Soft Fragmentation on the Celestial Sphere

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Fragmentation.

We wish to understand an object

Fragmentation.

- Take the object, and randomly break it.
- What do the pieces look like?

Fragmentation.

- Take the object, and randomly break it.
- What do the pieces look like?
- For QCD, the object is the total momentum of a hadron or hard interaction.
- The pieces are partons or hadrons.

Why Soft Fragmentation?

- Soft physics dominates, so phenomenology.

Motivation?

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- Space-Time Duality in QCD.

Motivation?

Why Soft Fragmentation?

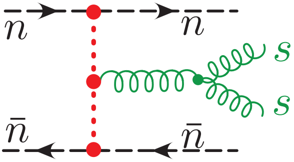
- Soft physics dominates, so phenomenology.
- Space-Time Duality in QCD.
- Frustration: what is angular ordering & MLLA *really?*

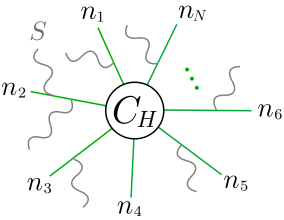
Space-Time Duality in QCD

Intuition: All phenomena in forward scattering or initial state physics has an exact counterpart in jet physics, with perhaps calculable conformal anomalies. Examples:

- Banfi-Marchesini-Smye Eq. and BK/B-JIMWLK Eqs. [**Hatta 0810.0889**]
- Rapidity and Threshold Anomalous Dimensions [**Li, Zhu 1604.01404, Vladimirov 1610.05791**]
- Asymptotics of Non-Global Logarithms to Gluon Saturation [**DN 1610.02031**]
- Reciprocity of Space-like and Time-like Fragmentation [**Basso, Korchemsky hep-th/0612247, Dokshitzer et al. hep-ph/0511302**]

Space-Time Duality in QCD



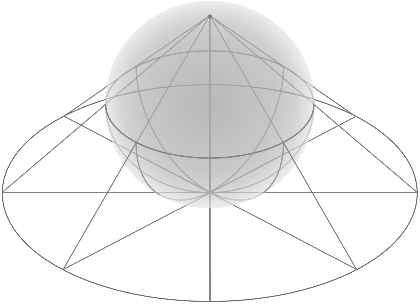


Soft processes in forward scattering

$\leftrightarrow$

Soft process in jet physics.

Stereographic projection of $S^2 \to \mathbb{R}^2$

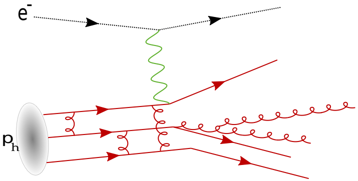


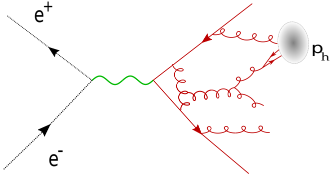
Eikonal lines of n and $\bar{n}$-collinear directions piercing the transverse plane of forward scattering,

$\leftrightarrow$

Eikonal lines emanating from a hard scattering vertex.

Then how is this duality realized for space-like and time-like parton branching?





That is, fragmentation.

Space-like and Time-like Fragmentation

Oldest Factorizations in QCD ($Q^2 = |q^2|$):

$$PDF: \; F(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} f_{i/h}\Big(\frac{x}{z},\mu^2\Big) C_i^S\Big(z,\mu^2,Q^2\Big), \quad x = \frac{Q^2}{2P_h\cdot q}$$

$$FF: \; D(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} d_{h/i}\Big(\frac{x}{z},\mu^2\Big) C_i^T\Big(z,\mu^2,Q^2\Big), \quad x = \frac{2P_h\cdot q}{Q^2}$$

- f : randomly break a hadron into partons.
- d : randomly break a parton into hadrons.
- x is the momentum fraction.

If we accept the Space-Time Duality,
$f\&d$ must map to each other, at least when $x \to 0$.

Moments

Often we will switch to moment space:

$$\bar{g}(n) = \int_0^1 dx x^n g(x),$$

$$xg(x) = \int_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} \bar{g}(n)$$

$$x \to 0 \text{ means } n \to 0$$

$$\frac{1}{n^\ell} \leftrightarrow \frac{1}{x} \ln^\ell x$$

$x \to 0$ for PDF

For PDF, soft limit is understood:

- **Rapidity Factorization** with Glauber modes by introducing "un-integrated" parton densities.
- "un-integrated" parton density obeys BFKL and DGLAP equations.
- Consistency resums DGLAP kernel.

Derivable in SCET + Glauber Potentials of Rothstein and Stewart.

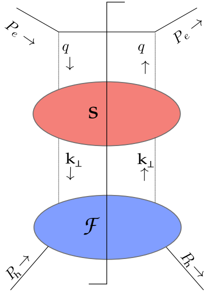
[DN, Pathak, Rothstein, Stewart, unpublished].

See talks in previous SCET workshops by Pathak.

Details for $x \to 0$ for PDF

Un-integrated PDF: $\mathcal{F}(x, k_\perp^2)$ density of collinear partons.

- Carrying momentum xP_h and at transverse scale $k_\perp$ *probed by potential Glauber exchanges.*
- Rapidity factorization between S and $\mathcal{F}$ when $x \to 0$.
- $f(x, \mu^2) \sim \int^{\mu^2} dk_\perp^2 \mathcal{F}(x, k_\perp^2)$.
- $x\frac{d}{dx}\mathcal{F}(x, k_\perp^2) = -\mathcal{F} - \frac{\alpha_s C_A}{\pi} K \otimes \mathcal{F}(x, k_\perp^2)$, K BFKL kernel.



Details for $x \to 0$ for PDF

Solving BFKL

- Power-law eigen-functions: $K \otimes \frac{1}{k_\perp^2}(k_\perp^2)^\gamma = \frac{\chi(\gamma)}{k_\perp^2}(k_\perp^2)^\gamma$
- Eigen-values: $\chi(\gamma) = -2\gamma_E - \psi(\gamma) - \psi(1-\gamma)$.
- DGLAP gives power-laws!
- Moment space: $\mu^2 \frac{d}{d\mu^2}\bar{f}(n,\mu^2) = \gamma^S(n)\bar{f}(n,\mu^2)$

Therefore:

$$1 = \frac{\alpha_s C_A}{\pi\, n}\chi(\gamma^S(n))$$

$x \to 0$ for PDF

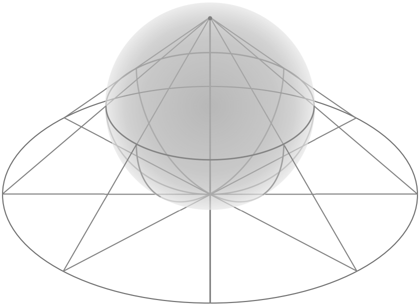
$$1 = \frac{\alpha_s C_A}{\pi\, n} \chi(\gamma^S(n))$$

Gives constraint on γ^S, resumming all poles:

$$\gamma^S(n) = \sum_{\ell=1}^{\infty} c_\ell \left(\frac{\alpha_s C_A}{\pi\, n} \right)^\ell ,$$

Once γ^S is resummed:
We can then evolve PDF in small-x region.
We can claim victory!

Stereographic projection of $S^2 \to \mathbb{R}^2$



BMS equation, soft branching for final states.

$\leftrightarrow$

BK/B-JIMWLK equations, branching for forward physics,

more or less exactly.

- Linearization to BFKL, [**DN 1610.02031**].
- Try un-integrated fragmentation function?

$x \to 0$ for FF, rinse and repeat?

$\mathcal{D}(x,\theta^2)$ density of eikonal lines.

- energy scale xQ with θ angle between eikonal lines
- $d(x,\mu^2) \sim \int^{\mu^2} d\theta_\perp^2 \mathcal{D}(x,\theta^2)$.
- $x\frac{d}{dx}\mathcal{D}(x,\theta^2) = -\mathcal{D} - \frac{\alpha_s C_A}{\pi} K \otimes \mathcal{D}(x,\theta^2)$, K BFKL kernel.
- Power-law eigen-functions: $K \otimes (\theta^2)^\gamma = \chi(\gamma)(\theta^2)^\gamma$
- DGLAP gives power-laws!
- Moment space: $\mu^2 \frac{d}{d\mu^2}\bar{d}(n,\mu^2) = \gamma^T(n)\bar{d}(n,\mu^2)$

Therefore:

$$1 = \frac{\alpha_s C_A}{\pi\, n}\chi(\gamma^T(n))????$$

Brief Aside.

NOTE: We do not need to use stereographic projection to get "un-integrated fragmentation function."

- Derive directly from large-N_c limit of soft matrix elements, following BMS arguments. **[BMS hep-ph/0206076]**.
- Still conclude: $x\frac{d}{dx}\mathcal{D}(x,\theta^2) = -\mathcal{D} - \frac{\alpha_s C_A}{\pi} K \otimes \mathcal{D}(x,\theta^2)$, K BFKL kernel.

So $\gamma^S(n) = \gamma^T(n)$ as $n \to 0$?

No.

Logarithmic structure of $\gamma^T(n)$ as $x \to 0$

Leading log double logarithmic:

$$\gamma^T(n) = \frac{\alpha_s C_A}{\pi\, n} \sum_{\ell=0}^{\infty} c_\ell \left(\frac{\alpha_s C_A}{\pi\, n^2} \right)^\ell,$$

Not single logarithmic:

$$\gamma^S(n) = \sum_{\ell=1}^{\infty} c_\ell \left(\frac{\alpha_s C_A}{\pi\, n} \right)^\ell,$$

This is responsible for very different small-x pheno between PDF and FF!

What failed?

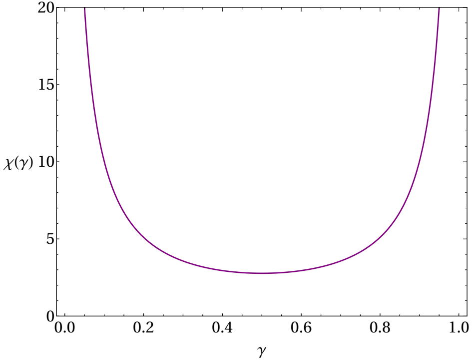
What has failed?

Our analysis of PDF was too fast:
DGLAP anomalous is to be generated by BFKL evolution.

- $x\frac{d}{dx}\mathcal{F}(x,k_\perp^2) = -\mathcal{F} - \frac{\alpha_s C_A}{\pi} K \otimes \mathcal{F}(x,k_\perp^2)$, K BFKL kernel.
- Power-law eigen-functions: $K \otimes \frac{1}{k_\perp^2}(k_\perp^2)^\gamma = \frac{\chi(\gamma)}{k_\perp^2}(k_\perp^2)^\gamma$
- But the initial conditions to BFKL given by perturbation theory is $\gamma = 0$.
- Eigen-function with twist-two scaling (free theory).

Concerning χ.

- χ diverges at the initial condition for $\mathcal{F}$.



- SCET_G: initial conditions dictated by factorization.

Solution.

Work BFKL to all orders in ϵ:

- $x\frac{d}{dx}\mathcal{F}(x,k_\perp^2) = -\mathcal{F} - \frac{\alpha_s C_A}{\pi} K \otimes_{2-2\epsilon} \mathcal{F}(x,k_\perp^2)$.

Then in moment space:

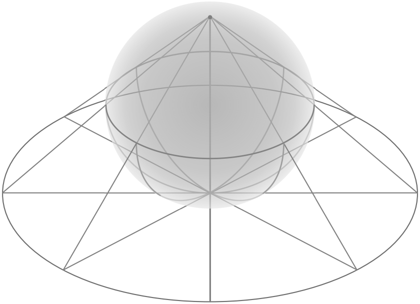
$$\bar{\mathcal{F}} = \gamma^S(\alpha_s,n)R^S(\alpha_s,n)\Big(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\Big)^{\gamma^S(\alpha_s,n)}\exp\Big(\int_0^{\alpha_s}\frac{d\alpha}{\beta(\alpha,\epsilon)}\gamma^S(\alpha,n)\Big)$$

- IR divergences accounted for.
- Essential scaling properties *unchanged*: $1 = \frac{\alpha_s C_A}{\pi\, n}\chi(\gamma^S(n))$
- Naive picture still “accurate.”
- Realized by **[Catani, Hautmann hep-ph/9405388]** as necessary for coefficient function resummation.

Simple explanation in SCET+Glaubers: define rapidity counterterms to all orders in ϵ.
[Chui, Jain, DN, Rothstein 1202.0814],
[DN, Pathak, Rothstein, Stewart, unpublished].

Returning to Space-Time Duality

Stereographic projection of $S^2 \to \mathbb{R}^2$



Dimensionless Angles

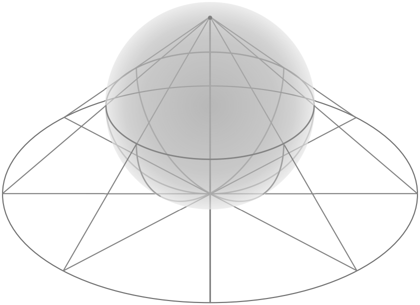
$\leftrightarrow$

Dimensionful Positions

Matters in $2-2\epsilon$, even at Leading Log or Conformal limits!

Returning to Space-Time Duality

Stereographic projection of $S^{2-2\epsilon} \to \mathbb{R}^{2-2\epsilon}$



$$x\theta Q \leftrightarrow \vec{b}_\perp$$

We are mapping celestial transverse scales to forward scattering transverse positions.

Solution

Work "Celestial" BFKL to all orders in ϵ:

$$x\frac{d}{dx}\mathcal{D}(x,\theta^2) = -(1+2\epsilon)\mathcal{D} - \frac{\alpha_s C_A}{\pi}\left(\frac{\mu^2}{x^2Q^2}\right)^{\epsilon} K \otimes_{2-2\epsilon} \mathcal{D}(x,\theta^2)$$

Then:

$$\bar{\mathcal{D}} = \gamma^T(\alpha_s,n) R^T(\alpha_s,n)\left(\frac{Q^2\theta^2}{\mu^2}\right)^{\gamma^T(\alpha_s,n)} \exp\left(\int_0^{\alpha_s}\frac{d\alpha}{\beta(\alpha,\epsilon)}\gamma^T(\alpha,n)\right)$$

(moment space)

- IR divergences accounted for.
- Essential scaling properties *achieved*, with celestial BFKL:

$$\text{Resummation: } 1 = \frac{\alpha_s C_A}{\pi\, n}\chi(\gamma^S(n))$$

$$\text{Reciprocity: } \gamma^S(n + 2\gamma^T(n)) = \gamma^T(n)$$

Reciprocity gives correct double logarithmic resummation for $\gamma^T(n)$.

Celestial BFKL Derivation.

$$x\frac{d}{dx}\mathcal{D}(x,\theta^2) = -(1+2\epsilon)\mathcal{D} - \frac{\alpha_s C_A}{\pi}\left(\frac{\mu^2}{x^2Q^2}\right)^{\epsilon} K \otimes_{2-2\epsilon} \mathcal{D}(x,\theta^2)$$

- Derive directly from large-N_c limit of soft matrix elements.
- Do not take $\epsilon \to 0$ until after evolution.
- Dimensional Regularization regulates $x \to 0$: *no rapidity factorization.*
- RG origins of celestial BFKL?

Summary and Questions

- BFKL factorization for the FF exists, *is* geometrically dual to PDF.
- But initial conditions (twist-two) ill defined.
- Regularization for initial conditions leads to *reciprocity* between γ^T and γ^S.
- What is the EFT for Celestial BFKL? Wilsonian RG in d-dimensions?
- Duality for SCET_G?
- Angular-Ordered DGLAP and soft resummation.

BFKL Equation

Momentum Space:

$$x\frac{d}{dx}\mathcal{F}\Big(x,\vec{k}_\perp\Big)=-\mathcal{F}\Big(x,\vec{k}_\perp\Big)$$
$$-2\frac{\alpha_s C_A}{\pi}(4\pi e^{-\gamma_E})^{-\epsilon}\mu^{2\epsilon}\int\frac{d^{2-2\epsilon}\vec{q}_\perp}{(2\pi)^{1-2\epsilon}}\Big\{\frac{\mathcal{F}\Big(x,\vec{q}_\perp\Big)}{(\vec{q}_\perp-\vec{k}_\perp)^2}-\frac{\vec{k}_\perp^{\,2}}{2\vec{q}_\perp^{\,2}(\vec{q}_\perp-\vec{k}_\perp)^2}\mathcal{F}\Big(x,\vec{k}_\perp\Big)\Big\},$$
$$x\frac{d}{dx}\mathcal{F}\Big(x,\vec{k}_\perp\Big)=-\mathcal{F}\Big(x,\vec{k}_\perp\Big)-\frac{\alpha_s C_A}{\pi}K\otimes_{2-2\epsilon}\mathcal{F}\Big(x,\vec{k}_\perp\Big)$$

Position Space:

$$x\frac{d}{dx}\mathcal{F}(x,\vec{b}_{\perp ab}^{\,2})=-\mathcal{F}(x,\vec{b}_{\perp ab}^{\,2})$$
$$-\Big(\frac{\mu e^{\frac{\gamma_E}{2}}}{Q}\Big)^{2\epsilon}\frac{\alpha_s C_A}{\pi}\int\frac{d^{2-2\epsilon}\vec{b}_{\perp j}}{2\pi^{1-\epsilon}}\frac{\vec{b}_{\perp ab}^{\,2}}{\vec{b}_{\perp aj}^{\,2}\vec{b}_{\perp jb}^{\,2}}\Big(\mathcal{F}(x,\vec{b}_{\perp aj}^{\,2})+\mathcal{F}(x,\vec{b}_{\perp jb}^{\,2})-\mathcal{F}(x,\vec{b}_{\perp ab}^{\,2})\Big).$$

$\vec{b}_{\perp ab}=\vec{b}_{\perp a}-\vec{b}_{\perp b}$ conjugate to $\vec{k}_\perp$.

"Celestial" BFKL Equation

On "angular" space:

$$x\frac{d}{dx}\mathcal{D}(x, n_a\cdot n_b) \\ = -(1+2\epsilon)\mathcal{D}(x, n_a\cdot n_b) \\ \quad -\Big(\frac{\mu e^{\frac{\gamma_E}{2}}}{xQ}\Big)^{2\epsilon}\frac{\alpha_s C_A}{\pi}\int\frac{d^{2-2\epsilon}\Omega_{\hat{j}}}{4\pi^{1-\epsilon}}\frac{n_a\cdot n_b}{n_a\cdot n_j\; n_j\cdot n_b}\Big(\mathcal{D}(x, n_a\cdot n_j)+\mathcal{D}(x, n_b\cdot n_j)-\mathcal{D}(x, n_a\cdot n_b)\Big),$$

Null vectors: $n=(1,\hat{n})$. n_a, n_b, n_j are null vectors parametrizing eikonal lines in a large N_c-dipole.

Small angles:

$$x\frac{d}{dx}\mathcal{D}(x, \vec{\theta}^{\,2}_{ab}) = -(1+2\epsilon)\mathcal{D}(x, \vec{\theta}^{\,2}_{ab}) \\ -\Big(\frac{\mu e^{\frac{\gamma_E}{2}}}{xQ}\Big)^{2\epsilon}\frac{\alpha_s C_A}{\pi}\int\frac{d^{2-2\epsilon}\vec{\theta}_j}{2\pi^{1-\epsilon}}\frac{\vec{\theta}^{\,2}_{ab}}{\vec{\theta}^{\,2}_{aj}\vec{\theta}^{\,2}_{jb}}\Big(\mathcal{D}(x, \vec{\theta}^{\,2}_{aj})+\mathcal{D}(x, \vec{\theta}^{\,2}_{jb})-\mathcal{D}(x, \vec{\theta}^{\,2}_{ab})\Big)$$

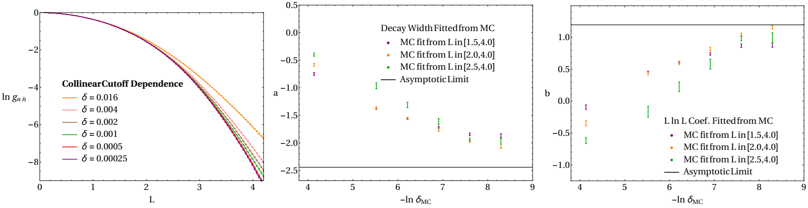
$\vec{\theta}_{ab}$ tangent vector on sphere, pointing from n_b to n_a.

BFKL and Non-global Logarithms

- NGLs corresponds to saturation, $\gamma \gg 0$ and $\epsilon = 0$, BFKL exactly the same in time and space cases, [**DN 1610.02031**].
- $L = \frac{\alpha_s C_A}{\pi} \ln\left(\frac{m_H}{m_L}\right) \to \infty$

From BFKL equation, sets asymptotics of NGL distribution, with a, b predicted from $\chi(\gamma)$:

$$\ln g_{n\bar{n}}(L) \sim aL^2 + bL\ln L + cL$$



MC runs with collinear cutoff on all emmissions.
Must extrapolate to zero.