The MasterCode and Precision Calculations

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Our tool:

The "MasterCode"

⇒ collaborative effort of theorists and experimentalists

[Buchmüller, Cavanaugh, De Roeck, Ellis, Flächer, Hahn, SH, Isidori, Olive, Paradisi, Rogerson, Ronga, Weiglein]

Über-code for the combination of different tools:

- Über-code original in Fortran, now re-written in C++
- tools are included as subroutines
- compatibility ensured by collaboration of authors of "MasterCode" and authors of "sub tools" /SLHA(2)
- sub-codes in Fortran or C++

⇒ evaluate observables of one parameter point consistently with various tools

cern.ch/mastercode

Status of the "MasterCode":

– one model: (MFV) MSSM (see next section)

– tools included:
  – B-physics observables [*SuFla*]
  – more B-physics observables [*SuperIso*]
  – Higgs related observables, $(g-2)_\mu$ [*FeynHiggs*]
  – Electroweak precision observables [*FeynWZ (SUSYPope)*]
  – Dark Matter observables [*MicrOMEGAs, DarkSUSY*]
  – for GUT scale models: RGE running [*SoftSusy*]

⇒ all most-up-to-date codes on the market!

– added: χ^2 analysis code [*Minuit*]

– currently being implemented:
  – Higgs constraints (for χ^2 contributions . . .) [*HiggsBounds*]

– planned: inclusion of more tools / more models

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(Some) Electroweak precision observables in the MasterCode

($\rightarrow$ as for blue band analysis, except Γ_W)

1. M_W (LEP/Tevatron)

2. A^e_{LR} (SLD)

3. A^b_{FB} (LEP)

4. A^c_{FB} (LEP)

5. A^l_{FB}

6. A_b, A_c

7. R_b, R_c

8. σ^0_{had}

$\Rightarrow$ largest impact: (1), (2), (3)

(Some) B/K physics observables in the MasterCode

1. $\text{BR}(b \to s\gamma)$ (MSSM/SM)

2. $\text{BR}(B_s \to \mu^+\mu^-)$

3. ΔM_s

4. $\text{R}(\Delta M_s/\Delta M_d)$

5. $\text{BR}(B_u \to \tau\nu_\tau)$ (MSSM/SM)

6. $\text{BR}(B \to X_x\ell^+\ell^-)$

7. $\text{BR}(K \to \ell\nu)$ (MSSM/SM)

8. $\text{BR}(\Delta M_K)$ (MSSM/SM)

$\Rightarrow$ largest impact: (1) and (2)

Further low-energy observables

– anomalous magnetic moment of the muon: $(g-2)_\mu$

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Higgs physics observables in the MasterCode

– lightest Higgs mass: M_h $\Leftarrow$ more details in a moment

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SM parameters

– top mass: m_t

– Z boson mass: M_Z

– hadronic contribution to fine structure constant: $\Delta\alpha_{\text{had}}$

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

Enlarged Higgs sector: Two Higgs doublets with $\mathcal{CP}$ violation

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

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gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

2 $\mathcal{CP}$-violating phases: ξ, arg(m_{12}) $\Rightarrow$ can be set/rotated to zero

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_{H^\pm}^2$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \qquad \tan(2\alpha) = \tan(2\beta)\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Three Goldstone bosons (as in SM): G^0, $G^\pm$

$\longrightarrow$ longitudinal components of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \qquad \tan(2\alpha) = \tan(2\beta)\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

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$\Rightarrow$ m_h, m_H, mixing angle α, $m_{\mathrm{H}^\pm}$: no free parameters, can be predicted

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} m^2_{\phi_1} & m^2_{\phi_1\phi_2} \\ m^2_{\phi_1\phi_2} & m^2_{\phi_2} \end{pmatrix} =$$

$$\begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h, m_H:

$$m^2_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_h \leq M_Z$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{\mathrm{e}\, m_t}{2 M_W s_\mathrm{W}}, \frac{\mathrm{e}\, m_t^2}{M_W s_\mathrm{W}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections
> (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete 1-loop and 'almost complete' 2-loop and very leading 3-loop result available

What is a mass

Definition: The mass of a particle is the pole of the propagator

Example: scalar particle

Propagator:

$$\frac{i}{q^2 - m^2}$$

q^2 : four-momentum squared
m^2: constant in the Lagrangian

If one chooses $q^2 = m^2$ then the propagator has a pole.
This q^2 is then the mass of the particle.

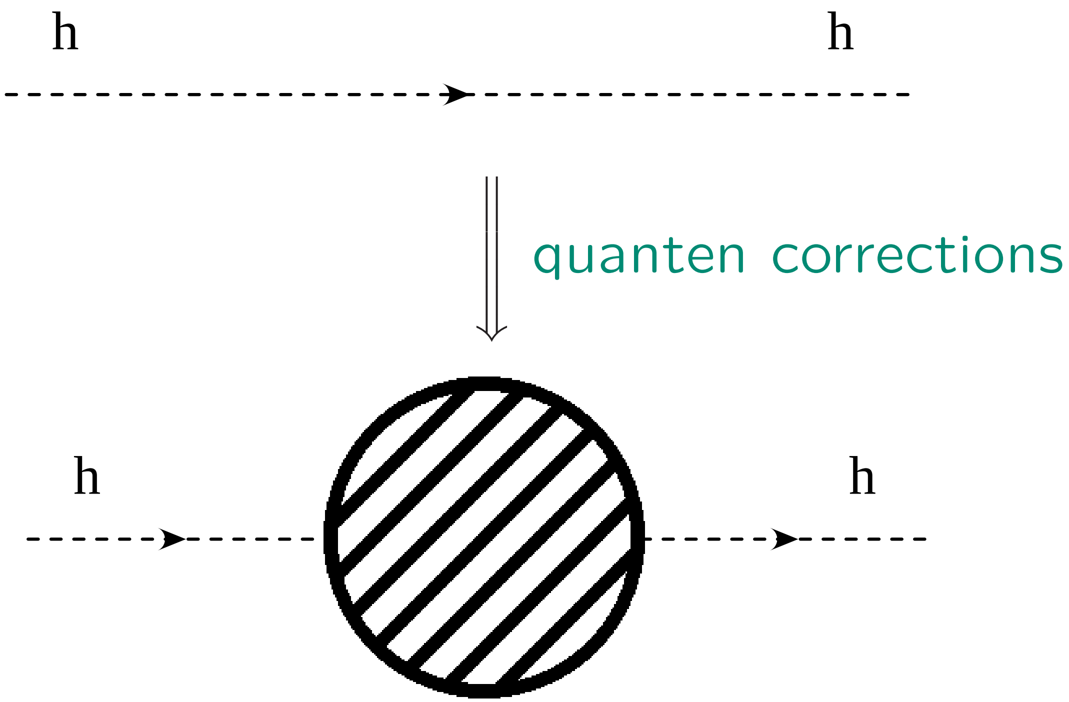
$\Rightarrow$ Pole of the propagator corresponds to zeroth of the inverse propagator.

Inverse propagator:

$$-i(q^2 - m^2)$$

Problem: quantum corrections

Higgs propagator:

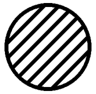


Inverse propagator:

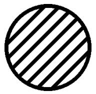
$$-i(q^2 - m^2) \quad \longrightarrow \quad -i\left(q^2 - m^2 + \hat{\Sigma}_h(q^2)\right)$$

$\hat{\Sigma}_h(q^2)$: renormalized Higgs self-energy

Calculation of the blob:

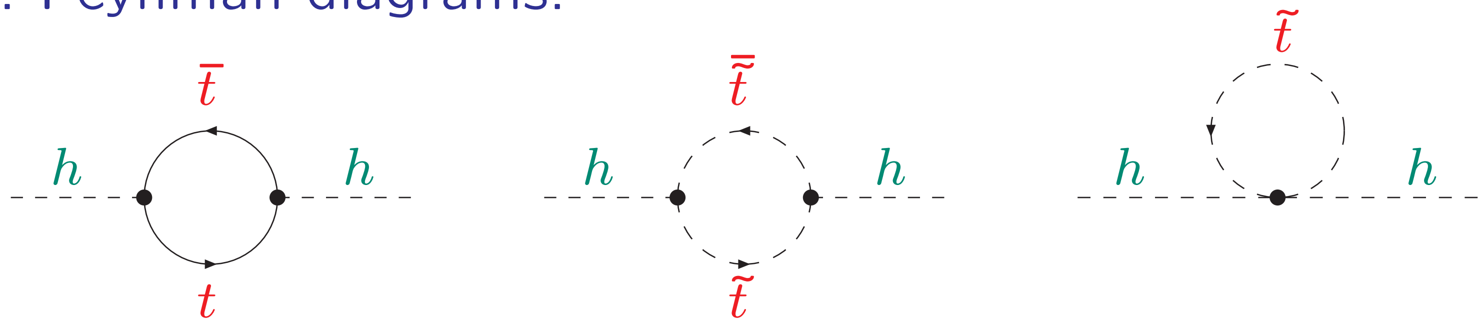


$$= \hat{\Sigma}(q^2) = \hat{\Sigma}^{(1)}(q^2) + \hat{\Sigma}^{(2)}(q^2) + \ldots$$



: all MSSM particles contribute
main contribution: $t/\tilde{t}$ sector ($\tilde{t}$: scalar top, SUSY partner of the t)

1-Loop: Feynman diagrams:



Dominant 1-loop corrections: $\Delta m_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

size of the corrections: $\mathcal{O}$(50 GeV)

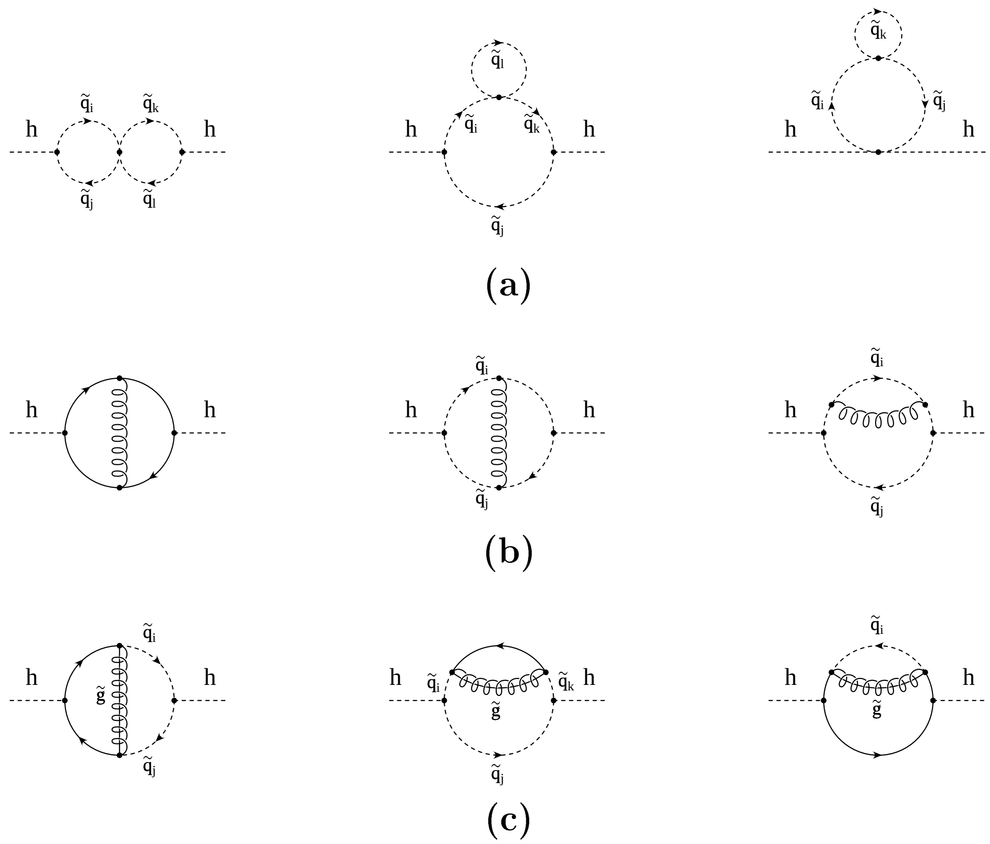
$\Rightarrow$ 2-Loop calculation necessary!

2-loop: $\hat{\Sigma}^{(2)}(0)$

[S. H., W. Hollik, G. Weiglein '98]

dominant contributions of $\mathcal{O}(\alpha_t \alpha_s)$:

(a) pure scalar diagrams
(b) diagrams with gluonexchange
(c) diagrams with gluinoexchange



Quite complicated calculation . . .
⇒ Need for computer algebra
programms

['98 - '09:] ⇒ many more corrections
calculated!

End of excursion: Higgs mass calculations

Upper bound on M_h in the MSSM:

"Unconstrained MSSM" :

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$M_h \lesssim 135 \text{ GeV}$$

for $m_t = 173.3 \pm 1.1\,\text{GeV}$

(including theoretical uncertainties from unknown higher orders)
⇒ observable at the LHC

Obtained with:

FeynHiggs

[*S.H., W. Hollik, G. Weiglein '98 – '02*]

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '03 – '09*]

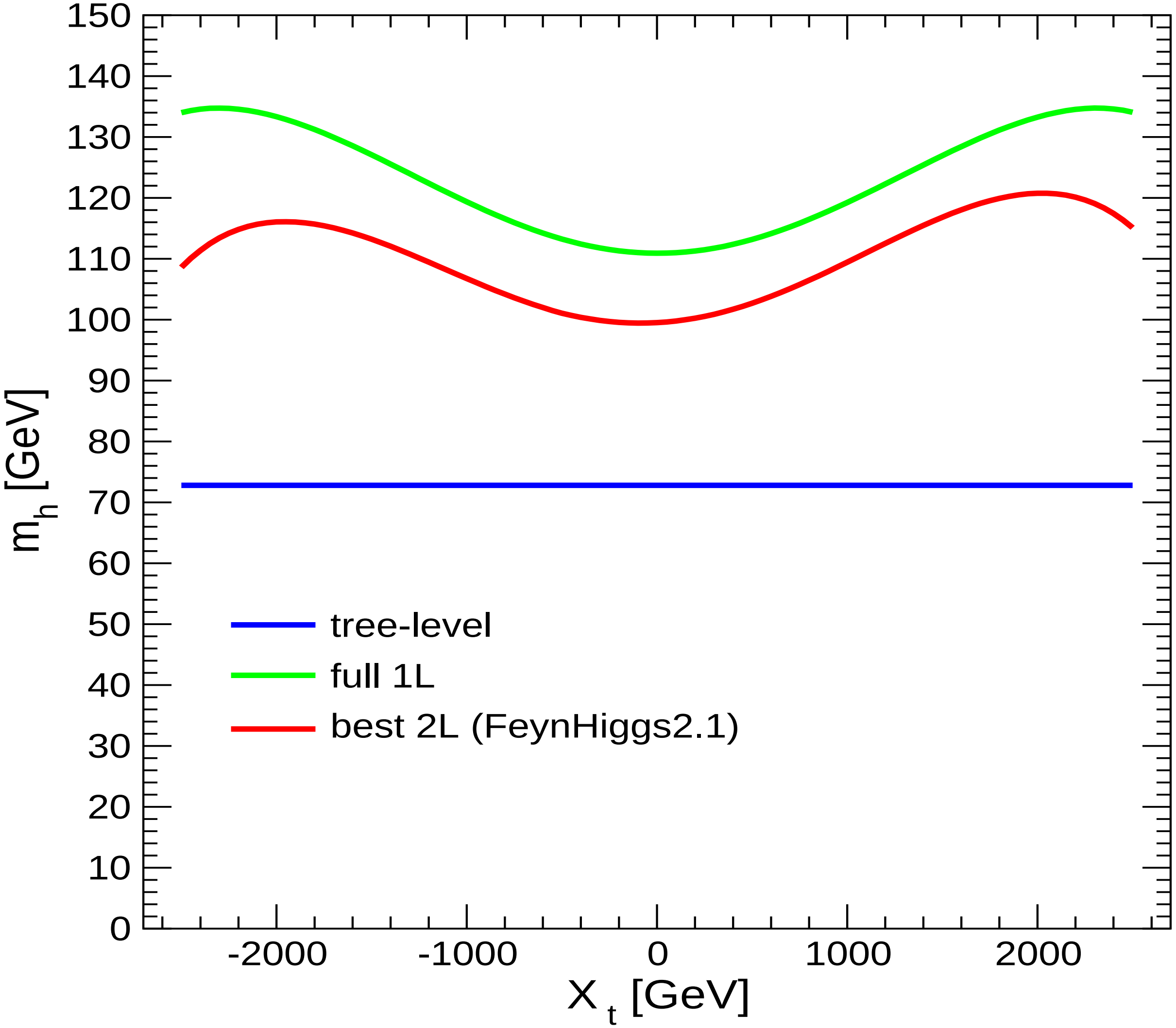
[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein, K. Williams '10*]

`www.feynhiggs.de`

→ all Higgs masses, couplings, BRs (easy to link, easy to use :-)

Effects of the two-loop corrections to the lightest Higgs mass:

Example for one set of MSSM parameters



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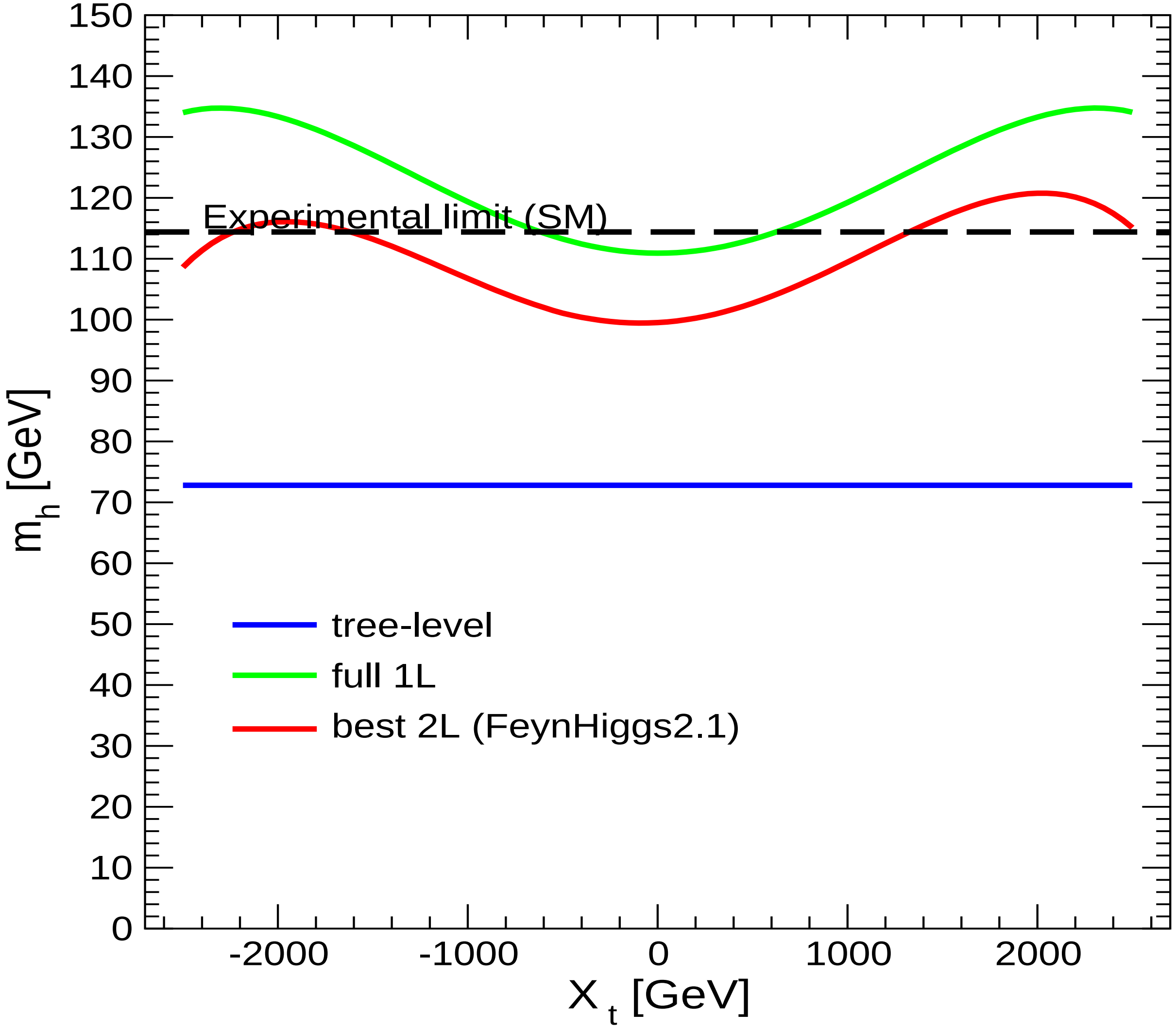

Comparison with experimental limits

$\Rightarrow$ strong impact on bound on SUSY parameters

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher-order corrections:
$\Rightarrow \Delta M_h \approx 2 - 3$ GeV

– From uncertainties in input parameters

$m_t, \ldots, M_A, \tan\beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$

$\Delta m_t \approx 1$ GeV $\Rightarrow \Delta M_h \approx 1$ GeV

$\Rightarrow$ crucial for (future) SUSY fits!