Enhanced violation of Leggett-Garg Inequality in three flavour neutrino oscillations via non-standard interactions

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1. Idea

We explore if violation of Leggett-Garg Inequality (LGI) can be enhanced in the context of three flavor neutrino oscillations. We consider the senario of non-standard neutrino interactions (NSI) which refer to a wide class of new physics scenarios parameterised in a model-independent way.

2. Non-standard interactions

Y. Farzan and M. Tortola, Front. Phys. 6, 10 (2018)

- Effective Hamiltonian for neutrino propagation

$$\mathcal{H} = \underbrace{\frac{1}{2E}\mathcal{U}\begin{pmatrix}0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}\mathcal{U}^\dagger}_{\mathcal{H}_{vac}} + \underbrace{\frac{A(x)}{2E}\begin{pmatrix}1 & & \\ & 0 & \\ & & 0\end{pmatrix}}_{\mathcal{H}_{SI}} + \underbrace{\frac{A(x)}{2E}\begin{pmatrix}\varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^\star & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^\star & \varepsilon_{\mu\tau}^\star & \varepsilon_{\tau\tau}\end{pmatrix}}_{\mathcal{H}_{NSI}}$$

- $\Delta m^2_{ij} = m^2_i - m^2_j$
- Mixing matrix in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) parameterisation

$$\mathcal{U} = \begin{pmatrix}1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1\end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$.
- $A(x) = 2E\sqrt{2}G_F n_e(x)$ is the standard charged-current potential
- $\varepsilon_{\alpha\beta}$ are the dimensionless NSI parameters

$$\varepsilon_{\alpha\beta} = \sum_{f=e,u,d}\frac{n_f}{n_e}\varepsilon^f_{\alpha\beta}$$

n_f is the density of fermion f in medium traversed by the neutrino

3. Leggett-Garg Inequalities

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985).

- LGI allow for testing the applicability of quantum mechanics in the macroscopic world.
- LGI rely on two assumptions:
  - **Macroscopic realism** which implies that the performance of a measurement on a macroscopic system reveals a well-defined pre-existing value.
  - **Non-invasive measurability** which states that in principle, we can measure this value without disturbing the system.
- In quantum mechanics, both these assumptions are violated as it is based on superposition principle and collapse of wave function under measurement.

4. Quantifying the violation of LGI

- We consider a dichotomic observable $Q = \pm 1$, which is measured at different times t_i

- Two time correlation functions $C_{ij} = \frac{1}{N}\sum_{q=1}^{N}\langle Q^q_i Q^q_j\rangle$
- Observable to quantify violation of LGI

$$K_3 = C_{12} + C_{23} - C_{13} = \langle Q_1Q_2\rangle + \langle Q_2Q_3\rangle - \langle Q_1Q_3\rangle$$

$$K_3 = \begin{cases}1 + 0 = 1 \\ -1 + (\pm 2) = 1 \quad \text{or} -3\end{cases}$$

$$K_4 = C_{12} + C_{23} + C_{34} - C_{14}$$

$$-3 \le K_3 \le 1 \quad ; \quad -2 \le K_4 \le 2$$

- In general

$$-n \le K_n \le (n-2) \qquad 3 \le n, \text{odd};$$
$$-(n-2) \le K_n \le (n-2) \qquad 4 \le n, \text{even}$$

C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 016001 (2013)

- The maximum quantum-mechanical value of the LGI correlator

$$K_3^{max} = \frac{3}{2} \quad \text{and} \quad K_4^{max} = 2\sqrt{2}$$

Luder's bound or temporal Tsirelson's bound

5. Neutrino oscillations and LGI violation

- Initial state $\rightarrow |\nu_e\rangle$ (Beta beam set up)
- Our dicotomic observable

$$Q = \begin{cases}+1 & \text{for } \nu_e \\ -1 & \text{for } \nu_\mu \text{ (or } \nu_\tau)\end{cases}$$

- For two-flavor neutrino oscillation

$$C_{12} = \mathbb{P}_{\nu_e\nu_e}(L_1, L_2) - \mathbb{P}_{\nu_e\nu_\mu}(L_1, L_2) - \mathbb{P}_{\nu_\mu\nu_e}(L_1, L_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(L_1, L_2)$$

where $\mathbb{P}_{\nu_\alpha\nu_\beta}(L_i, L_j) = P_{\nu_e\rightarrow\nu_\alpha}(L_i)P_{\nu_\alpha\rightarrow\nu_\beta}(L_j) \rightarrow$ joint probability.

$$C_{12} = 1 - 2\sin^2 2\theta \sin^2\left(\frac{\lambda}{2}\Delta L\right) \qquad K_4 = 2 - 2\sin^2 2\theta\left[3\sin^2\left(\frac{\lambda}{2}\Delta L\right) - \sin^2\left(\frac{3\lambda}{2}\Delta L\right)\right]$$

$$\Delta L = L_2 - L_1$$

D. Gangopadhyay, et. al Phys. Rev. A88(2), 022115 (2013)

- Three-flavor case

$$C_{12} = \mathbb{P}_{\nu_e\nu_e}(L_1, L_2) - \mathbb{P}_{\nu_e\nu_\mu}(L_1, L_2) - \mathbb{P}_{\nu_e\nu_\tau}(L_1, L_2) - \mathbb{P}_{\nu_\mu\nu_e}(L_1, L_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(L_1, L_2)$$
$$+ \mathbb{P}_{\nu_\mu\nu_\tau}(L_1, L_2) - \mathbb{P}_{\nu_\tau\nu_e}(L_1, L_2) + \mathbb{P}_{\nu_\tau\nu_\mu}(L_1, L_2) + \mathbb{P}_{\nu_\tau\nu_\tau}(L_1, L_2)$$

- Joint probabilities depend on the parameters of the neutrino Hamiltonian (both SI and NSI).
- We take E = 1 GeV, $L_1 = 140.15$ km, CP-phase $\delta = 3\pi/2$ and $(L_2 - L_1) = (L_3 - L_2) = (L_4 - L_3) = \Delta L$.

6. Impact of NSI on K_4

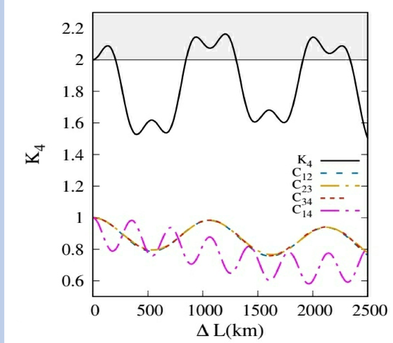

Figure: K_4 is plotted as a function of ΔL for the three flavour neutrino oscillations in matter with SI. The contribution of various C_{ij}'s is also depicted.

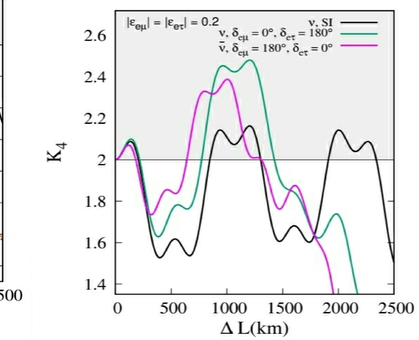

Figure: K_4 is plotted as a function of ΔL for three flavour oscillations for neutrinos and anti-neutrinos in presence of NSI.

7. Conclusion

- Neutrinos have extremely large mean free path and exhibit sustained coherence over astrophysical length scales.This provides for a unique opportunity to employ neutrinos as a useful quantum resource.
- Around 6σ violation of LGI has been reported in data from MINOS and Daya Bay using a two-flavor analysis.

J. A. Formaggio,et. al. Phys. Rev. Lett. 117(5), 050402 (2016)

Q. Fu and X. Chen, Eur. Phys. J. C77(11), 775 (2017)

- For appropriate choice of NSI parameters we report an enhancement in the violation of LGI.
- Even though we see enhancement, $K_4^m < 2\sqrt{2}$ (Luder's bound).

Scenario	K_4^m	ΔL (km)	(%)
ν, SI	2.163	1200	0
ν, NSI ($\lvert\varepsilon_{e\mu}\rvert = 0.1$)	2.240	1210	3.56
ν, NSI ($\lvert\varepsilon_{e\mu}\rvert = 0.2$)	2.331	1210	7.77
ν, NSI ($\lvert\varepsilon_{e\mu}\rvert = \lvert\varepsilon_{e\tau}\rvert = 0.2$, $\delta_{e\mu} = 0$, $\delta_{e\tau} = \pi$)	**2.481**	1200	**14.7**
$\bar{\nu}$, NSI ($\lvert\varepsilon_{e\mu}\rvert = \lvert\varepsilon_{e\tau}\rvert = 0.2$, $\delta_{e\mu} = \pi$, $\delta_{e\tau} = 0$)	2.388	1000	10.40

Table: Maximum value of K_4 and corresponding value of ΔL for different scenarios. The relative change with respect to SI case (for neutrinos) is also mentioned in the table.

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