

Question: is Minimal Flavor Violation compatible with Positivity bounds?

Setting up

Positivity bounds from unitarity and analyticity in the UV

$$\frac{1}{2} \left. \frac{d^2 \mathcal{A}(s)}{ds^2} \right|_{s=0} = \frac{1}{2\pi i} \oint_C \frac{ds}{s^3} \mathcal{A}(s) > 0$$

SM Effective Field Theory

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_{n \geq 5} \frac{c_n}{\Lambda^{n-4}} \mathcal{O}^{(n)}$$

Minimal Flavor Violation (MFV)

$Y_u, Y_d, Y_e \rightarrow$ spurions of $SU(3)_{\text{flavor}}^5$



$$\mathcal{L}_{\text{SMEFT}} \in \mathbf{1} \text{ of } SU(3)_{\text{flavor}}^5$$

4- Fermi operators $\sim c_{mnpq} \partial^2 (\bar{\psi}_m \Gamma \psi_n) (\bar{\psi}_p \Gamma \psi_q)$

$$M \equiv Y_u Y_u^\dagger \in \mathbf{1}_Q \oplus \mathbf{8}_Q, \quad \tilde{M} \equiv Y_u^\dagger Y_u \in \mathbf{1}_u \oplus \mathbf{8}_u$$



$$c(\xi)_{mnpq}^{u,i} = \xi_1^{u,i} (\delta_{mn} \delta_{pq}) + \xi_2^{u,i} (\tilde{M}_{mn} \delta_{pq} + \delta_{mn} \tilde{M}_{pq}) + \xi_3^{u,i} (\delta_{mq} \delta_{pn}) + \xi_4^{u,i} (\tilde{M}_{mq} \delta_{pn} + \delta_{mq} \tilde{M}_{pn})$$

Studying the bounds

Generic form $\alpha_m \alpha_q^* \beta_n \beta_p^* c(\xi, Y)_{mnpq} > 0$, with arbitrary $\alpha_n, \beta_n \in \mathbb{C}^{N_f}$.

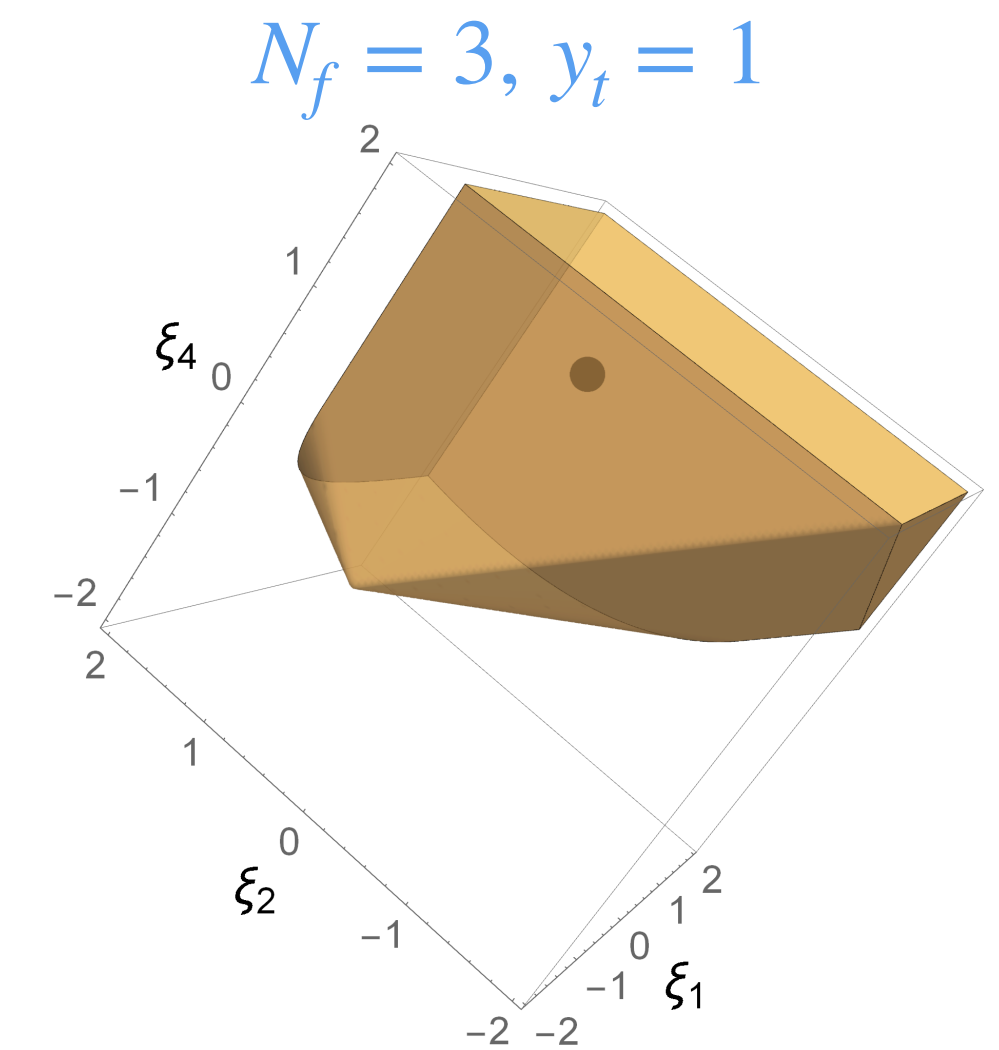
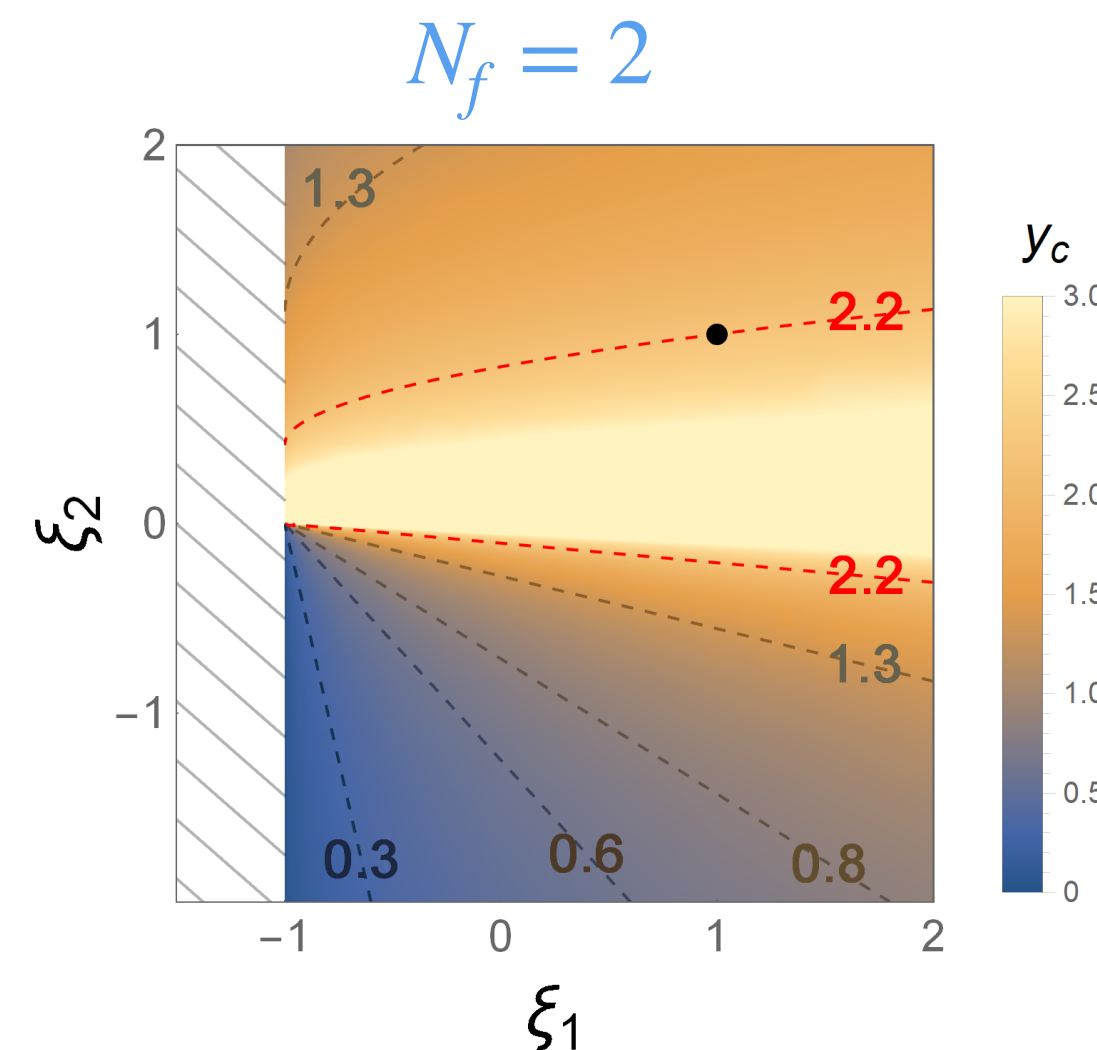
We need to remove α_n, β_n to get bounds on flavor-blind ξ 's, which depend on the biggest Yukawa.

Smaller Yukawa and CKM elements \Rightarrow sub-leading contributions.

Define $C(\beta)_{mq} = c_{mnpq} \beta_n \beta_p^*$, the bounds require it is positive definite



Equivalent to ask $C(\beta)_{mq}$ eigenvalues, depending on β_n only, to be positive

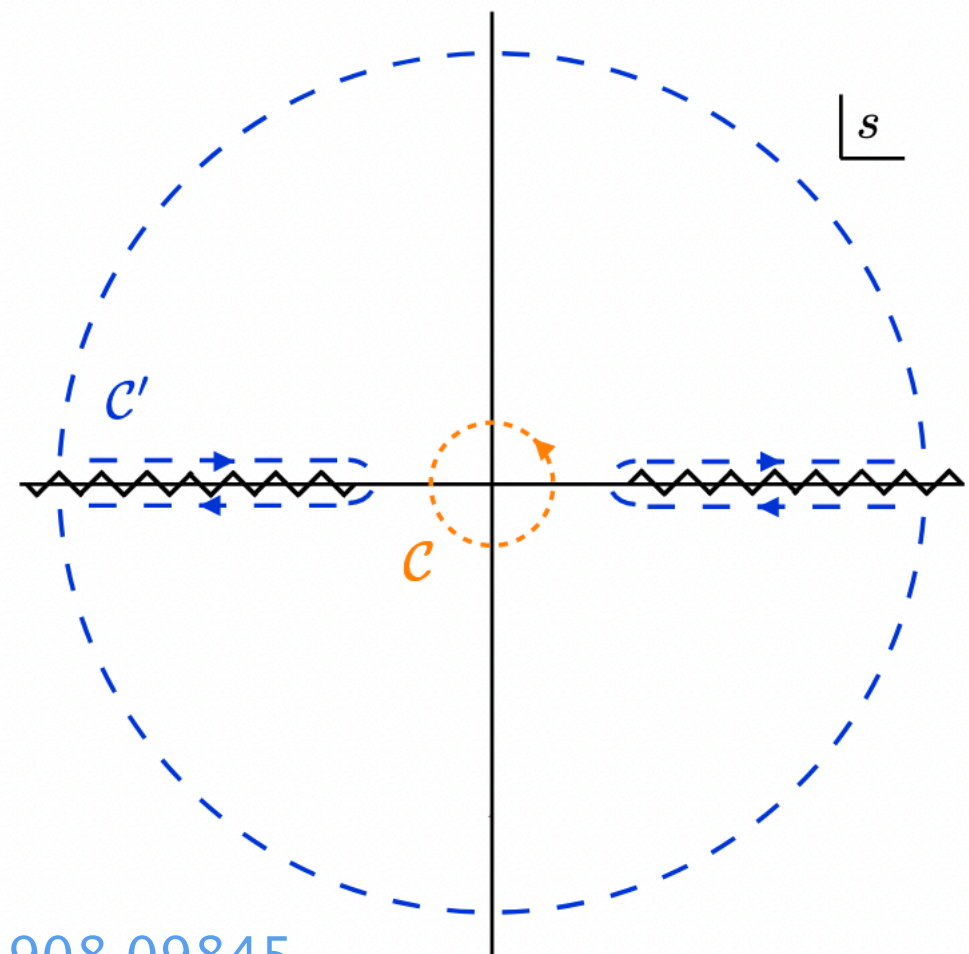


Minimal Flavor Violation and Positivity bounds are compatible!

Positivity bounds: example

Scalar with shift symmetry $\rightarrow \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{c}{M^4} (\partial_\mu \phi)^4$

2 \rightarrow 2 scattering in the forward limit $t \rightarrow 0 \rightarrow \mathcal{A}(s) \equiv \mathcal{M}(s, t = 0) = \frac{4cs^2}{M^4}$



$$\mathcal{A}(s) = \sum_{n=0}^{\infty} \lambda_n s^n$$

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$$\begin{aligned} \lambda_2 &= \frac{1}{2\pi i} \oint_C \frac{\mathcal{A}(s)}{s^3} ds = \frac{4c}{M^4} \\ &= C_R + \frac{1}{\pi i} \int_{s_d}^{+\infty} [\mathcal{A}(s + i\epsilon) - \mathcal{A}(s - i\epsilon)] ds \\ &= \frac{2}{\pi} \int_{s_d}^{+\infty} \frac{ds}{s^2} \sigma(s) > 0 \end{aligned} \rightarrow c > 0$$

Full list of dimension 8 4-Fermi operators

Type	Content	Operator	Symmetry		
self-quartic	(4-u)	$\mathcal{O}_1[u] = c_{mnpq}^{u,1} \partial_\mu (\bar{u}_m \gamma_\nu u_n) \partial^\mu (\bar{u}_p \gamma^\nu u_q)$	$c_{mnpq} = c_{pqmn}$ $c_{mnpq}^* = c_{nmqp}$		
		$\mathcal{O}_3[u] = c_{mnpq}^{u,3} \partial_\mu (\bar{u}_m T^a \gamma_\nu u_n) \partial^\mu (\bar{u}_p T^a \gamma^\nu u_q)$			
	(4-Q)	$\mathcal{O}_1[Q] = c_{mnpq}^{Q,1} \partial_\mu (\bar{Q}_m \gamma_\nu Q_n) \partial^\mu (\bar{Q}_p \gamma^\nu Q_q)$			
		$\mathcal{O}_2[Q] = c_{mnpq}^{Q,2} \partial_\mu (\bar{Q}_m \tau^I \gamma_\nu Q_n) \partial^\mu (\bar{Q}_p \tau^I \gamma^\nu Q_q)$			
		$\mathcal{O}_3[Q] = c_{mnpq}^{Q,3} \partial_\mu (\bar{Q}_m T^a \gamma_\nu Q_n) \partial^\mu (\bar{Q}_p T^a \gamma^\nu Q_q)$			
		$\mathcal{O}_4[Q] = c_{mnpq}^{Q,4} \partial_\mu (\bar{Q}_m T^a \tau^I \gamma_\nu Q_n) \partial^\mu (\bar{Q}_p T^a \tau^I \gamma^\nu Q_q)$			
	(4-d)	$\mathcal{O}_1[d] = c_{mnpq}^{d,1} \partial_\mu (\bar{d}_m \gamma_\nu d_n) \partial^\mu (\bar{d}_p \gamma^\nu d_q)$			
		$\mathcal{O}_3[d] = c_{mnpq}^{d,3} \partial_\mu (\bar{d}_m T^a \gamma_\nu d_n) \partial^\mu (\bar{d}_p T^a \gamma^\nu d_q)$			
	cross-quartic	(2-u)(2-Q)		$\mathcal{O}_{K1}[u, Q] = -a_{mnpq}^{uQ,1} (\bar{u}_m \gamma_\mu \partial_\nu u_q) (\bar{Q}_n \gamma^\nu \partial^\mu Q_p)$	$a_{mnpq}^{\psi\chi} = a_{pqnm}^{\chi\psi}$ $a_{mnpq}^* = a_{qpnm}^*$
				$\mathcal{O}_{K3}[u, Q] = -a_{mnpq}^{uQ,3} (\bar{u}_m T^a \gamma_\mu \partial_\nu u_q) (\bar{Q}_n T^a \gamma^\nu \partial^\mu Q_p)$	
		(2-d)(2-Q)		$\mathcal{O}_{K1}[d, Q] = -a_{mnpq}^{dQ,1} (\bar{d}_m \gamma_\mu \partial_\nu d_q) (\bar{Q}_n \gamma^\nu \partial^\mu Q_p)$	
				$\mathcal{O}_{K3}[d, Q] = -a_{mnpq}^{dQ,3} (\bar{d}_m T^a \gamma_\mu \partial_\nu d_q) (\bar{Q}_n T^a \gamma^\nu \partial^\mu Q_p)$	
(2-d)(2-u)		$\mathcal{O}_{K1}[d, u] = -a_{mnpq}^{du,1} (\bar{d}_m \gamma_\mu \partial_\nu d_q) (\bar{u}_n \gamma^\nu \partial^\mu u_p)$			
		$\mathcal{O}_{K3}[d, u] = -a_{mnpq}^{du,3} (\bar{d}_m T^a \gamma_\mu \partial_\nu d_q) (\bar{u}_n T^a \gamma^\nu \partial^\mu u_p)$			

There are other operators of the form

$$\mathcal{O} = \partial_\mu (\bar{\psi}_m \gamma_\nu \psi_n) \partial^\mu (\bar{\chi}_p \gamma^\nu \chi_q), \quad \psi \neq \chi$$

but their contribution to the amplitude vanishes as $t \rightarrow 0$ so we cannot bound them

How do we get the bounds?

Scatter flavor superpositions as initial and final states. E.g. scattering right-handed up-type quarks:

$$\begin{aligned} |\psi_1\rangle &= \alpha_{mi} |\bar{u}_{mi}\rangle, & |\psi_2\rangle &= \beta_{mi} |u_{mi}\rangle \\ |\psi_3\rangle &= \beta_{mi}^* |\bar{u}_{mi}\rangle, & |\psi_4\rangle &= \alpha_{mi}^* |u_{mi}\rangle \end{aligned}$$

we get

$$\mathcal{A} = 4s^2 \left[\left(c_{mnpq}^{u,1} - \frac{1}{6} c_{mnpq}^{u,3} \right) \alpha_{mi}^* \beta_{ni} \beta_{pj}^* \alpha_{qj} + \frac{1}{2} c_{mnpq}^{u,3} \alpha_{mi}^* \beta_{nj} \beta_{pj}^* \alpha_{qi} \right]$$

producing the two bounds

$$\begin{aligned} \alpha_m \alpha_q^* \beta_n \beta_p^* \left(c_{mnpq}^{u,1} + \frac{1}{3} c_{mnpq}^{u,3} \right) &> 0, \\ \alpha_m \alpha_q^* \beta_n \beta_p^* c_{mnpq}^{u,3} &> 0 \end{aligned}$$

then we can perform a linear redefinition on the flavor-blind coefficients ξ to take them both into the form

$$\alpha_m \alpha_q^* \beta_n \beta_p^* c(\xi)_{mnpq}^{u,i} > 0, \quad i = 1, 3$$

Approximations

We take only the largest up Yukawa to be non-zero, for both $N_f = 2$ and $N_f = 3$, and study the bounds as a function of it.

The down Yukawa matrix is set to zero.

$$Y_u \rightarrow \begin{pmatrix} \cancel{y_u} & 0 & 0 \\ 0 & \cancel{y_c} & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad Y_d \rightarrow 0$$

Some remarks

- To account for y_t being $\mathcal{O}(1)$, one needs to resum and set $y_t = 1$ in the end.
- As we can always rotate in the Lagrangian to have diagonal Y_u and the CKM matrix entirely in Y_d , the CKM does not show up at leading order in the bounds. To observe it we need to expand further to include both Y_u and Y_d contributions.
- Alternatively, we can scatter initial and final states limited to two flavors. In this case the bounds are as in $N_f = 2$ but depend on the CKM entries (in the Wolfenstein parametrization) through the combination $\sigma \equiv A^2 \lambda^4$.