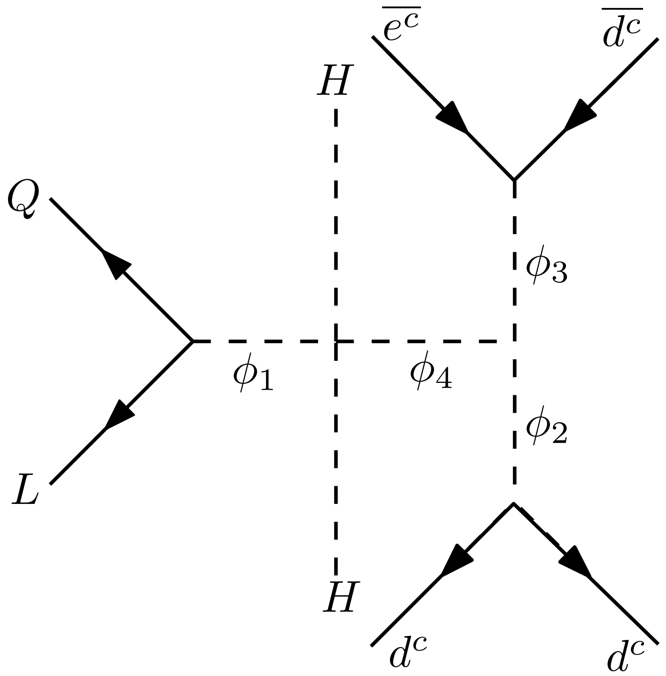
Most of the time, 'yes'! The reason is as follows. CPT invariance (at leading order) implies, for 100% polarized Majorana fermions,

$$\frac{d\Gamma(\nu_4 \to \mu^+\pi^-)}{d\cos\theta} \propto (1 + \alpha\cos\theta) \quad \text{while} \quad \frac{d\Gamma(\nu_4 \to \mu^-\pi^+)}{d\cos\theta} \propto (1 - \alpha\cos\theta)$$

so the **charge-blind sum of the two is also isotropic**. This is not the case for Dirac neutrinos as long as the production of neutrinos and antineutrinos is asymmetric, which is usually the case.

Can this be done in practice? We don't know – homework assignment

[arXiv:0708.1344 [hep-ph]]



Order-One Coupled, Weak Scale Physics Can Also Explain Naturally Small Majorana Neutrino Masses:

Multi-loop neutrino masses from lepton number violating new physics.

$$-\mathcal{L}_{\nu\mathrm{SM}} \supset \sum_{i=1}^{4} M_i\phi_i\bar{\phi}_i + iy_1 QL\phi_1 + y_2 d^c d^c\phi_2 + y_3 e^c d^c\phi_3 + \lambda_{14}\bar{\phi}_1\phi_4 HH + \lambda_{234}M\phi_2\bar{\phi}_3\phi_4 + h.c.$$

$m_\nu \propto (y_1 y_2 y_3 \lambda_{234})\lambda_{14}/(16\pi)^4 \rightarrow$ neutrino masses at 4 loops, requires $M_i \sim 100$ GeV!

WARNING: For illustrative purposes only. Scenario almost certainly ruled out by searches for charged-lepton flavor-violation and high-energy collider data.