An on-shell perspective on neutrino oscillations and non-standard interactions

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CAPES

Abstract

The on-shell amplitude techniques are applied to the study of neutrino oscillations in vacuum. We determine the 3-point amplitude involving one neutrino, one charged lepton and one W boson and the results obtained contain terms generated at all orders in an expansion in the cut-off scale of the theory. We then use this amplitude to construct the 4-point amplitude in which neutrinos are exchanged in the s-channel, giving rise to the oscillation phenomena. We also study in detail how flavor enters in the amplitudes, and how the PMNS matrix emerges from the on-shell perspective.

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Introduction

The fact that the neutrino flavor and mass eigenstates are misaligned gives rise to a phenomenon known as neutrino oscillations [1]. In order to compare experiments with theoretical predictions, one needs to compute the probability of a given neutrino flavor to evolve into a different neutrino flavor. This is usually done within the formalism of quantum mechanics through the Schrödinger equation. The same computation can be performed in Quantum Field Theory by computing the oscillation amplitude of the scattering process [2].

In the last decades a new way of facing amplitudes was developed, in which we abandon the Lagrangian formalism and focus directly on on-shell amplitudes [3,4]. This formalism, named on-shell formalism, offers a new and complementary way of understanding neutrino oscillation amplitudes.

In this project we applied on-shell methods to study neutrino oscillations and, in particular, derived the form of the $\nu l^+ W^-$ amplitude and computed the oscillation amplitude for $W^- l_i^+ \to W^+ l_j^-$.

Flavor structure

In order to study neutrino oscillations, we need first to describe flavor in this formalism. We begin by splitting the theory in the massless and massive regimes, which are connected at the electroweak scale. When all leptons are massless, the space of one-particle states for each type of lepton can be rotated by a $SU(3) \times U(1)^2$ transformation, since no quantum number distinguishes the generations ($SO(3)$ for Majorana neutrinos). As the leptons become massive, the non-abelian part of this symmetry group is lost, hence the symmetry group of flavor transformations is reduced to

$$SU(3)^2 \times U(1)^4 \to U(1)^4. \tag{1}$$

By performing the matching between the massless amplitude with the massive one at the electroweak scale, we obtain that the leading contribution to the massive amplitude is given by the standard PMNS matrix with the correct number of parameters (3 angles+1 phase for Dirac and 3+3 for Majorana neutrinos).

Connection with Effective Theories

The massive amplitude for the $\nu l^+ W^-$ vertex is

$$\mathcal{M}\left(\nu_1 l_2^+ W_3^-\right) = a\,\langle \mathbf{13}\rangle \langle \mathbf{23}\rangle + b\,\langle \mathbf{13}\rangle [\mathbf{23}] + c\,[\mathbf{13}] \langle \mathbf{23}\rangle + d\,[\mathbf{13}]\,[\mathbf{23}]\,, \tag{2}$$

where all coefficients are matrices in flavor space, in particular $b \propto U_{\text{PMNS}}$, and the numbers correspond to the momenta of each particle. The different terms in the amplitude above take into account all possible kinematical structures allowed by this particle content. The other coefficients do not receive contributions from the renormalizable Standard Model, but only from effective operators. For instance, the term proportional to a can be generated only by effective operators, for example via the dimension six operator

$$a\,\langle \mathbf{13}\rangle \langle \mathbf{23}\rangle \sim a\,\bar{l}_R \sigma_{\mu\nu} \nu_L W^-_{\mu\nu}, \tag{3}$$

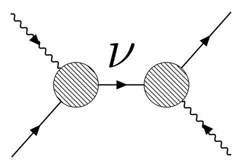
which corresponds to the dipole operators. Since the amplitude in Eq. (2) fixes the kinematical structures instead of the power counting of the couplings, we can write a as

$$a = a^{(0)} + \frac{v}{\Lambda} a^{(1)} + \left(\frac{v}{\Lambda}\right)^2 a^{(2)} + \cdots\,, \tag{4}$$

where v is the electroweak scale and Λ the UV cut-off. Therefore, all coefficients of Eq. (2) are an all-oder expansion in the Effective Theory point of view. This means that all possible modifications to the $\nu l^+ W^-$ vertex are parametrised by Eq. (2).

Oscillation amplitude

The oscillation amplitude for $W^- l_i^+ \to W^+ l_j^-$ can be computed at tree-level with pology [3]. This amplitude factorises only in the s-channel with a neutrino being exchanged as the intermediate state for each of the flavors. The residue in each channel is given by

$R_i =$  $= \lim_{P^2 = s \to m^2_{\nu_i}} \mathcal{M}(\nu_P l_1^+ W_2^-) \mathcal{M}(\bar{\nu}_{-P} l_3^- W_4^+).$ (5)

The total amplitude is obtained by summing over mass-eigenstates and dividing by the corresponding propagator [4]:

$$\mathcal{M}(W^- l^+ \to W^+ l^-) = \sum_i \frac{R_i}{s - m_i^2}. \tag{6}$$

The amplitude above leaves the near on-shellness of the propagating neutrino explicit and includes all possible modifications of new physics coming from the vertices.

Conclusions

The on-shell formalism applied in the description of neutrino oscillations allowed us to easily identify the kinematical structures that might appear within the Standard Model and beyond. Furthermore, we could recover the PMNS matrix from a symmetry approach, connect the on-shell formalism with effective field theories and write down the most general amplitude for $W^- l_i^+ \to W^+ l_j^-$.

References

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