


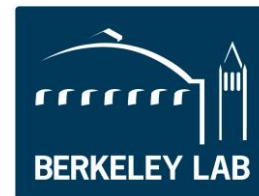
Connecting fluctuation measurements in heavy-ion collisions with the grand-canonical susceptibilities

Volodymyr Vovchenko (LBNL)

 **SHINE** NA61/SHINE theory meeting (virtual)

May 7, 2020

with Oleh Savchuk, Roman Poberezhnyuk, Mark Gorenstein, and Volker Koch, [arXiv:2003.13905](https://arxiv.org/abs/2003.13905)



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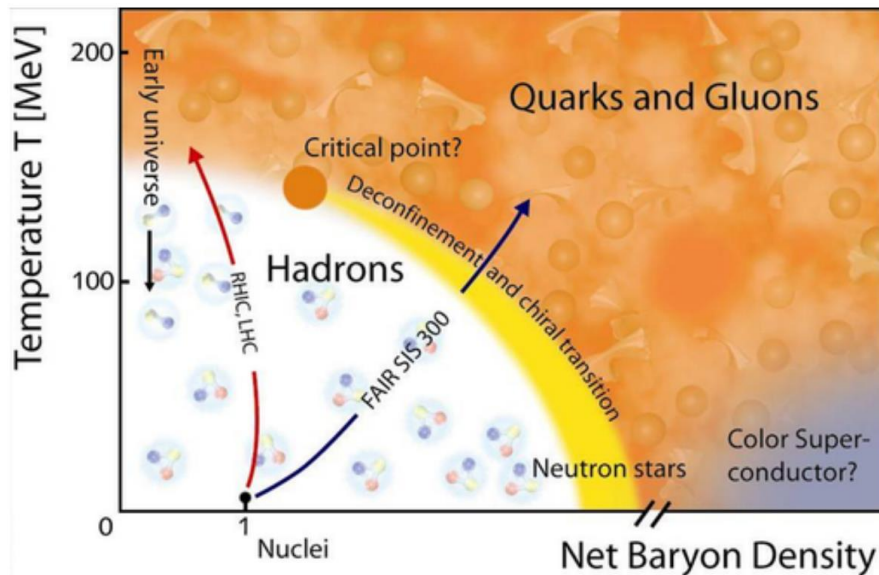
Alexander von Humboldt
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Strongly interacting matter

- Theory of strong interactions: *Quantum Chromodynamics* (QCD)

$$\mathcal{L} = \sum_{q=u,d,s,\dots} \bar{q} \left[i\gamma^\mu (\partial_\mu - igA_\mu^a \lambda_a) - m_q \right] q - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Basic degrees of freedom: quarks and gluons
- At smaller energies confined into baryons (qqq) and mesons ($q\bar{q}$)



Where is it relevant?

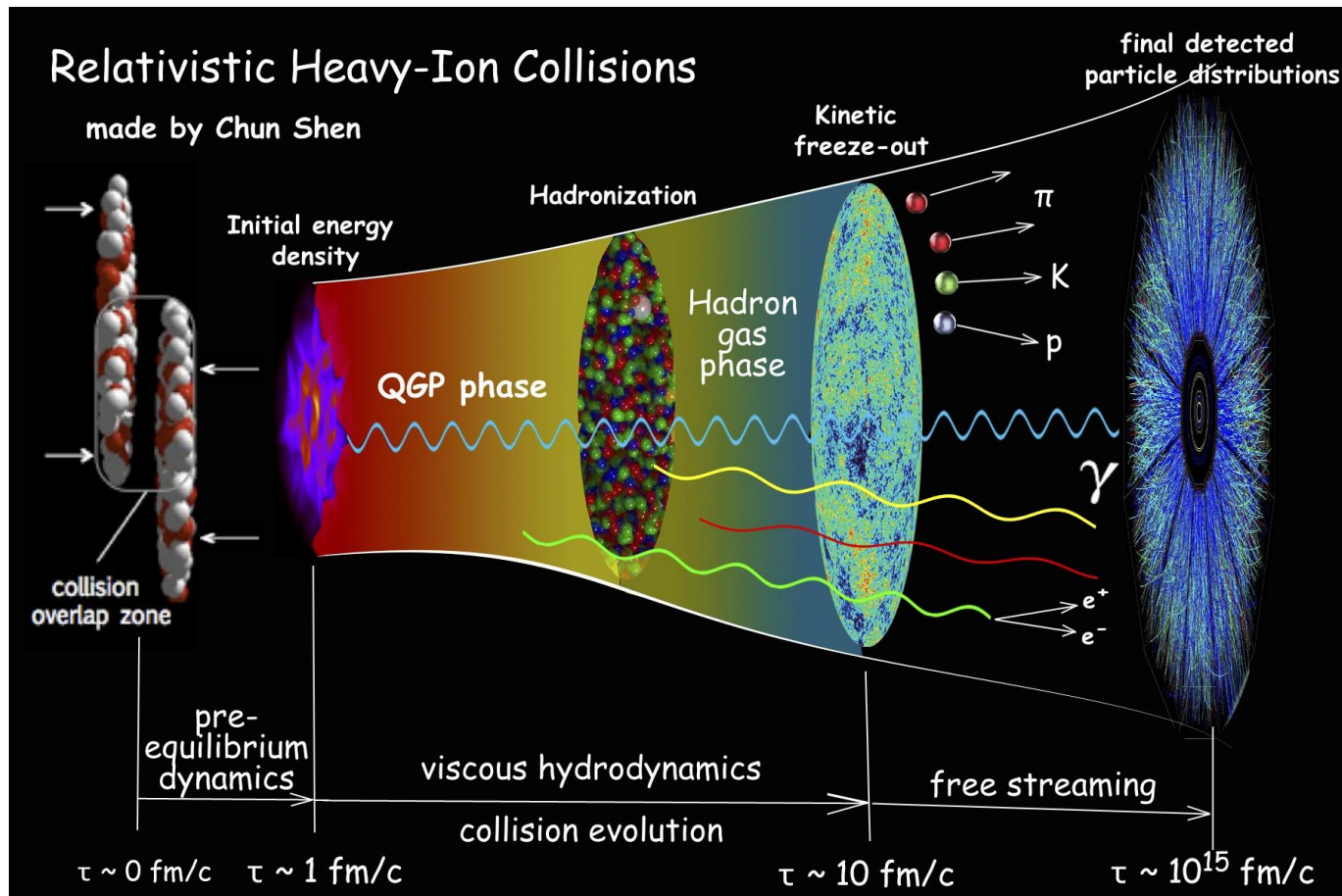
- Early universe
- Neutron star (mergers)
- **Heavy-ion collisions** (lab!)

Length scale: $1 \text{ fm} = 10^{-15} \text{ m}$

Energy scale: $100 \text{ MeV} = 10^{12} \text{ K}$

$$\hbar = c = k_B = 1$$

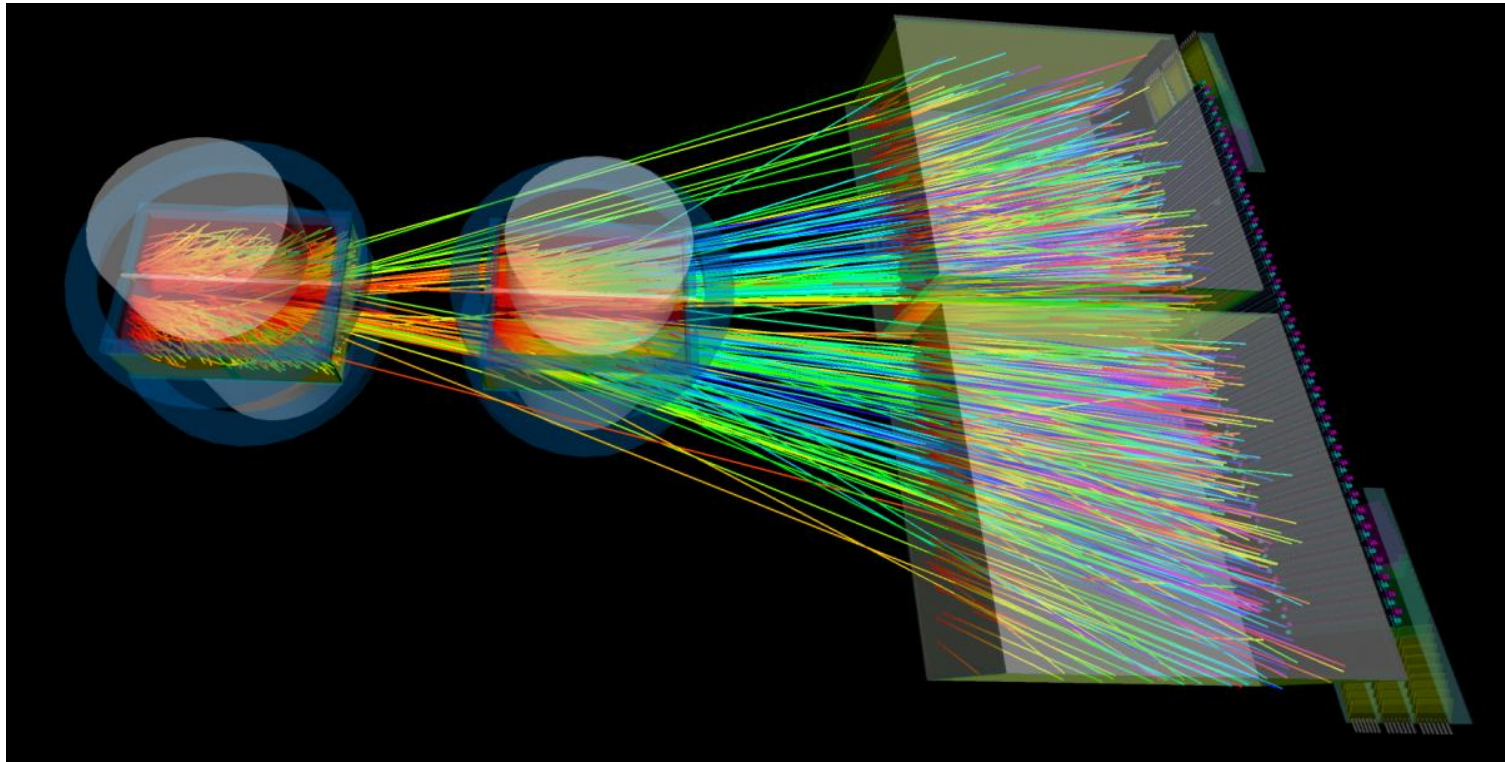
Relativistic heavy-ion collisions



Heavy-ion collision experiments study properties of strongly interacting matter (QCD) at extreme temperatures and densities, recreate conditions present in the Early Universe

Facilities: CERN-LHC, BNL-RHIC, CERN-SPS, FAIR-GSI, ...

Relativistic heavy-ion collisions



Xe+La collision at 150A GeV/c (© NA61/SHINE collaboration)

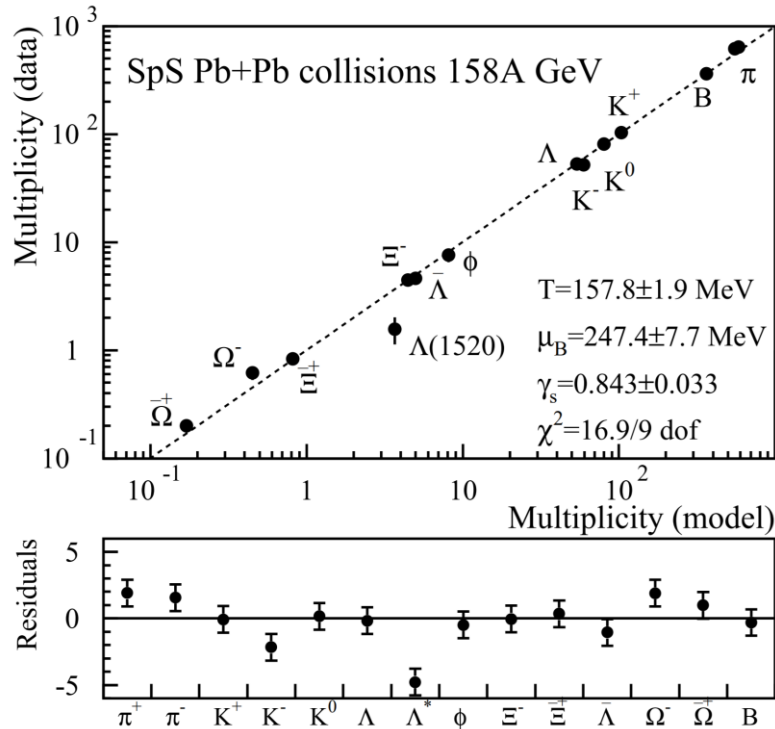
Large number of particles created in relativistic heavy-ion collisions



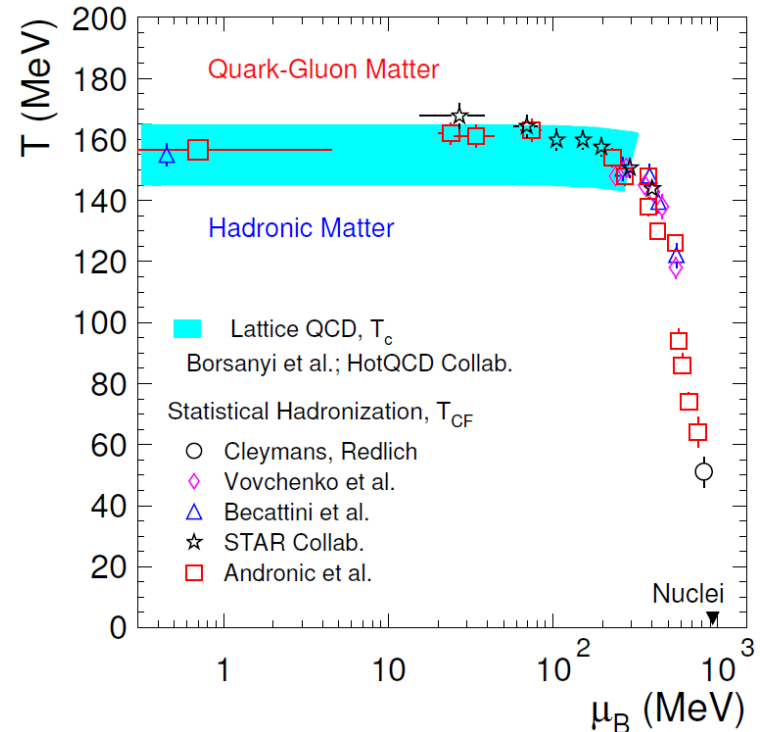
Apply concepts of statistical mechanics

Thermal hadron yields

HRG model: Equation of state of hadronic matter as a multi-component non-interacting gas of known hadrons and resonances



Becattini, Gazdzicki, Keranen, Manninen, Stock, PRC '04



A. Andronic et al., Nature 561, 321 (2018)

Analysis of hadron yields establishes evidence for chemical equilibration of matter produced in heavy-ion collisions and maps the experiments on the QCD phase diagram

From yields to event-by-event fluctuations

Hadron multiplicities can be computed in HRG, but not in (lattice) QCD. Analysis is dependent on the HRG model accuracy and assumptions/parameters in it. HRG analysis cannot give a direct evidence for chiral/deconfinement/QGP transition.

From yields to event-by-event fluctuations

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Event-by-event fluctuations:

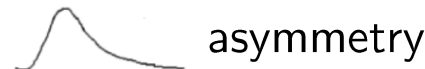
Consider not only first moments (yields) of N but also its *fluctuations*

Cumulants:
$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

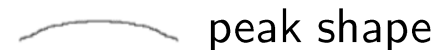
variance $\kappa_2 = \langle (\Delta N)^2 \rangle = \sigma^2$



skewness $\kappa_3 = \langle (\Delta N)^3 \rangle$



kurtosis $\kappa_4 = \langle (\Delta N)^4 \rangle - 3 \langle (\Delta N^2) \rangle^2$



Grand-canonical ensemble:
$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N} Z^{\text{ce}}(T, V, N) \right], \quad \kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial (\mu_N)^n}$$

Fluctuations probe finer details of the (QCD) equation of state

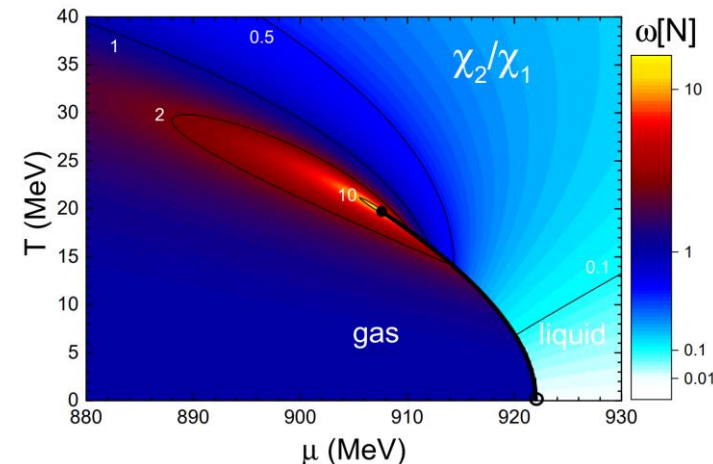
Event-by-event fluctuations: Motivations

Grand-canonical ensemble: $\kappa_n = \frac{1}{VT^3} \chi_B^n(T, \mu), \quad \chi_B^n(T, \mu) = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n}$

- Search for the QCD critical point $\kappa_2 \sim \xi^2, \quad \xi \rightarrow \infty$ [M. Stephanov, PRL '09]

- Scan $T - \mu_B$ looking for a “hill in fluctuations”
NA61/SHINE program

- Look for non-monotonic energy dependence of higher-order cumulants
STAR-BES program



V.V. et al, PRC '15

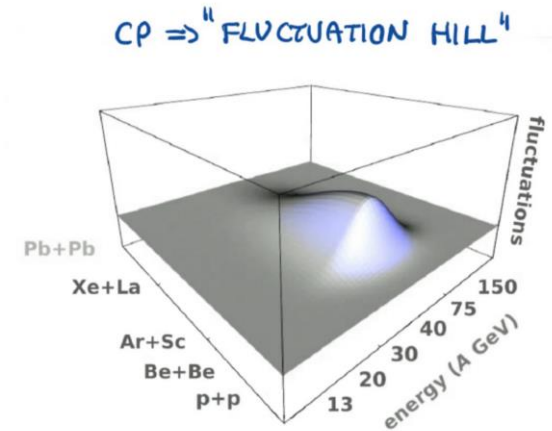
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M. Gazdzicki, CPOD2016

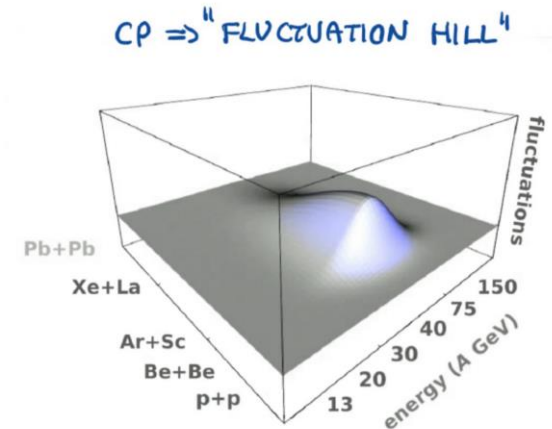
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M. Gazdzicki, CPOD2016

- Observables that can be compared with first-principle lattice QCD predictions (fluctuations of conserved charges)
 - Direct comparisons of experimental data with grand-canonical fluctuations from different theories is commonplace: lattice QCD (Wuppertal-Budapest; HotQCD), HRG (Houston group; Nahrgang, Bluhm;...), effective QCD approaches (Fischer et al.; Pawłowski et al.), ...

Theory vs experiment: Caveats

- proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)
Asakawa, Kitazawa, PRC '12; **V.V.**, Jiang, Gorenstein, Stoecker, PRC '18
- volume fluctuations
Gorenstein, Gazdzicki, PRC '11; Skokov, Friman, Redlich, PRC '13;
Braun-Munzinger, Rustamov, Stachel, NPA '17
- non-equilibrium (memory) effects
Mukherjee, Venugopalan, Yin, PRC '15
- final-state interactions in the hadronic phase
Steinheimer, **V.V.**, Aichelin, Bleicher, Stoecker, PLB '18
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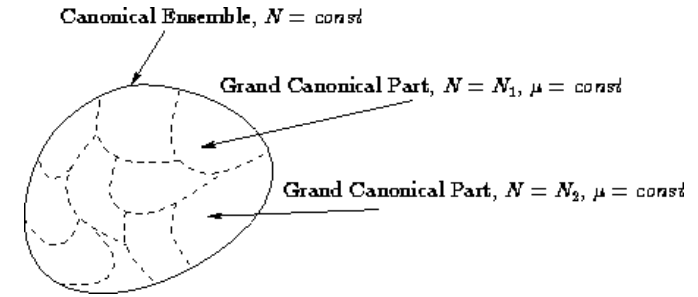
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Canonical vs grand-canonical

Grand-canonical ensemble: the system exchanges conserved charges with a heat bath

Canonical ensemble: conserved charges fixed to a same set of values in all microstates

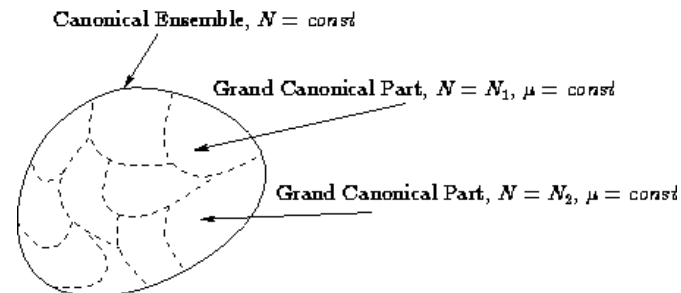
Thermodynamic equivalence: in the limit $V \rightarrow \infty$ all statistical ensembles are equivalent wrt to all average quantities, e.g. $\langle N \rangle_{GCE} = N_{CE}$



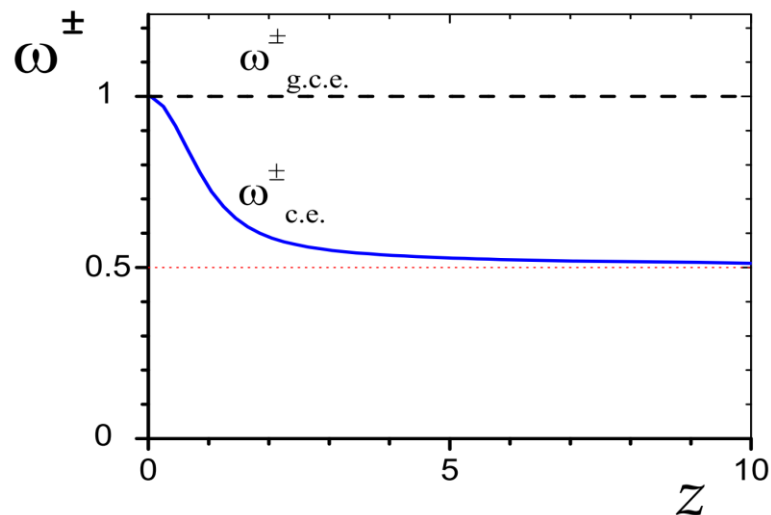
Canonical vs grand-canonical

Grand-canonical ensemble: the system exchanges conserved charges with a heat bath

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Begun, Gorenstein, Gazdzicki, Zozulya, PRC '04

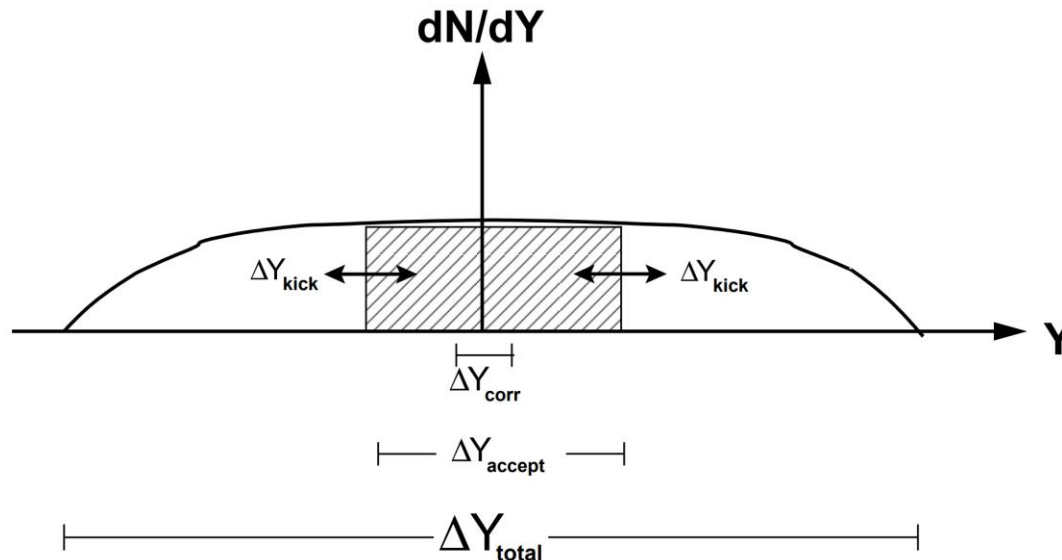
Thermodynamic equivalence does *not* extend to **fluctuations**. The results are **ensemble-dependent** in the limit $V \rightarrow \infty$

So what ensemble should one use?

Canonical? Grand-canonical?
Something else?

Applicability of the GCE in heavy-ion collisions

Experiments measure fluctuations in a finite momentum acceptance



V. Koch, 0810.2520

GCE applies if $\Delta Y_{\text{total}} \gg \Delta Y_{\text{accept}} \gg \Delta Y_{\text{kick}}, \Delta Y_{\text{corr}}$ and momentum-space correlation is strong (e.g. Bjorken flow)

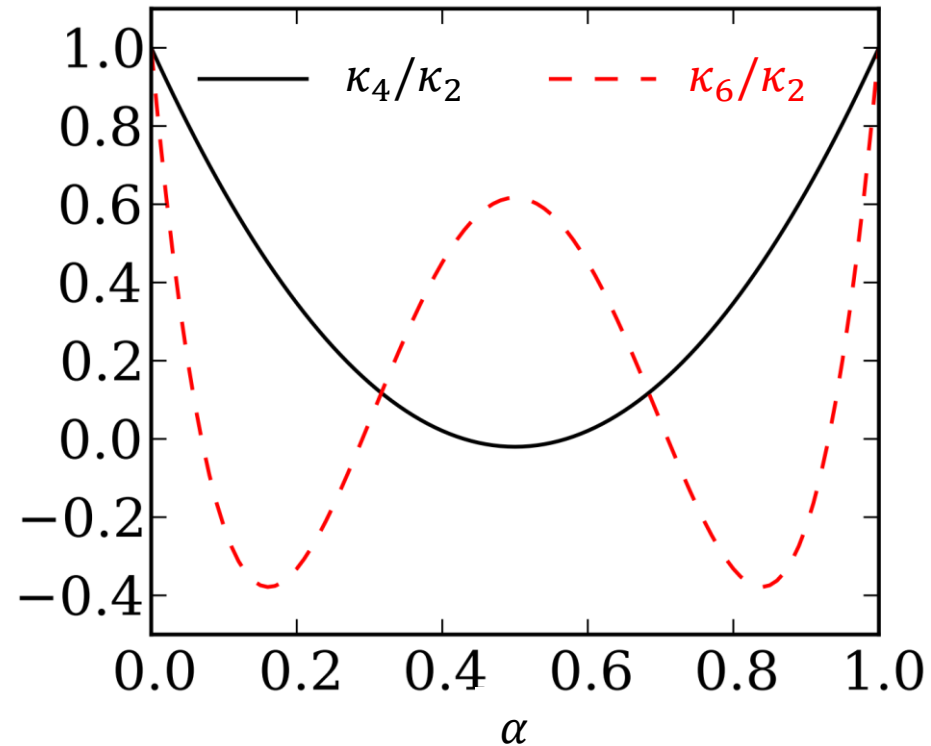
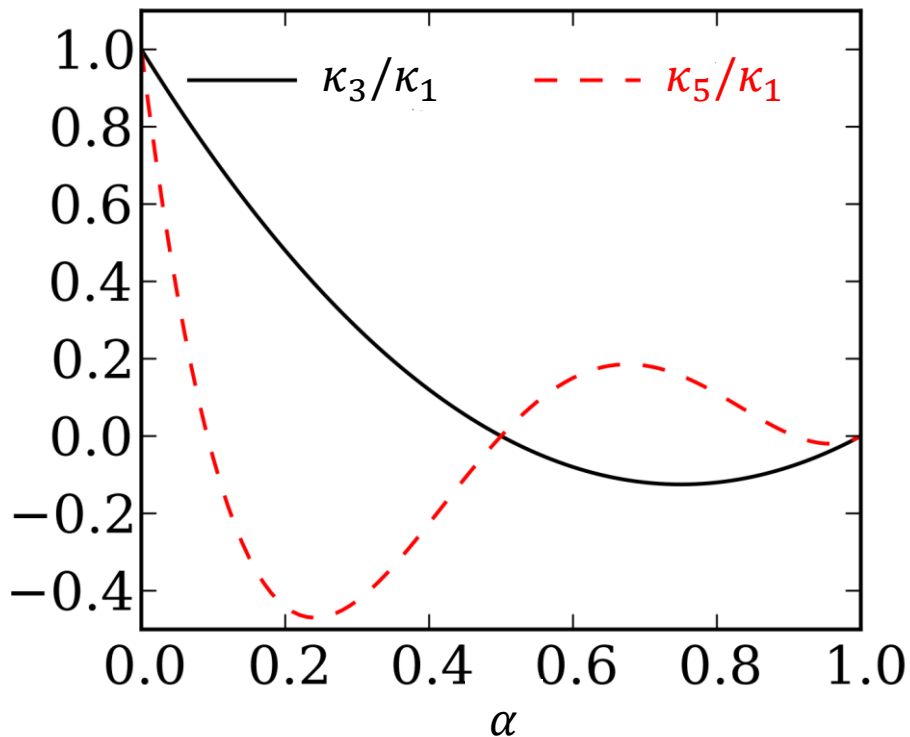
In practice difficult to satisfy all conditions simultaneously...

If $\Delta Y_{\text{total}} \gg \Delta Y_{\text{accept}}$ does not hold, corrections from global conservation appear

Baryon number conservation in an ideal gas

Pilot study for an ideal gas of baryons and antibaryons [Bzdak, Skokov, Koch, PRC '13]

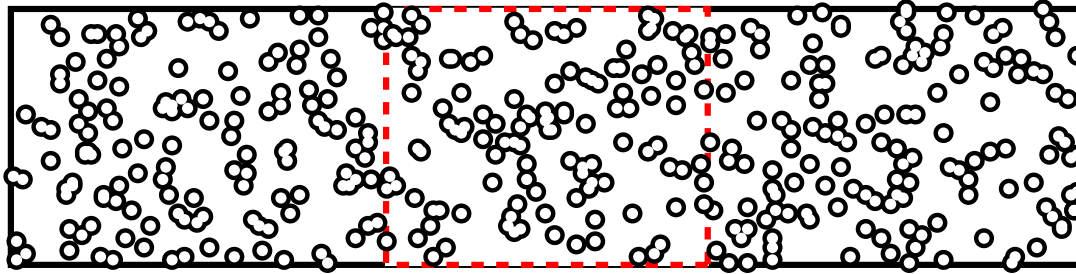
No spatial correlations \Rightarrow Binomial filter applies



$\langle N_B \rangle = 400$, $\langle N_{\bar{B}} \rangle = 100$, $\alpha = \text{fraction of the full acceptance} = \Delta Y_{\text{accept}} / \Delta Y_{\text{total}}$

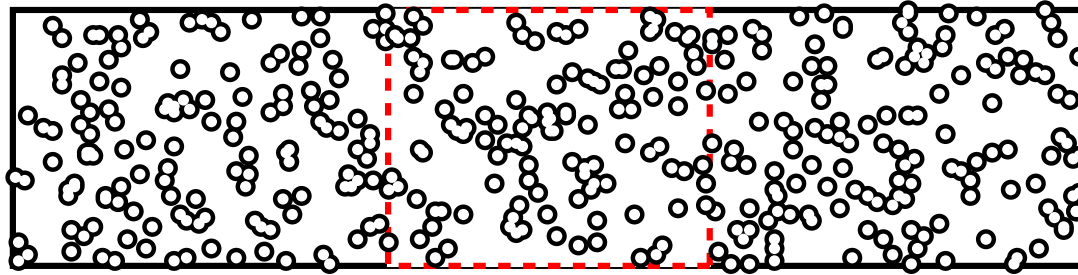
Global conservation effects are huge for higher-order cumulants already at “small” α

Binomial acceptance vs actual acceptance



Binomial acceptance: accept each particle (charge) with a probability α independently from all other particles

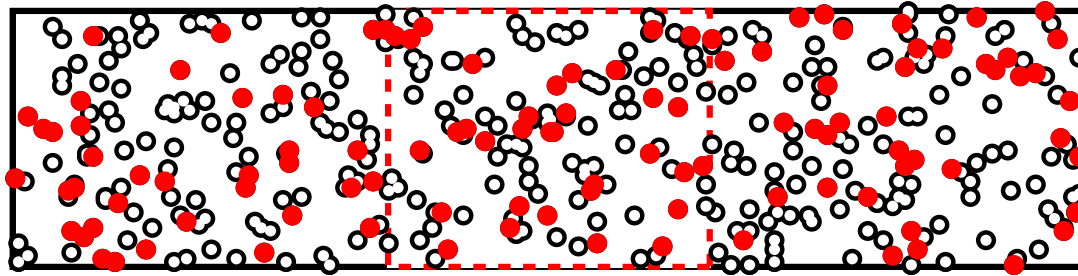
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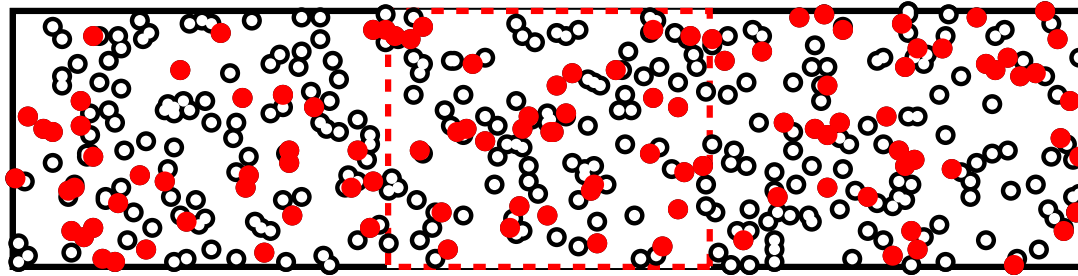
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The binomial acceptance will not provide the correct result (except for a gas of uncorrelated particles)

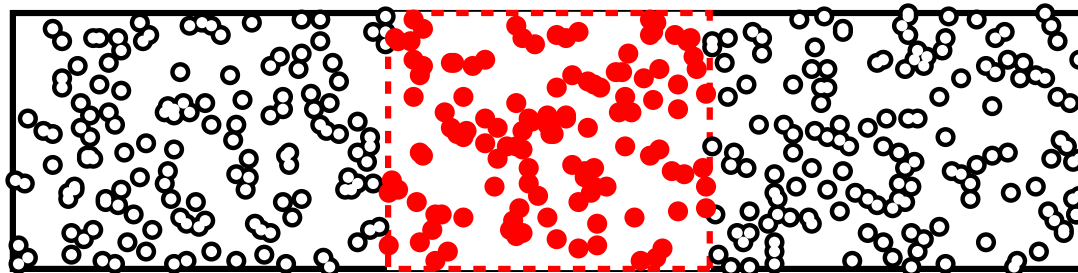
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What we really need is



Subensemble acceptance method

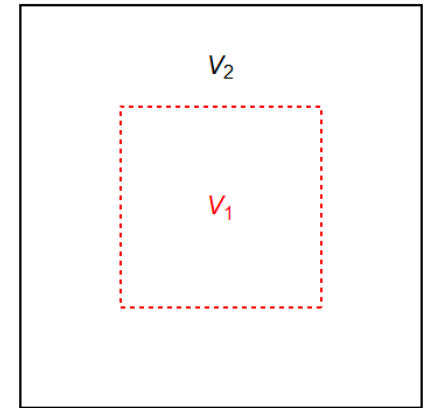
Partition a thermal system with a globally conserved charge B (*canonical ensemble*) into two subsystems which can exchange the charge

Neglect surface effects:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{V}_{1,2} \approx \hat{H}_1 + \hat{H}_2$$

The canonical partition function then reads:

$$Z^{\text{ce}}(T, V, B) = \text{Tr} e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{\text{ce}}(T, V_1, B_1) Z^{\text{ce}}(T, V - V_1, B - B_1)$$



$$V_1 + V_2 = V$$

The probability to have charge B_1 is:

$$P(B_1) \propto Z^{\text{ce}}(T, \alpha V, B_1) Z^{\text{ce}}(T, (1 - \alpha)V, B - B_1), \quad \alpha \equiv V_1/V$$

If the canonical partition function known, B_1 -cumulants can be calculated explicitly

Subensemble acceptance: Thermodynamic limit

In the thermodynamic limit, $V \rightarrow \infty$, Z^{ce} expressed through free energy density

$$Z^{ce}(T, V, B) \stackrel{V \rightarrow \infty}{\simeq} \exp \left[-\frac{V}{T} f(T, \rho_B) \right]$$

B_1 cumulant generating function:

$$G_{B_1}(t) \equiv \ln \langle e^{t B_1} \rangle = \ln \left\{ \sum_{B_1} e^{t B_1} \exp \left[-\frac{\alpha V}{T} f(T, \rho_{B_1}) \right] \exp \left[-\frac{\beta V}{T} f(T, \rho_{B_2}) \right] \right\} + \tilde{C}$$

B_1 cumulants:

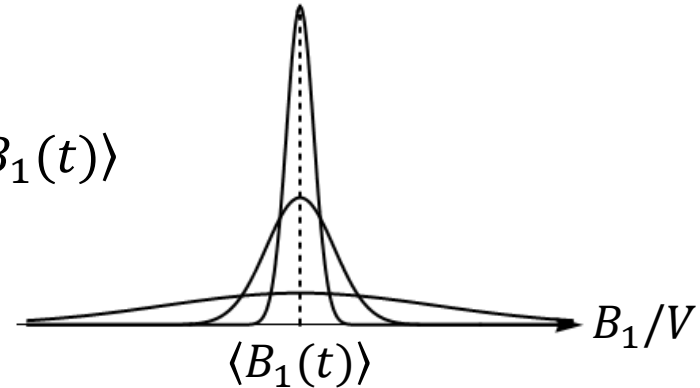
$$\kappa_n[B_1] = \left. \frac{\partial^n G_{B_1}(t)}{\partial t^n} \right|_{t=0} \equiv \tilde{\kappa}_n[B_1(t)]|_{t=0} \quad \text{or} \quad \kappa_n[B_1] = \left. \frac{\partial^{n-1} \tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}} \right|_{t=0}$$

All κ_n can be calculated by determining the t -dependent first cumulant $\tilde{\kappa}_1[B_1(t)]$

Subensemble acceptance: Thermodynamic limit

$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1 \tilde{P}(B_1; t)}{\sum_{B_1} \tilde{P}(B_1; t)} \equiv \langle B_1(t) \rangle \quad \text{with} \quad \tilde{P}(B_1; t) = \exp \left\{ tB_1 - V \frac{\alpha f(T, \rho_{B_1}) + \beta f(T, \rho_{B_2})}{T} \right\}.$$

Thermodynamic limit: $\tilde{P}(B_1; t)$ highly peaked at $\langle B_1(t) \rangle$



$\langle B_1(t) \rangle$ is a solution to equation $d\tilde{P}/dB_1 = 0$:

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)]$$

where $\hat{\mu}_B \equiv \mu_B/T$, $\mu_B(T, \rho_B) = \partial f(T, \rho_B)/\partial \rho_B$

$t = 0$: $\rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. conserved charge uniformly distributed between the two subsystems

Subensemble acceptance: $\kappa_2[B_1]$

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)] \quad (*)$$

$$\frac{\partial(*)}{\partial t} : \quad 1 = \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B_1}} \right)_T \left(\frac{\partial \rho_{B_1}}{\partial \langle B_1 \rangle} \right)_V \frac{\partial \langle B_1 \rangle}{\partial t} - \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B_2}} \right)_T \left(\frac{\partial \rho_{B_2}}{\partial \langle B_2 \rangle} \right)_V \frac{\partial \langle B_2 \rangle}{\partial \langle B_1 \rangle} \frac{\partial \langle B_1 \rangle}{\partial t}$$

$$\left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B_{1,2}}} \right)_T \equiv [\chi_2^B(T, \rho_{B_{1,2}}) T^3]^{-1}, \quad \rho_{B_1} \equiv \frac{\langle B_1 \rangle}{\alpha V}, \quad \rho_{B_2} \equiv \frac{\langle B_2 \rangle}{(1-\alpha)V}, \quad \langle B_2 \rangle = B - \langle B_1 \rangle, \quad \frac{\partial \langle B_1 \rangle}{\partial t} \equiv \tilde{\kappa}_2[B_1(t)]$$

Solve the equation for $\tilde{\kappa}_2$:

$$\tilde{\kappa}_2[B_1(t)] = \frac{V T^3}{[\alpha \chi_2^B(T, \rho_{B_1})]^{-1} + [(1-\alpha) \chi_2^B(T, \rho_{B_2})]^{-1}}$$

$t = 0$:

$$\kappa_2[B_1] = \alpha (1 - \alpha) V T^3 \chi_2^B$$

Higher-order cumulants: iteratively differentiate $\tilde{\kappa}_2$ w.r.t. t

Subensemble acceptance: Full result up to κ_6

$$\kappa_1[B_1] = \alpha VT^3 \chi_1^B$$

$$\kappa_2[B_1] = \alpha VT^3 \beta \chi_2^B$$

$$\kappa_3[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \chi_3^B$$

$$\kappa_4[B_1] = \alpha VT^3 \beta \left[\chi_4^B - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B \chi_4^B}{\chi_2^B} \right]$$

$$\kappa_5[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \left\{ [1 - 2\beta\alpha] \chi_5^B - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B} \right\}$$

$$\kappa_6[B_1] = \alpha VT^3 \beta [1 - 5\alpha\beta(1 - \alpha\beta)] \chi_6^B + 5 VT^3 \alpha^2 \beta^2 \left\{ 9\alpha\beta \frac{(\chi_3^B)^2 \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} \right. \\ \left. - 2(1 - 2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1 - 3\beta\alpha] \frac{\chi_3^B \chi_5^B}{\chi_2^B} \right\}$$

$$\beta = 1 - \alpha$$

$$\chi_n^B = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n} \quad - \text{grand-canonical susceptibilities}$$

Subensemble acceptance: Cumulant ratios

Some common cumulant ratios:

scaled variance $\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1 - \alpha) \frac{\chi_2^B}{\chi_1^B},$

skewness $\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1 - 2\alpha) \frac{\chi_3^B}{\chi_2^B},$

kurtosis $\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta \left(\frac{\chi_3^B}{\chi_2^B} \right)^2 .$

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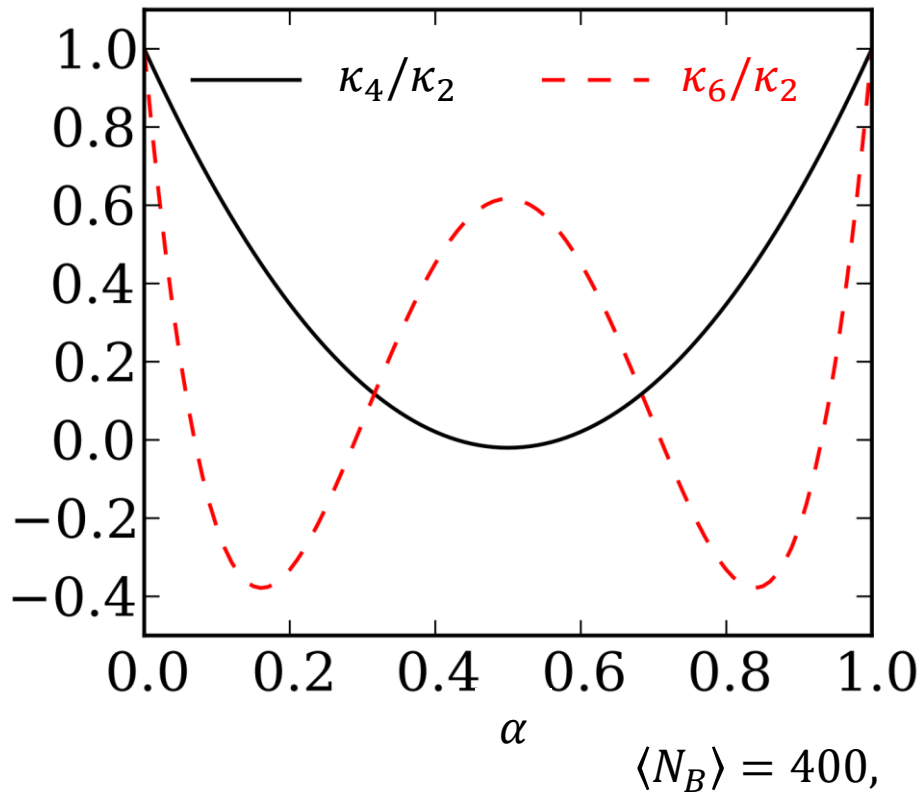
kurtosis $\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta \left(\frac{\chi_3^B}{\chi_2^B} \right)^2.$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \rightarrow 0$ – GCE limit
- $\alpha \rightarrow 1$ – CE limit
- Charge conservation suppresses the magnitude of scaled variance and skewness

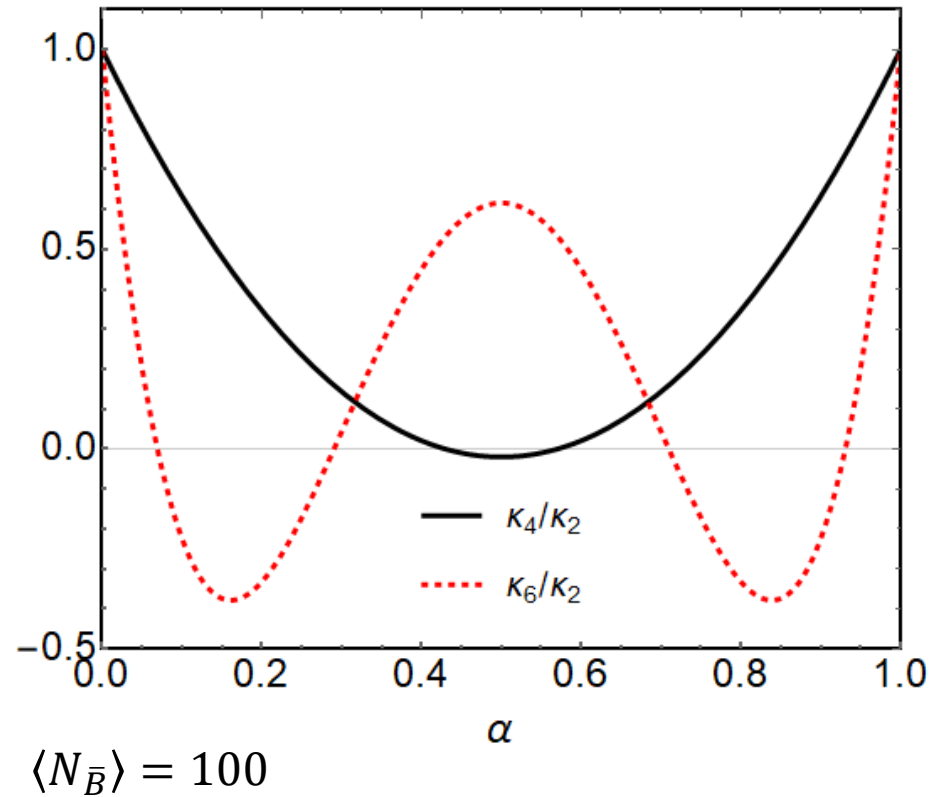
Subensemble acceptance: ideal gas

Ideal gas of baryons and antibaryons: $\chi_{2n}^B \propto \langle N_B \rangle + \langle N_{\bar{B}} \rangle$, $\chi_{2n-1}^B \propto \langle N_B \rangle - \langle N_{\bar{B}} \rangle$

Binomial acceptance [Bzdak et al., PRC '13]



Subensemble acceptance [V.V. et al., 2003.13905]

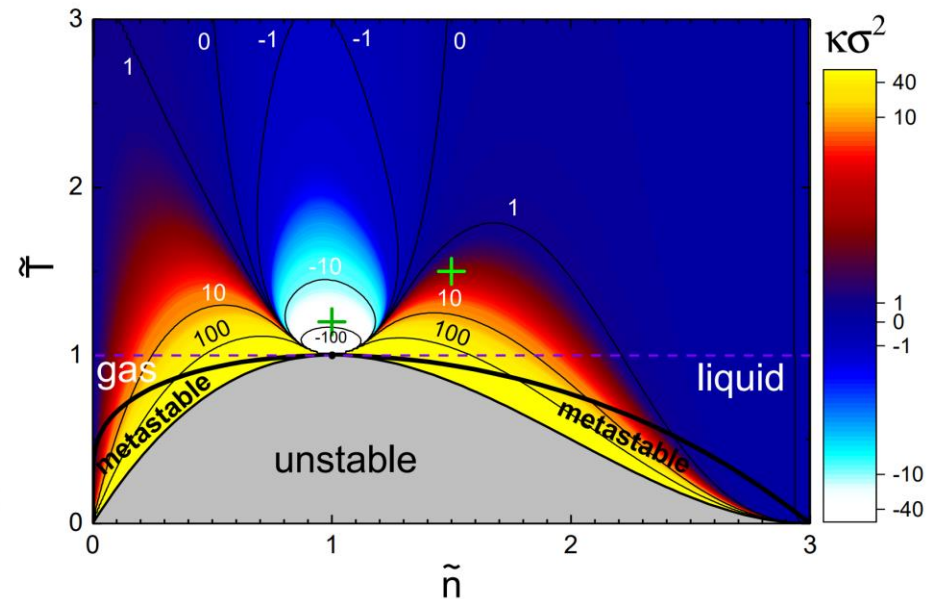
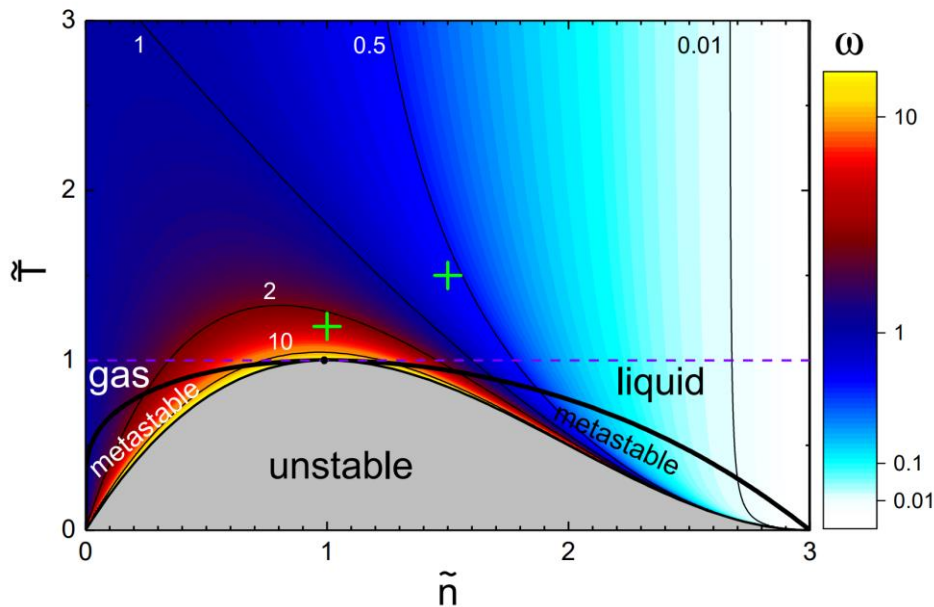


Subensemble acceptance: van der Waals fluid

van der Waals equation of state: first-order phase transition and a **critical point**

$$Z_{\text{vdW}}^{\text{ce}}(T, V, N) = Z_{\text{id}}^{\text{ce}}(T, V - bN, N) \exp\left(\frac{aN^2}{VT}\right) \theta(V - bN)$$

Rich structures in cumulant ratios close to the CP



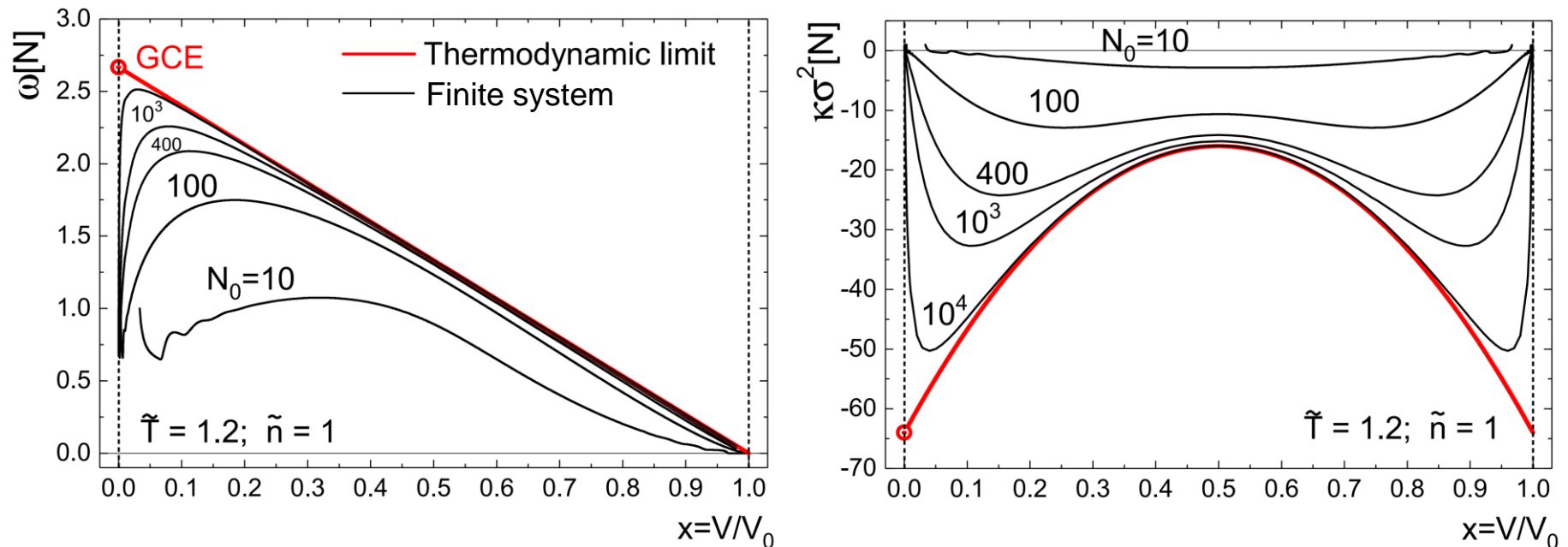
V.V., Poberezhnyuk, Anchishkin, Gorenstein, 1507.06537

Subensemble acceptance: van der Waals fluid

Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z_{\text{vdW}}^{\text{ce}}(T, xV_0, N) Z_{\text{vdW}}^{\text{ce}}(T, (1-x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results

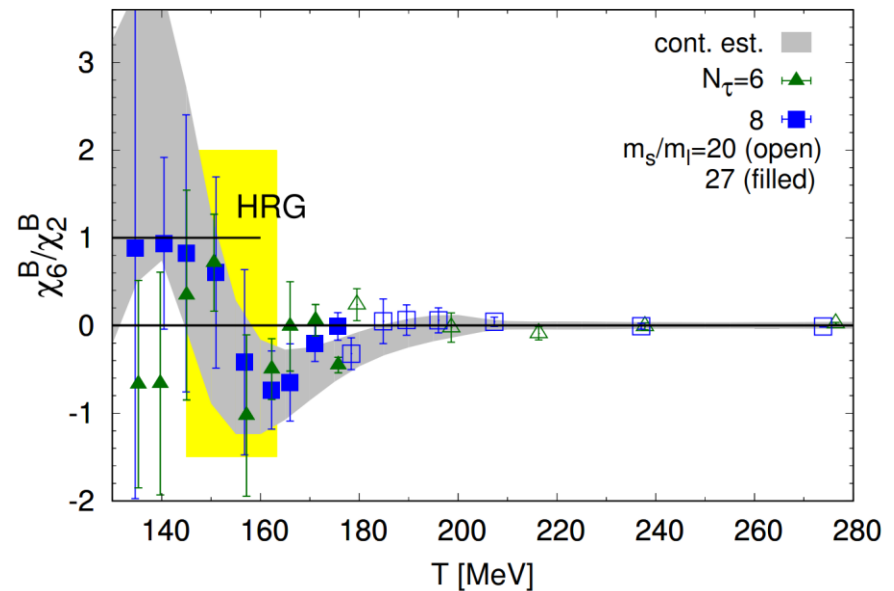
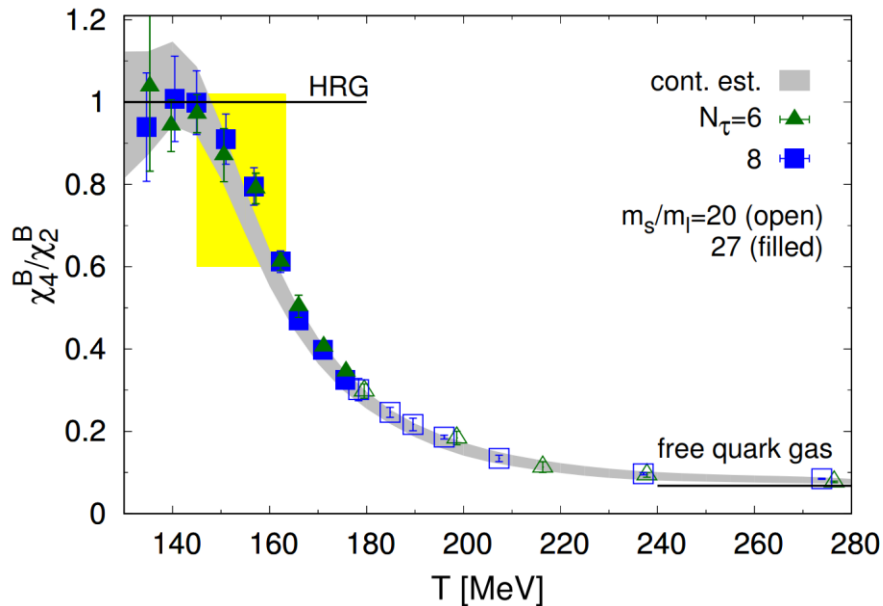


Results agree with subensemble acceptance in thermodynamic limit ($N_0 \rightarrow \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length ξ

Net baryon fluctuations at LHC and top RHIC

Freeze-out temperatures $T_{ch} \approx 155-165$ MeV at LHC and RHIC are very close to the pseudocritical chiral crossover transition temperature from lattice



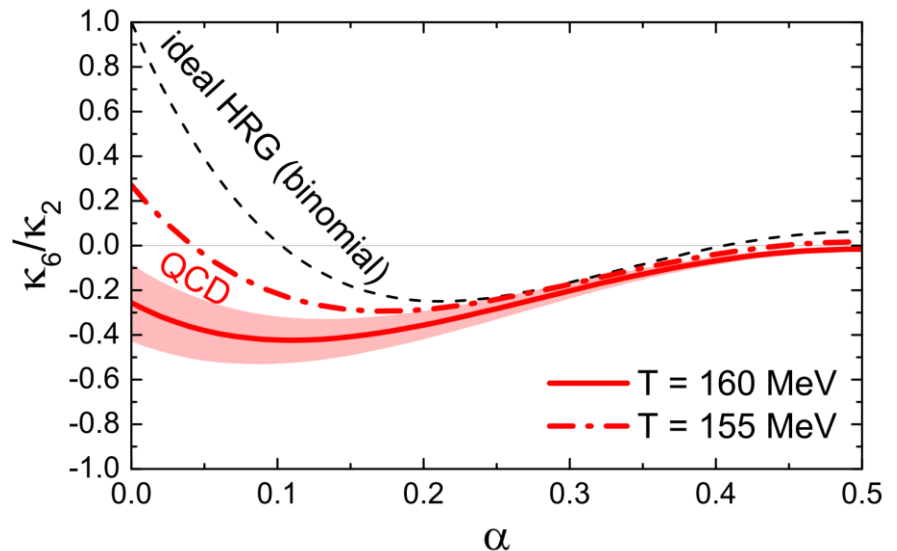
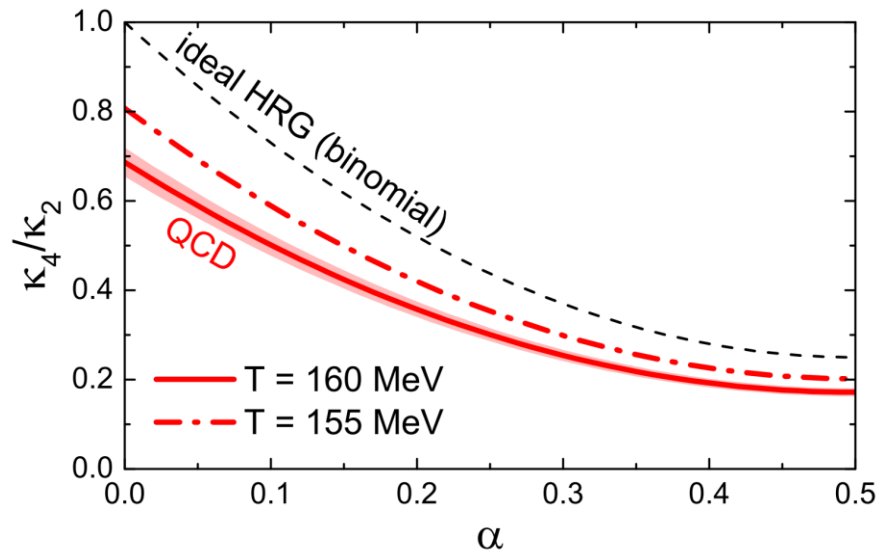
Bazavov et al. (HotQCD), 1701.04325

Dip in χ_6^B / χ_2^B down to negative values observed at T_{pc} , possibly related remnants of chiral criticality [Friman, Karsch, Skokov, Redlich, EPJC '11]

Can this be measured and if so, distinguished from baryon number conservation?

Net baryon fluctuations at LHC and top RHIC

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B} \quad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1 - 5\alpha\beta(1 - \alpha\beta)] \frac{\chi_6^B}{\chi_2^B} - 10\alpha(1 - 2\alpha)^2\beta \left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$



Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from [Borsanyi et al., 1805.04445](#)

For $\alpha > 0.2$ difficult to distinguish effects of the EoS and baryon conservation in χ_6^B/χ_2^B , $\alpha \leq 0.1$ is a sweet spot where measurements are mainly sensitive to the EoS

Estimates: $\alpha \approx 0.1$ corresponds to $\Delta Y_{acc} \approx 2(1)$ at LHC (RHIC)

Summary

- Subensemble acceptance procedure is a method to correct cumulants of distributions in heavy-ion collisions for global charge conservation
- The method works for an arbitrary equation of state and sufficiently large systems, such as created in central collisions of heavy ions
- Measurements of χ_6^B / χ_2^B at LHC and RHIC in acceptances up to $\alpha \approx 0.1$ reflect the grand-canonical QCD susceptibilities and will probe the chiral crossover transition

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Thanks for your attention!