

# Higgs Production at NNLL' + NNLO using Rapidity-Dependent Jet Vetoess.

World SCET 2020

SHIREEN GANGAL

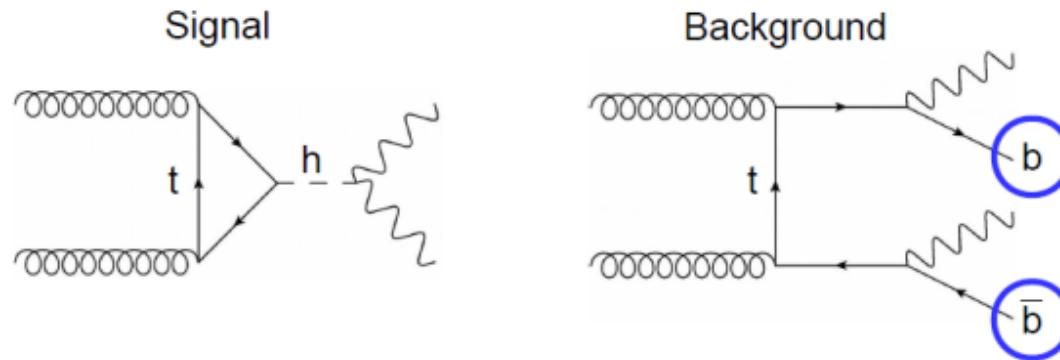
TIFR, MUMBAI



Based on: SG, J. Gaunt, F. Tackmann , E. Vryon diou : JHEP 2005 054.

# Introduction

- Jet vetoes are important at the LHC for separating signal from background.



eg:  $H \rightarrow WW$ : Jet binning into 0-jet, 1-jet, 2-jet.

0-jet bin: Hard jets with  $p_T^{jet} > p_T^{cut} \sim 20 - 30$  GeV are vetoed.

Jet selection cuts and vetoes on additional emissions induce Sudakov double logarithms.

$$\text{Eg: } gg \rightarrow H + 0 \text{ jet: } \sigma_0(p_T^{cut}) \propto \sigma_B \left( 1 - 2 \frac{\alpha_s C_A}{\pi} \log^2 \frac{p_T^{cut}}{m_H} + \dots \right)$$

- For tighter jet vetoes ( $p_T^{cut} \ll m_H$ ), these logs become large and dominate the perturbative series → increased theoretical uncertainties.
- These logs must be systematically resummed to obtain reliable theory predictions.

# Rapidity- dependent Jet Veto

**Using the  $p_T$  of the jet as a veto:** In harsh pile up, low  $p_T$  jets hard to identify in forward region of detector  $|\eta| > 2.5$ .

- Can try to get around this by a hard rapidity cut: vetoing only jets with  $|\eta| < 2.5$  .
- Or raise  $p_T^{cut}$  everywhere : Lose the utility of a tight central jet veto.
- Use a different jet veto that depends smoothly on the rapidity of jets.

$$\mathcal{T}_{fj} = |\vec{p}_{Tj}| f(y_j)$$

Tackmann, Walsh, Zuberi '12 ,  
SG, Stahlhofen, Tackmann '14.

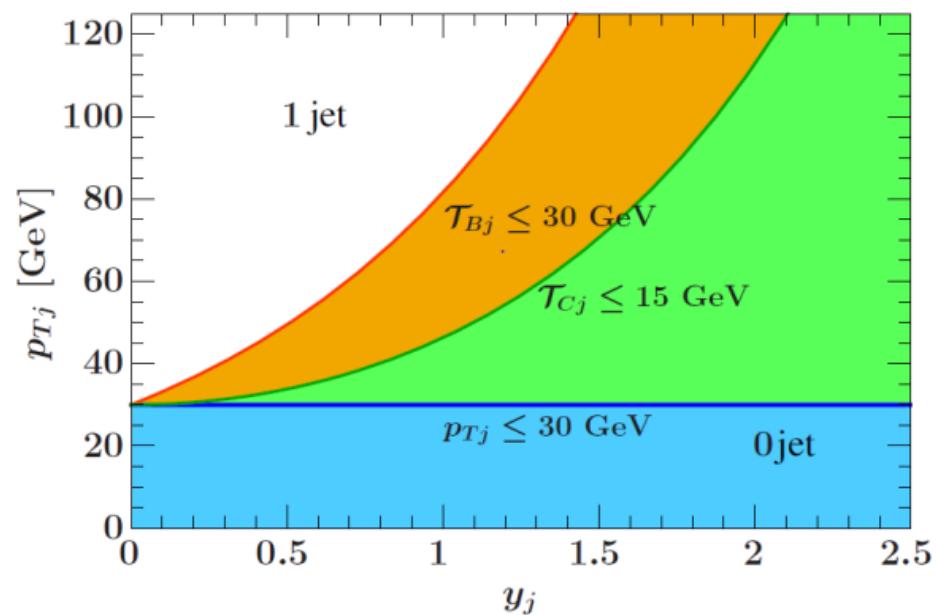
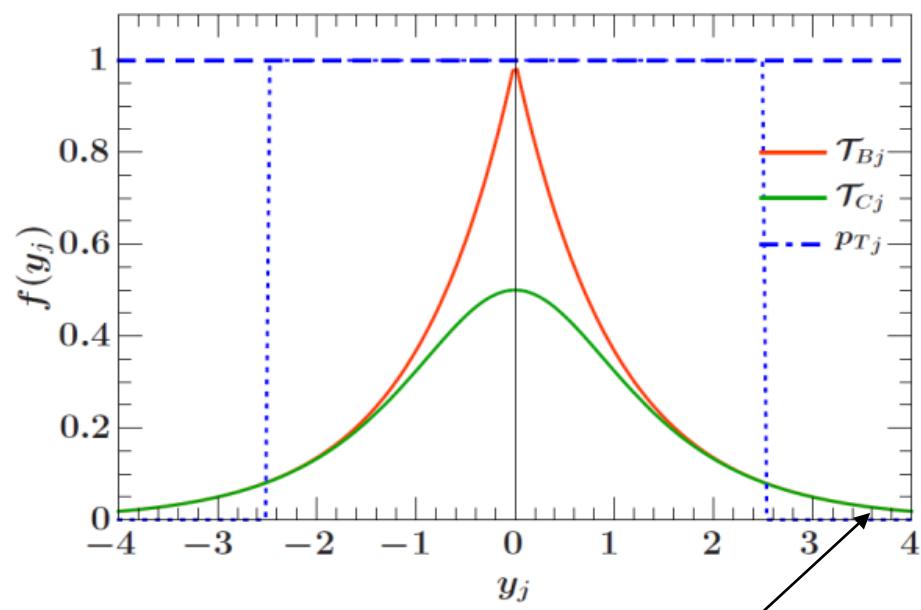
$f(y_j)$  : decreasing function of  $|y_j|$ .  
Tight veto at small rapidity and  
loose at large rapidity.

$$p_{Tj} < \frac{\mathcal{T}^{cut}}{f(y_j)} , \quad \mathcal{T}_f^{\text{jet}} = \max_{j \in R} \mathcal{T}_{fj}$$

# Rapidity-dependent Jet Veto

Two observables:  $\mathcal{T}_B^j : f(y_j) = e^{-|y_j - Y|}$  ,  $\mathcal{T}_{B\text{cm}}^j : f(y_j) = e^{-|y_j|}$

$\mathcal{T}_C^j : f(y_j) = \frac{1}{2 \cosh(y_j - Y)}$  ,  $\mathcal{T}_{C\text{cm}}^j : f(y_j) = \frac{1}{2 \cosh(y_j)}$



Two observables become equal  
at forward rapidities.

# Factorization for H+0-jet

$$\frac{d\sigma_0}{dY}(\mathcal{T}_f^{\text{jet}} < \mathcal{T}^{\text{cut}}) = \frac{d\sigma_0^{\text{resum}}}{dY}(\mathcal{T}_f^{\text{jet}} < \mathcal{T}^{\text{cut}}) + \frac{d\sigma_0^{\text{nons}}}{dY}(\mathcal{T}_f^{\text{jet}} < \mathcal{T}^{\text{cut}})$$

$$\frac{d\sigma_0^{\text{resum}}}{dY}(\mathcal{T}_{B,C}^{\text{jet}} < \mathcal{T}^{\text{cut}}) = \sigma_B H_{gg}(m_t, m_H^2, \mu_H) B_g(m_H \mathcal{T}^{\text{cut}}, x_a, R, \mu_B) B_g(m_H \mathcal{T}^{\text{cut}}, x_b, R, \mu_B)$$

$$S_{gg}^{B,C}(\mathcal{T}^{\text{cut}}, R, \mu_S) + O(R^2)$$

(Tackmann, Walsh, Zuberi '12)  
(Stewart, Tackmann, Waalewijn '09)

$$d\sigma \sim |C_{ggH}|^2 \langle p_a p_b | O_{ggH}^\dagger \hat{\mathcal{M}} O_{ggH} | p_a p_b \rangle$$

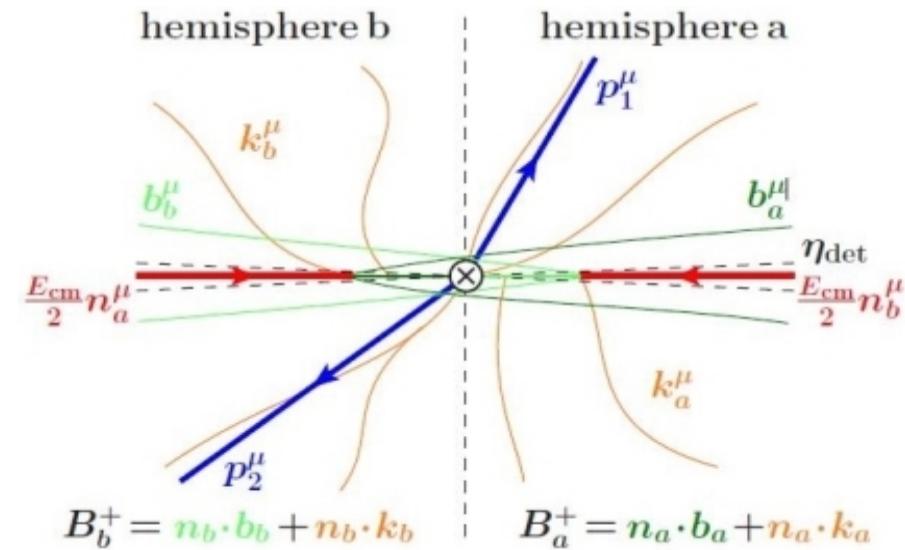
$\sim \mathcal{O}_a \mathcal{O}_s \mathcal{O}_b$       Measurement Function

$$O_{ggH} = H \mathcal{B}_{n_a \perp}^\mu T[\mathcal{Y}_{n_a}^\dagger \mathcal{Y}_{n_b}] \mathcal{B}_{n_b \perp}^\mu$$

$$\mathcal{M}^{\text{jet}} \sim \mathcal{M}_a \times \mathcal{M}_b \times \mathcal{M}_s + \delta \mathcal{M}_{SC}^{\text{jet}}$$

$$B_a(\mu) = \langle p_a | O_a^\dagger \hat{\mathcal{M}}_a O_a | p_a \rangle$$

$$S(\mu) = \langle 0 | O_s^\dagger \hat{\mathcal{M}}_s O_b | 0 \rangle$$



# Factorization in SCET

$$B_g(t^{\text{cut}}, x, \mu) = \langle p_n(P^-) | \mathcal{B}_{n\perp\mu}(0) \mathcal{M}^{\text{jet}}[\delta(\omega - \bar{\mathcal{P}}_n) \mathcal{B}_{n\perp\mu}(0)] | p_n(P^-) \rangle$$

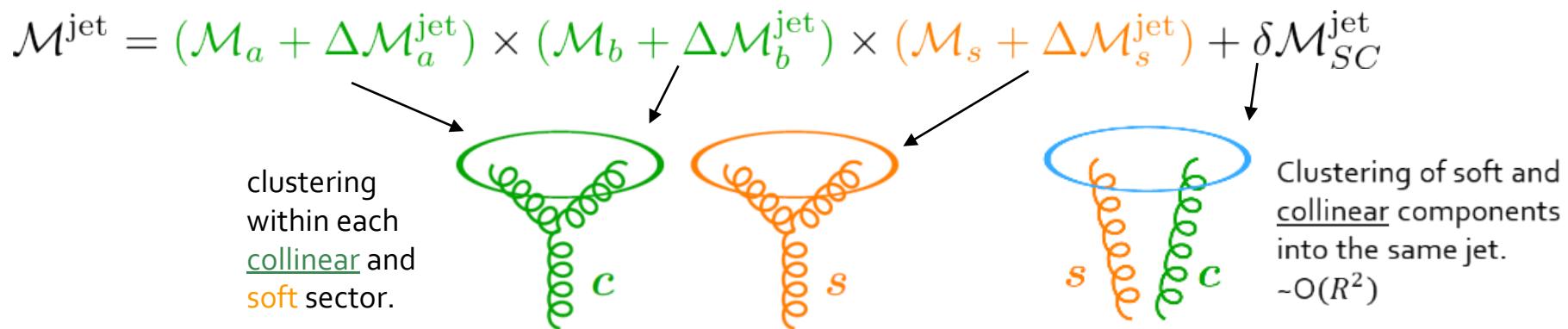
↓  $\omega/P^-$   
↓ Collinear Gluon field

Stewart, Tackmann, Waalewijn '10

Measurement function:  $\mathcal{M}_B^{\text{jet}}(\mathcal{T}_B^{\text{cut}}) = \theta(\mathcal{T}_B^{\text{jet}} < \mathcal{T}_B^{\text{cut}}) = \prod_{j \in J(R)} \theta(k_j^+ < \mathcal{T}_B^{\text{cut}})$

For  $m_H \mathcal{T}^{\text{cut}} \gg \Lambda_{QCD}^2$ :  $B_g(t^{\text{cut}}, x, \mu) \sim \sum_j \int_x^1 \frac{dz}{z} \mathcal{I}_{gj}\left(t^{\text{cut}}, z, \mu, R\right) f_j\left(\frac{x}{z}, \mu\right)$

Soft Collinear factorization implies: (Tackmann, Walsh, Zuberi '12)



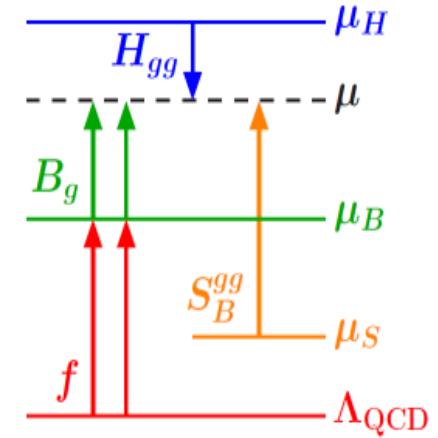
# Resummation in SCET

Natural Scale choice:  $\mu_H \simeq m_H$ ,  $\mu_B \simeq \sqrt{\mathcal{T}^{\text{cut}} m_H}$ ,  $\mu_S \simeq \mathcal{T}^{\text{cut}}$

$$\ln^2 \frac{\mathcal{T}^{\text{cut}}}{m_H} = 2 \ln^2 \frac{m_H}{\mu} - \ln^2 \frac{\mathcal{T}^{\text{cut}} m_H}{\mu^2} + 2 \ln^2 \frac{\mathcal{T}^{\text{cut}}}{\mu}$$

Beam Function RGE:  $\mu \frac{d}{d\mu} \ln [B_g(t^{\text{cut}}, x, R, \mu)] = \gamma_B^g(t^{\text{cut}}, R, \mu)$

$$\begin{aligned} \sigma_0(\mathcal{T}^{\text{cut}}) &= H_{ggH}(m_H, \mu_H) U_H(m_H, \mu_H, \mu) \\ &\times B_g(m_H \mathcal{T}^{\text{cut}}, x_a, R, \mu_B) B_g(m_H \mathcal{T}^{\text{cut}}, x_b, R, \mu_B) U_B(m_H \mathcal{T}^{\text{cut}}, \mu_B, \mu)^2 \\ &\times S_{gg}^{B,C}(\mathcal{T}^{\text{cut}}, R, \mu_S) U_S(\mu_S, \mu) \end{aligned}$$



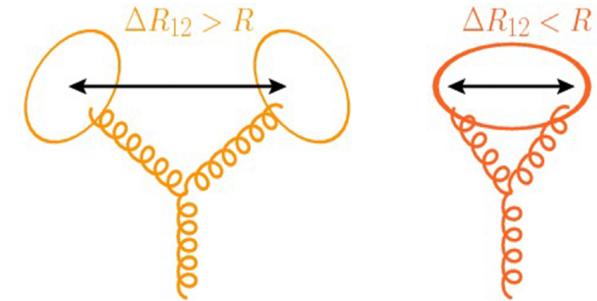
Log counting:	Fixed-order corrections matching	nonsingular	$\gamma_{H,B,S}^\mu$	$\Gamma_{\text{cusp}}$	$\beta$
NLL	1	-	1-loop	2-loop	2-loop
NLL'	NLO	-	1-loop	2-loop	2-loop
NLL'+ NLO	NLO	NLO	1-loop	2-loop	2-loop
NNLL'+NNLO	NNLO	NNLO	2-loop	3-loop	3-loop

# Jet Clustering Corrections

We need finite parts of 2-loop beam and soft functions.

Jet Dependent Measurement Function:

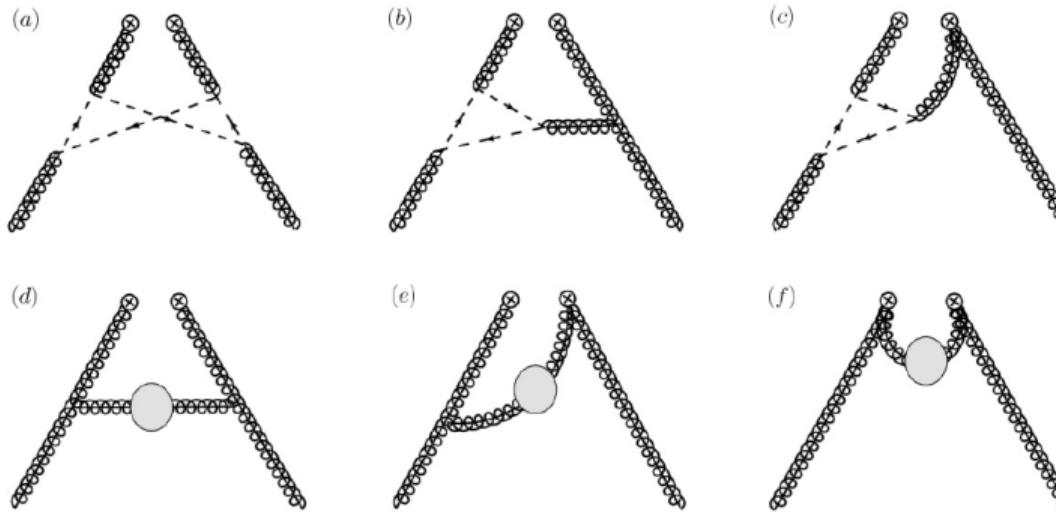
$$\begin{aligned} \mathcal{M}_i^{\text{jet}}(\mathcal{T}^{\text{cut}}, R) = & \theta(\Delta R > R)\theta(\mathcal{T}_{B1} < \mathcal{T}^{\text{cut}})\theta(\mathcal{T}_{B2} < \mathcal{T}^{\text{cut}}) \\ & + \theta(\Delta R < R)\theta(\mathcal{T}_B^{\text{jet}} < \mathcal{T}^{\text{cut}}) \end{aligned}$$



- Inclusive 2 loop Beam and soft function calculation available.
- Compute difference to these inclusive (jet-algorithm independent global ) Beam and Soft functions.
- Global reference measurement we use coincide with jet-dependent at 1 loop  
→ We only need to consider double real emission amplitudes.
- All IR Divergences disappear in the difference.

$$\begin{aligned} \Delta\mathcal{M}_i^{\text{jet}}(\mathcal{T}^{\text{cut}}) = & \theta(\Delta R > R)\theta(\mathcal{T}_{B1} < \mathcal{T}^{\text{cut}})\theta(\mathcal{T}_{B2} < \mathcal{T}^{\text{cut}}) + \theta(\Delta R < R)\theta(\mathcal{T}_B^{\text{jet}} < \mathcal{T}^{\text{cut}}) \\ & - \theta(\mathcal{T}_{B1} + \mathcal{T}_{B2} < \mathcal{T}^{\text{cut}}) \\ = & \theta(\Delta R > R) \left[ \theta(k_1^+ < \mathcal{T}^{\text{cut}})\theta(k_2^+ < \mathcal{T}^{\text{cut}}) - \theta(k_1^+ + k_2^+ < \mathcal{T}^{\text{cut}}) \right]. \end{aligned}$$

# 2-loop Beam Function



J. Gaunt,  
M.Stahlhofen,  
F.Tackmann '14

$$B_{ij}(t^{\text{cut}}, x, \mu, R) = B_{G,ij}(t^{\text{cut}}, x, \mu) + \Delta B_{ij}(t^{\text{cut}}, x, \mu, R)$$

- Consider double real amplitudes:  $\mathcal{A}(k_1, k_2, x) = \mathcal{A}_A(k_1, k_2, x) + \mathcal{A}_B(k_1, k_2, x)$ 
  - Obtained by taking soft limit of one of the partons  $\mathcal{A}_A = \mathcal{A}_i^{S(1)} \mathcal{A}_{ij}^{B(1)}$
  - “Uncorrelated emission”
  - “Correlated emission”  $\log(R), \log^2(R)$

Use the additional measurement constraint :  $\delta(k_1^- + k_2^- - (1-x)p^-)$

$$k_1^+ = z\mathcal{T}_T, \quad k_2^+ = (1-z)\mathcal{T}_T, \quad \cos \Delta\phi = \frac{k_1^\perp \cdot k_2^\perp}{|k_1^\perp||k_2^\perp|}, \quad y_t = y_1 + y_2, \quad \Delta y = y_1 - y_2$$

# 2-loop Beam Function

Integration of  $\mathcal{A}_A$  with  $(1 - \theta(\Delta R < R))$ : Done analytically. Result as divergent as  $1/\epsilon^2$  and  $R^2/\epsilon$ .

J. Gaunt, SG, M.Stahlhofen et al '17

Integration of  $\mathcal{A}_B$ : Expand in small  $\Delta R$  limit to extract log R terms analytically.

Three 1D functions determined numerically by fitting :  $f(R)$ ,  $g(R)$  and  $h(x)$  in correlated. All # : determined analytically.

$$\Delta B_{ij}(t^{\text{cut}}, x, R) = \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\mu^2}{t^{\text{cut}}}\right)^\epsilon \left[ \delta(1-x) \left\{ \frac{1}{\epsilon} \left[ \# \log R + f(R) \right] + \right. \right.$$

Correlated Contribution

$$\left. \left. \left[ \# \log [R]^2 + \# \log R + g(R) \right] \right\} + \right]$$

Expansion of

$$(1-x)^{-1-2\epsilon} \longrightarrow \mathcal{L}_0(1-x) \{ \#(x) \log R + h(x) + \#(x)R^2 + \#(x)R^4 + \dots \}$$

$$\left. \left\{ \frac{1}{\epsilon^2} \#(x) + \frac{1}{\epsilon} (\#(x) + \#(x)R^2) + \#(x)R^2 + \#(x)R^2 \log R + \dots \right\} \right]$$

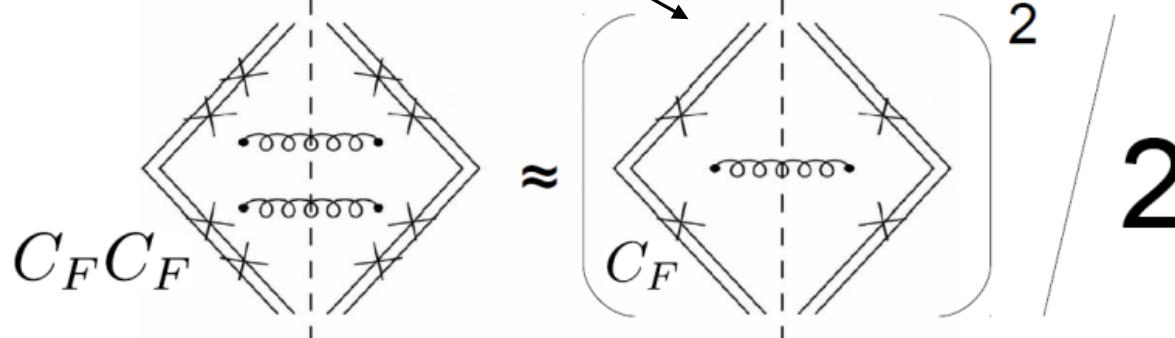
Uncorrelated Contribution

$$\mathcal{I}_{gj}^{(2)}(t^{\text{cut}}, x, \mu, R) = \mathcal{I}_{G,gj}^{(2)}(t^{\text{cut}}, x, \mu) + \Delta \mathcal{I}_{gj}^{(2)}(t^{\text{cut}}, x, \mu, R) + \Delta \mathbb{I}_{gj}^{(2)}(t^{\text{cut}}, x, \mu, R)$$

# 2-loop Soft Function

Treat Abelian ( $C_F C_F$ ) uncorrelated piece in different way to Non-Abelian ( $C_F C_A$ ,  $C_F T_F$ ) .

$$S_f^{\text{bare}(2, C_F^2)}(\mathcal{T}^{\text{cut}}, R) = \frac{1}{2} \left[ S^{\text{bare}(1)}(\mathcal{T}^{\text{cut}}) \right]^2 + \Delta S_{f, \text{indep}}^{\text{bare}(2)}(\mathcal{T}^{\text{cut}}, R)$$



SG, Gaunt, Stahlhofen,  
Tackmann '17

$$\mathcal{M}_i^{\text{jet}}(\mathcal{T}^{\text{cut}}, R) = \theta(\Delta R > R) \theta(\mathcal{T}_{B1} < \mathcal{T}^{\text{cut}}) \theta(\mathcal{T}_{B2} < \mathcal{T}^{\text{cut}}) + \theta(\Delta R < R) \theta(\mathcal{T}_B^{\text{jet}} < \mathcal{T}^{\text{cut}})$$

$$\begin{aligned} \Delta \mathcal{M}_f^{\text{indep}}(\mathcal{T}^{\text{cut}}, R) &= \mathcal{M}_i^{\text{jet}}(\mathcal{T}^{\text{cut}}, R) - \theta(\mathcal{T}_{f1} < \mathcal{T}^{\text{cut}}) \theta(\mathcal{T}_{f2} < \mathcal{T}^{\text{cut}}) \\ &= \theta(\Delta R < R) [\theta(\mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}) - \theta(\mathcal{T}_{f1} < \mathcal{T}^{\text{cut}}) \theta(\mathcal{T}_{f2} < \mathcal{T}^{\text{cut}})] \end{aligned}$$

Results proportional to  $R^2/\varepsilon$  and  $\mathcal{O}(R^4)$ .

# 2-loop Soft Function

$C_F C_A, C_f T_f n_f$  channel: Correction computed via difference from inclusive soft.

$$\Delta \mathcal{M}_f = \theta(\Delta R > R) \theta(\mathcal{T}_{f1} < \mathcal{T}^{\text{cut}}) \theta(\mathcal{T}_{f2} < \mathcal{T}^{\text{cut}}) + \theta(\Delta R < R) \theta(\mathcal{T}_f^{\text{jet}} < \mathcal{T}^{\text{cut}})$$

$$- \theta(\mathcal{T}_{f1} + \mathcal{T}_{f2} < \mathcal{T}^{\text{cut}})$$

$$\Delta S_{B/C}^{(2)}(\mathcal{T}^{\text{cut}}, R) = \Delta S_{\text{base}}^{(2)}(\mathcal{T}^{\text{cut}}, R) + \Delta S_{\text{rest}}^{(2)}(\mathcal{T}^{\text{cut}}, R)$$

↓                            ↓

Captures remaining  $1/\varepsilon$  divergence.                            Finite, numerically integrate.

$\Delta S_{\text{base}}$  :  $\log R, \log^2 R$  terms

analytically

$\Delta S_{\text{rest}} = \Delta S_f - \Delta S_{\text{base}}$  :      Computed by integrating numerically

$$S_f^{(2)}(\mathcal{T}^{\text{cut}}, R, \mu) = S_{G,f}(\mathcal{T}^{\text{cut}}, \mu) + \frac{1}{2}[S_f^{(1)}(\mathcal{T}^{\text{cut}}, \mu)]^2 + \Delta S_f^{(2)}(\mathcal{T}^{\text{cut}}, R, \mu)$$

$$+ \Delta \mathbb{S}_f^{(2)}(\mathcal{T}^{\text{cut}}, R, \mu)$$

Anom. Dimension:  $\gamma_{(S,B)}(\mathcal{T}^{\text{cut}}, \mu, R) = \gamma_{G,(S,B)}(\mathcal{T}^{\text{cut}}, \mu) + \Delta \gamma_{(S,B)}(\mathcal{T}^{\text{cut}}, \mu, R)$

J. Gaunt, SG, M.Stahlhofen, F.Tackmann '17

# Non Singular at NNLO

$$\sigma_0^{\text{nons,NNLO}}(\mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}, \mu_{\text{FO}}) = \sigma_0^{\text{FO,NNLO}}(\mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}) - \sigma_0^{\text{resum,NNLL'}}(\mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}, \mu_B = \mu_S = \mu_H = \mu_{\text{FO}})$$

$$\sigma_0^{\text{FO,NNLO}}(\mathcal{T}_{fj} < \mathcal{T}^{\text{cut}}) = \sigma_{\geq 0}^{\text{FO,NNLO}} - \sigma_{\geq 1}^{\text{FO,NLO}}(\mathcal{T}_{fj} > \mathcal{T}^{\text{cut}})$$

Full NNLO inclusive

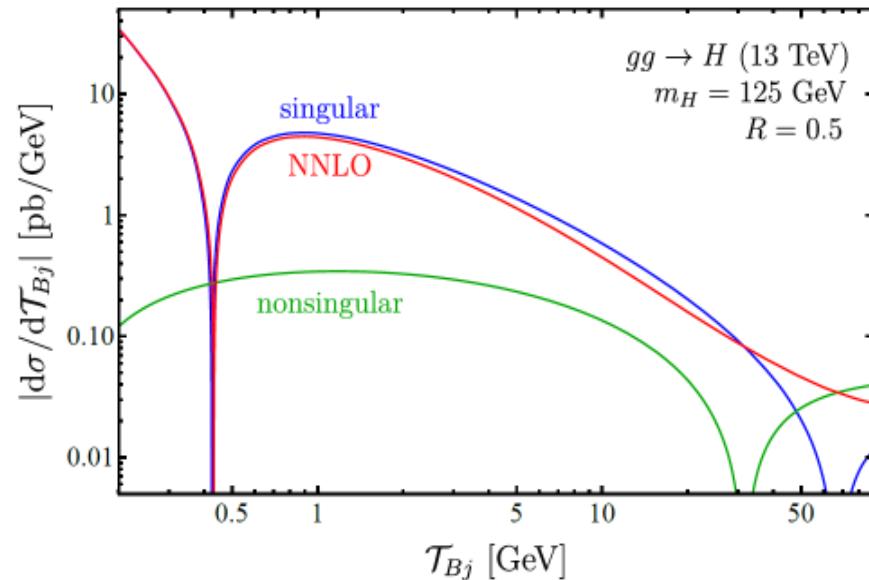
Catani, Grazzini: HNNLO

Resummation region: Logs are large and resummed:

$$|\mu_H| \sim m_H, \mu_S \sim \mathcal{T}^{\text{cut}}, \mu_B \sim \sqrt{m_H \mathcal{T}^{\text{cut}}}$$

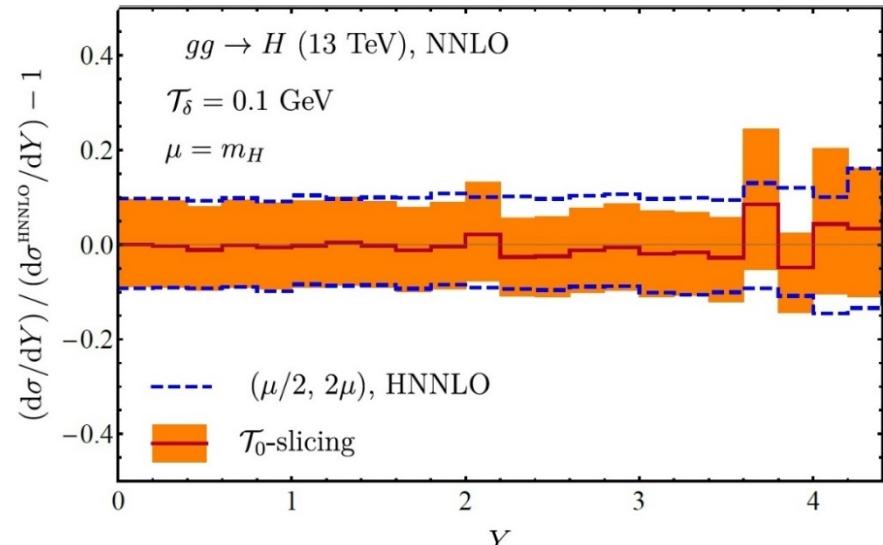
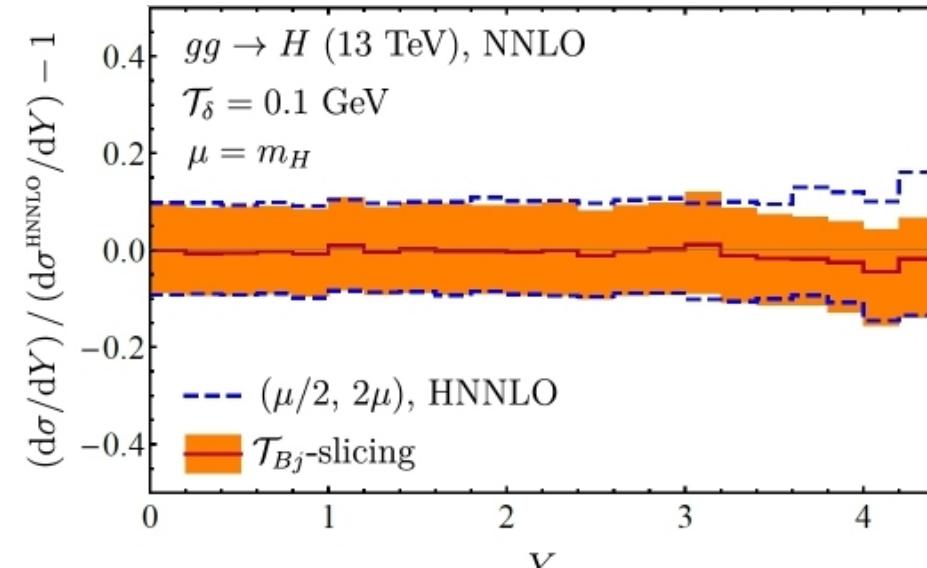
Fixed Order region: Resummation is turned off:

$$\mu_B, \mu_S \rightarrow \mu_{\text{FO}} \sim m_H$$



Transition region: Profiles for  $\mu_B, \mu_S$  provide smooth transition from resummation to fixed-order region.

# NNLO Slicing Cross-check



$$(\text{FO, NNLO}) = \text{NNLL}'(\mathcal{T} < \mathcal{T}^{\text{cut}}) + \text{nonsing}(\mathcal{T} < \mathcal{T}^{\text{cut}}) + \text{NLO 1-jet}(\mathcal{T} > \mathcal{T}^{\text{cut}})$$

SCET Resummed  
results

$\sim 0$  for cut = 0.1 GeV

Madgraph H+1-jet

- Reproduce right FO HNNLO cross section for both B and C observables.
- Good cross check for 2-loop B and S including constant terms in the resummed prediction.

J. Gaunt, M. Stahlhofen, F. Tackmann et al '15

# Uncertainties using Profile Scale Variations

Central Profile Scales:  $\mu_H = -i\mu_{FO}$  ,  $\mu_S(\mathcal{T}^{\text{cut}}) = \mu_{FO} f_{\text{run}}(\mathcal{T}^{\text{cut}}/m_H)$ ,

$$\mu_B(\mathcal{T}^{\text{cut}}) = \mu_{FO} \sqrt{f_{\text{run}}(\mathcal{T}^{\text{cut}}/m_H)}$$

Stewart, Tackmann, Walsh  
et al '13

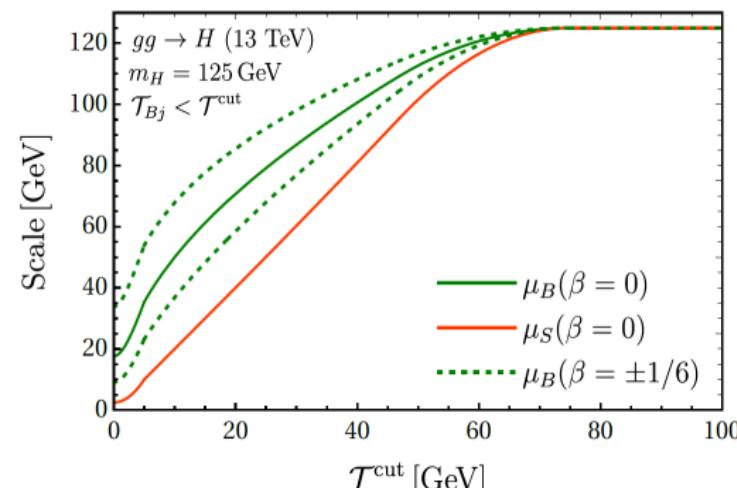
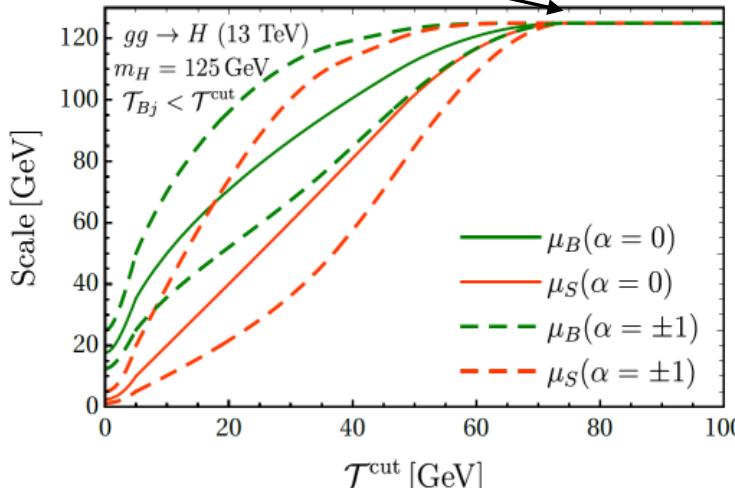
Resummation Region:  $f_{\text{run}}(x) = r_s x$

Total Uncertainty :

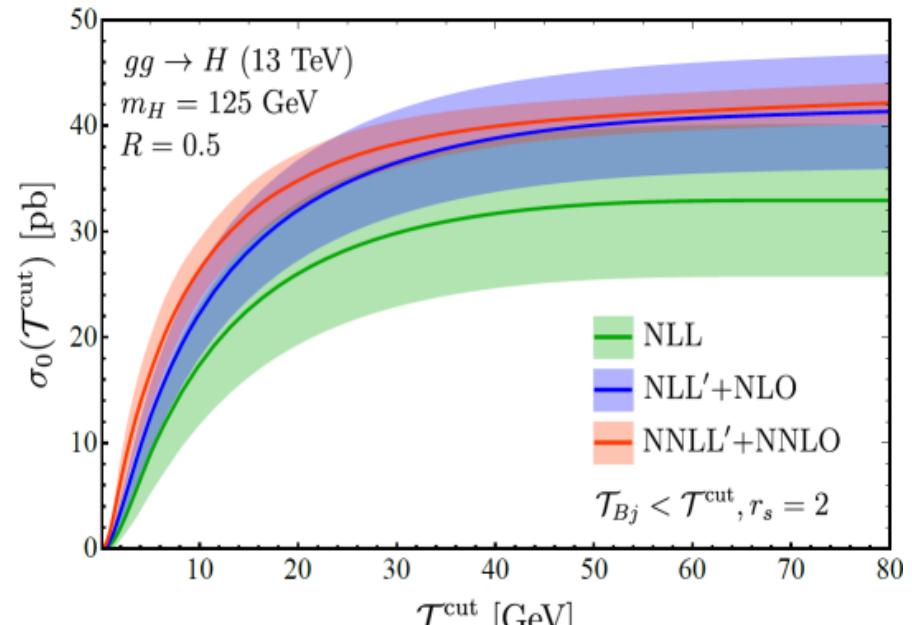
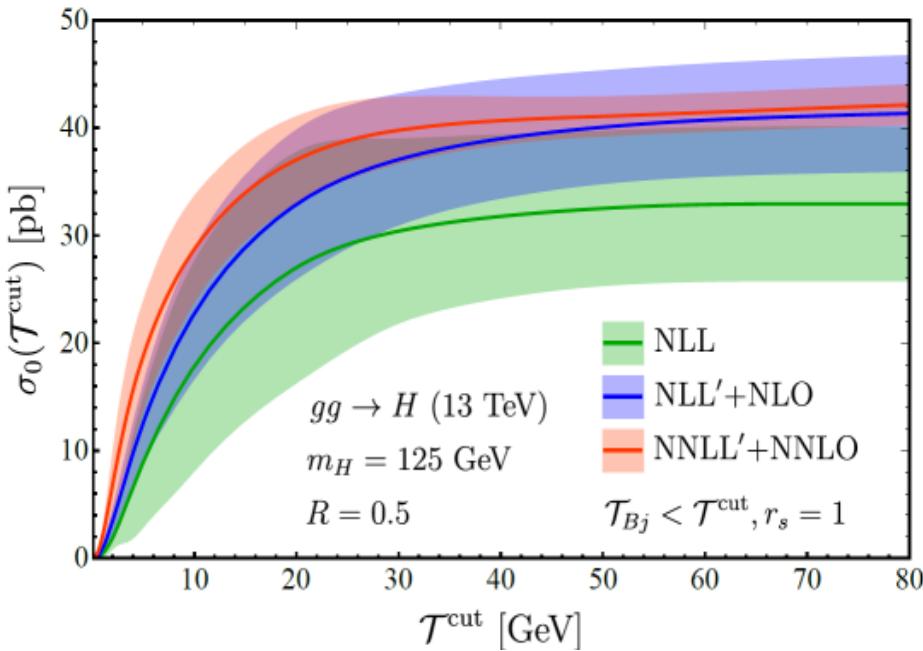
$$\Delta_0^2 = [\Delta^{\text{FO}}(\mathcal{T}^{\text{cut}})]^2 + [\Delta^{\text{resum}}(\mathcal{T}^{\text{cut}})]^2.$$

Fixed Order:  $(\mathcal{T}^{\text{cut}} \sim Q) : \Delta_0^y = \Delta^{\text{FO}}$

Resummation :  $(\mathcal{T}^{\text{cut}} \ll Q) : \Delta^{\text{cut}} = \Delta^{\text{resum}}$

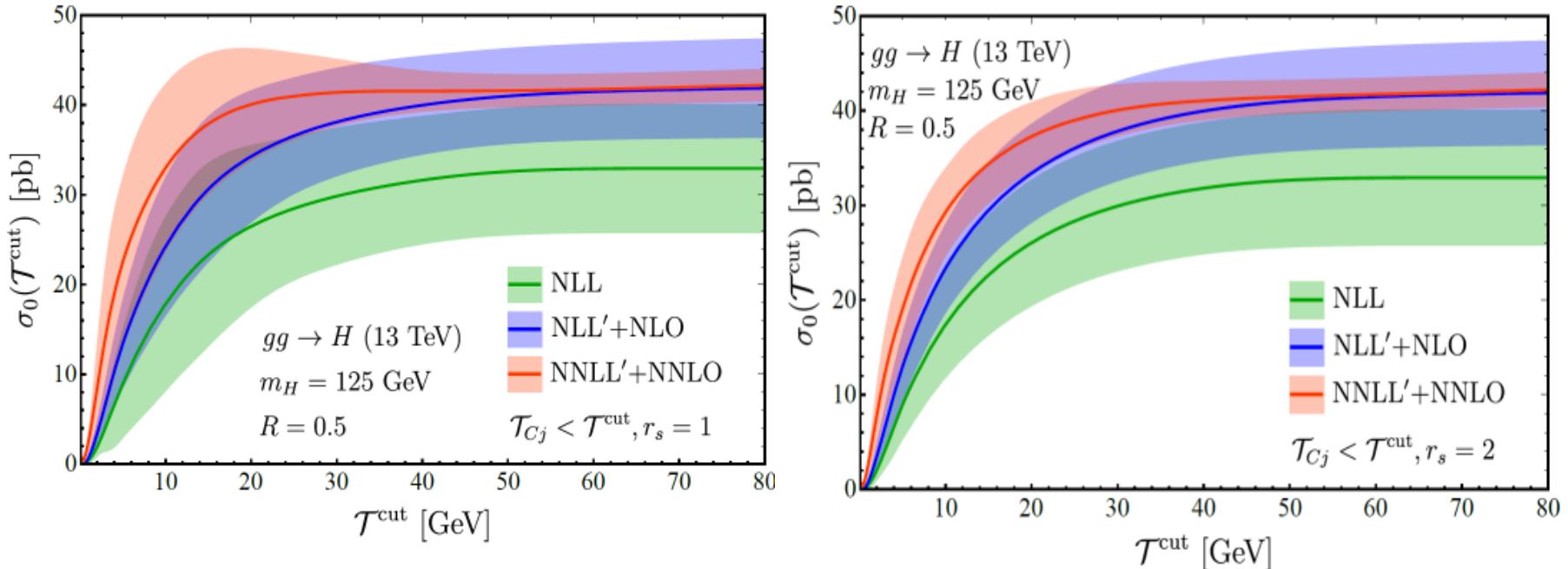


# H+o-jet cross section at NNLL' + NNLO



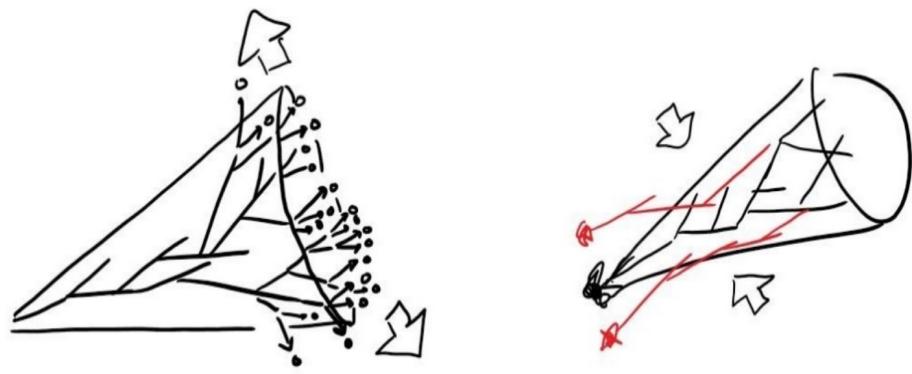
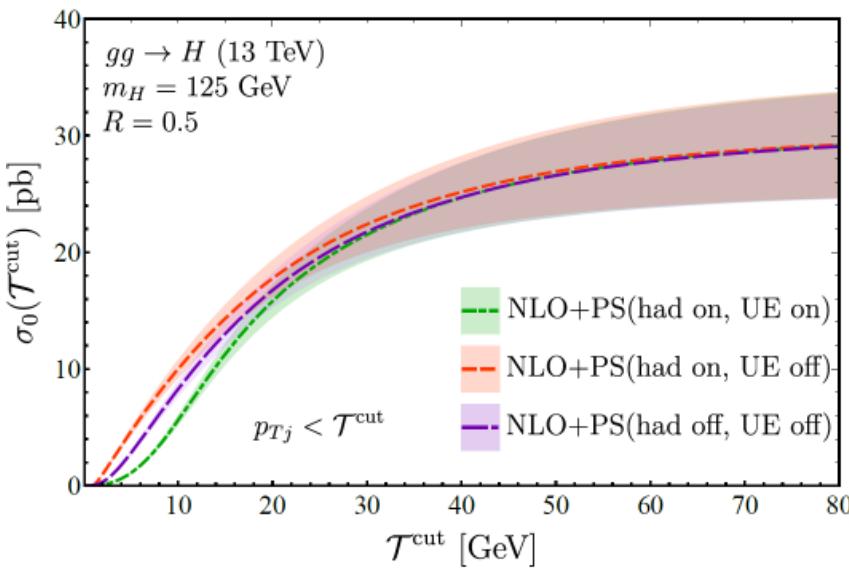
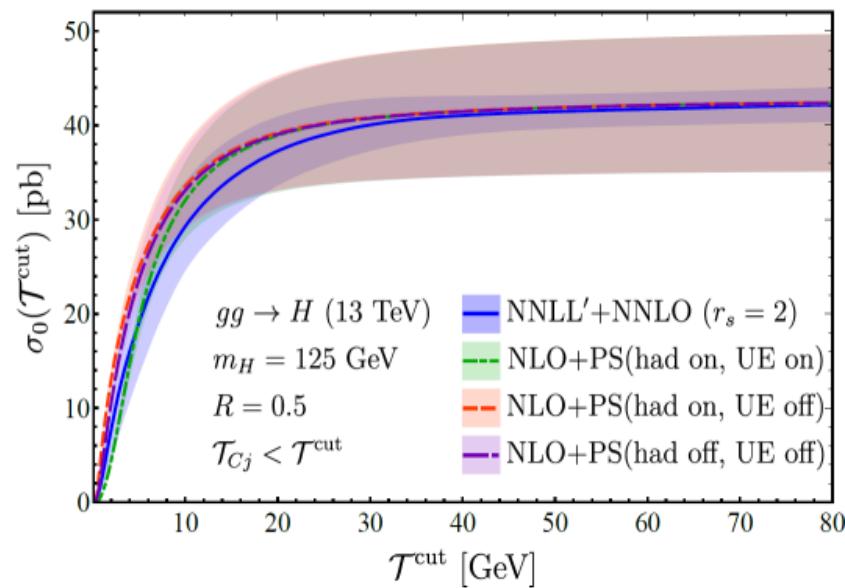
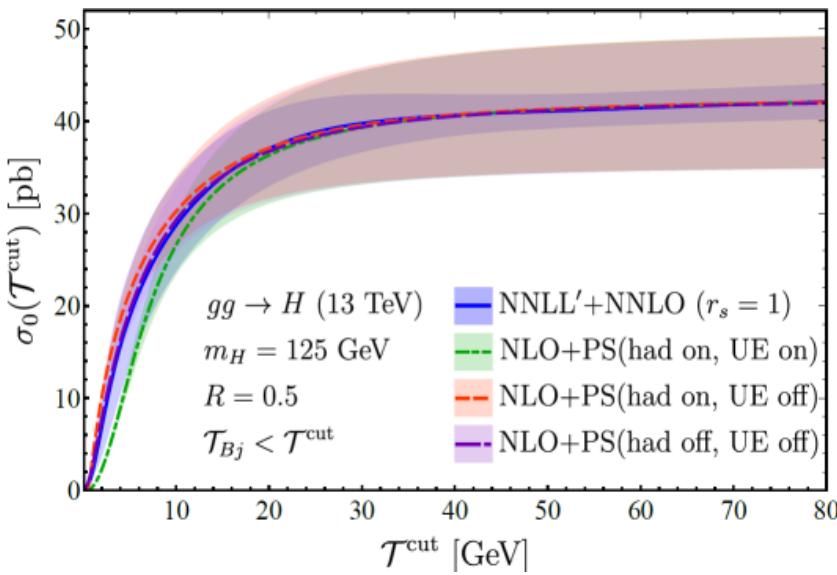
	$\sigma_0(\mathcal{T}^{\text{cut}})$ [pb] ( $r_s = 1$ )	$\sigma_0(\mathcal{T}^{\text{cut}})$ [pb] ( $r_s = 2$ )
NLL'+NLO		
$\mathcal{T}_{Bj} < \mathcal{T}^{\text{cut}} = 20$ GeV	$32.88 \pm 6.95$ (21.2%)	$32.02 \pm 4.75$ (14.8%)
$\mathcal{T}_{Bj} < \mathcal{T}^{\text{cut}} = 30$ GeV	$37.05 \pm 6.12$ (16.5%)	$36.50 \pm 4.96$ (13.6%)
NNLL'+NNLO		
$\mathcal{T}_{Bj} < \mathcal{T}^{\text{cut}} = 20$ GeV	$37.03 \pm 4.06$ (10.9%)	$34.81 \pm 2.57$ (7.39%)
$\mathcal{T}_{Bj} < \mathcal{T}^{\text{cut}} = 30$ GeV	$39.77 \pm 3.11$ (7.82%)	$38.30 \pm 2.23$ (5.82%)

# H+o-jet cross section at NNLL' + NNLO

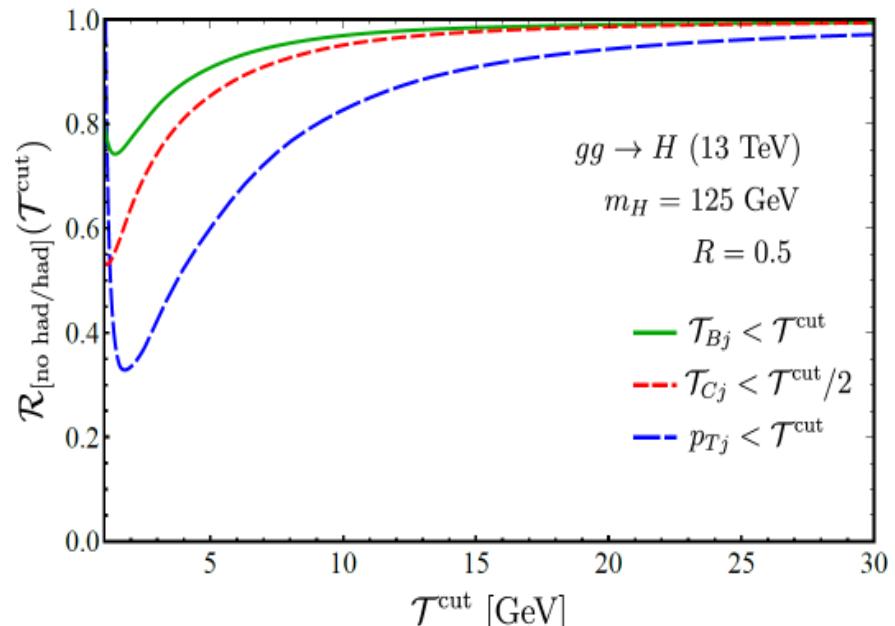
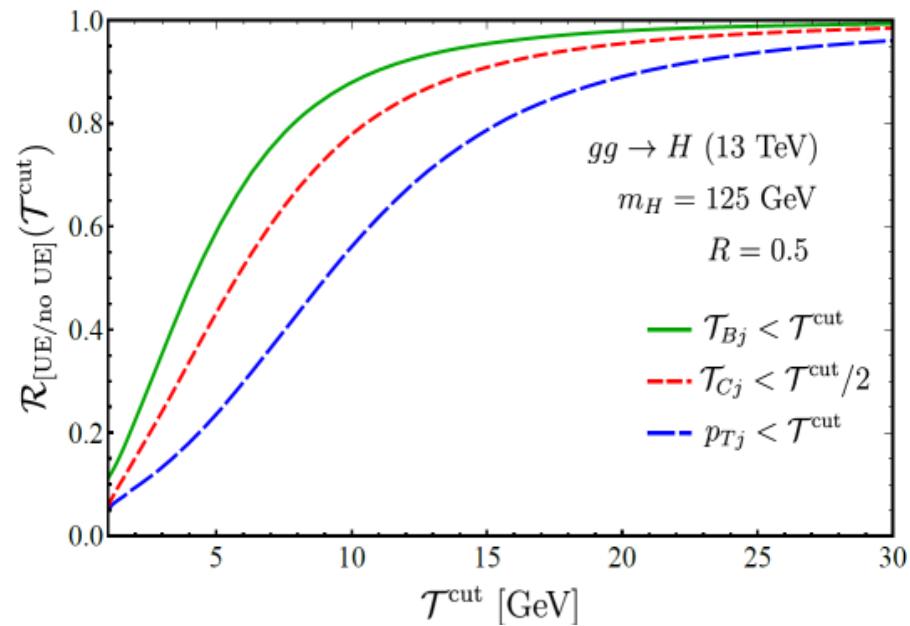


NLL'+NLO			
$\mathcal{T}_{Cj} < \mathcal{T}^{\text{cut}} = 20$ GeV		$34.28 \pm 7.37$ (21.5%)	$33.40 \pm 5.24$ (15.7%)
$\mathcal{T}_{Cj} < \mathcal{T}^{\text{cut}} = 30$ GeV		$38.10 \pm 6.05$ (15.8%)	$37.82 \pm 5.27$ (13.9%)
NNLL'+NNLO			
$\mathcal{T}_{Cj} < \mathcal{T}^{\text{cut}} = 20$ GeV		$40.05 \pm 6.28$ (15.69%)	$37.27 \pm 3.64$ (9.77%)
$\mathcal{T}_{Cj} < \mathcal{T}^{\text{cut}} = 30$ GeV		$41.39 \pm 3.75$ (9.07%)	$40.05 \pm 2.75$ (6.88%)

# Effects of Underlying Event and Hadronisation



# Effects of Underlying Event and Hadronisation



$$\mathcal{R}_{(\text{UE/no UE})}(\mathcal{T}^{\text{cut}}) = \frac{\sigma_0(\mathcal{T}^{\text{cut}})|_{\text{had on, UE on}}}{\sigma_0(\mathcal{T}^{\text{cut}})|_{\text{had on, UE off}}}$$

$$\mathcal{R}_{(\text{no had/had})}(\mathcal{T}^{\text{cut}}) = \frac{\sigma_0(\mathcal{T}^{\text{cut}})|_{\text{had off, UE off}}}{\sigma_0(\mathcal{T}^{\text{cut}})|_{\text{had on, UE off}}}$$

# Summary.

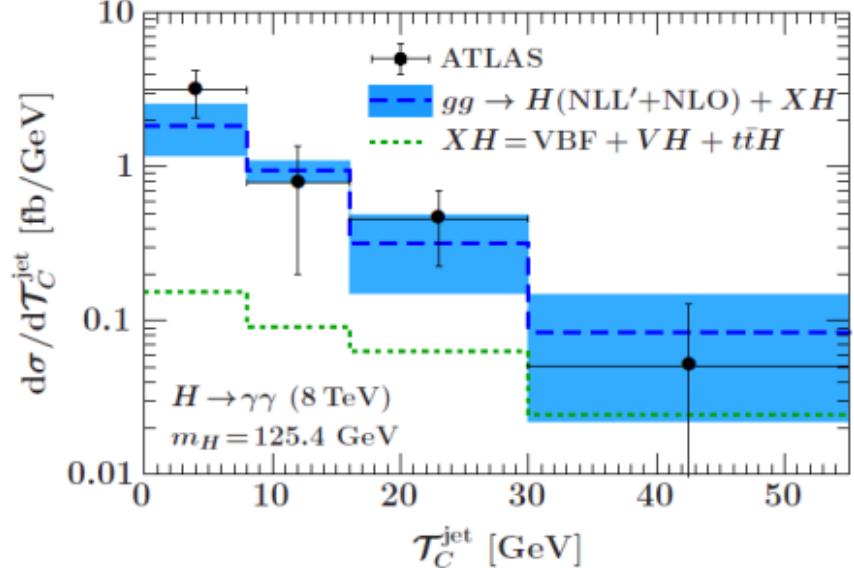
- Rapidity-dependent Jet vetoes : Efficient to veto central jets.  
Can avoid experimental and theoretical limitations  $p_{Tj}$  veto.  
Can provide complementary ways to perform jet binning.
- Computed 2-loop beam and soft functions for the rapidity-dependent jet vetoes, including all parton and color channels.
- Computed H+ 0-jet cross section at NNLL' + NNLO for Rapidity - dependent jet vetoes. Precision of resummation to same level as  $p_{Tj}$  .
- The cross sections with Rapidity dependent jet vetoes have a reduced sensitivity to underlying event and hadronisation effects compared to that with a  $p_{Tj}$  veto .
- Future Directions: Drell Yan cross section at NNLL' + NNLO.

# H+o-jet at NLL' + NLO

Comparison of our NLL'+ NLO predictions in bins of  $\mathcal{T}_C^{\text{jet}}$

with the ATLAS measurements in the  $H \rightarrow \gamma\gamma$  analysis.

SG, Tackmann, Stahlhofen '14



Ingredients at NNLL':

$$H_{gg} = |C_{ggH}^{(0)}|^2 + \alpha_s \text{Re} \left[ C_{ggH}^{(1)} C_{ggH}^{(0)*} \right] + \alpha_s^2 \left( 2 \text{Re} \left[ C_{ggH}^{(2)} C_{ggH}^{(0)*} \right] + |C_{ggH}^{(1)}|^2 \right)$$

Harlander, Kant, '05  
Berger, Marcantonini,  
Stewart et al '11

$$\begin{aligned} \sigma_0^{\text{Rsub}} &= \frac{\alpha_s^2(\mu_{\text{avg}})}{(4\pi)^2} H_{gg}^{(0)} U_{\text{total}}(\mathcal{T}^{\text{cut}}, \mu_H, \mu_B, \mu_S, \mu_{\text{FO}}) \times \left[ \left\{ f_g(x_a, \mu_B) f_j(x_b, \mu_B) \right. \right. \\ &\quad \otimes \left( \Delta \mathbb{I}_{gj}^{(2)}(x_b, \mu_{\text{avg}}, R) + S C_{gj}^{(2)}(x_b, \mu_{\text{avg}}, R) \right) + (x_a \leftrightarrow x_b) \Big\} \\ &\quad \left. + f_g(x_a, \mu_B) f_g(x_b, \mu_B) \Delta \mathbb{S}_f^{(2)}(\mathcal{T}^{\text{cut}}, \mu_{\text{avg}}, R) \right] \end{aligned}$$

Stewart, Tackmann, et al '13  
Banfi, Monni, Salam et al '12

$$f_{\text{run}}(x) = \begin{cases} x_0 \left[ 1 + (2r_s - 1)(x/x_0)^2/4 \right] & x \leq 2x_0, \\ r_s x & 2x_0 \leq x \leq x_1, \\ r_s x + \frac{(2-r_s x_2 - r_s x_3)(x-x_1)^2}{2(x_2-x_1)(x_3-x_1)} & x_1 \leq x \leq x_2, \\ 1 - \frac{(2-r_s x_1 - r_s x_2)(x-x_3)^2}{2(x_3-x_1)(x_3-x_2)} & x_2 \leq x \leq x_3, \\ 1 & x_3 \leq x. \end{cases}$$

$$\mu_S^{\text{vary}}(x,\alpha)=\mu_{FO}\,f_{\text{vary}}^\alpha(x)\,f_{\text{run}}(x)\,,$$

$$\mu_B^{\text{vary}}(x,\alpha,\beta)=\mu_S^{vary}(x,\alpha)^{1/2-\beta}\mu_{FO}^{1/2+\beta}=\mu_{FO}\left[f_{\text{vary}}^\alpha(x)\,f_{\text{run}}(x)\right]^{1/2-\beta}$$

$$C = \frac{3}{2} \frac{1}{(\sum_i \vec{p}_i)^2} \sum_{ij} [\vec{p}_i \vec{p}_j] \sin^2 \theta_{ij} \quad \quad \quad C = \frac{3}{Q} \sum_{j \in c,s} \frac{p_j^\perp}{\cosh \eta_j}$$

$$\Delta \mathcal{M}_{\text{base}} = 2\theta(y_t>0)\theta(\Delta R>R)[\theta(k_1^+<\mathcal{T}^{\text{cut}})\theta(k_2^+<\mathcal{T}^{\text{cut}})-\theta(k_1^++k_2^+<\mathcal{T}^{\text{cut}})$$

$$\begin{aligned} \Delta S_f(\mathcal{T}^{\text{cut}}, R, \mu) &= \Delta \gamma_{S1}^i(R) \log \frac{\mu}{\mathcal{T}^{\text{cut}}} + \Delta s_{2f}^i(R) \\ \Delta \mathcal{I}_{ij}(t^{\text{cut}}, x, R, \mu) &= \delta_{ij} \frac{\Delta \gamma_{S1}^i(R)}{4} \left[ \delta(1-x) \log \frac{\mathcal{T}^{\text{cut}}}{\mu^2} + \mathcal{L}_0(1-x) \right] + \Delta \mathcal{I}_{ij,run}(t^{\text{cut}}, x, \mu) \\ &\quad + \Delta I_{ij}(x, R) \end{aligned}$$

# Uncorrelated and Soft-Collinear Mixing

$$A_{A,ij} = \frac{1}{2} A_{ij}^{B(1)}(k_2, x) A_i^{S(1)}(k_1)$$

$$A_{ij}^{B(1)}(k_2, x) = \frac{2g^2 P_{ij}^{(0)}(1-x)p^-}{k_2^+ k_2^-} , \quad A_i^{S(1)}(k_1) = \frac{4g^2 C_i}{k_1^+ k_1^-}$$

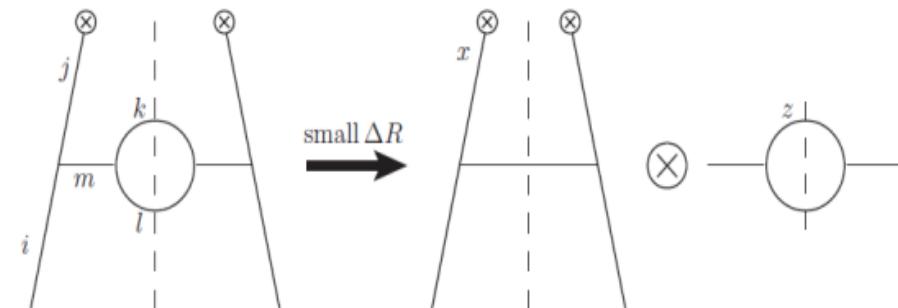
$$\Delta\sigma_{CC} \sim \int d\Delta y \, d\Delta\phi \, dz \, \theta(\Delta R < R) \theta[\mathcal{T}^{\text{cut}} < \mathcal{T}_T < \frac{\mathcal{T}^{\text{cut}}}{\max[z, (1-z)]}] \delta(k_1^- + k_2^- = (1-x)p^-) P_{ij}^{(0)}(1-x)p^-$$

$$\Delta\sigma_{CC}^{\text{indep}} = -\sigma_0 \frac{1}{\epsilon} \left( \frac{\alpha_s C_A}{\pi} \right)^2 \left( \frac{\mu^2}{m_H \mathcal{T}^{\text{cut}}} \right)^{2\epsilon} \frac{\pi^2}{12} R^2 \longrightarrow \text{Uncorrelated CC - Zero bin}$$

$$\Delta M_{as} = \theta(\Delta R < R) [\theta(\mathcal{T}_c + \mathcal{T}_s < \mathcal{T}^{\text{cut}}) - \theta(\mathcal{T}_c < \mathcal{T}^{\text{cut}}) \theta(\mathcal{T}_s < \mathcal{T}^{\text{cut}})]$$

$$\Delta\sigma_{SC} = \sigma_0 \frac{1}{\epsilon} \left( \frac{\alpha_s C_A}{\pi} \right)^2 \left( \frac{\mu^2}{m_H \mathcal{T}^{\text{cut}}} \right)^{2\epsilon} \frac{\pi^2}{6} R^2$$

Log(R) Terms: Only Bubble Insertion Graphs



$$A_R = \hat{P}_{ij \rightarrow q}(x, \epsilon) \hat{P}_{q \rightarrow kl}(z, \epsilon) \frac{2g^4}{\Delta R^2 (p^- \mathcal{T}_T)^2 (1-x)(z(1-z))}$$