The Subleading Power Rapidity RG

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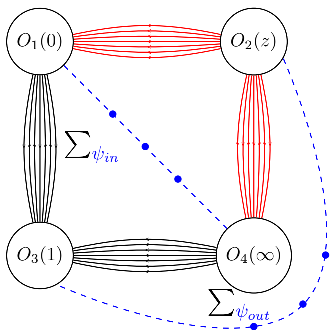
SLAC

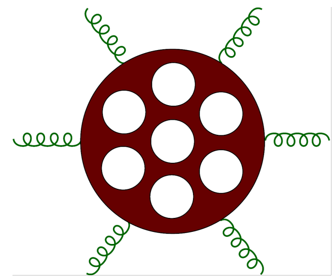
w/:
- Vita, Yan
- Dixon, Zhu
- YuJiao Zhu, HuaXing Zhu

Motivation: Bootstrap

- There has been significant recent progress in bootstrapping correlation functions and amplitudes.
  See e.g. "What can we learn about QCD and Collider Physics from $\mathcal{N} = 4$" by Henn

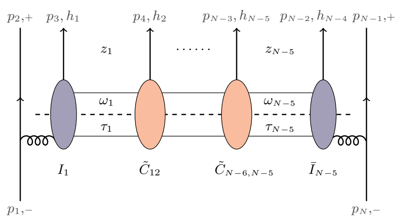


[Coronado]



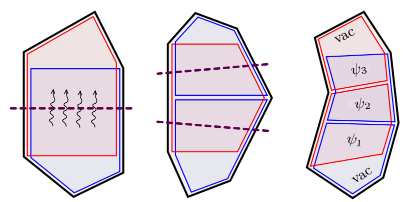
[Dixon et al.]

Regge:



[Del Duca, Duhr et al.]

Collinear:



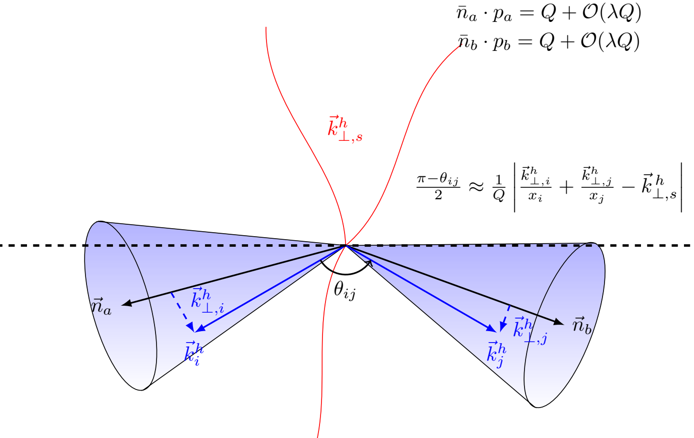
[Basso, Sever, Vieira]

See Greg Ridgway's Talk

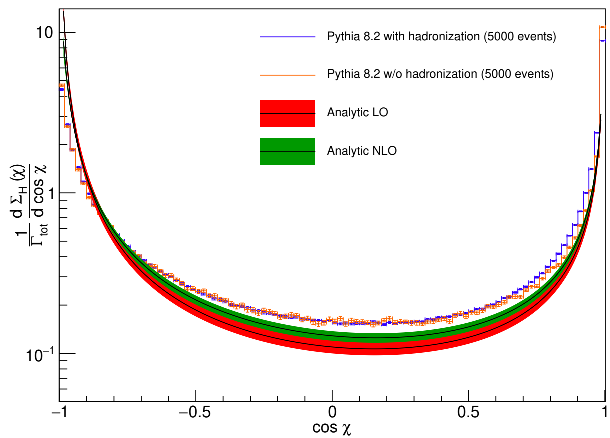
- Exploits all orders understanding of kinematic limits and functional/analytic properties of these objects.

Motivation: Bootstrap

- For this audience, the objects of interest are cross section level (event shape) observables.



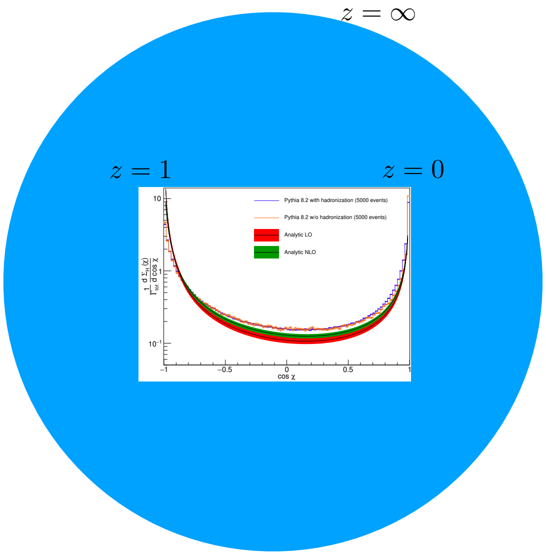
From [Luo, Shtabovenko, Yang, Zhu]



- SCET offers a formalism for systematically expanding event shapes about their kinematic limits. Useful phenomenologically.
- Can one do better and fully bootstrap (or globally approximate) event shape observables? even in $\mathcal{N}=4$?
- What observables are amenable to this, and what information is needed from SCET?

Outline

- Motivation: Global Structure and Asymptotics of the EEC
  - Four Loop Rapidity Anomalous Dimension

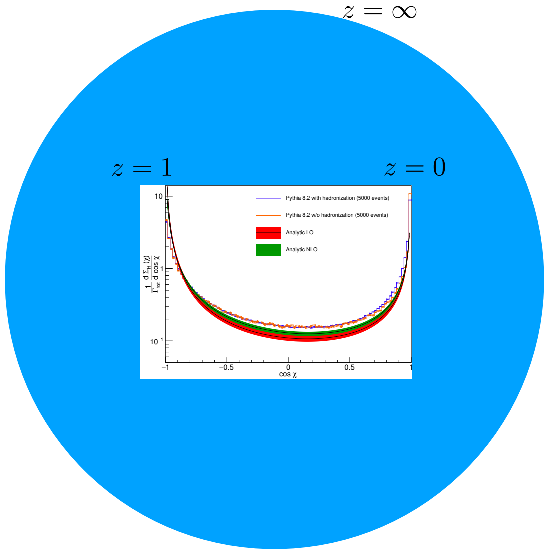


- Expansion of the EEC in the Sudakov Region from the Subleading Rapidity RG

"Dawson's Sudakov"

$$\mathrm{EEC}^{(2)} = -\sqrt{2a_s}\, D\left[\sqrt{\frac{\Gamma^{\mathrm{cusp}}}{2}}\log(1-z)\right]$$

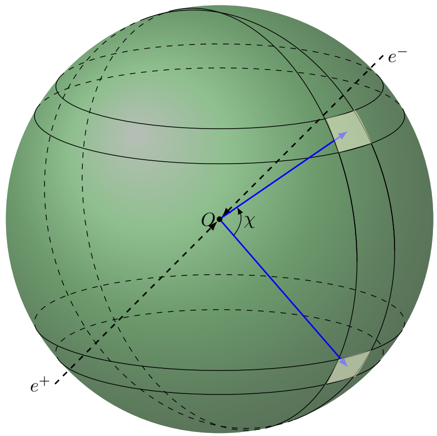
Global Structure and Asymptotics of the EEC

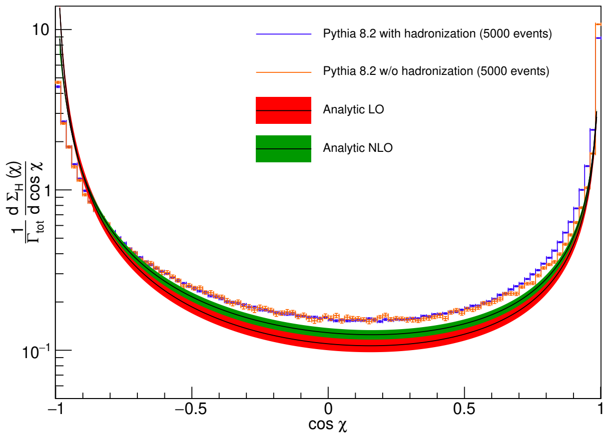


Energy-Energy Correlators

- To understand the structure of event shape observables, one should start with those that are most closely tied to simple field theoretic objects (no algorithms).
- Arguably the simplest is the two-point correlator, which is called the Energy-Energy Correlator.

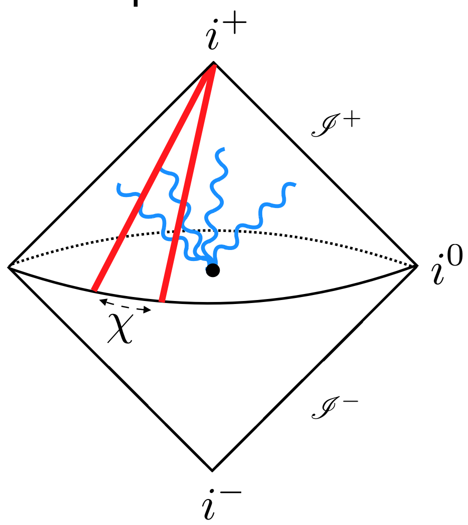
$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1 - \cos\chi_{ij}}{2}\right)$$





Energy-Energy Correlators

- The EEC admits an alternative formulation as a four point function of light ray operators



$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dz} = \frac{\int d^4x\, e^{iq\cdot x}\langle \mathcal{O}(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{O}^\dagger(0)\rangle}{\int d^4x\, e^{iq\cdot x}\langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\rangle}$$

[Korchemsky; Maldacena, Hofman]

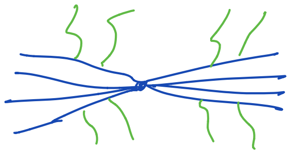
- Simplest extension of a standard four point correlator of local operators $\implies$ has led to significant recent progress.

[Chicherin, Henn, Sokatchev, Yan,Simmons Duffin, Kologlu, Kravchuk, Zhiboedov,Korchemsky, Moult, Dixon, Zhu,...]

- Useful for understanding properties of event shapes.

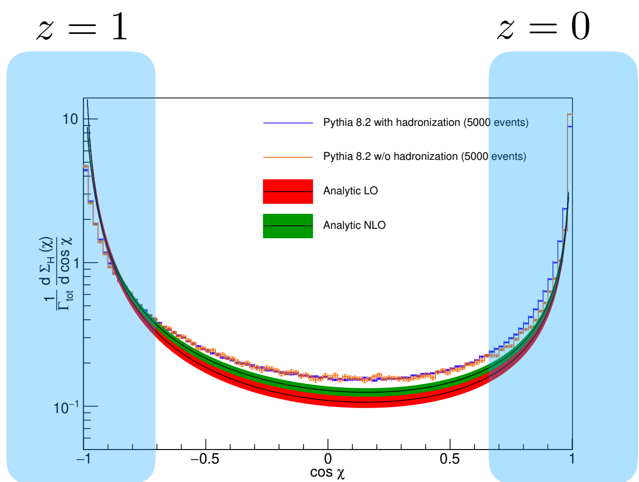
Limits of The Energy-Energy Correlator

- The EEC has two kinematic limits in its distribution:



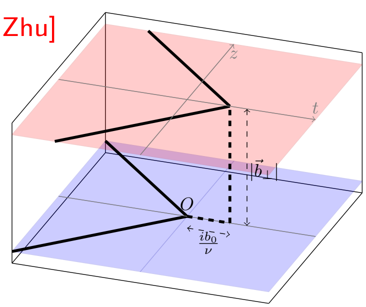
Sudakov

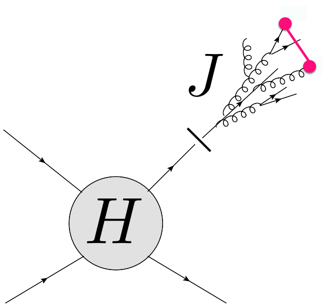
(rapidity factorization)



Collinear

[Moult, Zhu]





[Dixon, Moult, Zhu; Korchemsky; Simmons Duffin, Kologlu, Kravchuk, Zhiboedov]

Leading Power Asymptotics

- The leading power asymptotics in both limits are known to 4 loops.

In Progress: [Moult, Zhu, Zhu]

Wilson Lines

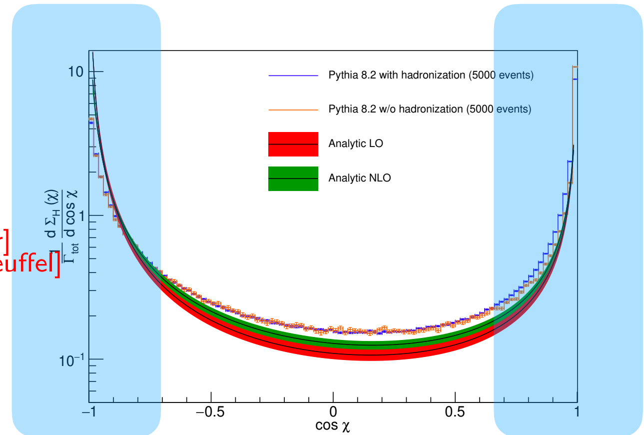
$\Gamma^{\rm cusp}$

[Henn, Korchemsky, Mistlberger]
[Schabinger, Panzer, von Manteuffel]

$$\gamma^R(\epsilon^*) = \gamma^S$$

[Vladimirov]

$z = 1$ $z = 0$

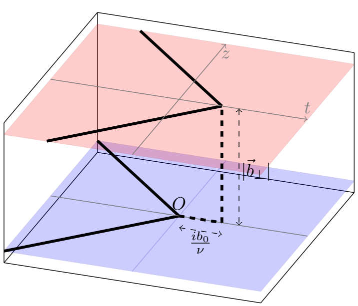


In Progress: [Dixon, Moult, Zhu]

Twist 2 Spin-j

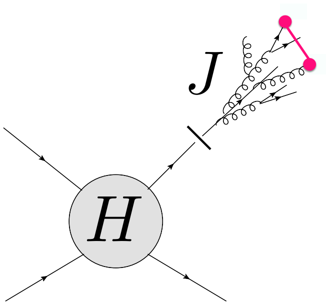
$\gamma_T(3)$

[Moch, Vermaserren, Vogt, ...]



For jet functions/ constants

see talk by Tongzhi Yang



Four Loop Rapidity Anomalous Dimension

[IM, YuJiao Zhu, Hua Xing Zhu]

$$\gamma_3^R = \Big(-\frac{1}{2} b\left(C_A C_F^2 N_f\right) - \frac{1}{4} b\left(C_F^3 N_f\right) - \frac{1}{48} b(d_{FF}^{(4)}) - \frac{2146}{9}\zeta(3)^2 + \frac{520}{3}\zeta(2)\zeta(3)$$

$$-\frac{61913}{81}\zeta(3) - \frac{91067)}{486}\zeta(2 + \frac{10906}{27}\zeta(4) - \frac{4484}{27}\zeta(5) + \frac{791}{54}\zeta(6) + \frac{10761379}{11664}\Big) C_A^2 C_F N_f$$

$$+\Big(b\left(C_A C_F^2 N_f\right) + \frac{1700}{3}\zeta(3)^2 - \frac{2216}{9}\zeta(2)\zeta(3) - \frac{473}{9}\zeta(3) + \frac{2561}{54}\zeta(2) - \frac{30554}{27}\zeta(4)$$

$$+\frac{5476}{9}\zeta(5) - 359\zeta(6) + \frac{2149049}{1944}\Big) C_A C_F^2 N_f + \Big(\frac{1}{48} f(d_{FA}^{(4)}) - \frac{1694}{3}\zeta(3)^2$$

$$-\frac{6190}{9}\zeta(2)\zeta(3) + 360\zeta(4)\zeta(3) + \frac{301900}{81}\zeta(3) + \frac{396427}{486}\zeta(2) + \frac{2065}{9}\zeta(4) + 208\zeta(2)\zeta(5)$$

$$-\frac{15122}{9}\zeta(5) - \frac{2387}{4}\zeta(6) + 850\zeta(7) - \ldots$$

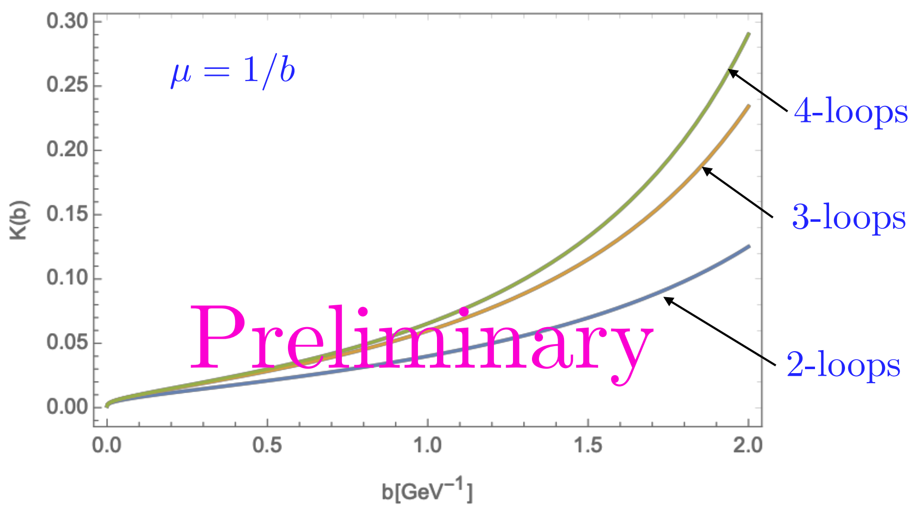
$$-\frac{5564}{81}\zeta(3) + \frac{40}{9}\zeta(4) + \frac{368}{9}\zeta(5) - \ldots$$

$$+\frac{256}{3}\zeta(2)\zeta(3) + \frac{560}{9}\zeta(3) - 162\zeta(2) + \ldots$$

$$+\Big(b(d_{FF}^{(4)}) - \frac{608}{3}\zeta(3)^2 - 64\zeta(2)\zeta(3) + \ldots$$

$$-\frac{592}{9}\zeta(6) + 192\Big) N_f \frac{d_F^{abcd} d_F^{abcd}}{N_c}\Big) + \Big(\ldots$$

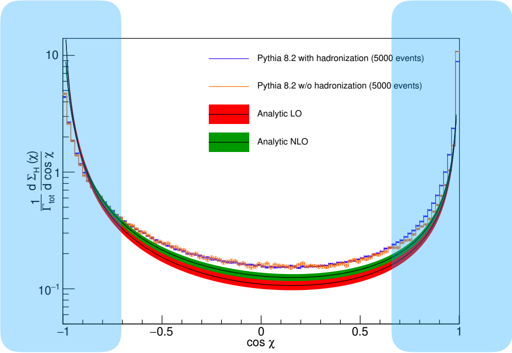
$$+\frac{40}{3}\zeta(4) + 8\zeta(5) - \frac{110059}{972}\Big) C_F^2 N_f^2 - \ldots$$



Into the Bulk of the Distribution

- Leading power singular behavior known for the EEC is known at 4 loops at both ends! How can we make our way into the bulk of the distribution? No numerical fixed order codes.
- Attempt to perform power expansions about the two ends:

$z = 1$ $\quad\quad$ $z = 0$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{1}{1-z}\sum c_n^{(0)} \log^n(1-z) + \sum c_n^{(2)} \log^n(1-z) + (1-z)\sum c_n^{(4)} \log^n(1-z) + \cdots$$

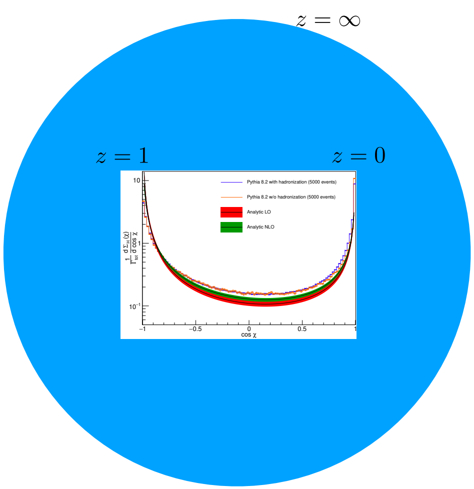
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{1}{z}\sum d_n^{(0)} \log^n(z) + \sum d_n^{(2)} \log^n(z) + (z)\sum d_n^{(4)} \log^n(z) + \cdots$$

Into the Bulk of the Distribution

- From the representation in terms of the discontinuity of the four point correlator, one can show

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{1}{z^2(1-z)} f(z)\,, \qquad f(z)|_{z\to\infty} \to c\log(z)$$

- Behavior also observed in QCD calculation of Zhu, Dixon et al.



- Naive power expansions are inconsistent with this behavior:

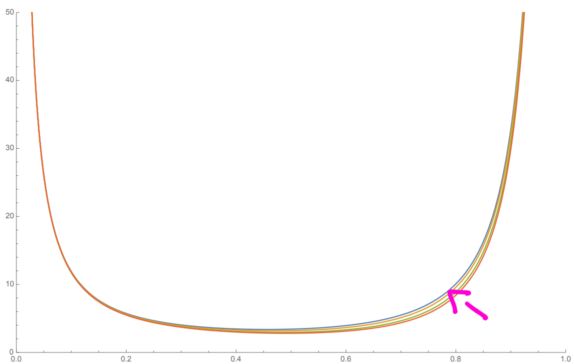
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{1}{z^2(1-z)} \left(\log(1-z) + (1-z)\log(1-z) + \cdots\right)$$

- Places very strong constraints!

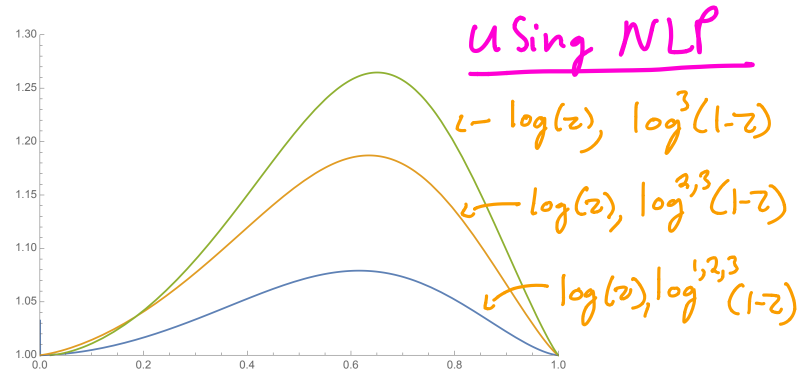
Into the Bulk of the Distribution

- Must expand in terms of functions that obey all $z \to 0, 1, \infty$ constraints.(involves further structural knowledge of functions not discussed here) $\implies$ "globally approximate" or fully bootstrap.

EEC at $\mathcal{O}(\alpha_s^2)$



Percent Error

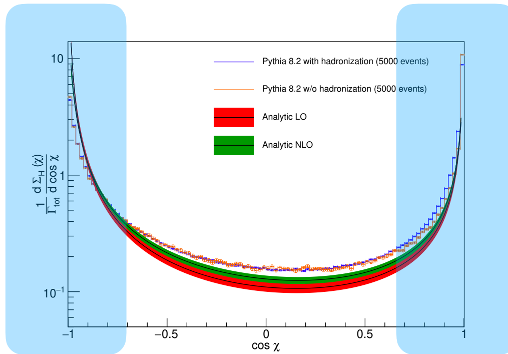


- Integral at a given order can then be used to fix endpoint contributions via Ward Identities. $\int dz\, z \frac{d\sigma}{dz} = \int dz (1-z) \frac{d\sigma}{dz} = \frac{\sigma}{2}$
- Very promising that event shape observables that have analytic structure can be "globally approximated" or fully bootstrapped.

Subleading Power Asymptotics

- Motivates detailed understanding of subleading asymptotics.

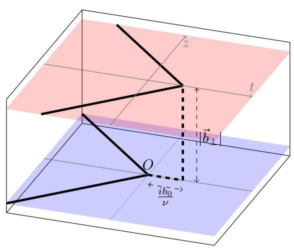
$z = 1$ $z = 0$

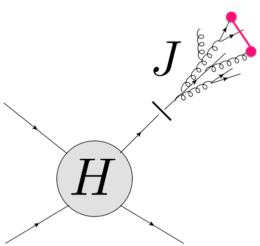


?

Light Ray OPE

[Braun, Balitsky]
[Simmons Duffin et al.]





- No understanding of subleading $z \to 1$ asymptotics in either CFT or EFT language.

The Subleading Power Rapidity RG

$$\boxed{\mathrm{EEC}^{(2)} = -\sqrt{2a_s}\; D\left[\sqrt{\frac{\Gamma^{\mathrm{cusp}}}{2}}\log(1-z)\right]}$$

[IM, Vita, Yan]

Perturbative Data

[Chicherin, Henn, Sokatchev, Yan]

- EEC is known to 3 loops in $\mathcal{N} = 4$ for generic angles.
- Can extract the NLP series in Sudakov region:

$$
\begin{aligned}
\mathsf{EEC}^{(2)} &= -2a_s \log(1-z) \\
&+ a_s^2 \left[\frac{8}{3}\log^3(1-z) + 3\log^2(1-z) + (4+16\zeta_2)\log(1-z) + (-12 - 2\zeta_2 + 36\zeta_2\log(2) + 5\zeta_3) \right] \\
&+ a_s^3 \left[-\frac{32}{15}\log^5(1-z) - \frac{16}{3}\log^4(1-z) - \left(\frac{8}{3} + 24\zeta_2\right)\log^3(1-z) + (4 - 36\zeta_2 - 50\zeta_3)\log^2(1-z) \right. \\
&- \left(\frac{131}{2} + 4\zeta_2 + 372\zeta_4 + 12\zeta_3\right)\log(1-z) \\
&- \frac{3061}{2}\zeta_5 - 96\zeta_2\zeta_3 - \frac{4888}{27}\zeta_4\pi\sqrt{3} + 192\sqrt{3}I_{2,3} + 1482\zeta_4\log(2) - 256\zeta_2\log^3(2) \\
&- \frac{64}{5}\log^5(2) + 1536\mathrm{Li}_5\left(\frac{1}{2}\right) - 544\zeta_4 + 192\zeta_2\log^2(2) + 16\log^4(2) + 384\mathrm{Li}_4\left(\frac{1}{2}\right) \\
&- 288\zeta_2\log(2) + 158\zeta_3 + 55\zeta_2 + \frac{533}{2}
\end{aligned}
$$

- Excellent playground for understanding subleading power rapidity factorization.

Perturbative Data: Leading Logarithms

- Here we will focus on understanding the leading logarithmic series:

$$\mathrm{EEC}^{(2)} = -2a_s \log(1-z) + \frac{8}{3}a_s^2 \log^3(1-z) - \frac{32}{15}a_s^3 \log^5(1-z) + \frac{128}{105}a_s^4 \log^7(1-z)$$
$$- \frac{512}{945}a_s^5 \log^9(1-z) + \frac{2048}{10395}a_s^6 \log^{11}(1-z) - \frac{8192}{135135}a_s^7 \log^{13}(1-z) + \cdots$$

- Will show that this can be written as

$$\boxed{\mathrm{EEC}^{(2)} = -\sqrt{2a_s}\; D\left[\sqrt{\frac{\Gamma^{\mathrm{cusp}}}{2}}\log(1-z)\right]}$$

- We will call this "Dawson's Sudakov" after Dawson's function

$$D(x) = \frac{1}{2}\sqrt{\pi}e^{-x^2}\mathrm{erfi}(x)$$

- Already an interesting structure in the LLs at NLP!

General Approach

$$\mu \frac{d}{d\mu}\begin{pmatrix} S^{(2)} \\ S_\theta^{(2)} \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ 0 & \gamma_{22} \end{pmatrix}\begin{pmatrix} S^{(2)} \\ S_\theta^{(2)} \end{pmatrix}$$

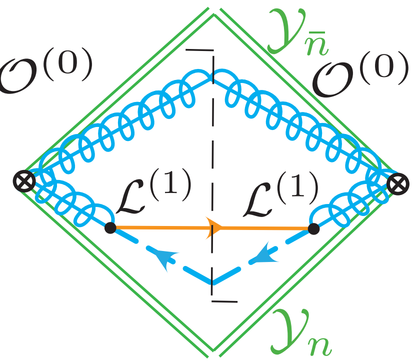
- In the case of SCET$_I$ we found at subleading power a non-trivial mixing structure with "identity operators".

$$S_{g,\theta}^{(2)}(k,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}_{\bar{n}}^T(0)\mathcal{Y}_n(0)\theta(k-\hat{\mathcal{T}})\mathcal{Y}_n^T(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- Would like to understand their analogues for the rapidity RG.
- To explore the structure of consistent rapidity RGs at NLP we will take the following approach:
  - Assume a naive factorization (without endpoint divergences) at subleading power. $H^{(0)} \otimes J^{(0)} \otimes J^{(0)} \otimes S^{(2)} + H^{(0)} \otimes J^{(0)} \otimes J^{(2)} \otimes S^{(0)} + \cdots$
  - Understand mixing structure into "identity operators"
  - Use known behavior of leading power functions combined with RG consistency in $\mu\frac{d}{d\mu}$, $\nu\frac{d}{d\nu}$, and $\left[\frac{d}{d\mu}, \frac{d}{d\nu}\right] = 0$ to derive anomalous dimensions of "identity operators".

Caveats

- In general one must worry about the presence of endpoint divergences (see talks by Zelong Liu and Bianka Mecaj).
- These definitely appear for event shapes when both quarks and gluons are present:



$$\to -4C_F\left(\frac{\alpha_s}{4\pi}\right)\log(\tau)e^{-4C_F\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)}\left[\frac{\left(1-e^{-4(C_A-C_F)\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)}\right)}{4(C_A-C_F)\left(\frac{\alpha_s}{4\pi}\right)\log^2(\tau)}\right]$$

- SUSY saves you since you have a supersymmetric relation between soft quarks and soft gluons.
- Intuitively, one does not expect such problems to appear in $\mathcal{N}=4$ at LL. Supported by the fact that our RG will predict the $\mathcal{O}(\alpha_s^3)$ coefficient of the $\mathcal{N}=4$ result.

An Illustrative Example

- To initiate understanding of subleading rapidity RG, consider leading power soft (or jet) function:

$$S(\vec{p}_T) = \frac{1}{N_c^2 - 1} \langle 0 \big| \mathrm{Tr}\big\{ \mathrm{T}\big[\mathcal{S}_{\bar{n}}^\dagger \mathcal{S}_n\big] \delta^{(2)}(\vec{p}_T - \mathcal{P}_\perp) \overline{\mathrm{T}}\big[\mathcal{S}_n^\dagger \mathcal{S}_{\bar{n}}\big] \big\} \big| 0 \rangle$$

$$\nu \frac{d}{d\nu} S(\vec{p}_T) = \int d\vec{q}_T \gamma_\nu^S (p_T - q_T) S(\vec{q}_T)\,,$$

$$\mu \frac{d}{d\mu} S(\vec{p}_T) = \gamma_\mu^S S(\vec{p}_T)$$

$$\gamma_\nu^S = 2\Gamma^{\mathrm{cusp}}(\alpha_s)\mathcal{L}_0\left(\vec{p}_T, \mu\right)$$

$$\gamma_\mu^S = 4\Gamma^{\mathrm{cusp}}(\alpha_s)\log\left(\frac{\mu}{\nu}\right)$$

- What is the RG of the subleading power soft function:

$$S^{(2)}_{p_T^2}(\vec{p}_T) = \vec{p}_T^{\,2} S(\vec{p}_T)$$

An Illustrative Example

- μ RGE is unaffected, since it is multiplicative:

$$\mu \frac{d}{d\mu} S^{(2)}_{p_T^2}(\vec{p}_T) = \gamma_S^{\mu} S^{(2)}_{p_T^2}(\vec{p}_T)$$

- ν RGE is more non-trivial. Using the identity

$$\vec{p}_T^{\,2} = (\vec{p}_T - \vec{q}_T)^2 + \vec{q}_T^{\,2} + 2(\vec{p}_T - \vec{q}_T)\cdot\vec{q}_T$$

we obtain

$$\nu \frac{d}{d\nu} S^{(2)}_{p_T^2}(\vec{p}_T) = \int d\vec{q}_T (\vec{p}_T - \vec{q}_T)^2 \gamma_S(p_T - q_T) S(q_T)$$
$$+ \int d\vec{q}_T \gamma_S(p_T - q_T)\left[2(\vec{p}_T - \vec{q}_T)\cdot\vec{q}_T S(q_T)\right] + \int d\vec{q}_T \gamma_S(p_T - q_T)\left[\vec{q}_T^{\,2} S(\vec{q}_T)\right]$$

which simplifies to

$$\nu \frac{d}{d\nu} S^{(2)}_{p_T^2}(\vec{p}_T) = 2\Gamma^{\text{cusp}}\mathbb{I}_S$$
$$+ \int d\vec{q}_T \gamma_S(p_T - q_T) 2(\vec{p}_T - \vec{q}_T)\cdot\vec{S}^{(1)}(q_T) + \int d\vec{q}_T \gamma_S(p_T - q_T) S^{(2)}_{p_T^2}(\vec{q}_T)\,.$$

where

=1+....

$$\mathbb{I}_S \propto \int d^2\vec{q}_T\; S(\vec{q}_T)\,, \qquad \vec{S}^{(1)}(\vec{p}_T) = \vec{p}_T S^{(0)}(\vec{p}_T)$$

Rapidity Identity Operators

- Unlike in SCET$_I$ case, the identity soft function needs a regulator to be defined.

$$\mathbb{I}_S \propto \int d^2\vec{q}_T \; S(\vec{q}_T)$$

- Its LL renormalization group equations can be derived by applying $\mu\frac{d}{d\mu}$, $\nu\frac{d}{d\nu}$, and $\left[\frac{d}{d\mu}, \frac{d}{d\nu}\right] = 0$ to the NLP factorization formula.
- Interestingly, one finds two possible consistent solutions.

$$\boxed{\mathbb{I}_S^{\nu}}$$

$$\mu\frac{d}{d\mu}\mathbb{I}_S^{\nu} = \gamma_\mu^S \mathbb{I}_S^{\nu}$$

$$\nu\frac{d}{d\nu}\mathbb{I}_S^{\nu}(\mu,\nu) = -\gamma_\mu^S \mathbb{I}_S^{\nu}(\mu,\nu)\,,$$

At LL:

$$\mathbb{I}_S^{\nu}(\mu,\nu) = \int_0^{\nu^2} d^2\vec{q}_T \; S(\vec{q}_T)$$

$$\boxed{\mathbb{I}_S^{p_T^2}}$$

$$\mu\frac{d}{d\mu}\mathbb{I}_S^{p_T^2} = \gamma_\mu^S \mathbb{I}_S^{p_T^2}$$

$$\nu\frac{d}{d\nu}\mathbb{I}_S^{p_T^2}(p_T^2,\mu,\nu) = 2\Gamma^{\rm cusp}\log\left(\frac{p_T^2}{\mu^2}\right)\mathbb{I}_S^{p_T^2}(p_T^2,\mu,\nu)$$

At LL:

$$\mathbb{I}_S^{p_T^2}(p_T^2,\mu,\nu) = \int_0^{p_T^2} d^2\vec{q}_T \; S(\vec{q}_T)$$

Rapidity Identity Operators

- These operators are distinguished by their ν-scaling (boost) properties at the scale $\mu^2 = p_T^2$:

$$\nu \frac{d}{d\nu} \mathbb{I}_S^{\nu}(\mu = p_T, \nu) = -2\Gamma^{\text{cusp}}(\alpha_s) \log\left(\frac{p_T^2}{\nu^2}\right) \mathbb{I}_S^{\nu}(\mu = p_T, \nu)\,, \qquad \nu \frac{d}{d\nu} \mathbb{I}_S^{p_T^2}(\mu = p_T, \nu) = 0$$

- For the particular case of $S^{(2)}_{p_T^2} = \vec{p}_T^{\,2} S(\vec{p}_T)$, only $\mathbb{I}_S^{p_T^2}$ appears:

$$\nu \frac{d}{d\nu} S^{(2)}_{p_T^2}(\vec{p}_T) = 2\Gamma^{\text{cusp}} \mathbb{I}_S^{p_T^2} + \int d\vec{q}_T \gamma_S(p_T - q_T) S^{(2)}_{p_T^2}(\vec{q}_T)\,.$$

- More general soft/ jet functions at NLP involve $n, \bar{n} \cdot B_S$, $n, \bar{n} \cdot \partial_S$ further modify boost properties as compared to Wilson lines.
- Would be interesting to understand through explicit two loop calculations of NLP soft/ jet functions.

RG Solutions

- We can now consider the solution of the RG equations for these functions. At LL, sufficient to consider the solutions on the hyperbola $\mu^2 = p_T^2$.
- Consider first $\mathbb{I}_S^{p_T^2}$:
$$\nu \frac{d}{d\nu} \begin{pmatrix} S^{(2)} \\ \mathbb{I}_S^{p_T^2} \end{pmatrix} = \begin{pmatrix} 0 & \gamma_{\delta \mathbb{I}} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} S^{(2)} \\ \mathbb{I}_S^{p_T^2} \end{pmatrix}$$

  with the boundary conditions
$$\mathbb{I}_S^{p_T^2}(\mu = p_T, \nu = p_T) = 1\,, \qquad S^{(2)}(\mu = p_T, \nu = p_T) = 0$$

$$\boxed{S^{(2)}(\mu = p_T, \nu = Q) = \gamma_{\delta \mathbb{I}} \log(p_T/Q) \mathbb{I}_S^{p_T^2}(\mu = p_T, \nu = p_T)}$$

- Will generate standard Sudakov when combined with leading power jet/ hard functions.

RG Solutions

- $\mathbb{I}_S^\nu$ exhibits a more non-trivial RG:

$$\nu \frac{d}{d\nu} \begin{pmatrix} S^{(2)} \\ \mathbb{I}_S^\nu \end{pmatrix} = \begin{pmatrix} 0 & \gamma_{\delta\mathbb{I}} \\ 0 & \gamma_S^\mu \end{pmatrix} \begin{pmatrix} S^{(2)} \\ \mathbb{I}_S^\nu \end{pmatrix} ,$$

with the boundary conditions

$$\mathbb{I}_S^\nu(\mu = p_T, \nu = p_T) = 1 , \qquad S^{(2)}(\mu = p_T, \nu = p_T) = 0$$

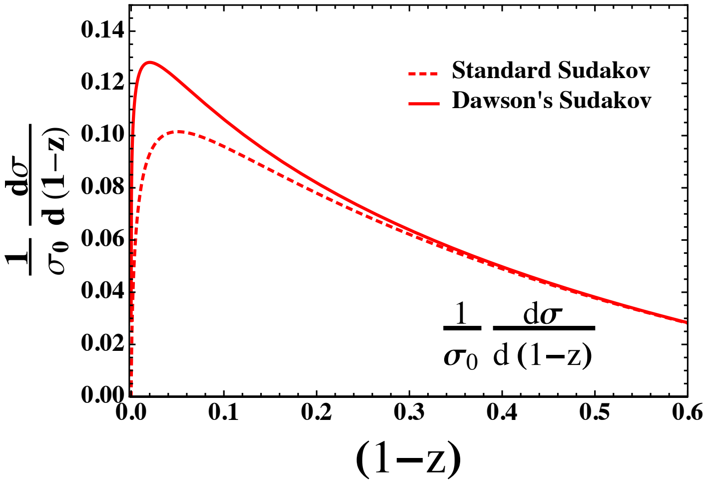
$$\boxed{S^{(2)}(\mu = p_T, \nu = Q) = -\frac{\sqrt{\pi}\gamma_{\delta\mathbb{I}}}{\sqrt{\tilde{\gamma}}} \mathrm{erfi}\left[\sqrt{\tilde{\gamma}} \log(p_T/Q)\right] \mathbb{I}_S^\nu(\mu = p_T, \nu = p_T)}$$

- Will generate "Dawson's Sudakov" when combined with hard function.

EEC in $\mathcal{N} = 4$ at LL

- Combine factors for EEC and resum to LL:

$$\text{EEC}^{(2)} = -\frac{\sqrt{\pi} a_s}{\sqrt{2 a_s}} \text{erfi}\left[\sqrt{2 a_s} \log(1-z)\right] \exp\left[-2 a_s \log(1-z)^2\right]$$



$$\text{EEC}^{(2)} = -\sqrt{2 a_s}\; D\left[\sqrt{\frac{\Gamma^{\text{cusp}}}{2}} \log(1-z)\right]$$

- Agrees with expansion of the three loop result!

$$\text{EEC}^{(2)}\Big|_{\text{LL}} = -2 a_s \log(1-z) + \frac{8}{3} a_s^2 \log^3(1-z) - \frac{32}{15} a_s^3 \log^5(1-z)$$

A Guess for Yang-Mills

- Analytic results for the EEC are known to two loops in QCD for both e^+e^- annihilation and H decay.
- Naively would not expect endpoint divergences in pure Yang Mills at LL.
- Under this assumption, one obtains for the LL series

$$\mathrm{EEC}^{(2)}\Big|_{\text{Yang-Mills}} = 2a_s \log(1-z) \exp\left[-2a_s \log^2(1-z)\right] - 4\sqrt{2a_s}\, D\left[\sqrt{\frac{\Gamma^{\text{cusp}}}{2}} \log(1-z)\right]$$

- Two loops is not high enough to test the interplay of these two series or for the presence of endpoint divergences.

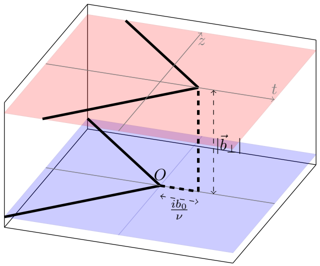
Open Questions

- This provides first (very preliminary) glimpse of the structure of the rapidity RG at subleading power (and it seems interesting!).
- Many interesting questions:
  - Systematic operator analysis.
  - Interplay with endpoint divergences.
  - Extension to NLL.
- In the CFT literature, an OPE for the four point correlator in this limit is not known? What can SCET teach us about it?
- In a CFT, the rapidity anomalous dimensions is redundant, does this persist at NLP?

- Much more understanding needed!

Summary

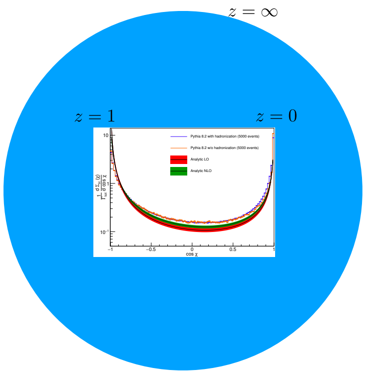
- Four Loop Leading Power Asymptotics of the EEC are known in QCD. (N^4LL resummation on both ends)



- Dawson's Sudakov describes LL soft gluon radiation in rapidity factorization at NLP

$$\text{EEC}^{(2)} = -\sqrt{2a_s}\; D\left[\sqrt{\frac{\Gamma^{\text{cusp}}}{2}}\log(1-z)\right]$$

- Systematic power expansion about asymptotic limits combined with knowledge of global structure offers a powerful means to approximate or fully bootstrap event shape observables.




THANKS
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