Enhanced violation of Leggett-Garg Inequality in three flavour neutrino oscillations via non-standard interactions

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Introduction

- Neutrino oscillate among themselves and these oscillations have their origin in the non-zero neutrino masses and mixing among the neutrino flavors.
- The standard paradigm of neutrino oscillations involves three flavours of neutrinos which are superpositions of the mass states carrying well-defined masses.
- Effective Hamiltonian for neutrino propagation

$$\mathcal{H} = \mathcal{H}_{\rm vac} + \mathcal{H}_{\rm SI} + \mathcal{H}_{\rm NSI}$$

  where $\mathcal{H}_{\rm vac}$ is the vacuum Hamiltonian and $\mathcal{H}_{\rm SI}, \mathcal{H}_{\rm NSI}$ are the effective Hamiltonians in presence of standard interaction (SI) and non-standard interaction (NSI) respectively.
- NSI refer to a wide class of new physics scenarios parameterised in a model-independent way.

Leggett-Garg Inequalities

Leggett and Garg derived a class of inequalities which provide a way to test the applicability of quantum mechanics in the macroscopic world. Leggett-Garg Inequalities(LGI) are based on the following assumptions:

Macroscopic realism (MR):

A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states.

Non-Invasive measurability (NIM):

It is possible, in principle, to determine which of the states the system is in, without affecting the states itself or the system's subsequent dynamics.

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985)

- We consider a dichotomic observable $Q = \pm 1$

- Two time correlation functions $C_{ij} = \frac{1}{N}\sum_{q=1}^{N}\langle Q_i^q Q_j^q \rangle$
- Observable to quantify violation of LGI:

$$K_3 = C_{12} + C_{23} - C_{13} \quad = \quad \langle Q_1 Q_2 \rangle + \langle Q_2 Q_3 \rangle - \langle Q_1 Q_3 \rangle$$

$$K_3 \quad = \quad \begin{cases} 1 + 0 = 1 \\ -1 + (\pm 2) = 1 \quad \text{or} - 3 \end{cases}$$

$$-3 \leq K_3 \leq 1$$

- In general we have

$$-n \leq K_n \leq (n-2) \qquad 3 \leq n, \mathrm{odd};$$

$$-(n-2) \leq K_n \leq (n-2) \qquad 4 \leq n, \mathrm{even}$$

- The maximum quantum-mechanical value of the LGI correlator referred to as Luder's bound or temporal Tsirelson's bound is

$$K_3^{max} = \frac{3}{2} \quad \text{and} \quad K_4^{max} = 2\sqrt{2}$$

C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 016001 (2013)

- Initial state $\rightarrow |\nu_e\rangle$
- Our dicotomic observable

$$Q = \begin{cases} +1 & \text{for } \nu_e \\ -1 & \text{for } \nu_\mu \text{ (or } \nu_\tau) \end{cases}$$

- For two-flavor neutrino oscillation

$$C_{12} = \mathbb{P}_{\nu_e\nu_e}(L_1, L_2) - \mathbb{P}_{\nu_e\nu_\mu}(L_1, L_2) - \mathbb{P}_{\nu_\mu\nu_e}(L_1, L_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(L_1, L_2)$$

where $\mathbb{P}_{\nu_\alpha\nu_\beta}(L_i, L_j) = P_{\nu_e\rightarrow\nu_\alpha}(L_i)P_{\nu_\alpha\rightarrow\nu_\beta}(L_j) \rightarrow$ joint probability.

$$C_{12} = 1 - 2\sin^2 2\theta \sin^2\left(\frac{\lambda}{2}\Delta L\right)$$

$$K_4 = 2 - 2\sin^2 2\theta \left[3\sin^2\left(\frac{\lambda}{2}\Delta L\right) - \sin^2\left(\frac{3\lambda}{2}\Delta L\right)\right]$$

$\Delta L = L_2 - L_1$

D. Gangopadhyay, et. al Phys. Rev. A88(2), 022115 (2013)

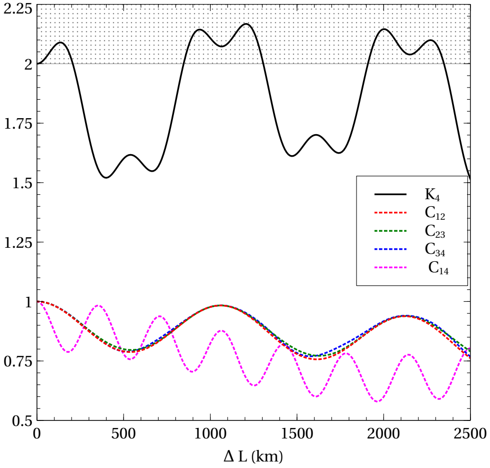
- Three-flavor case

$$\begin{aligned} C_{12} &= \mathbb{P}_{\nu_e\nu_e}(L_1,L_2) - \mathbb{P}_{\nu_e\nu_\mu}(L_1,L_2) - \mathbb{P}_{\nu_e\nu_\tau}(L_1,L_2) \\ &- \mathbb{P}_{\nu_\mu\nu_e}(L_1,L_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(L_1,L_2) + \mathbb{P}_{\nu_\mu\nu_\tau}(L_1,L_2) \\ &- \mathbb{P}_{\nu_\tau\nu_e}(L_1,L_2) + \mathbb{P}_{\nu_\tau\nu_\mu}(L_1,L_2) + \mathbb{P}_{\nu_\tau\nu_\tau}(L_1,L_2) \end{aligned}$$

- Joint probabilities depend on the parameters of the neutrino Hamiltonian (both SI and NSI).

- We take E = 1 GeV, $L_1 = 140.15$ km, CP-phase $\delta = 3\pi/2$ and $(L_2 - L_1) = (L_3 - L_2) = (L_4 - L_3) = \Delta L$.

Results



$K_4 = C_{12} + C_{23} + C_{34} - C_{41}$

$-2 \leq K_4 \leq 2$

Figure: SI-K_4

$$\mathcal{H} = \frac{1}{2E}\mathcal{U}\begin{pmatrix} 0 & & \\ & \delta m^2_{21} & \\ & & \delta m^2_{31} \end{pmatrix}\mathcal{U}^\dagger$$

$$+\frac{A(x)}{2E}\begin{pmatrix} 1+\varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}{}^\star & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}{}^\star & \varepsilon_{\mu\tau}{}^\star & \varepsilon_{\tau\tau} \end{pmatrix}$$

where $A(x) = 2E\sqrt{2}G_F n_e(x)$ is the standard charged-current potential, $\mathcal{U}$ is the mixing matrix and $\varepsilon_{\alpha\beta}$ are the dimensionless NSI parameters.

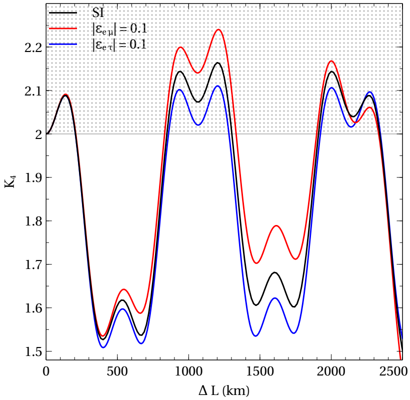


Figure: Non-standard senario

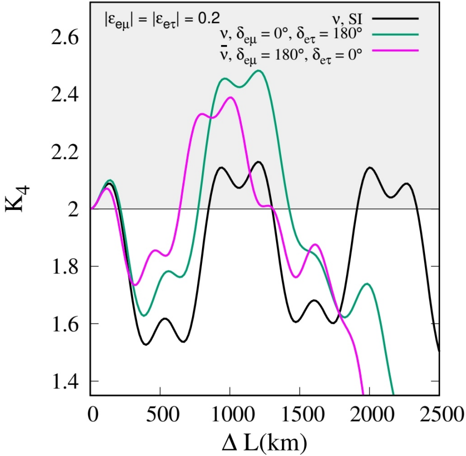


Figure: SI and NSI

Summary

Scenario	K_4^m	ΔL (km)	(%)
ν, SI	2.163	1200	0
ν, NSI ($\|\varepsilon_{e\mu}\| = 0.1$)	2.240	1210	3.56
ν, NSI ($\|\varepsilon_{e\mu}\| = 0.2$)	2.331	1210	7.77
ν, NSI ($\|\varepsilon_{e\mu}\| = \|\varepsilon_{e\tau}\| = 0.2,\ \delta_{e\mu} = 0,\ \delta_{e\tau} = \pi$)	**2.481**	1200	**14.7**
$\bar{\nu}$, NSI ($\|\varepsilon_{e\mu}\| = \|\varepsilon_{e\tau}\| = 0.2,\ \delta_{e\mu} = \pi,\ \delta_{e\tau} = 0$)	2.388	1000	10.40

Table: Maximum value of K_4 and corresponding value of ΔL for different scenarios.

- We note a significant enhancement in the violation of LGI in the NSI senario as compared to the SI senario but the value of K_4 does not exceed the Luder's bound.

Thank You